Problem 2:

$$\log h_\theta(x^i) = \log \frac{1}{1+e^{-\theta^T x^i}}$$
$$= -\log(1+e^{-\theta^T x^i})$$

$$\log(1-h_\theta(x^i)) = \log\left(1 - \frac{1}{1+e^{-\theta^T x^i}}\right)$$
$$= \log(e^{-\theta^T x^i}) - \log(1+e^{-\theta^T x^i})$$
$$= -\theta^T x^i - \log(1+e^{-\theta^T x^i})$$

Insert these two terms in $J(\theta)$:

$$J(\theta) = -\frac{1}{m}\sum_{i=1}^{m}\left[-y^i(\log(1+e^{-\theta^T x^i})) + (1-y^i)\cdot(-\theta^T x^i - \log(1+e^{-\theta^T x^i}))\right]$$

$$= -\frac{1}{m}\sum_{i=1}^{m}\left[y^i\theta^T x^i - \theta^T x^i - \log(1+e^{-\theta^T x^i})\right]$$

$$= -\frac{1}{m}\sum_{i=1}^{m}\left[\underbrace{y^i\theta^T x^i}_{a} - \underbrace{\log(1+e^{\theta^T x^i})}_{b}\right]$$

Partial derivatives of a and b to θ:

$$a = y_i x^i$$
$$b = \frac{x^i e^{\theta^T x^i}}{1+e^{\theta^T x^i}}$$
$$= x^i h_\theta(x^i)$$

$$\frac{\partial}{\partial\theta_j} J(\theta) = -\frac{1}{m}\sum_{i=1}^{m}\left[y^i x^i - x^i h_\theta(x^i)\right]$$

$$= -\frac{1}{m}\sum\left[x^i(h_\theta(x^i) - y^i)\right]$$