

### Exercise Sheet 3

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Group G

Problem 1:

Split A:  $(300, 100)$  and  $(100, 300) \hat{=} (0,75; 0,25)$  and  $(0,25; 0,75)$

$$\text{Impurity: } 1 - 0,75^2 - 0,25^2 = 0,38$$

$$1 - 0,25^2 - 0,75^2 = 0,38$$

$$\text{Entropy: } - 0,75 \cdot \log(0,75) - 0,25 \cdot \log(0,25) = 0,56$$

$$- 0,25 \cdot \log(0,25) - 0,75 \cdot \log(0,75) = 0,56$$

$$\text{Mistclass.: } 1 - 0,75 = 0,25$$

$$1 - 0,25 = 0,25$$

Split B:  $(200, 0)$  and  $(200, 400) \hat{=} (1; 0)$  and  $(\left(\frac{1}{3}\right); \left(\frac{2}{3}\right))$

$$\text{Impurity: } 1 - 1^2 - 0^2 = 0$$

$$1 - \left(\frac{1}{3}\right)^2 - \left(\frac{2}{3}\right)^2 = 0,44$$

$$\text{Entropy: } - 1 \cdot \log(1) - 0 \cdot \log(0) = 0$$

$$- \frac{1}{3} \cdot \log\left(\frac{1}{3}\right) - \frac{2}{3} \cdot \log\left(\frac{2}{3}\right) = 0,64$$

$$\text{Mistclass.: } 1 - 1 = 0$$

$$1 - \frac{2}{3} = \frac{1}{3}$$

Best split for Impurity: B

Entropy: B

Mistclass.: B

Problem 2:

a) OR:  $x_1 w + x_2 w + \dots + x_n w + b$

b) AND:  $(x_1 w) \cdot (x_2 w) \cdot \dots \cdot (x_n w) + b$

c)

Colour	$N_1$	$N_2$	Desired	$Weight_1 + Weight_2 = -1$
blue	1	1	0	$Bias = 0$
red	1	0	0	
yellow	0	1	0	$a(z) = \begin{cases} 1 & \text{if } z \geq 0 \\ 0 & \text{if } z < 0 \end{cases}$
green	0	0	1	

$$N_1 W_1 + N_2 W_2 + Bias$$

$$1 \cdot (-1) + 1 \cdot (-1) + 0 = -2$$

$$1 \cdot (-1) + 0 \cdot (-1) + 0 = -1$$

$$0 \cdot (-1) + 1 \cdot (-1) + 0 = -1$$

$$0 \cdot (-1) + 0 \cdot (-1) + 0 = 0$$

Insert into  $a(z)$

$$\left. \begin{array}{l} a(-2) = 0 \stackrel{?}{=} \text{blue} \\ a(-1) = 0 \stackrel{?}{=} \text{red} \\ a(-1) = 0 \stackrel{?}{=} \text{yellow} \\ a(0) = 1 \stackrel{?}{=} \text{green} \end{array} \right\} \text{Desired output}$$

d)

Colour	$N_1$	$N_2$	Desired	Weight <sub>1</sub> = 1
blue	1	1	0	Weight <sub>2</sub> = -1
red	1	0	1	Bias = -1
yellow	0	1	0	
green	0	0	0	

$$N_1 W_1 + N_2 W_2 + \text{Bias}$$

$$1 \cdot 1 + 1 \cdot (-1) + (-1) = -1$$

$$1 \cdot 1 + 0 \cdot (-1) + (-1) = 0$$

$$0 \cdot 1 + 1 \cdot (-1) + (-1) = -2$$

$$0 \cdot 1 + 0 \cdot (-1) + (-1) = -1$$

Insert into  $a(z)$

$$a(-1) = 0 \hat{=} \text{blue}$$

$$a(0) = 1 \hat{=} \text{red}$$

$$a(-2) = 0 \hat{=} \text{yellow}$$

$$a(-1) = 0 \hat{=} \text{green}$$

Desired output

e) /