Lecture 2 Follow Up

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1. Explain the difference between the underlying space of values for a classical bit versus a quantum bit.

Classic bit: State of either 0 or 1, no other possibilities

Quantum Bit: Any possible Super position Combination of 0, 1

2. Explain the same for a three bit system (classical versus quantum).

3 Bit Classic: 2^3 possible states {000 001 010 011 100 101 110 111} (only one at a time)

3 Bit Quantum: superposition of all 8 states at the same time

3. What are the four bit quantum basis vectors?

The four basis vectors are

$|00\rangle$

$|01\rangle$

$|10\rangle$

$|11\rangle$

4. What is the result of applying a Hadamard gate on a 0 basis vector? A 1 basis vector? A general quantum bit?

zero basis:

- You get a state that is a superposition of 0 & 1 with equal probability & a phase difference

$$H|0\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}\begin{pmatrix} 1 \\ 0 \end{pmatrix} = \frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle$$

One Basis

$$H|1\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}\begin{pmatrix} 0 \\ 1 \end{pmatrix} = \frac{1}{\sqrt{2}}|0\rangle - \frac{1}{\sqrt{2}}|1\rangle$$

General Quantum Bit

$$H|A\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}\begin{pmatrix} \alpha \\ \beta \end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix} \alpha+\beta \\ 0 \end{pmatrix} + \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ \alpha-\beta \end{pmatrix} = \frac{\alpha+\beta}{\sqrt{2}}|0\rangle + \frac{\alpha-\beta}{\sqrt{2}}|1\rangle$$

5. What are all possible classical functions of a two bit system? Present these cleanly in a table.

2^n possible binary functions $\rightarrow 2^{(2^2)} = 2^4 = 16$

Input (AB)	F0	F1	F2	F3	F4	F5	F6	F7	F8	F9	F10	F11	F12	F13	F14	F15
00	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1
01	0	0	1	0	1	0	1	1	0	0	1	0	1	0	1	1
10	0	1	0	0	0	1	1	1	0	0	0	1	1	1	0	1
11	0	1	1	1	0	0	0	1	0	1	1	1	0	0	0	1
Func. Desc	Always 0	Always A	Always B	A AND B	NOT A AND B	A AND NOT B	A XOR B	A OR B	NOT (A OR B)	NOT (A XOR B)	NOT (A AND NOT B)	NOT (NOT A AND B)	NOT (A AND B)	Always NOT B	Always NOT A	Always 1

6. What is the difference between (a) the space of all possible 2 quantum bits, (b) the space of entangled 2-quantum bit values , (c) the space of values for all 2-quantum bits formed via the tensor product?

a) The space of all 2 quantum bits are all linear combinations of the following basis states: |00> |01> |10> |11>
b) The space of entangled 2-quantum bit values is a subset of the space of all possible 2 quantum bits.
c) The spaces of values for all 2 quantum bits formed via tensor product is the same thing as the space of all 2 quantum bits (a). The tensor product of 2 qubits forms a 4D complex vector space with the basis states: |00> |01> |10> |11>

7. Express the basis values of a 2-quantum bit system in terms of tensor products of 1-quantum bit values.

Basis states of 2 qubit system = tensor products of basis states of two individual qubits

$|00\rangle = |0\rangle \otimes |0\rangle$

$|10\rangle = |1\rangle \otimes |0\rangle$

$|01\rangle = |0\rangle \otimes |1\rangle$

$|11\rangle = |1\rangle \otimes |1\rangle$

$|11\rangle =$

8. If I have the 2-quantum bit system where there is a 50% chance of measuring 00 and 11, express this using the ket notation, and the tensor product. If I measure one of these bits to be a 1, what will the other bit be measured as, and with what probability? Repeat for the case of a 0 measured on the first bit

$$|\Psi\rangle = \frac{1}{\sqrt{2}}|00\rangle + \frac{1}{\sqrt{2}}|11\rangle$$

if measure one bit as 1, the other collapses to 1 with 100% probability

" " 0 " " 0 with 100% probability

9. If I have the 2-quantum bit system where there is a 1/4 chance of measuring any of the four basis values, and I measure one of the bits to be 1 what are the probabilities of measuring the various values of the other bits. Compare this situation with that of Q8.

$$|\Psi\rangle = \frac{1}{\sqrt{4}}\left(|00\rangle + |01\rangle + |10\rangle + |11\rangle\right)$$

if one of the bits is 1, you have this prob. dist:

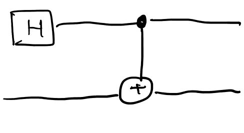
$\|00\rangle$	0%
$\|01\rangle$	33%
$\|10\rangle$	33%
$\|11\rangle$	33%

Compared to Q8 we do not have 100% certainty of the values of other bits.

10. Construct an entangled state for the 2 quantum bit system, that differs from question 8.

$$|\Psi\rangle = \frac{1}{\sqrt{2}}\left(|01\rangle - |10\rangle\right)$$

11. Suppose you have a CNOT gate. Apply the result of the Hadamard gate on the control bit of the CNOT, and a general bit value x to the other input. Show clearly (i.e., show your work) what the output will be.



Hadamard Gate		CNOT Gate
In	Out	
$\|0\rangle$	$\frac{1}{\sqrt{2}}(\|0\rangle + \|1\rangle)$	$\frac{1}{\sqrt{2}}(\|00\rangle + \|11\rangle)$
$\|1\rangle$	$\frac{1}{\sqrt{2}}(\|0\rangle - \|1\rangle)$	$\frac{1}{\sqrt{2}}(\|01\rangle + \|10\rangle)$

CNOT gate flips target qubit if control qubit is $|1\rangle$