

Problem 2:

$$\begin{aligned}\log h_{\theta}(x^i) &= \log \frac{1}{1 + e^{-\theta^T x^i}} \\ &= -\log(1 + e^{-\theta^T x^i})\end{aligned}$$

$$\begin{aligned}\log(1 - h_{\theta}(x^i)) &= \log\left(1 - \frac{1}{1 + e^{-\theta^T x^i}}\right) \\ &= \log(e^{-\theta^T x^i}) - \log(1 + e^{-\theta^T x^i}) \\ &= -\theta^T x^i - \log(1 + e^{-\theta^T x^i})\end{aligned}$$

Insert these two terms in $J(\theta)$:

$$\begin{aligned}J(\theta) &= -\frac{1}{m} \sum_{i=1}^m \left[-y^i (\log(1 + e^{-\theta^T x^i})) + (1 - y^i) \cdot (-\theta^T x^i - \log(1 + e^{-\theta^T x^i})) \right] \\ &= -\frac{1}{m} \sum_{i=1}^m \left[y^i \theta^T x^i - \theta^T x^i - \log(1 + e^{-\theta^T x^i}) \right] \\ &= -\frac{1}{m} \sum_{i=1}^m \left[\underbrace{y^i \theta^T x^i}_{(a)} - \underbrace{\log(1 + e^{-\theta^T x^i})}_{(b)} \right]\end{aligned}$$

Partial derivatives of (a) and (b) to θ :

$$\begin{aligned}(a) &= y^i x^i \\ (b) &= \frac{x^i e^{\theta^T x^i}}{1 + e^{\theta^T x^i}} \\ &= x^i h_{\theta}(x^i)\end{aligned}$$

$$\begin{aligned}\frac{\partial}{\partial \theta_j} J(\theta) &= -\frac{1}{m} \sum_{i=1}^m \left[y^i x^i - x^i h_{\theta}(x^i) \right] \\ &= -\frac{1}{m} \sum \left[x^i (h_{\theta}(x^i) - y^i) \right]\end{aligned}$$