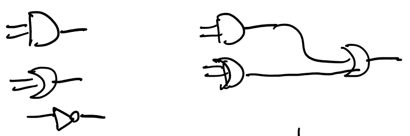
Week 2

Saturday, May 13, 2023 10:43 AM

• test is open Book

Last week

Classical Computation



Gate

circuit

any classical Algo
⇒ Circuit of above gates

w/o quantum RAM we can't use higher level
so need to build w quantum circuits

CC
• Values: 0, 1] bits → composite Data:
'stringing Bits' in a place-value System

• operators
↳ AND, OR, NOT

0, 1, 11101 ⇒ 16+8+4+1
16 8 4 2 1 = 29

QC equivalents!

Quantum Physics:

Notion of value of a State — Pre measure / After measure

in classical physics
$\boxed{x}$ — 1
$x \in \{0,1\}$
• 1 bit
$x \in \{0,1,2,3\}$
• 2 bits

QP

$\boxed{x}$
state

2 possible state

0 1
up dn
0.5Å 1.5Å

• After measurement: $x=0$ or $x=1$
• Before measurement: $x = \{0, 1,$ any lin. superposition of '0', '1'$\}$
↳ unobservable
↳ but can still put in a circuit & have it interact w quantum

x: 'amount of '0'ness' / amt of '1'ness } vector

$\underline{x} = \begin{pmatrix} \alpha \\ \beta \end{pmatrix}$ ← vector

Two state Q. object
Q Bit
Qubit

in C.C. this would be a bit {0,1}

$$\boxed{\begin{matrix} |\alpha|^2 + |\beta|^2 = 1 \\ \alpha, \beta \in \mathbb{C} \end{matrix}}$$

→ why complex will be discussed later

Qubits (2 state) $\in \begin{pmatrix} \alpha \\ \beta \end{pmatrix} \in \mathbb{C}^2$ | $|\alpha|^2 + |\beta|^2 = 1$

$|\alpha|^2 \sim$ Pr of observing '0' $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$

$|\beta|^2 \sim$ Pr of observing '1' $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$

QBit: Vector

Dirac notation
$\underline{x} \equiv |x\rangle$ ← col. vectors
ket-x

observing '0' $\binom{1}{0}$

QBit : Vector Dirac notation $\underline{x} \equiv |x\rangle$ ~ col. vectors
ket-x

$|\beta|^2$ ~ Pr of observing '1' $\binom{0}{1}$

OPerator : matrix

$$\underbrace{\begin{pmatrix} a & b \\ c & d \end{pmatrix}}_{\text{Any matrix st. output is also a Qubit}} \underbrace{\begin{pmatrix} \alpha \\ \beta \end{pmatrix}}_{\text{Qubit}} = \underbrace{\begin{pmatrix} k \\ l \end{pmatrix}}_{\text{Qubit}}$$

Any matrix st. output is also a Qubit ↪ Space of unity matrices $\equiv$ Q. Opr

$$U^{-1} = U^{\dagger} = \underbrace{(U^*)^T}_{\text{Conj. transpose}} \qquad |\lambda| = 1$$

Gate

$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ $\qquad I\,|x\rangle = |x\rangle$

$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ $\qquad \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} \alpha \\ \beta \end{pmatrix} = \begin{pmatrix} \beta \\ \alpha \end{pmatrix}$ ~ Not 'X'

$\underbrace{\frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}}_{\text{H gate}}$ $\qquad \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}\begin{pmatrix} 1 \\ 0 \end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 1/\sqrt{2} \\ 1/\sqrt{2} \end{pmatrix} = \frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle$

H gate
Hadamard

$\binom{1}{0}$: 100% in '0' $|0\rangle$ $\qquad$ 50% in $|0\rangle$, 50% in $|1\rangle$ $\qquad \binom{1}{0} \qquad \binom{0}{1}$

$$\frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}\begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ -1 \end{pmatrix}\frac{1}{\sqrt{2}} = \frac{1}{\sqrt{2}}|0\rangle - \frac{1}{\sqrt{2}}|1\rangle$$

$\binom{0}{1}$: 100% in '1'
$|1\rangle$

2 State $|x\rangle = \binom{a}{b}$

2 State $|y\rangle = \binom{c}{d}$ → interact (unsupervised)

CP/CC:
$x \in \{0,1\}$
$y \in \{0,1\}$

$xy \in \{00, 01, 10, 11\}$

$$|xy\rangle = \begin{pmatrix} w \\ x \\ y \\ z \end{pmatrix} \quad \underbrace{\boxed{\begin{matrix} |w|^2 \\ |x|^2 \\ |y|^2 \\ |z|^2 \end{matrix}}}_{\Sigma = 1} \begin{matrix} = \Pr\ 0,0 \sim \Pr|x\rangle\ 0 \text{ AND } \Pr|y\rangle 0 = |a|^2|c|^2 \\ = \Pr\ 01 \sim \Pr|x\rangle\ 0 \text{ AND } \Pr|y\rangle 1 = |a|^2|d|^2 \\ = \Pr\ 10 \sim \Pr|x\rangle\ 1 \text{ AND } \Pr|y\rangle 0 = |b|^2|c|^2 \\ = \Pr\ 11 \sim \Pr|x\rangle\ 1 \text{ AND } \Pr|y\rangle 1 = |b|^2|d|^2 \end{matrix}$$

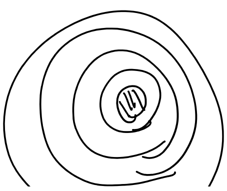
Tensor product

$$= \begin{pmatrix} ac \\ ad \\ bc \\ bd \end{pmatrix} \qquad \binom{a}{b} \otimes \binom{c}{d} = \underline{\underline{acbd}}$$

$|x\rangle \otimes |y\rangle$

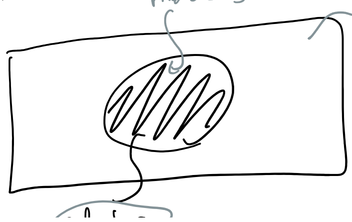
4 state quantum bit

$$\begin{pmatrix} a \\ b \\ c \\ d \end{pmatrix} \quad \mathbb{C}^4 \;\Big|\; |a|^2 + |b|^2 + |c|^2 + |d|^2 = 1$$

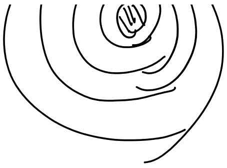


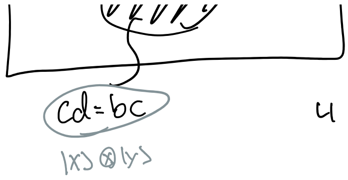
Product states

non-product states
'Entangled states'



$\mathbb{C}^4$
$\Sigma |\;|^2 = 1$





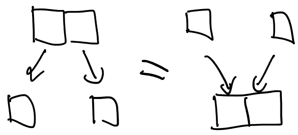
$\Sigma | \ |^2 = 1$

4 state Qbits

4 state classical Word

{00, 01, 10, 11}

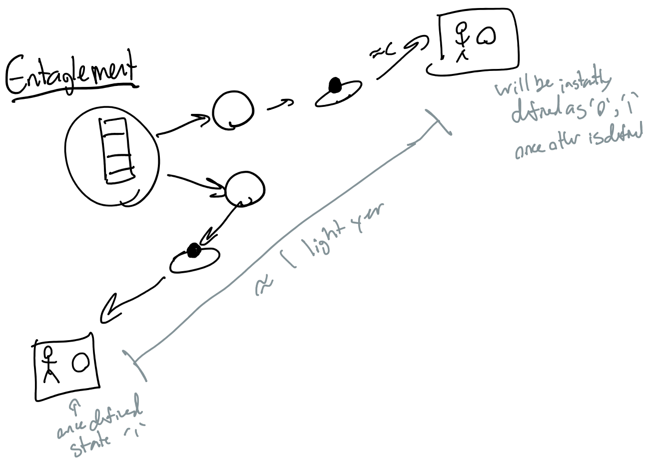
x=0 y=0 x=1 y=1



in classic
- Low level composes HL state & Vice Versa

in Q.
- Cannot, they are <u>entangled</u>

<u>Entanglement</u>



<u>Q Algns</u>

① All CC → QC ② QC Advantages

⇒ QC ≥ CC