Problem 1)

a. $\frac{1}{1+e^a} = 1 - \frac{1}{1+e^{-a}} \Rightarrow$ LCM $(e^a+1)(e^{-a}+1)$

$\Rightarrow \frac{(e^a+1)(e^{-a}+1)}{1+e^a} = (e^a+1)(e^{-a}+1) - (e^a+1)$

$\Rightarrow (e^a)^{-1} + 1 = (e^a+1)[(e^a)^{-1}+1) - (e^a+1)$

$e^a = y$

$\Rightarrow (y)^{-1} + 1 = (y+1)((y)^{-1}+1) - (y+1)$

i.e $y = 1$

$\Rightarrow$

$(1)^{-1} + 1 = 2.(1^{-1}+1) - 2$

$2 = 4 - 2$

$2 = 2$

r.d.a

(b) $\frac{d}{da}\sigma(a) = (1-\sigma(a))\sigma(a)$

since $\sigma(-a) = 1-\sigma(a)$

$\Rightarrow \frac{d\sigma(a)}{da} = \frac{d}{da}\frac{1}{1+e^{-1}} \Rightarrow \frac{d}{da}(1+e^{-1})^{-1}$

$\Rightarrow -(1+e^{-a})^{-2} \cdot (-e^{-a}) \Rightarrow \frac{e^{-a}}{(1+e^{-a})^2}$

$\Rightarrow \frac{1}{1+e^{-a}} \cdot \frac{e^{-a}}{1+e^{-a}} \Rightarrow \sigma(a) \cdot \sigma(-a)$

$\Rightarrow (1-\sigma(a)) \cdot \sigma(a)$

(c) $\sigma^{-1}(b) = \log \frac{b}{1-b}$