

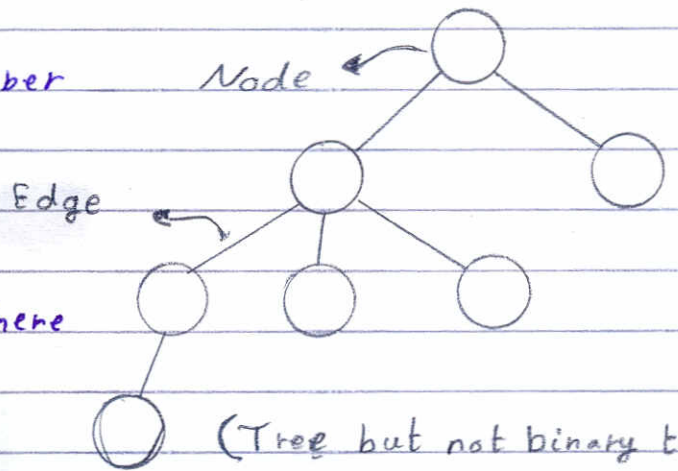
## Binary Tree

**Tree** is a connected acyclic graph. It's hierarchical.

• Every node can have any number of subtrees.

• It cannot be empty.

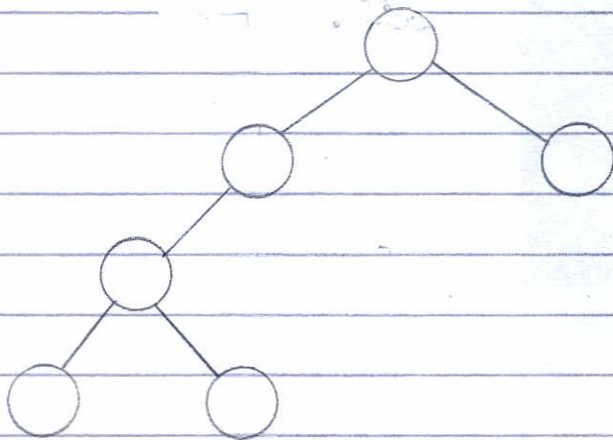
• For a tree with  $n$  nodes, there are  $(n-1)$  edges



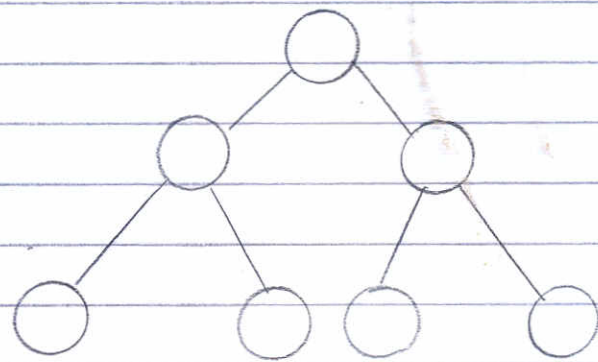
**Binary Tree** It's a special case of tree.

It's a finite set of nodes, and it could be empty.

Every node has **at most** two subtrees.



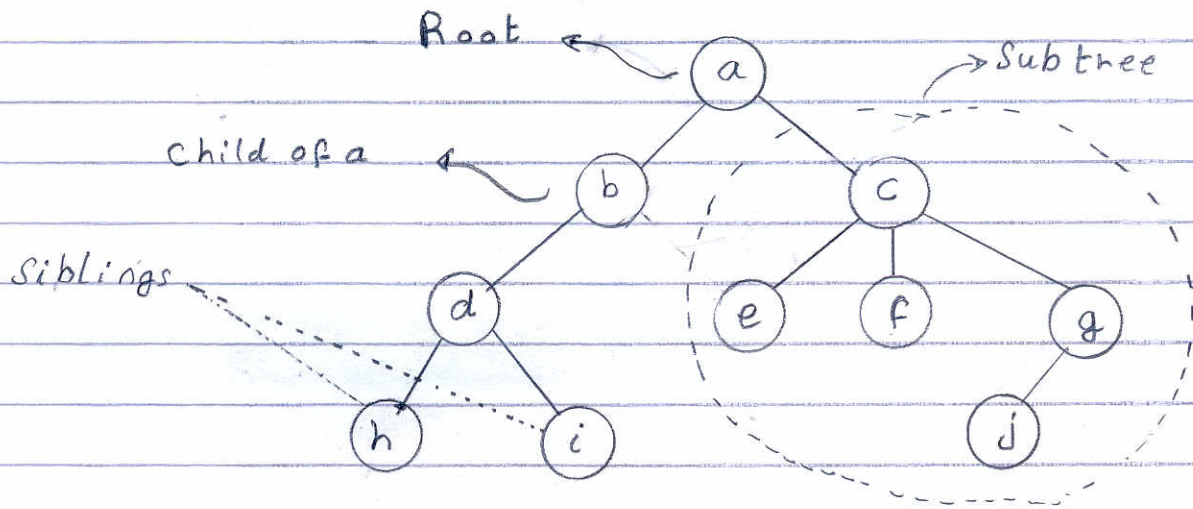
(Binary Tree)



(Binary Tree)

كل شجرة تكون متصلة فقط ، و لا تكون حلقية فقط ، و لا تكون على الإطلاق

## Terminologies



**Root** : is the node at the highest level of the hierarchy.

**Child** : is an element that has a superior element.  
(b is child of a)

**parent** : is an element that has a subordinate element.  
(g is parent of j)

**Siblings** : are child elements with the same parent.  
(h, i are siblings cause they share the same parent d)

**Descendants** : الأبناء  
(Descendants of c are e, f, g, j)

**Ancestors** : الأجداد  
(Ancestors of d are b, a)

**Subtree** : is a smaller tree that is contained within the original tree.

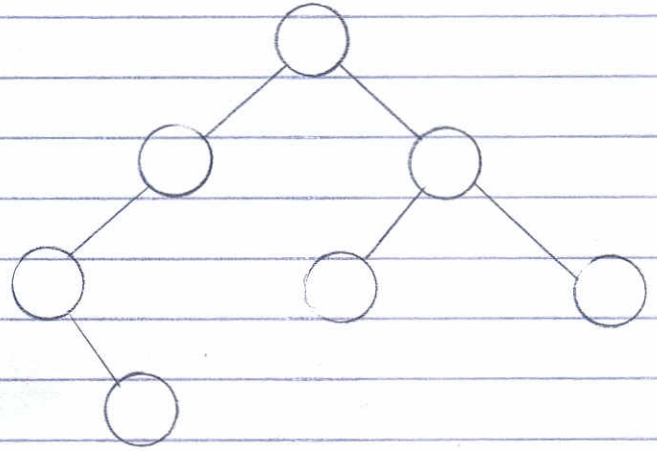
Height = 4

Level 1

Level 2

Level 3

Level 4



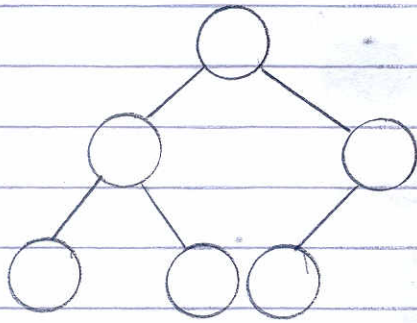
Height = maximum level

Maximum number of nodes in a level =  $2^{\text{Level} - 1}$

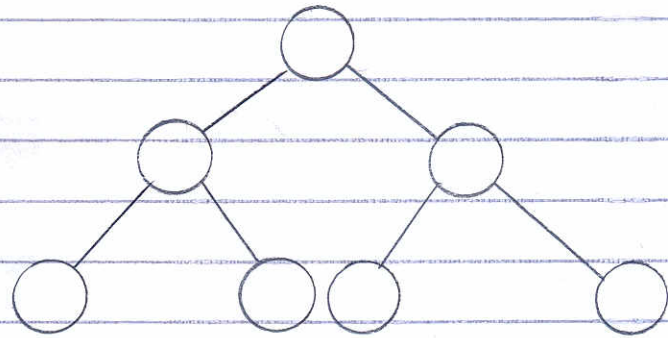


## The full binary tree

It's full binary tree if and only if the number of nodes in level equal to  $2^{\text{level} - 1}$



(Binary tree but not full)



(Full binary tree)

$$\text{Full number of nodes} = 2^{\text{height}} - 1$$

↪ متقانون المتوالية العددية

$$\text{Height} = \log_2 (n + 1)$$

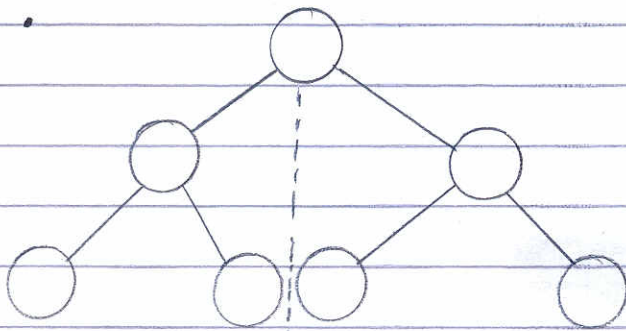
↪ Number of nodes

$$n = 2^h - 1$$

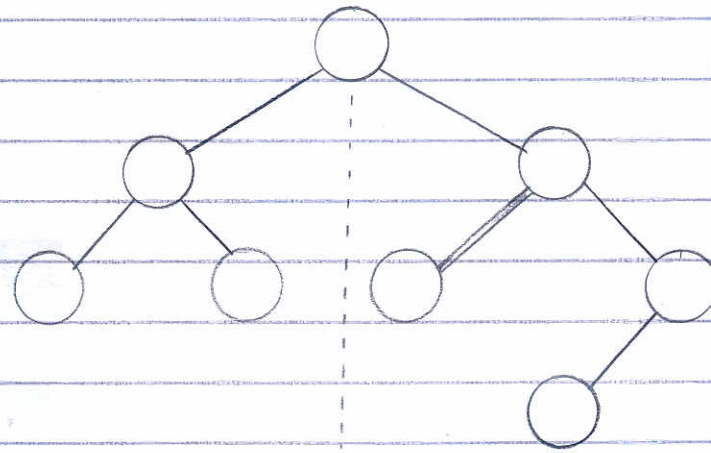
$$2^h = (n + 1)$$

$$h = \log_2 (n + 1)$$

## The balanced tree



Height = 3 | Height = 3  
(Balanced tree)



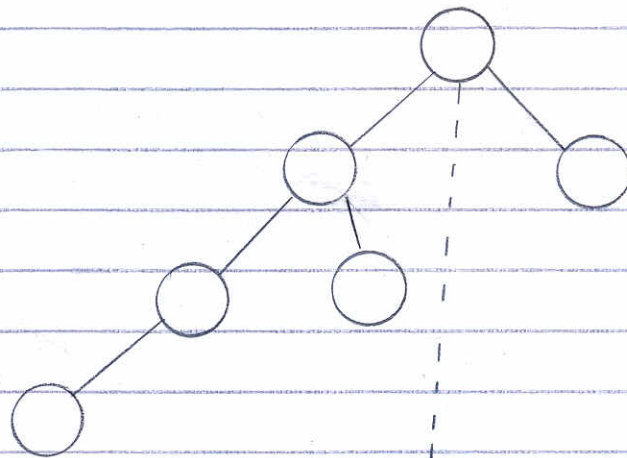
Height = 3 | Height = 4  
(Balanced tree)

**Balanced tree** is a property that the heights of the left and right

Subtrees differ at most by one level.  $|H_{\text{left}} - H_{\text{right}}| \leq 1$

Height

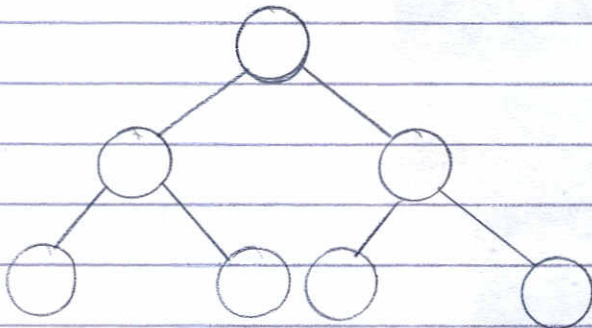
So full tree, Complete tree are also balanced tree.



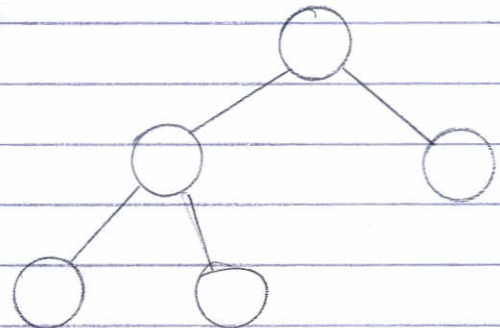
Height = 4 | Height = 2  
(Not balanced tree)

## Complete Binary Tree

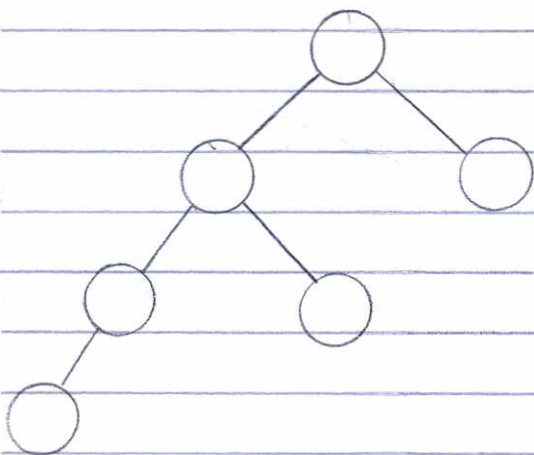
- It's a binary tree in which each level of the tree is completely filled
- But the bottom level could be incomplete, but the nodes should occupy from the leftmost positions.



(Binary tree)

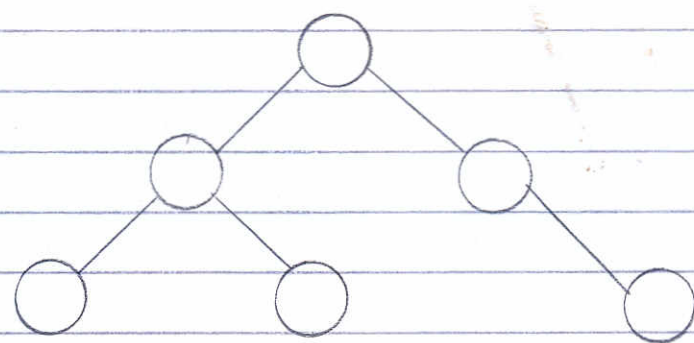


(Binary tree)



(Not binary tree)

cause the height of the left subtree is 4, and the right subtree is 2, so the difference is 2



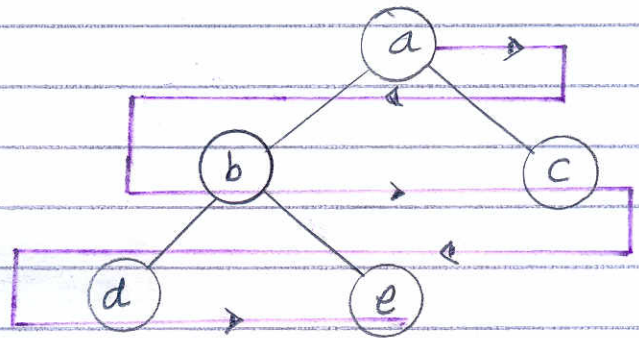
(Not binary tree)

cause a node is missing at the bottom level, and there's a node after it.



## Implementing a Complete binary tree as an array:

Level order



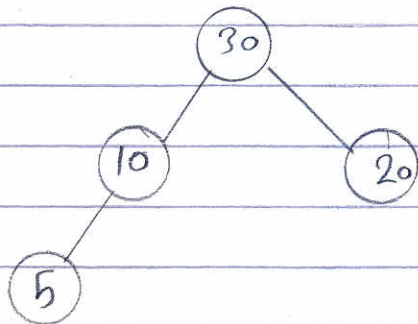
a	b	c	d	e	Array
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Assuming that values are stored starting at subscript 0 then:

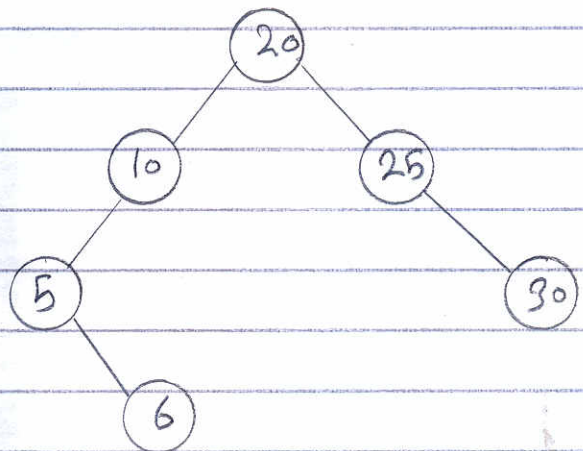
- Root of the complete binary tree is located at [0]
- Left child is located at  $[2 * \text{parent index} + 1]$
- Right child is located at  $[2 * \text{parent index} + 2]$
- Parent is located at either  $[\text{child's index} - 1 / 2]$

## Binary Search tree

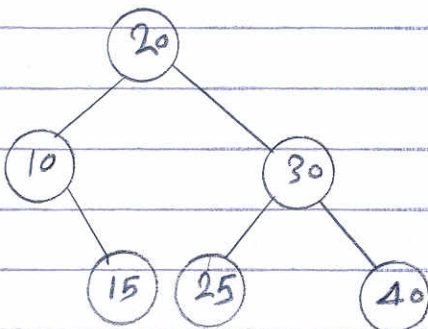
- It's a binary tree that stores key-data in its nodes
- No key duplication (each element is distinct)
- The elements in the left subtree of the root are less than the root
- The elements in the right subtree of the root are greater than the root
- The left and right subtrees of the root are also binary search trees



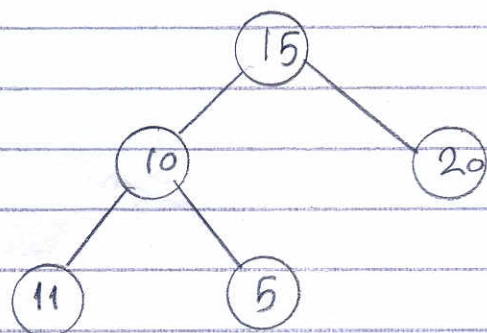
(Not Binary search tree)  
cause 20 is less than 30.



(Binary search tree)



(Binary search tree)



(Not Binary search tree)



## Tree Traversal

### Pre-Order (Depth-first Traversal)

Root - Left - Right

### In-Order

Left - Root - Right

### Post-Order

Left - Right - Root

### Level-order (Breadth-first traversal)

Top Level to Bottom starting from left to node to right.

### Reverse - pre-order

Root - Right - Left

### Reverse In-order

Right - Root - Left

### Reverse post-order

Right - Left - Root