Problem 1 (Properties of the Logistic Sigmoid) (5pt + 5pt + 5pt).
A more common notation for the logistic sigmoid function uses σ and is defined as

$$\sigma(a) = \frac{1}{1 + e^{-a}} \tag{1.1}$$

Show that the logistic sigmoid satisfies the following properties:

(a) $\sigma(-a) = 1 - \sigma(a)$

(b) $\frac{d}{da}\sigma(a) = (1 - \sigma(a))\sigma(a)$

(c) $\sigma^{-1}(b) = \log \frac{b}{1-b}$ where $\sigma^{-1}(\cdot)$ is the inverse function of logistic sigmoid

Problem 2 (Gradient of the Binary Cross-entropy Loss) (15 pt).
Show that the partial derivative of the logistic regression loss function (aka binary cross-entropy loss) is as follows

$$\frac{\partial}{\partial \theta_j} J(\theta) = \frac{1}{m} \sum_{i=1}^{m} (h_\theta(x^i) - y^i) x_j^i \tag{2.1}$$

where $J(\theta)$ and $h_\theta(x)$ is defined as (using the same notation as used in **cs229** lectures),

$$J(\theta) = \frac{1}{m} \sum_{i=1}^{m} -y^i \log(h_\theta(x^i)) - (1 - y^i) \log(1 - h_\theta(x^i)) \tag{2.2}$$

$$h_\theta(x) = g(\theta^T x) \tag{2.3}$$

$$g(z) = \frac{1}{1 + e^{-z}} \tag{2.4}$$