Assignment 2 - Khaled Gaber 1004144302

Required Installations

In [ ]:
```
!pip install qiskit
```

In [ ]:
```
!pip install tweedledum
```

In [ ]:
```
!pip install git+https://github.com/qiskit-community/qiskit-textbook.git#subdirecto
```

In [ ]:
```
#Libraries Needed
import matplotlib.pyplot as plt
import math
import numpy as np

from qiskit.circuit.library import PhaseOracle, GroverOperator
from qiskit import QuantumCircuit, ClassicalRegister, QuantumRegister, Aer, transpi
from qiskit.visualization import plot_histogram, array_to_latex
from qiskit.providers.ibmq import least_busy
```

Grover's Search Algorithm (Introduction) [Done]

Following textbook: https://learn.qiskit.org/course/introduction/grovers-search-algorithm

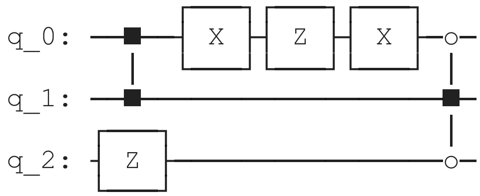
First Implementation

In [ ]:
```
with open('3sat.dimacs', 'r', encoding='utf8') as f:
    dimacs = f.read()
print(dimacs)  # let's check the file is as promised
```

```
c example DIMACS-CNF 3-SAT
p cnf 3 5
-1 -2 -3 0
1 -2 3 0
1 2 -3 0
1 -2 -3 0
-1 2 3 0
```

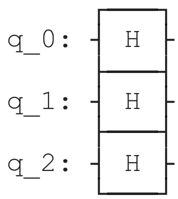
In [ ]:
```
oracle = PhaseOracle.from_dimacs_file('3sat.dimacs')
oracle.draw()
```

Out[ ]:



In [ ]:
```
init = QuantumCircuit(3)
init.h([0,1,2])
init.draw()
```

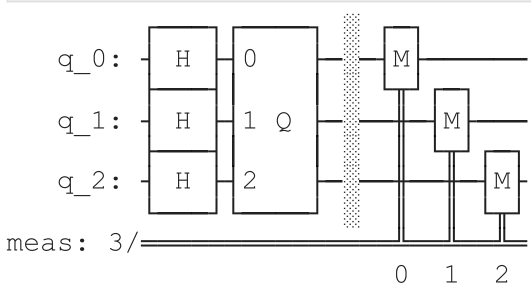
Out[ ]:



In [ ]:
```
grover_operator = GroverOperator(oracle)
```

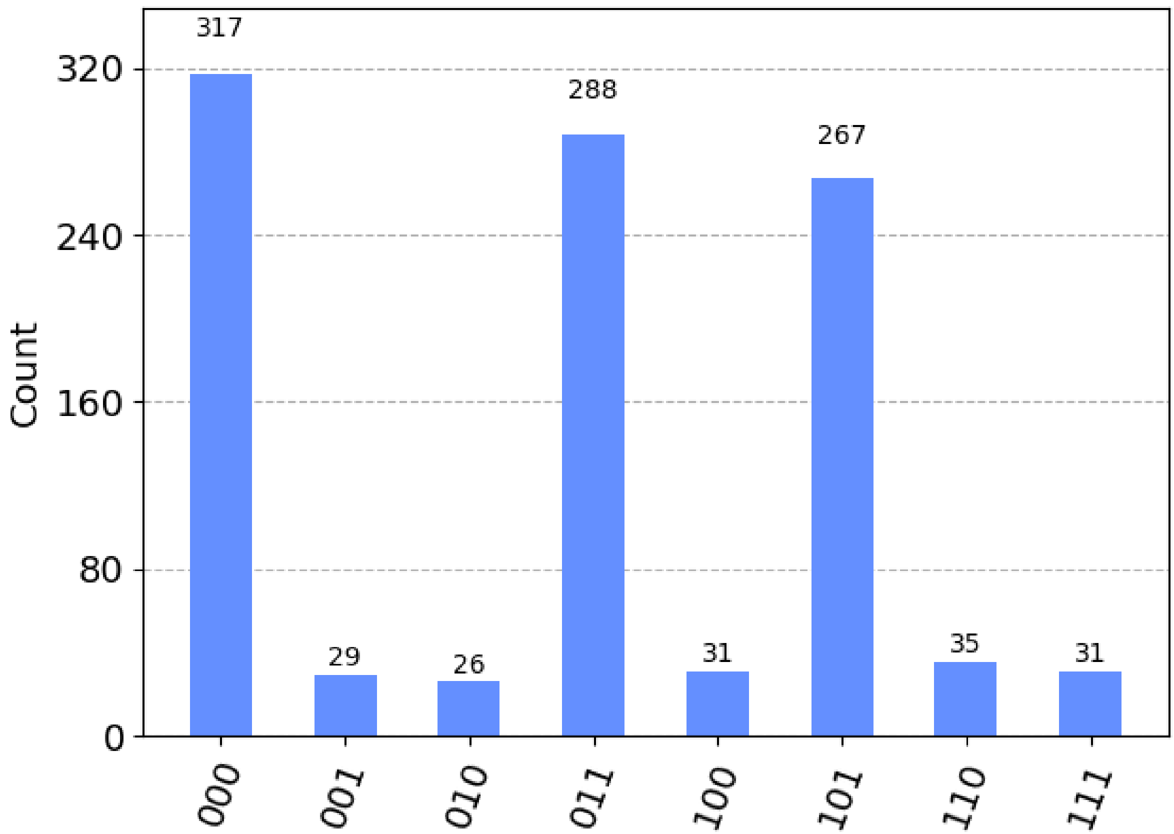
In [ ]:
```
qc = init.compose(grover_operator)
qc.measure_all()
qc.draw()
```

Out[ ]:



In [ ]:
```
# Simulate the circuit
sim = Aer.get_backend('aer_simulator')
t_qc = transpile(qc, sim)
counts = sim.run(t_qc).result().get_counts()

# plot the results
plot_histogram(counts)
```

Out[ ]:



Circuit Implementation

Basic Oracle implementation of a circuit that flips the phase of the state |11>

In [ ]:
```
oracle = QuantumCircuit(2)
oracle.cz(0,1)
oracle.draw()
```

Out[ ]:
```
q_0: ─■─
      │
q_1: ─■─
```

In [ ]:
```
def display_unitary(qc, prefix=""):
    """Simulates a simple circuit and display its matrix representation.
    Args:
        qc (QuantumCircuit): The circuit to compile to a unitary matrix
        prefix (str): Optional LaTeX to be displayed before the matrix
    Returns:
        None (displays matrix as side effect)
    """
    sim = Aer.get_backend('aer_simulator')
    # Next, we'll create a copy of the circuit and work on
    # that so we don't change anything as a side effect
    qc = qc.copy()
    # Tell the simulator to save the unitary matrix of this circuit
    qc.save_unitary()
```

```
    unitary = sim.run(qc).result().get_unitary()
    display(array_to_latex(unitary, prefix=prefix))
```

In [ ]:
```
display_unitary(oracle, "U_\\text{oracle}=")
```

$$U_\text{oracle} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix}$$

From Qiskit:

> Can you create 3 more oracle circuits that instead target the other 3 computational basis states (|00>, |01> and|10>)? Use display_unitary to check your answer.
>
> Hint: Try to create circuits that transform to and from the basis state you're targeting, can you then use these circuits with the cz gate?

Oracle |00>

In [ ]:
```
oracle_00 = QuantumCircuit(2)
oracle_00.cz(1, 0)
oracle_00.draw()
```

Out[ ]:
```
q_0: ─■─
      │
q_1: ─■─
```

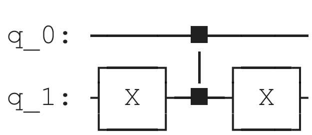
In [ ]:
```
display_unitary(oracle_00, "U_\\text{oracle 00}=")
```

$$U_\text{oracle 00} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix}$$

Oracle |01>

In [ ]:
```
oracle_01 = QuantumCircuit(2)
oracle_01.x(1)
oracle_01.cz(1, 0)
oracle_01.x(1)
oracle_01.draw()
```

Out[ ]:



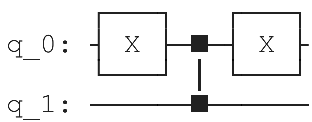
In [ ]:
```
display_unitary(oracle_01, "U_\\text{oracle 01}=")
```

$$U_{\text{oracle 01}} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Oracle |10>

In [ ]:
```
oracle_10 = QuantumCircuit(2)
oracle_10.x(0)
oracle_10.cz(1, 0)
oracle_10.x(0)
oracle_10.draw()
```

Out[ ]:



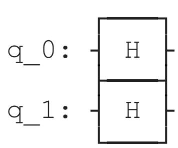
In [ ]:
```
display_unitary(oracle_10, "U_\\text{oracle 10}=")
```

$$U_{\text{oracle 10}} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Creating Diffuser

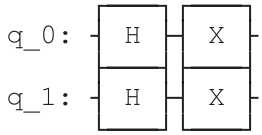
In [ ]:
```
diffuser = QuantumCircuit(2)
diffuser.h([0, 1])
diffuser.draw()
```

Out[ ]:



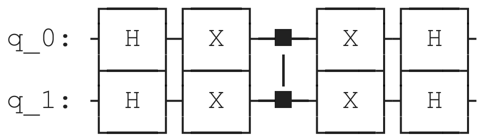
In [ ]:
```
diffuser.x([0,1])
diffuser.draw()
```

Out[ ]:



In [ ]:
```
diffuser.cz(0,1)
diffuser.x([0,1])
diffuser.h([0,1])
diffuser.draw()
```

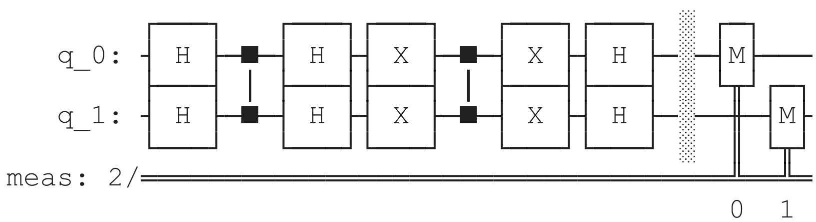
Out[ ]:



Putting it together

In [ ]:
```
grover = QuantumCircuit(2)
grover.h([0,1])  # initialise |s>
grover = grover.compose(oracle)
grover = grover.compose(diffuser)
grover.measure_all()
grover.draw()
```

Out[ ]:



In [ ]:
```
sim = Aer.get_backend('aer_simulator')
sim.run(grover).result().get_counts()
```

Out[ ]: {'11': 1024}

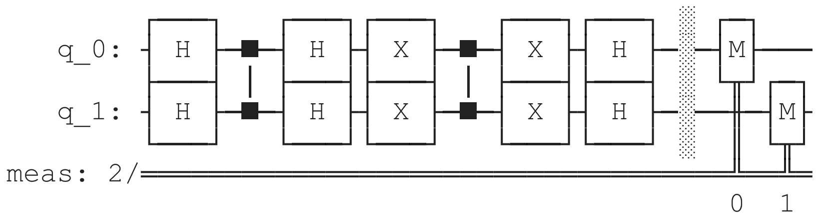
Following Qiskit

> Try replacing the oracle in this circuit with the different oracles you created above. Do you get the expected result?

Oracle |00>

In [ ]:
```
grover = QuantumCircuit(2)
grover.h([0,1])  # initialise |s>
grover = grover.compose(oracle_00)
grover = grover.compose(diffuser)
grover.measure_all()
grover.draw()
```

Out[ ]:



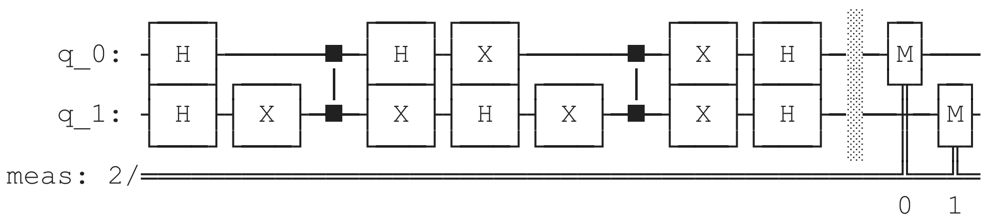
In [ ]:
```
sim = Aer.get_backend('aer_simulator')
sim.run(grover).result().get_counts()
```

Out[ ]: {'11': 1024}

Oracle |01>

In [ ]:
```
grover = QuantumCircuit(2)
grover.h([0,1])  # initialise |s>
grover = grover.compose(oracle_01)
grover = grover.compose(diffuser)
grover.measure_all()
grover.draw()
```

Out[ ]:



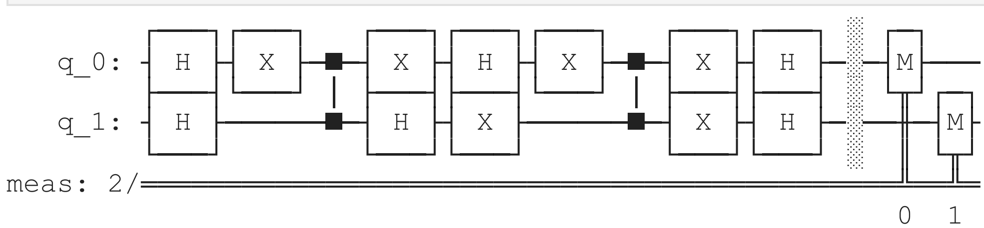
In [ ]:
```
sim = Aer.get_backend('aer_simulator')
sim.run(grover).result().get_counts()
```

Out[ ]: {'01': 1024}

Oracle |10>

In [ ]:
```
grover = QuantumCircuit(2)
grover.h([0,1])  # initialise |s>
grover = grover.compose(oracle_10)
grover = grover.compose(diffuser)
grover.measure_all()
grover.draw()
```

Out[ ]:



In [ ]:
```
sim = Aer.get_backend('aer_simulator')
sim.run(grover).result().get_counts()
```

Out[ ]: {'10': 1024}

Grover's Algorithm (V2) [Done]

Following Textbook: https://learn.qiskit.org/course/ch-algorithms/grovers-algorithm

Qiskit Implementation

In [ ]:
```
n = 2
grover_circuit = QuantumCircuit(n)
```

In [ ]:
```
def initialize_s(qc, qubits):
    """Apply a H-gate to 'qubits' in qc"""
    for q in qubits:
        qc.h(q)
    return qc
```

In [ ]:
```
grover_circuit = initialize_s(grover_circuit, [0,1])
grover_circuit.draw()
```

Out[ ]:
```
     ┌───┐
q_0: ┤ H ├
     ├───┤
q_1: ┤ H ├
     └───┘
```

In [ ]:
```
grover_circuit.cz(0,1) # Oracle
grover_circuit.draw()
```

Out[ ]:
```
     ┌───┐   
q_0: ┤ H ├─■─
     ├───┤ │ 
q_1: ┤ H ├─■─
     └───┘   
```

In [ ]:
```
# Diffusion operator (U_s)
grover_circuit.h([0,1])
grover_circuit.z([0,1])
grover_circuit.cz(0,1)
grover_circuit.h([0,1])
grover_circuit.draw()
```

Out[ ]:
```
     ┌───┐   ┌───┐┌───┐   ┌───┐
q_0: ┤ H ├─■─┤ H ├┤ Z ├─■─┤ H ├
     ├───┤ │ ├───┤├───┤ │ ├───┤
q_1: ┤ H ├─■─┤ H ├┤ Z ├─■─┤ H ├
     └───┘   └───┘└───┘   └───┘
```

Experimenting with Simulators

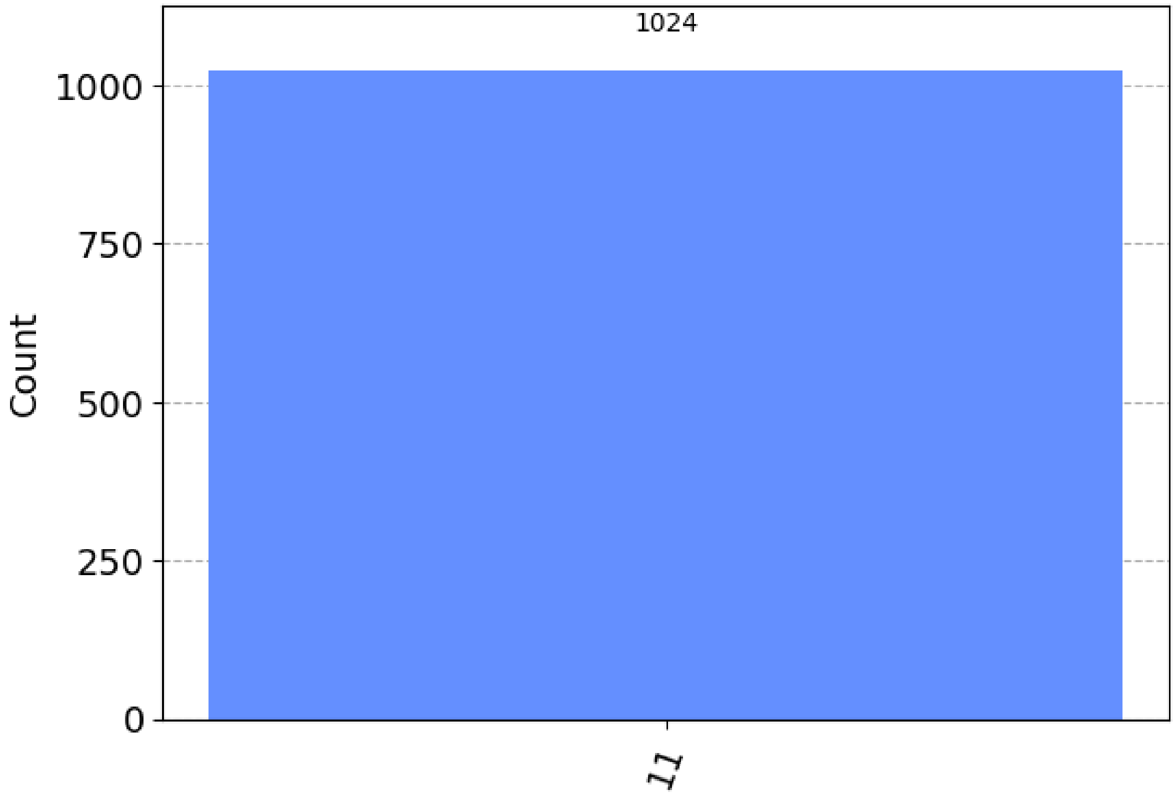
In [ ]:
```
sv_sim = Aer.get_backend('statevector_simulator')
result = sv_sim.run(grover_circuit).result()
statevec = result.get_statevector()
array_to_latex(statevec, prefix="|\\psi\\rangle =")
```

Out[ ]:

$$|\psi\rangle = \begin{bmatrix} 0 & 0 & 0 & 1 \end{bmatrix}$$

In [ ]:
```
grover_circuit.measure_all()

qasm_sim = Aer.get_backend('qasm_simulator')
result = qasm_sim.run(grover_circuit).result()
counts = result.get_counts()
plot_histogram(counts)
```

Out[ ]:



3. Example: 3 Qubits

Grover's algorithm for 3 qubits with two marked states|101> and |110>

In [ ]:
```
qc = QuantumCircuit(3)
qc.cz(0, 2)
qc.cz(1, 2)
oracle_ex3 = qc.to_gate()
oracle_ex3.name = "U$_\omega$"
```

In [ ]:
```
def diffuser(nqubits):
    qc = QuantumCircuit(nqubits)
    # Apply transformation |s> -> |00..0> (H-gates)
    for qubit in range(nqubits):
        qc.h(qubit)
    # Apply transformation |00..0> -> |11..1> (X-gates)
    for qubit in range(nqubits):
        qc.x(qubit)
```

```python
    # Do multi-controlled-Z gate
    qc.h(nqubits-1)
    qc.mct(list(range(nqubits-1)), nqubits-1)  # multi-controlled-toffoli
    qc.h(nqubits-1)
    # Apply transformation |11..1> -> |00..0>
    for qubit in range(nqubits):
        qc.x(qubit)
    # Apply transformation |00..0> -> |s>
    for qubit in range(nqubits):
        qc.h(qubit)
    # We will return the diffuser as a gate
    U_s = qc.to_gate()
    U_s.name = "U$_s$"
    return U_s
```

In [ ]:

```python
n = 3
grover_circuit = QuantumCircuit(n)
grover_circuit = initialize_s(grover_circuit, [0,1,2])
grover_circuit.append(oracle_ex3, [0,1,2])
grover_circuit.append(diffuser(n), [0,1,2])
grover_circuit.measure_all()
grover_circuit.draw()
```

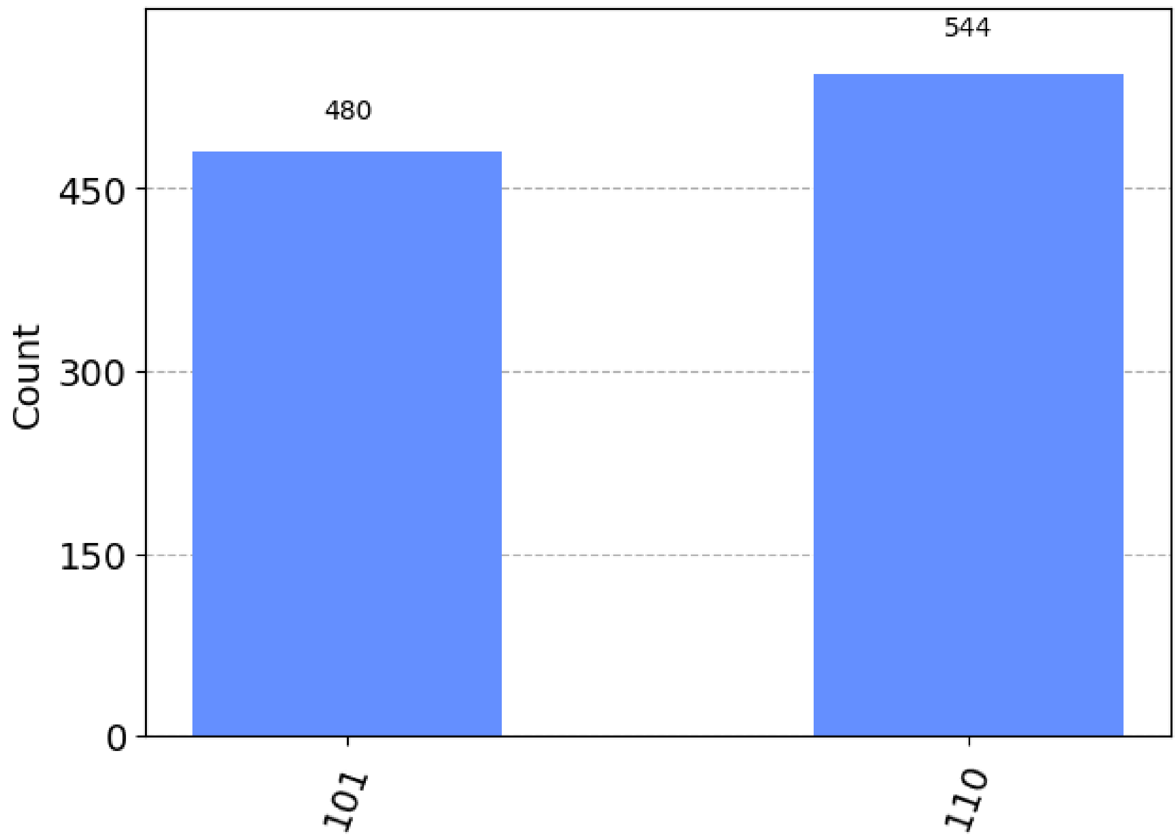
Out[ ]:

```
        ┌───┐┌──────────────┐┌──────────┐ ░ ┌─┐
   q_0: ┤ H ├┤0             ├┤0         ├─░─┤M├──────
        ├───┤│              ││          │ ░ └╥┘┌─┐
   q_1: ┤ H ├┤1 U$_\omega$  ├┤1 U$_s$   ├─░──╫─┤M├───
        ├───┤│              ││          │ ░  ║ └╥┘┌─┐
   q_2: ┤ H ├┤2             ├┤2         ├─░──╫──╫─┤M├
        └───┘└──────────────┘└──────────┘ ░  ║  ║ └╥┘
meas: 3/═════════════════════════════════════╩══╩══╩═
                                             0  1  2
```

In [ ]:

```python
qasm_sim = Aer.get_backend('qasm_simulator')
transpiled_grover_circuit = transpile(grover_circuit, qasm_sim)
results = qasm_sim.run(transpiled_grover_circuit).result()
counts = results.get_counts()
plot_histogram(counts)
```

Out[ ]:



4. Problems

In [ ]:

```
from qiskit_textbook.problems import grover_problem_oracle
## Example Usage
n = 4
oracle = grover_problem_oracle(n, variant=1)  # 0th variant of oracle, with n qubit
qc = QuantumCircuit(n)
qc.append(oracle, [0,1,2,3])
qc.draw()
```

Out[ ]:

```
q_0: ┤0              ├
     │               │
q_1: ┤1              ├
     │   Oracle
n=4, var=1 │
q_2: ┤2              ├
     │               │
q_3: ┤3              ├
     └───────────────┘
```

Question 1

> grover_problem_oracle(4, variant=2) uses 4 qubits and has 1 solution.

> a. How many iterations do we need to have a > 90% chance of measuring this solution?

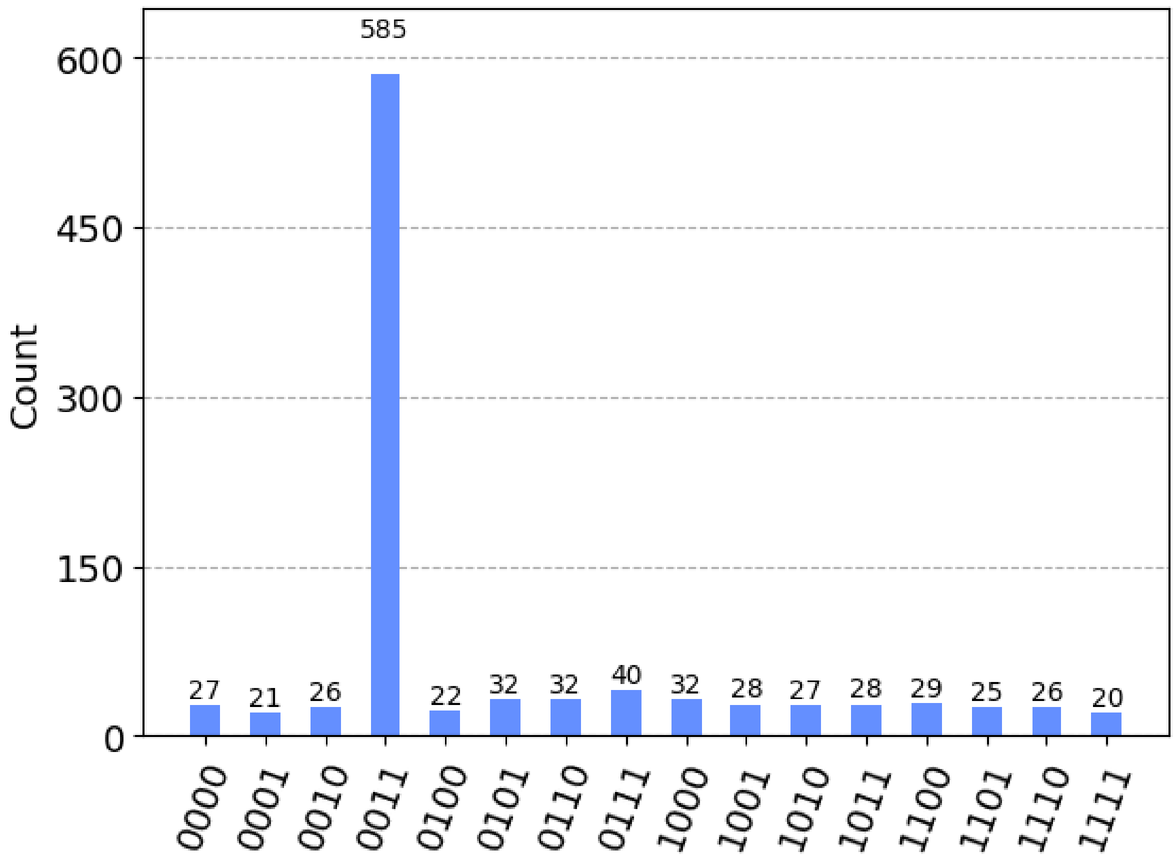
For 1 solution and n=4 qubits, optimal number of iterations is $\frac{\sqrt{2^n}}{2}$ = $\frac{\sqrt{2^4}}{2}$ =2

> b. Use Grover's algorithm to find this solution state.

In [ ]:
```
qc = QuantumCircuit(4, 4)
oracle_ex4 = grover_problem_oracle(4, variant=2)

# Initialize to superposition
qc.h([0,1,2,3])

# Apply Grover's iteration
qc.append(oracle_ex4, [0,1,2,3])
qc.append(diffuser(4), [0,1,2,3])

# Repeat the iteration
for _ in range(3):
    qc.append(oracle_ex4, [0,1,2,3])
    qc.append(diffuser(4), [0,1,2,3])

# Measure the qubits
qc.measure([0,1,2,3], [0,1,2,3])

# Execute the circuit
counts = execute(qc, Aer.get_backend('qasm_simulator'), shots=1000).result().get_co
print(counts)
plot_histogram(counts)
```

```
{'1100': 29, '0011': 585, '0110': 32, '0101': 32, '1001': 28, '1011': 28, '1000': 
32, '0111': 40, '0100': 22, '0010': 26, '0000': 27, '1110': 26, '0001': 21, '101
0': 27, '1111': 20, '1101': 25}
```

Out[ ]:



> c. What happens if we apply more iterations than the number we calculated in problem 1a above? Why?

We may decrease the chance of finding our solution because we have over-rotated and overshot our target state, so |s'> =/= |w>

This is shown below:

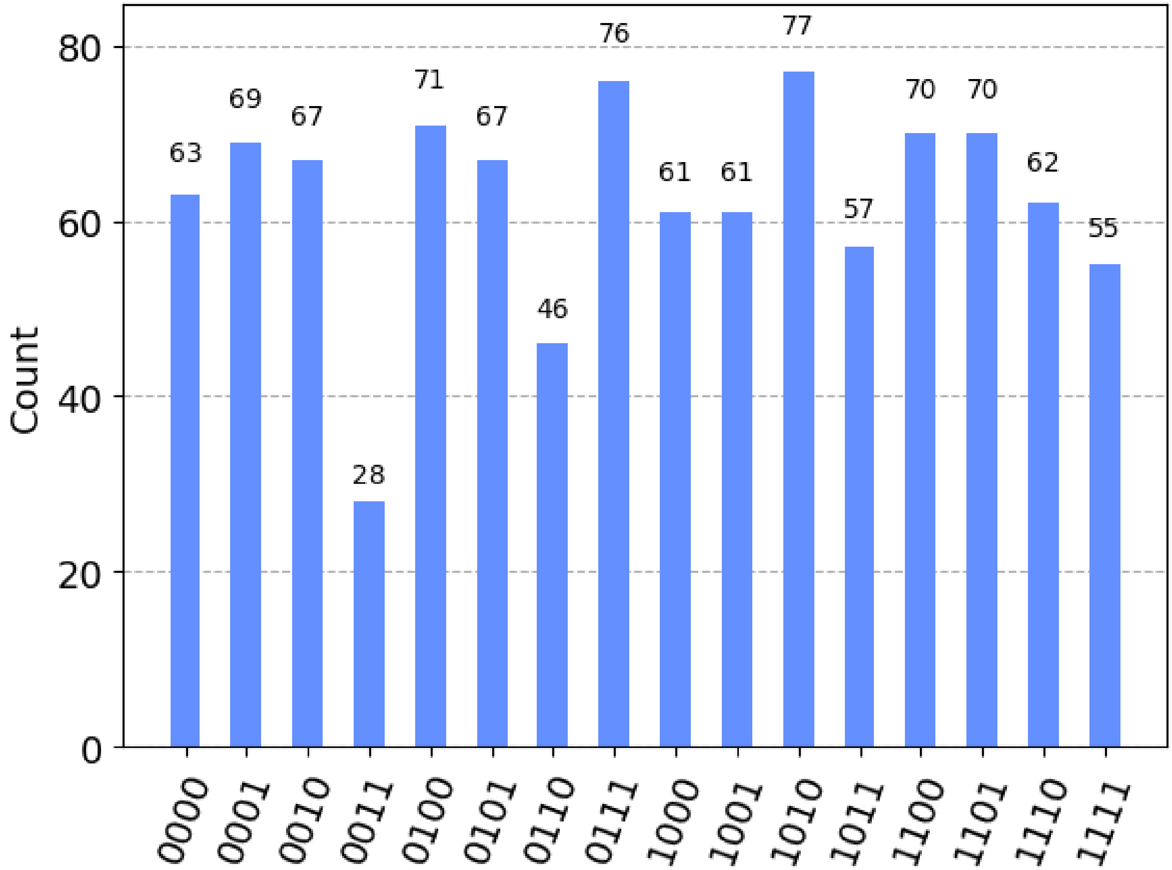
In [ ]:

```
qc = QuantumCircuit(4, 4)
oracle_ex4 = grover_problem_oracle(4, variant=2)

# Initialize to superposition
qc.h([0,1,2,3])

# Apply Grover's iteration
qc.append(oracle_ex4, [0,1,2,3])
qc.append(diffuser(4), [0,1,2,3])

# Repeat the iteration
for _ in range(5):
    qc.append(oracle_ex4, [0,1,2,3])
    qc.append(diffuser(4), [0,1,2,3])

# Measure the qubits
qc.measure([0,1,2,3], [0,1,2,3])

# Execute the circuit
```

```
counts = execute(qc, Aer.get_backend('qasm_simulator'), shots=1000).result().get_co
print(counts)
plot_histogram(counts)
```

```
{'0110': 46, '0101': 67, '0111': 76, '0001': 69, '1101': 70, '0100': 71, '0010': 6
7, '1000': 61, '1010': 77, '0011': 28, '1011': 57, '1110': 62, '1111': 55, '1100':
70, '0000': 63, '1001': 61}
```

Out[ ]:



Question 2

> 2. With 2 solutions and 4 qubits, how many iterations do we need for a >90% chance of measuring a solution? Test your answer using the oracle grover_problem_oracle(4, variant=1) (which has two solutions). Test your answer using the oracle grover_problem_oracle(4, variant=1) (which has two solutions).

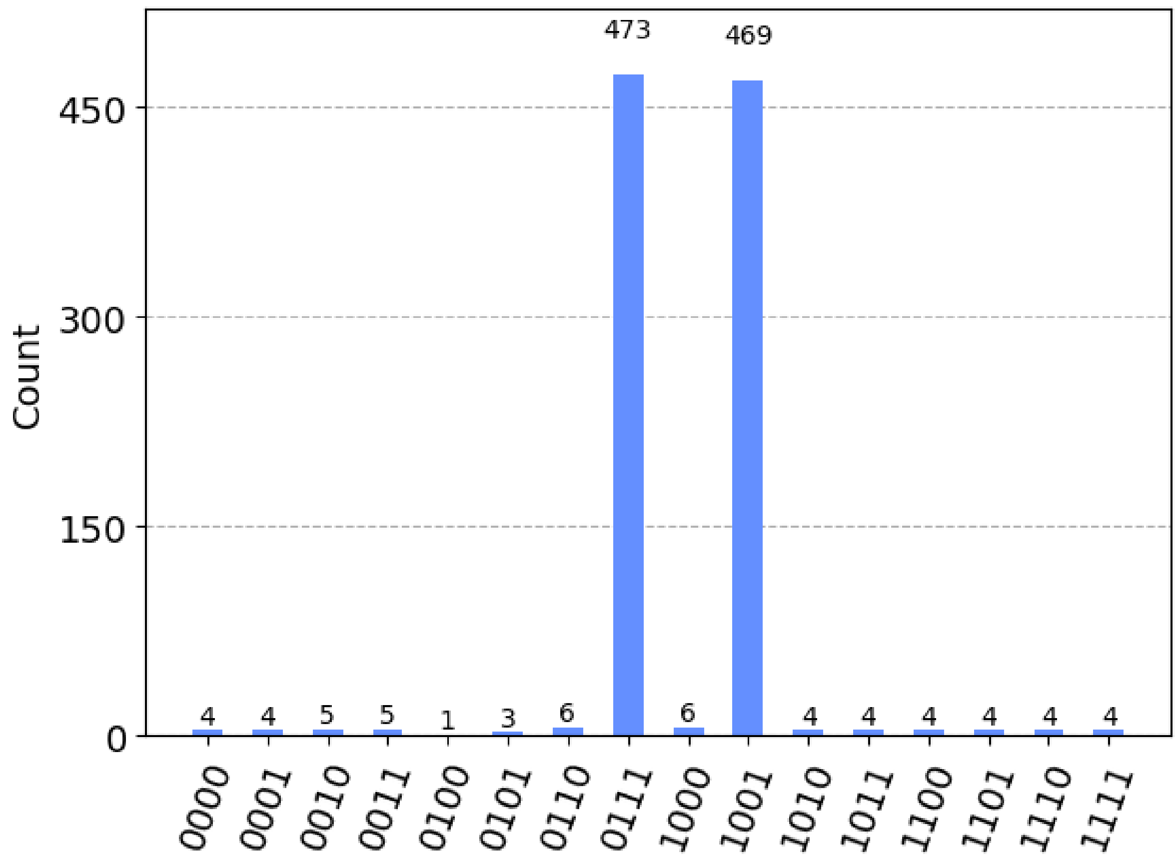
Iterations needed is $\sqrt{2}$ so ~1 iteration. I tried experimenting with 1 and 2, as shown below

In [ ]:
```
#1 Rotation
qc = QuantumCircuit(4, 4)
oracle_ex2_4 = grover_problem_oracle(4, variant=1)

# Initialize to superposition
qc.h([0,1,2,3])

# Apply Grover's iteration
qc.append(oracle_ex2_4, [0,1,2,3])
```

```
qc.append(diffuser(4), [0,1,2,3])

# Repeat the iteration
for _ in range(1):
    qc.append(oracle_ex2_4, [0,1,2,3])
    qc.append(diffuser(4), [0,1,2,3])

# Measure the qubits
qc.measure([0,1,2,3], [0,1,2,3])

# Execute the circuit
counts = execute(qc, Aer.get_backend('qasm_simulator'), shots=1000).result().get_co
print(counts)
plot_histogram(counts)
```

```
{'1010': 4, '1001': 469, '0111': 473, '0011': 5, '0101': 3, '1000': 6, '0000': 4,
'0110': 6, '1110': 4, '1111': 4, '0010': 5, '1100': 4, '1101': 4, '0001': 4, '101
1': 4, '0100': 1}
```

Out[ ]:

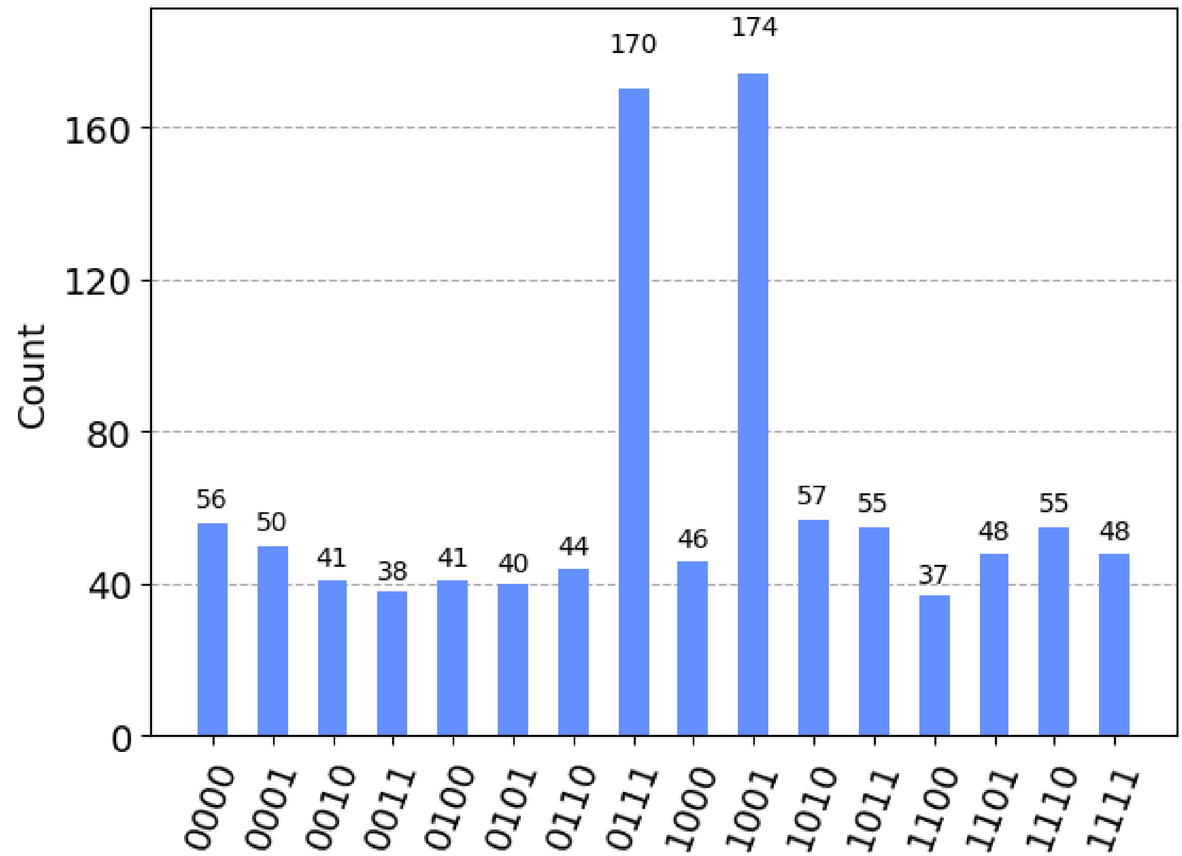


In [ ]:
```
#2 Rotations
qc = QuantumCircuit(4, 4)
oracle_ex2_4 = grover_problem_oracle(4, variant=1)

# Initialize to superposition
qc.h([0,1,2,3])

# Apply Grover's iteration
qc.append(oracle_ex2_4, [0,1,2,3])
qc.append(diffuser(4), [0,1,2,3])
```

```
# Repeat the iteration
for _ in range(2):
    qc.append(oracle_ex2_4, [0,1,2,3])
    qc.append(diffuser(4), [0,1,2,3])

# Measure the qubits
qc.measure([0,1,2,3], [0,1,2,3])

# Execute the circuit
counts = execute(qc, Aer.get_backend('qasm_simulator'), shots=1000).result().get_co
print(counts)
plot_histogram(counts)
```

```
{'1010': 57, '1001': 174, '1110': 55, '1000': 46, '0000': 56, '0011': 38, '0111': 
170, '1101': 48, '1111': 48, '0010': 41, '0110': 44, '1011': 55, '0101': 40, '000
1': 50, '1100': 37, '0100': 41}
```

Out[ ]:



We can see from comparing the two graphs, the probability is higher for 1 rotation.

Question 3

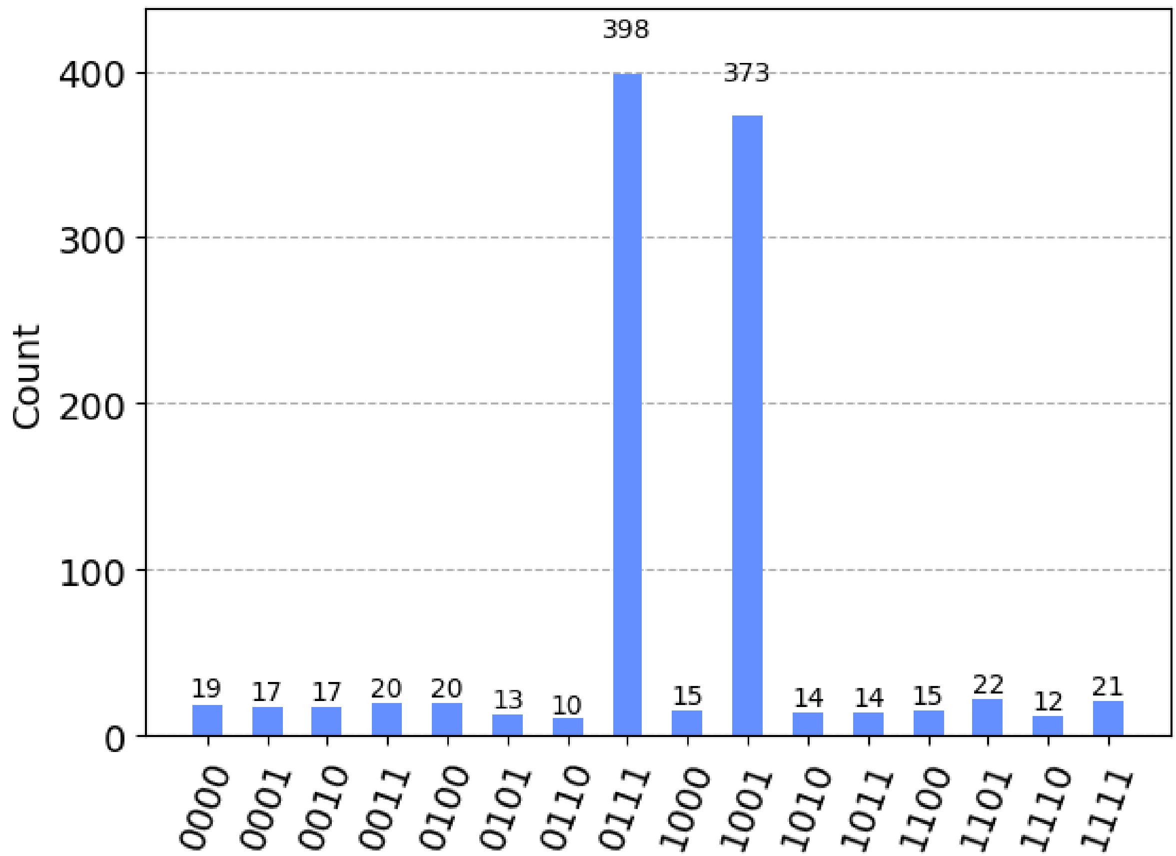
> Create a function, grover_solver(oracle, iterations) that takes as input:
>
> - A Grover oracle as a gate (oracle)
> - An integer number of iterations (iterations)
>
> and returns a QuantumCircuit that performs Grover's algorithm on the 'oracle' gate, with 'iterations' iterations.

In [ ]:
```
def grover_solver(oracle, iterations):
    n = oracle.num_qubits  # Get the number of qubits from the oracle
    qc = QuantumCircuit(n, n)

    # Initialize to superposition
    qc.h(range(n))

    # Apply Grover's iterations
    for _ in range(iterations):
        qc.append(oracle, range(n))
        qc.append(diffuser(n), range(n))

    # Measure the qubits
    qc.measure(range(n), range(n))

    return qc
```

In [ ]:
```
# Example Usage:
oracle = grover_problem_oracle(4, variant=1)
iterations = 1
qc = grover_solver(oracle, iterations)

# Execute the circuit
counts = execute(qc, Aer.get_backend('qasm_simulator'), shots=1000).result().get_co
print(counts)
plot_histogram(counts)
```

```
{'1101': 22, '1001': 373, '0111': 398, '1011': 14, '0010': 17, '1000': 15, '1111':
21, '1110': 12, '0001': 17, '0110': 10, '0101': 13, '1100': 15, '1010': 14, '001
1': 20, '0100': 20, '0000': 19}
```

Out[ ]:



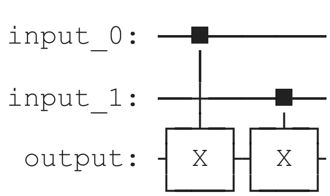
Solving Suduko using Grover's Algorithm

In [ ]:
```
clause_list = [ [0,1],
                [0,2],
                [1,3],
                [2,3] ]
```

In [ ]:
```
def XOR(qc, a, b, output):
    qc.cx(a, output)
    qc.cx(b, output)
```

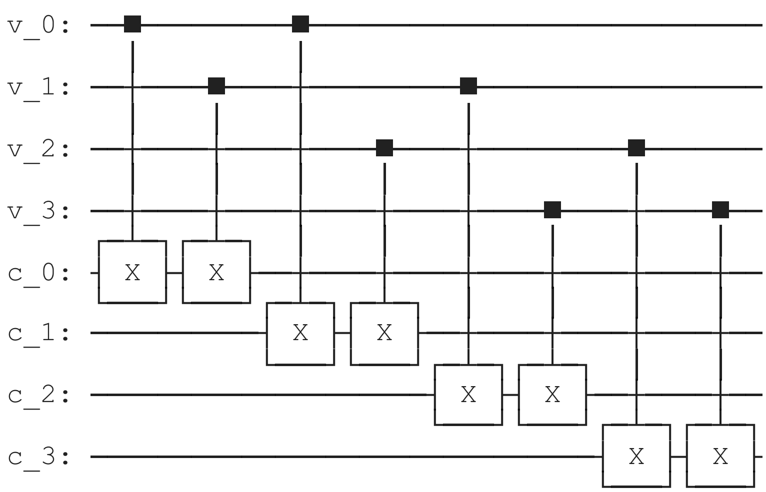
In [ ]:
```
# We will use separate registers to name the bits
in_qubits = QuantumRegister(2, name='input')
out_qubit = QuantumRegister(1, name='output')
qc = QuantumCircuit(in_qubits, out_qubit)
XOR(qc, in_qubits[0], in_qubits[1], out_qubit)
qc.draw()
```

Out[ ]:



In [ ]:
```
# Create separate registers to name bits
var_qubits = QuantumRegister(4, name='v')  # variable bits
clause_qubits = QuantumRegister(4, name='c')  # bits to store clause-checks

# Create quantum circuit
qc = QuantumCircuit(var_qubits, clause_qubits)

# Use XOR gate to check each clause
i = 0
for clause in clause_list:
    XOR(qc, clause[0], clause[1], clause_qubits[i])
    i += 1

# Create separate registers to name bits
var_qubits = QuantumRegister(4, name='v')
clause_qubits = QuantumRegister(4, name='c')
output_qubit = QuantumRegister(1, name='out')
qc = QuantumCircuit(var_qubits, clause_qubits, output_qubit)

# Compute clauses
i = 0
for clause in clause_list:
    XOR(qc, clause[0], clause[1], clause_qubits[i])
    i += 1

# Flip 'output' bit if all clauses are satisfied
qc.mct(clause_qubits, output_qubit)

qc.draw()
```

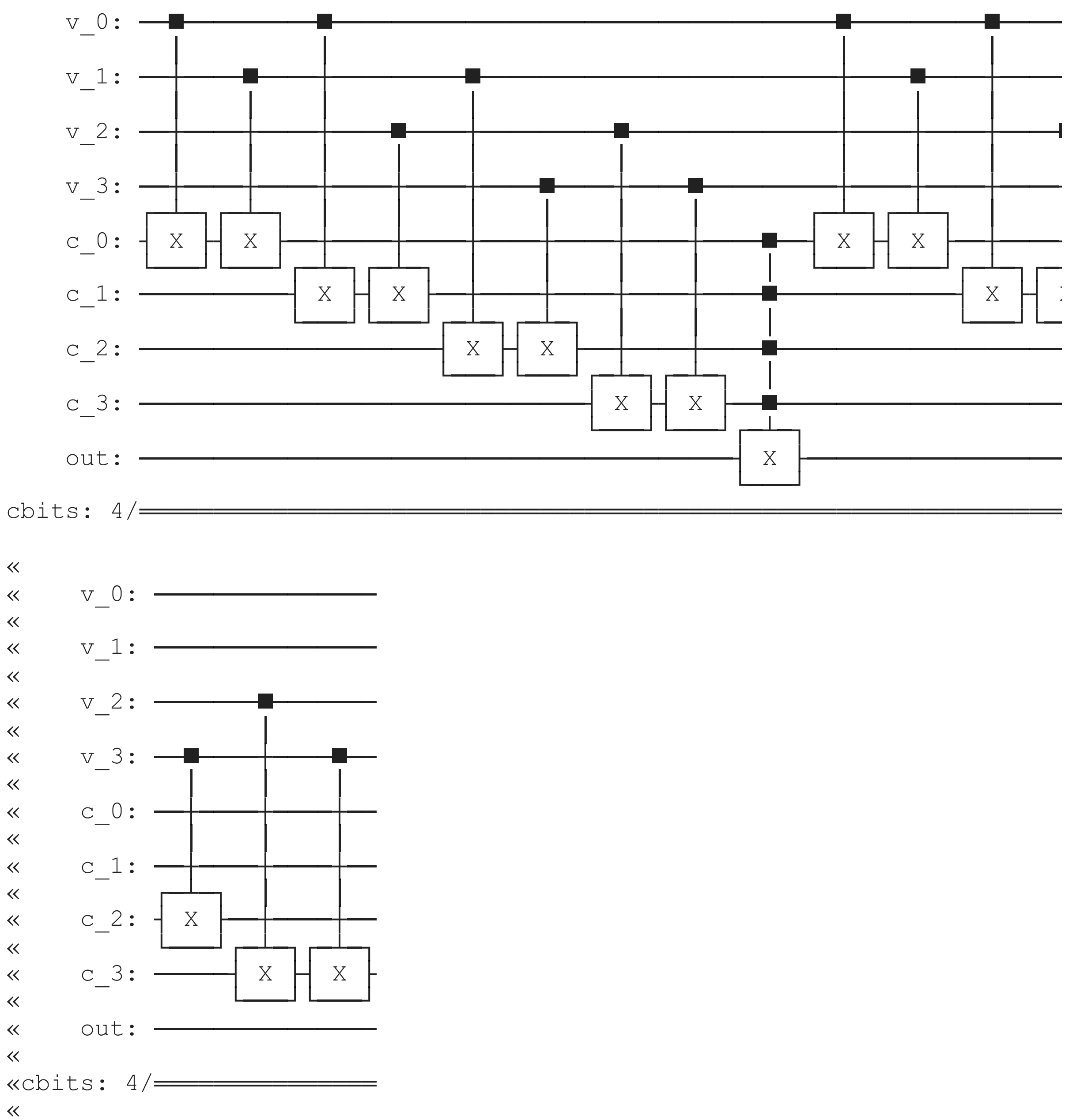
Out[ ]:



In [ ]:
```
var_qubits = QuantumRegister(4, name='v')
clause_qubits = QuantumRegister(4, name='c')
output_qubit = QuantumRegister(1, name='out')
cbits = ClassicalRegister(4, name='cbits')
qc = QuantumCircuit(var_qubits, clause_qubits, output_qubit, cbits)

def sudoku_oracle(qc, clause_list, clause_qubits):
    # Compute clauses
    i = 0
```

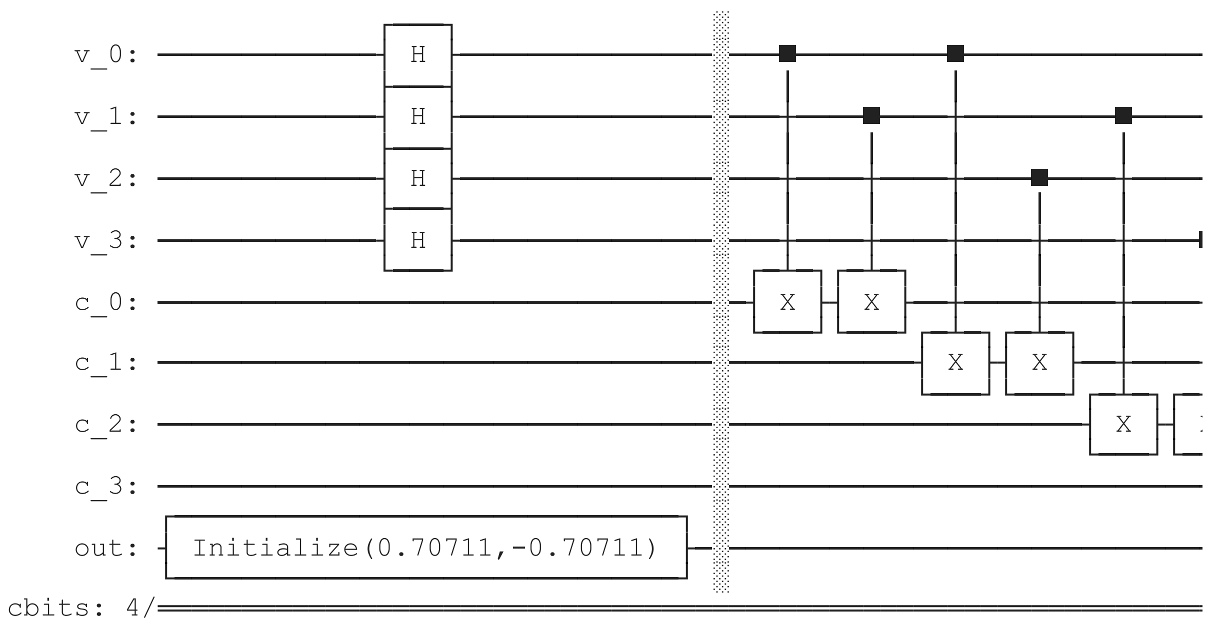
```
    for clause in clause_list:
        XOR(qc, clause[0], clause[1], clause_qubits[i])
        i += 1

    # Flip 'output' bit if all clauses are satisfied
    qc.mct(clause_qubits, output_qubit)

    # Uncompute clauses to reset clause-checking bits to 0
    i = 0
    for clause in clause_list:
        XOR(qc, clause[0], clause[1], clause_qubits[i])
        i += 1
sudoku_oracle(qc, clause_list, clause_qubits)
qc.draw()
```

Out[ ]:



In [ ]:
```python
var_qubits = QuantumRegister(4, name='v')
clause_qubits = QuantumRegister(4, name='c')
output_qubit = QuantumRegister(1, name='out')
cbits = ClassicalRegister(4, name='cbits')
qc = QuantumCircuit(var_qubits, clause_qubits, output_qubit, cbits)

# Initialize 'out0' in state |->
qc.initialize([1, -1]/np.sqrt(2), output_qubit)

# Initialize qubits in state |s>
qc.h(var_qubits)
qc.barrier()  # for visual separation

## First Iteration
# Apply our oracle
sudoku_oracle(qc, clause_list, clause_qubits)
qc.barrier()  # for visual separation
# Apply our diffuser
qc.append(diffuser(4), [0,1,2,3])

## Second Iteration
sudoku_oracle(qc, clause_list, clause_qubits)
qc.barrier()  # for visual separation
# Apply our diffuser
qc.append(diffuser(4), [0,1,2,3])

# Measure the variable qubits
qc.measure(var_qubits, cbits)

qc.draw(fold=-1)
```

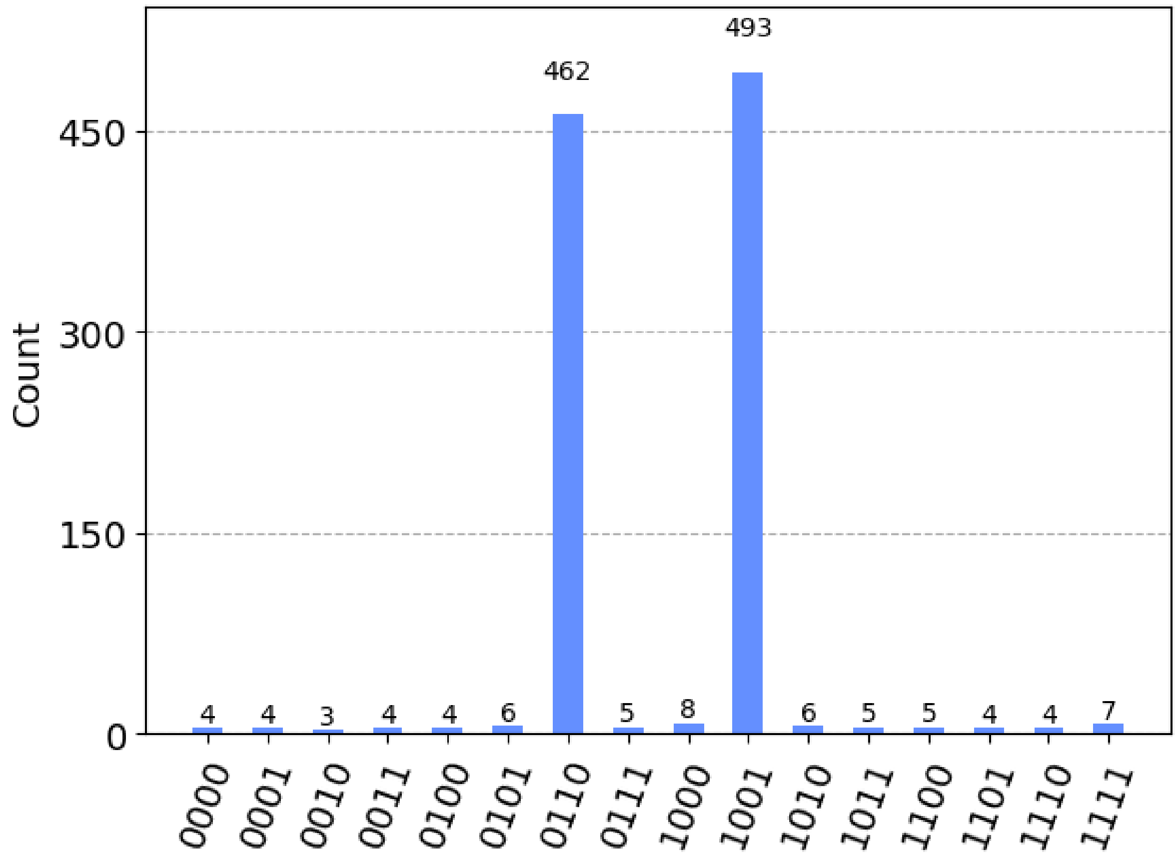
Out[ ]:



In [ ]:
```python
# Simulate and plot results
qasm_simulator = Aer.get_backend('qasm_simulator')
transpiled_qc = transpile(qc, qasm_simulator)
```

```
result = qasm_sim.run(transpiled_qc).result()
plot_histogram(result.get_counts())
```

Out[ ]:



Triangle Finding Problem Using Grover

In [ ]:

```
#Edges list
edges =[(0, 1), (0, 2), (1, 2), (2, 3)]
#Number of nodes
n_nodes = 4
```

In [ ]:

```
#We used the W state implementation from W state in reference 6
def control_rotation (qcir,cQbit,tQbit,theta):
    """ Create an intermediate controlled rotation using only unitary gate and cont

    Args:
    qcir: QuantumCircuit instance to apply the controlled rotation to.
    cQbit: control qubit.
    tQbit: target qubit.
    theta: rotation angle.

    Returns:
    A modified version of the QuantumCircuit instance with control rotation applied

    """
    theta_dash = math.asin(math.cos(math.radians(theta/2)))
    qcir.u(theta_dash,0,0,tQbit)
    qcir.cx(cQbit,tQbit)
```

```python
    qcir.u(-theta_dash,0,0,tQbit)
    return qcir
```

In [ ]:
```python
def wn (qcir,qbits):
    """ Create the W-state using the control-rotation function.

    Args:
    qcir: QuantumCircuit instance used to construct the W-state.
    qbits: the qubits used to construct the W-state.

    Returns:
    A modified version of the QuantumCircuit instance with the W-state construction

    """
    for i in range(len(qbits)):
        if i == 0:
            qcir.x(qbits[0])
            qcir.barrier()
        else:
            p = 1/(len(qbits)-(i-1))
            theta = math.degrees(math.acos(math.sqrt(p)))
            theta = 2* theta
            qcir = control_rotation(qcir,qbits[i-1],qbits[i],theta)
            qcir.cx(qbits[i],qbits[i-1])
            qcir.barrier()
    return qcir,qbits
```

In [ ]:
```python
sub_qbits = QuantumRegister(n_nodes)
sub_cir = QuantumCircuit(sub_qbits, name="state_prep")
sub_cir, sub_qbits = wn(sub_cir, sub_qbits)
sub_cir.x(sub_qbits)
stat_prep = sub_cir.to_instruction()
inv_stat_prep = sub_cir.inverse().to_instruction()
```

In [ ]:
```python
def edge_counter(qc,qubits,anc,flag_qubit,k):
    bin_k = bin(k)[2:][::-1]
    l = []
    for i in range(len(bin_k)):
        if int(bin_k[i]) == 1:
            l.append(qubits[i])
    qc.mct(l,flag_qubit,[anc])
```

In [ ]:
```python
def oracle(n_nodes, edges, qc, nodes_qubits, edge_anc, ancilla, neg_base):
    k = 3 #k is the number of edges, in case of a triangle, it's 3
    #1- edge counter
    #forward circuit
    qc.barrier()
    qc.ccx(nodes_qubits[edges[0][0] ],nodes_qubits[edges[0][1] ],edge_anc[0])
    for i in range(1,len(edges)):
        qc.mct([nodes_qubits[edges[i][0] ],nodes_qubits[edges[i][1] ],edge_anc[0] ]
        qc.ccx(nodes_qubits[edges[i][0] ],nodes_qubits[edges[i][1] ],edge_anc[0])
    #---------------------------------------------------------------------------
     #Edges check Qubit
    edg_k = int((k/2)*(k-1))
    edge_counter(qc,edge_anc,ancilla[0],neg_base[0],edg_k)
```

```
    #----------------------------------------------------------------------------

    #4- Reverse edge count
    for i in range(len(edges)-1,0,-1):
        qc.ccx(nodes_qubits[edges[i][0] ],nodes_qubits[edges[i][1] ],edge_anc[0])
        qc.mct([nodes_qubits[edges[i][0] ],nodes_qubits[edges[i][1] ],edge_anc[0] ]
    qc.ccx(nodes_qubits[edges[0][0] ],nodes_qubits[edges[0][1] ],edge_anc[0])
    qc.barrier()
```

In [ ]:
```
#Diffusion Operator
def cnz(qc, num_control, node, anc):
    """Construct a multi-controlled Z gate

    Args:
    num_control :  number of control qubits of cnz gate
    node :             node qubits
    anc :               ancillaly qubits
    """
    if num_control>2:
        qc.ccx(node[0], node[1], anc[0])
        for i in range(num_control-2):
            qc.ccx(node[i+2], anc[i], anc[i+1])
        qc.cz(anc[num_control-2], node[num_control])
        for i in range(num_control-2)[::-1]:
            qc.ccx(node[i+2], anc[i], anc[i+1])
        qc.ccx(node[0], node[1], anc[0])
    if num_control==2:
        qc.h(node[2])
        qc.ccx(node[0], node[1], node[2])
        qc.h(node[2])
    if num_control==1:
        qc.cz(node[0], node[1])
```

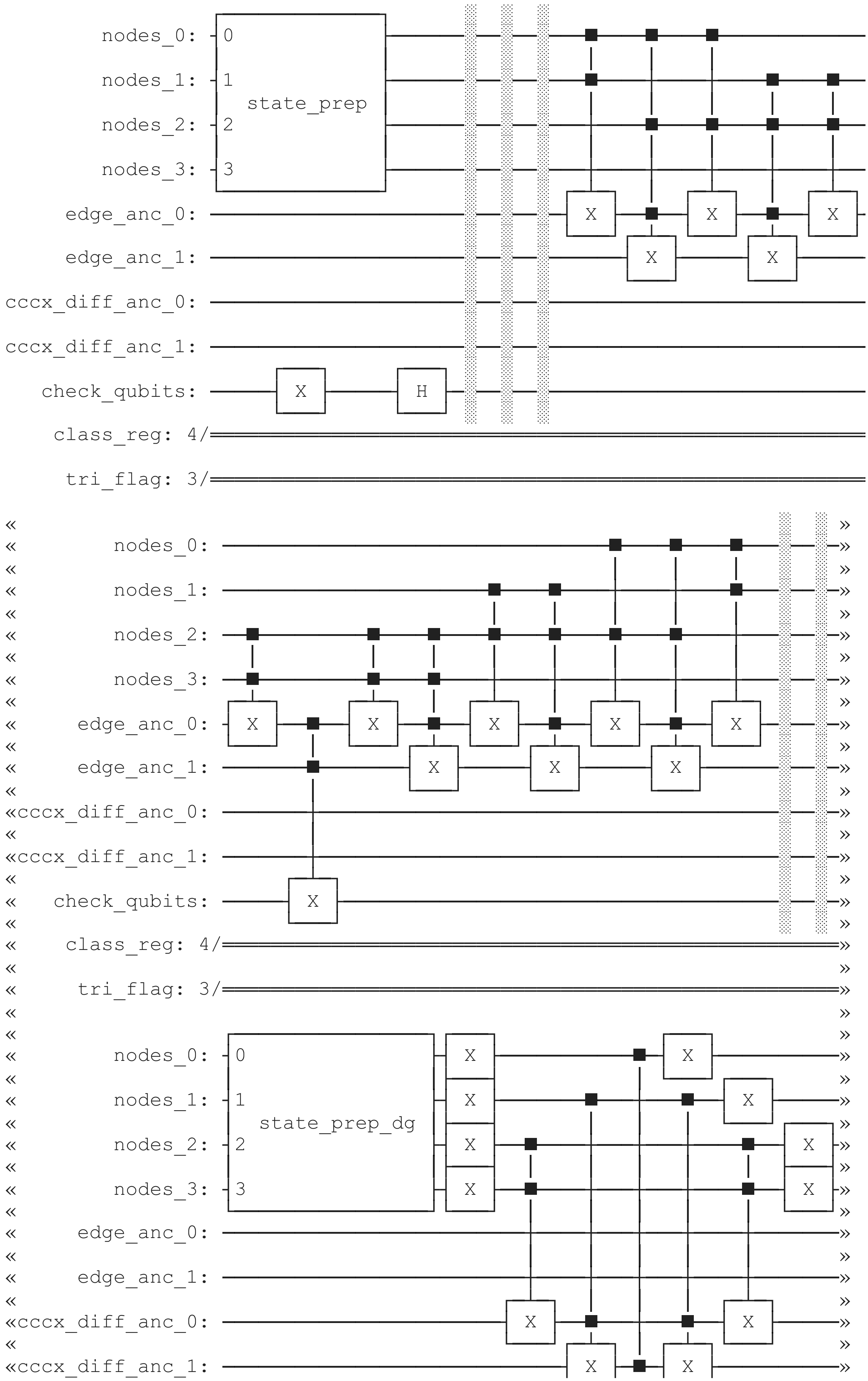
In [ ]:
```
def grover_diff(qc, nodes_qubits,edge_anc,ancilla,stat_prep,inv_stat_prep):
    qc.append(inv_stat_prep,qargs=nodes_qubits)
    qc.x(nodes_qubits)
    #====================================================
        #3 control qubits Z gate
    cnz(qc,len(nodes_qubits)-1,nodes_qubits[::-1],ancilla)
    #====================================================
    qc.x(nodes_qubits)
    qc.append(stat_prep,qargs=nodes_qubits)
```

In [ ]:
```
# Grover algo function
def grover(n_nodes,stat_prep,inv_stat_prep):
    #N = 2**n_nodes  # for optimal iterations count if the state prep is done using
    N = math.comb(n_nodes, 3) #Since we are using W-state to perform initial prepar
    nodes_qubits = QuantumRegister(n_nodes, name='nodes')
    edge_anc = QuantumRegister(2, name='edge_anc')
    ancilla = QuantumRegister(n_nodes-2, name = 'cccx_diff_anc')
    neg_base = QuantumRegister(1, name='check_qubits')
    class_bits = ClassicalRegister(n_nodes, name='class_reg')
    tri_flag = ClassicalRegister(3, name='tri_flag')
    qc = QuantumCircuit(nodes_qubits, edge_anc, ancilla, neg_base, class_bits, tri_
    # Initialize qunatum flag qubits in |-> state
```

```
    qc.x(neg_base[0])
    qc.h(neg_base[0])
    # Initializing i/p qubits in superposition
    qc.append(stat_prep,qargs=nodes_qubits)
    qc.barrier()
    # Calculate iteration count
    iterations = math.floor(math.pi/4*math.sqrt(N))
    # Calculate iteration count
    for i in np.arange(iterations):
        qc.barrier()
        oracle(n_nodes, edges, qc, nodes_qubits, edge_anc,  ancilla, neg_base)
        qc.barrier()
        grover_diff(qc, nodes_qubits,edge_anc,ancilla,stat_prep,inv_stat_prep)
    qc.measure(nodes_qubits,class_bits)
    return qc
```

In [ ]:
```
qc = grover(n_nodes,stat_prep,inv_stat_prep)
qc.draw()
```

Out[ ]:

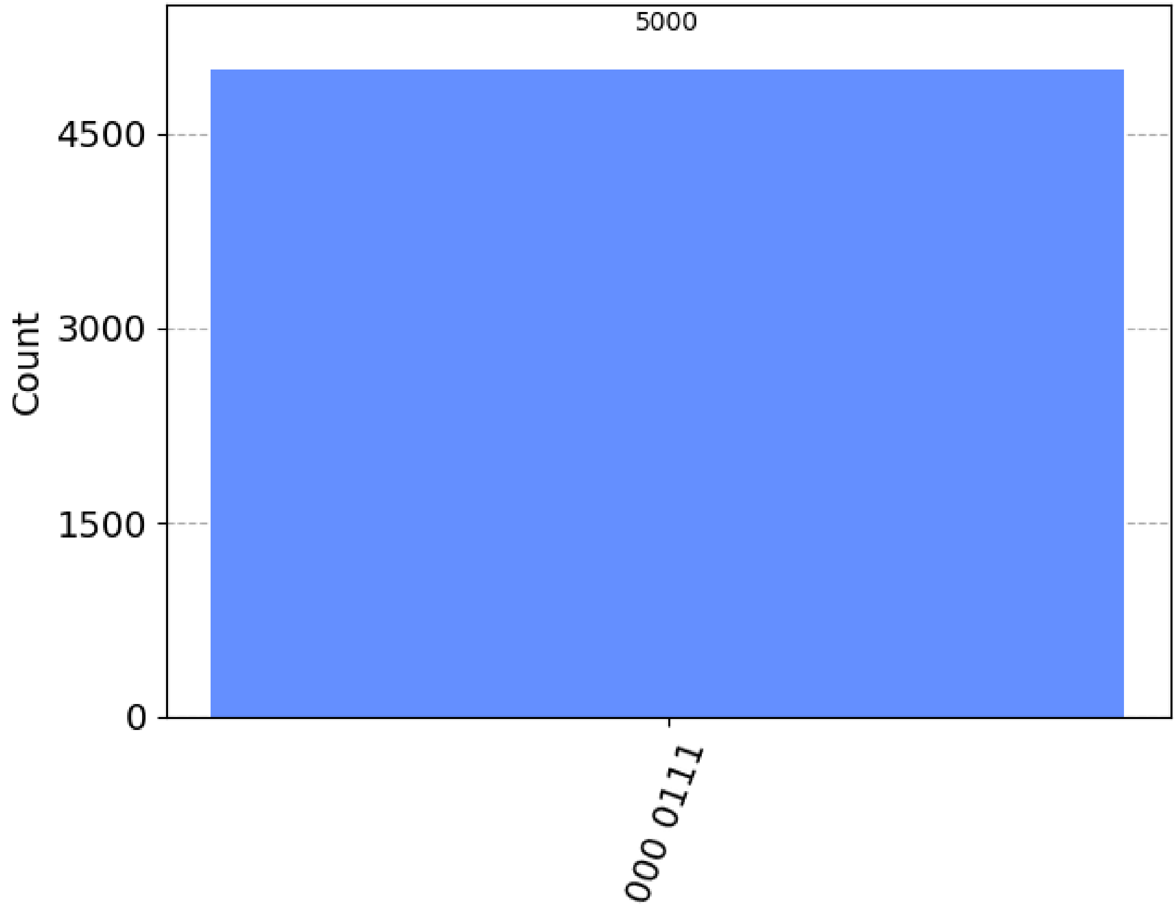

nodes_0:
nodes_1:
nodes_2:
nodes_3:
state_prep
edge_anc_0:
edge_anc_1:
cccx_diff_anc_0:
cccx_diff_anc_1:
check_qubits:
class_reg: 4/
tri_flag: 3/
X
H
state_prep_dg

```
«                                                      └───┘   └───┘          »
«   check_qubits: ────────────────────────────────────────────────────────────»
«                                                                             »
«    class_reg: 4/════════════════════════════════════════════════════════════»
«                                                                             »
«     tri_flag: 3/════════════════════════════════════════════════════════════»
«                                                                             »
«                ┌──────────────┐┌─┐
«        nodes_0: ┤0             ├┤M├───────────
«                │              │└╥┘┌─┐
«        nodes_1: ┤1             ├─╫─┤M├────────
«                │  state_prep  │ ║ └╥┘┌─┐
«        nodes_2: ┤2             ├─╫──╫─┤M├─────
«                │              │ ║  ║ └╥┘┌─┐
«        nodes_3: ┤3             ├─╫──╫──╫─┤M├─
«                └──────────────┘ ║  ║  ║ └╥┘
«     edge_anc_0: ─────────────────╫──╫──╫──╫──
«                                 ║  ║  ║  ║
«     edge_anc_1: ─────────────────╫──╫──╫──╫──
«                                 ║  ║  ║  ║
«cccx_diff_anc_0: ─────────────────╫──╫──╫──╫──
«                                 ║  ║  ║  ║
«cccx_diff_anc_1: ─────────────────╫──╫──╫──╫──
«                                 ║  ║  ║  ║
«   check_qubits: ─────────────────╫──╫──╫──╫──
«                                 ║  ║  ║  ║
«    class_reg: 4/═════════════════╩══╩══╩══╩══
«                                 0  1  2  3
«     tri_flag: 3/═══════════════════════════
```

In [ ]:

```
# Simulate and plot results
qasm_simulator = Aer.get_backend('qasm_simulator')
#transpiled_qc = transpile(qc, qasm_simulator)
# Execute circuit and show results
ex = execute(qc, qasm_simulator, shots = 5000)
res = ex.result().get_counts(qc)
plot_histogram(res)
```

Out[ ]:



Deutsh Jozsa Algorithm [Done]

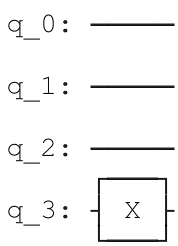
Following Textbook: https://learn.qiskit.org/course/ch-algorithms/deutsch-jozsa-algorithm

In [ ]:
```
# initialization
import numpy as np

# importing Qiskit
from qiskit import IBMQ, Aer
from qiskit.providers.ibmq import least_busy
from qiskit import QuantumCircuit, transpile

# import basic plot tools
from qiskit.visualization import plot_histogram
```

In [ ]:
```
# set the length of the n-bit input string.
n = 3

const_oracle = QuantumCircuit(n+1)

output = np.random.randint(2)
if output == 1:
    const_oracle.x(n)
```

```
const_oracle.draw()
```

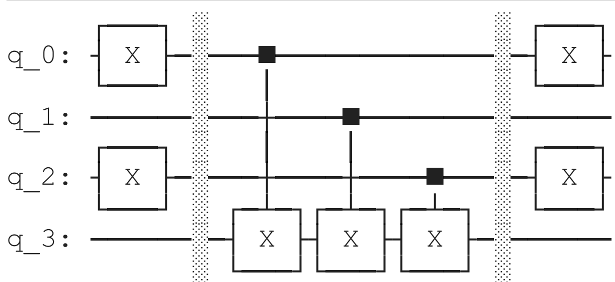
Out[ ]:



In [ ]:

```
balanced_oracle = QuantumCircuit(n+1)
b_str = "101"

# Place X-gates
for qubit in range(len(b_str)):
    if b_str[qubit] == '1':
        balanced_oracle.x(qubit)

# Use barrier as divider
balanced_oracle.barrier()

# Controlled-NOT gates
for qubit in range(n):
    balanced_oracle.cx(qubit, n)

balanced_oracle.barrier()

# Place X-gates
for qubit in range(len(b_str)):
    if b_str[qubit] == '1':
        balanced_oracle.x(qubit)

# Show oracle
balanced_oracle.draw()
```

Out[ ]:



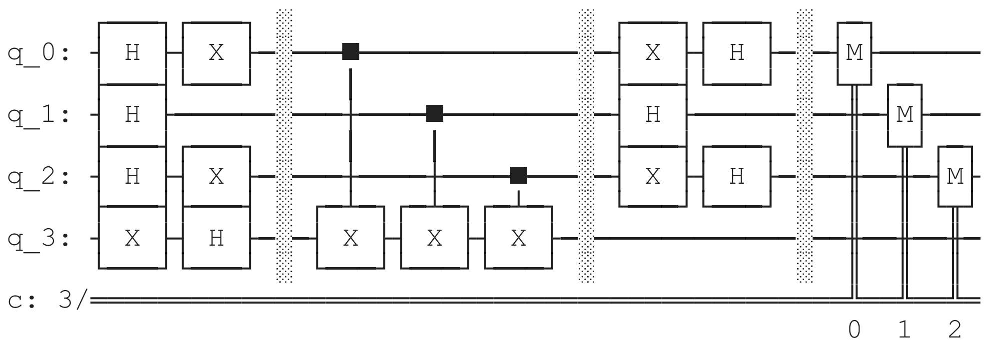
Full Algorithm

In [ ]:

```
dj_circuit = QuantumCircuit(n+1, n)

# Apply H-gates
for qubit in range(n):
    dj_circuit.h(qubit)

# Put qubit in state |->
```

```
dj_circuit.x(n)
dj_circuit.h(n)

# Add oracle
dj_circuit = dj_circuit.compose(balanced_oracle)

# Repeat H-gates
for qubit in range(n):
    dj_circuit.h(qubit)
dj_circuit.barrier()

# Measure
for i in range(n):
    dj_circuit.measure(i, i)

# Display circuit
dj_circuit.draw()
```

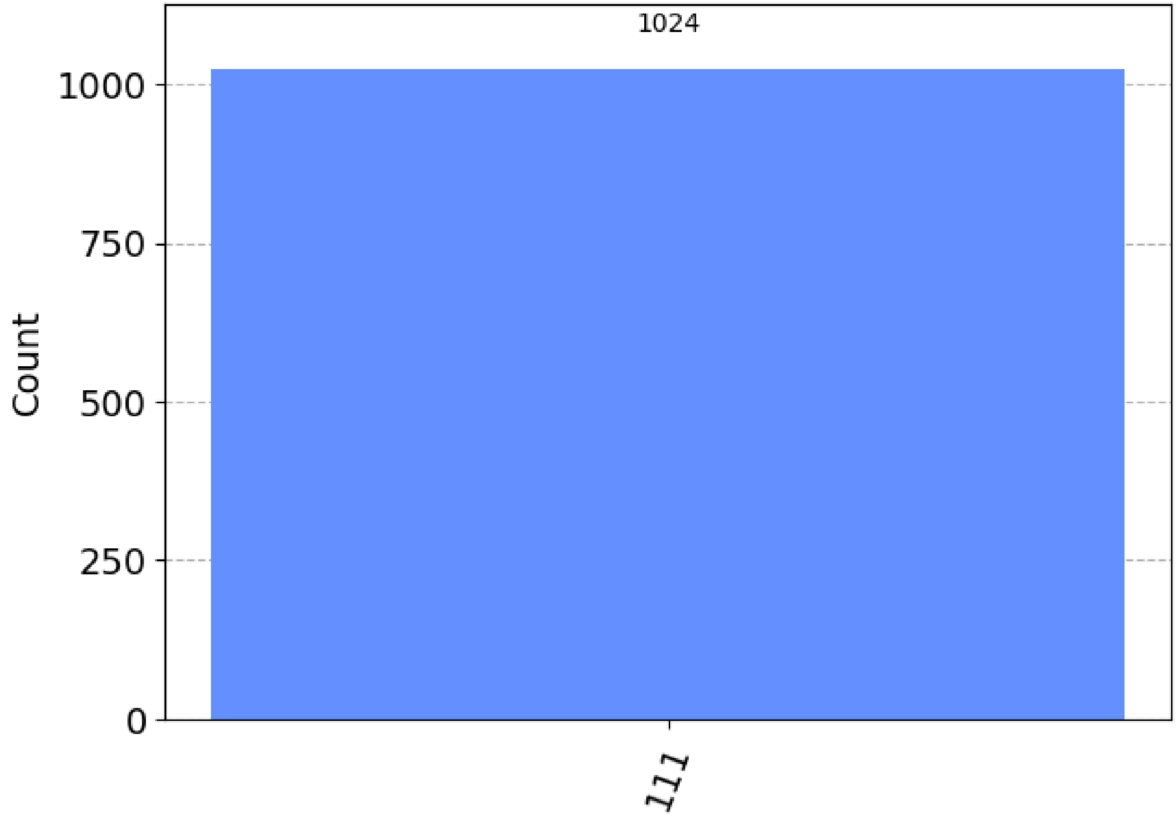
Out[ ]:



In [ ]:

```
# use local simulator
aer_sim = Aer.get_backend('aer_simulator')
results = aer_sim.run(dj_circuit).result()
answer = results.get_counts()

plot_histogram(answer)
```

Out[ ]:



Generalized Circuits

In [ ]:
```
def dj_oracle(case, n):
    # We need to make a QuantumCircuit object to return
    # This circuit has n+1 qubits: the size of the input,
    # plus one output qubit
    oracle_qc = QuantumCircuit(n+1)

    # First, let's deal with the case in which oracle is balanced
    if case == "balanced":
        # First generate a random number that tells us which CNOTs to
        # wrap in X-gates:
        b = np.random.randint(1,2**n)
        # Next, format 'b' as a binary string of length 'n', padded with zeros:
        b_str = format(b, '0'+str(n)+'b')
        # Next, we place the first X-gates. Each digit in our binary string
        # corresponds to a qubit, if the digit is 0, we do nothing, if it's 1
        # we apply an X-gate to that qubit:
        for qubit in range(len(b_str)):
            if b_str[qubit] == '1':
                oracle_qc.x(qubit)
        # Do the controlled-NOT gates for each qubit, using the output qubit
        # as the target:
        for qubit in range(n):
            oracle_qc.cx(qubit, n)
        # Next, place the final X-gates
        for qubit in range(len(b_str)):
            if b_str[qubit] == '1':
                oracle_qc.x(qubit)
```

```
    # Case in which oracle is constant
    if case == "constant":
        # First decide what the fixed output of the oracle will be
        # (either always 0 or always 1)
        output = np.random.randint(2)
        if output == 1:
            oracle_qc.x(n)

    oracle_gate = oracle_qc.to_gate()
    oracle_gate.name = "Oracle" # To show when we display the circuit
    return oracle_gate
```

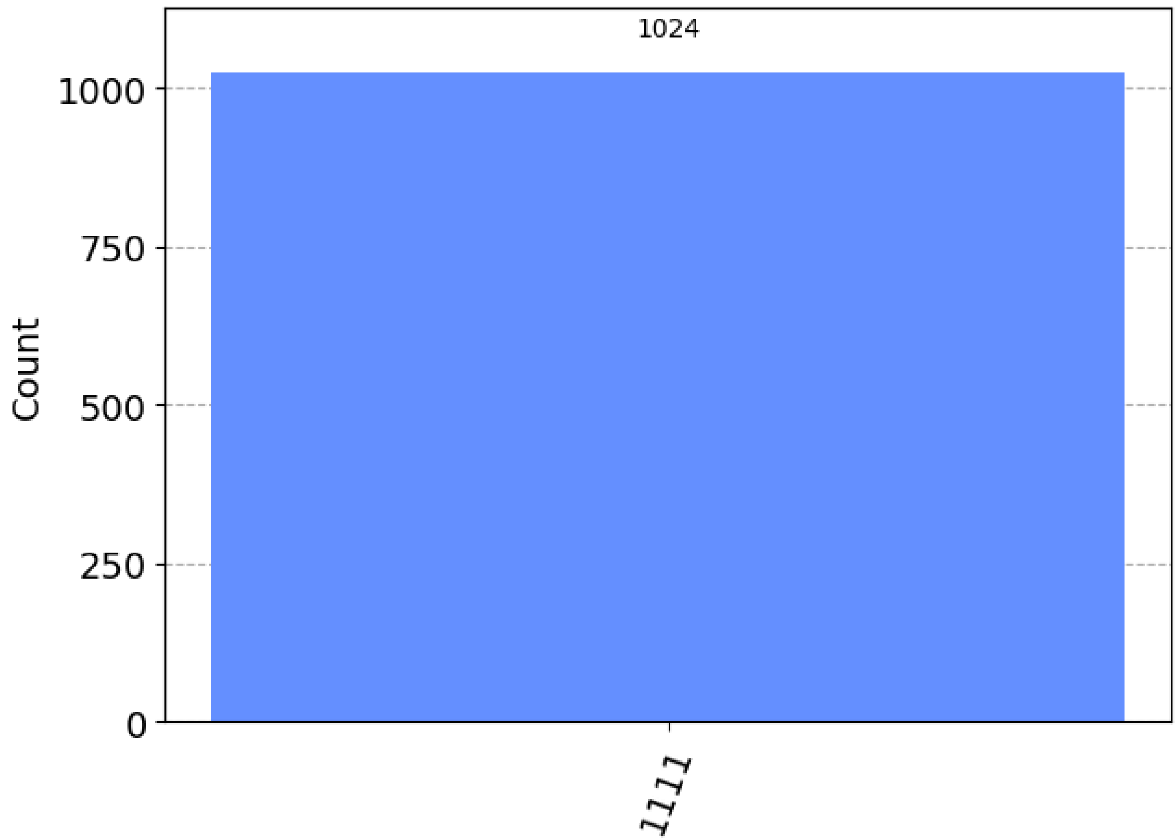
In [ ]:
```
def dj_algorithm(oracle, n):
    dj_circuit = QuantumCircuit(n+1, n)
    # Set up the output qubit:
    dj_circuit.x(n)
    dj_circuit.h(n)
    # And set up the input register:
    for qubit in range(n):
        dj_circuit.h(qubit)
    # Let's append the oracle gate to our circuit:
    dj_circuit.append(oracle, range(n+1))
    # Finally, perform the H-gates again and measure:
    for qubit in range(n):
        dj_circuit.h(qubit)

    for i in range(n):
        dj_circuit.measure(i, i)

    return dj_circuit
```

In [ ]:
```
n = 4
oracle_gate = dj_oracle('balanced', n)
dj_circuit = dj_algorithm(oracle_gate, n)
dj_circuit.draw()
```

Out[ ]:
```
     ┌───┐     ┌─────────┐┌───┐┌─┐
q_0: ┤ H ├─────┤0        ├┤ H ├┤M├─────────
     ├───┤     │         │├───┤└╥┘┌─┐
q_1: ┤ H ├─────┤1        ├┤ H ├─╫─┤M├──────
     ├───┤     │         │├───┤ ║ └╥┘┌─┐
q_2: ┤ H ├─────┤2 Oracle ├┤ H ├─╫──╫─┤M├───
     ├───┤     │         │├───┤ ║  ║ └╥┘┌─┐
q_3: ┤ H ├─────┤3        ├┤ H ├─╫──╫──╫─┤M├
     ├───┤┌───┐│         │└───┘ ║  ║  ║ └╥┘
q_4: ┤ X ├┤ H ├┤4        ├──────╫──╫──╫──╫─
     └───┘└───┘└─────────┘      ║  ║  ║  ║
c: 4/═══════════════════════════╩══╩══╩══╩═
                                0  1  2  3
```

In [ ]:
```
transpiled_dj_circuit = transpile(dj_circuit, aer_sim)
results = aer_sim.run(transpiled_dj_circuit).result()
answer = results.get_counts()
plot_histogram(answer)
```

Out[ ]:



Problems

> 1. Are you able to create a balanced or constant oracle of a different form?

Yes you can create different types of balanced or constant oracles. E.g. You can create a balanced oracle for a function that returns 1 for half of inputs and 0 for other half, but at a different pattern. For Constant oracles it always outputs the same value regardless of input, so there is less possible variation for how to implement that.

In [ ]:

```
# Example of a balanced oracle that flips output qubit if input has odd number of o

def balanced_oracle(n):
    oracle_qc = QuantumCircuit(n+1)
    for qubit in range(n):
        oracle_qc.cx(qubit, n)
    oracle_gate = oracle_qc.to_gate()
    oracle_gate.name = "Balanced Oracle"
    return oracle_gate
```

2. The function dj_problem_oracle (below) returns a Deutsch-Jozsa oracle for n = 4 in the form of a gate. The gate takes 5 qubits as input where the final qubit (q_4) is the output qubit (as with the example oracles above). You can get different oracles by giving dj_problem_oracle different integers between 1 and 5. Use the Deutsch-Jozsa algorithm

to decide whether each oracle is balanced or constant (Note: It is highly recommended you try this example using the aer_simulator instead of a real device).

In [ ]:
```
from qiskit_textbook.problems import dj_problem_oracle

for i in range(1, 6):
    oracle = dj_problem_oracle(i)
    dj_circuit = dj_algorithm(oracle, n)
    transpiled_dj_circuit = transpile(dj_circuit, aer_sim)
    results = aer_sim.run(transpiled_dj_circuit).result()
    answer = results.get_counts()
    print(f"Oracle {i} result distribution:")
    print(answer)
    if '0'*n in answer:
        print(f"constant")
    else:
        print(f"balanced")
    print("\n")
```

```
Oracle 1 result distribution:
{'1111': 1024}
balanced


Oracle 2 result distribution:
{'0000': 1024}
constant


Oracle 3 result distribution:
{'1111': 244, '0011': 278, '0001': 260, '1101': 242}
balanced


Oracle 4 result distribution:
{'0100': 1024}
balanced


There are only currently 4 oracles in this problem set, returning empty (balanced)
gate
Oracle 5 result distribution:
{'0000': 1024}
constant
```