# **Data Structures Graph**

#### Question 1

Which of the following is an advantage of adjacency list representation over adjacency matrix representation of a graph?



In adjacency list representation, space is saved for sparse graphs.

DFS and BSF can be done in O(V + E) time for adjacency list representation. These operations take



O(V^2) time in adjacency list representation. Here is V and E are number of vertices and edges respectively.



Adding a vertex in adjacency list representation is easier than adjacency matrix representation.



All of the above

## **Question 1 Explanation:**

See http://www.geeksforgeeks.org/graph-and-its-representations/

#### Question 2

The degree sequence of a simple graph is the sequence of the degrees of the nodes in the graph in decreasing order. Which of the following sequences can not be the degree sequence of any graph? I. 7, 6, 5, 4, 4, 3, 2, 1 II. 6, 6, 6, 6, 3, 3, 2, 2 III. 7, 6, 6, 4, 4, 3, 2, 2 IV. 8, 7, 7, 6, 4, 2, 1, 1



I and II



III and IV



IV only



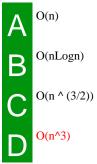
II and IV

### **Question 2 Explanation:**

See Question 3 of <a href="http://www.geeksforgeeks.org/data-structures-and-algorithms-set-25/">http://www.geeksforgeeks.org/data-structures-and-algorithms-set-25/</a> for explanation.

### Question 3

The time complexity of computing the transitive closure of a binary relation on a set of n elements is known to be:

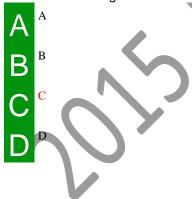


## **Question 3 Explanation:**

See question 3 of <a href="http://www.geeksforgeeks.org/data-structures-and-algorithms-set-22/">http://www.geeksforgeeks.org/data-structures-and-algorithms-set-22/</a>

### Question 4

The most efficient algorithm for finding the number of connected components in an undirected graph on n vertices and m edges has time complexity. (A) (n) (B) (m) (C) (m + n) (D) (mn)

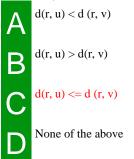


## **Question 4 Explanation:**

Connected components can be found in O(m + n) using <u>Tarjan's algorithm</u>. Once we have connected components, we can count them.

### **Question 5**

Consider an undirected unweighted graph G. Let a breadth-first traversal of G be done starting from a node r. Let d(r, u) and d(r, v) be the lengths of the shortest paths from r to u and v respectively, in G. If u is visited before v during the breadth-first traversal, which of the following statements is correct? (GATE CS 2001)

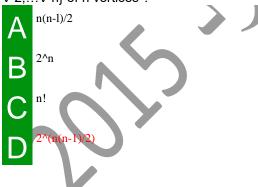


## **Question 5 Explanation:**

d(r, u) and d(r, v) will be equal when u and v are at same level, otherwise d(r, u) will be less than d(r, v)

#### Question 6

How many undirected graphs (not necessarily connected) can be constructed out of a given set  $V = \{V \mid 1, V \mid 2,...V \mid n\}$  of n vertices?

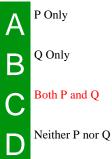


### **Question 6 Explanation:**

In an undirected graph, there can be maximum n(n-1)/2 edges. We can choose to have (or not have) any of the n(n-1)/2 edges. So, total number of undirected graphs with n vertices is  $2^{n}$  (n(n-1)/2).

#### Question 7

Which of the following statements is/are TRUE for an undirected graph? P: Number of odd degree vertices is even Q: Sum of degrees of all vertices is even

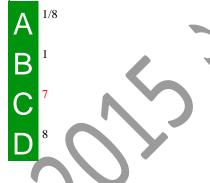


# **Question 7 Explanation:**

P is true for undirected graph as adding an edge always increases degree of two vertices by 1. Q is true: If we consider sum of degrees and subtract all even degrees, we get an even number because every edge increases the sum of degrees by 2. So total number of odd degree vertices must be even.

#### **Question 8**

Consider an undirected random graph of eight vertices. The probability that there is an edge between a pair of vertices is 1/2. What is the expected number of unordered cycles of length three?



### **Question 8 Explanation:**

A cycle of length 3 can be formed with 3 vertices. There can be total 8C3 ways to pick 3 vertices from 8. The probability that there is an edge between two vertices is 1/2. So expected number of unordered cycles of length  $3 = (8C3)^*(1/2)^3 = 7$ 

#### Question 9

Given an undirected graph G with V vertices and E edges, the sum of the degrees of all vertices is



## **Question 9 Explanation:**

Since the given graph is undirected, every edge contributes as 2 to sum of degrees. So the sum of degrees is 2E.

# **Question 10**

How many undirected graphs (not necessarily connected) can be constructed out of a given set  $V = \{v1, v2, ... vn\}$  of n vertices?

# **Question 10 Explanation:**

There are total  $n^*(n-1)/2$  possible edges. For every edge, there are to possible options, either we pick it or don't pick. So total number of possible graphs is  $2^{n(n-1)/2}$ .

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