Data Mining

Practical Machine Learning Tools and Techniques

Slides for Chapter 5 of *Data Mining* by I. H. Witten, E. Frank and M. A. Hall

Credibility: Evaluating what's been learned

- Issues: training, testing, tuning
- Predicting performance: confidence limits
- Holdout, cross-validation, bootstrap
- Comparing schemes: the t-test
- Predicting probabilities: loss functions
- Cost-sensitive measures
- Evaluating numeric prediction
- The Minimum Description Length principle

Evaluation: the key to success

- How predictive is the model we learned?
- Error on the training data is *not* a good indicator of performance on future data
  - Otherwise 1-NN would be the optimum classifier!
- Simple solution that can be used if lots of (labeled) data is available:
  - Split data into training and test set
- However: (labeled) data is usually limited
  - More sophisticated techniques need to be used

Issues in evaluation

- Statistical reliability of estimated differences in performance (→ significance tests)
- Choice of performance measure:
  - Number of correct classifications
  - Accuracy of probability estimates
  - Error in numeric predictions
- Costs assigned to different types of errors
  - Many practical applications involve costs

Training and testing I

- Natural performance measure for classification problems: *error rate*
  - *Success*: instance's class is predicted correctly
  - *Error*: instance's class is predicted incorrectly
  - Error rate: proportion of errors made over the whole set of instances
- *Resubstitution error:* error rate obtained from training data
- Resubstitution error is (hopelessly) optimistic!

Training and testing II

- *Test set*: independent instances that have played no part in formation of classifier
  - Assumption: both training data and test data are representative samples of the underlying problem
- Test and training data may differ in nature
  - Example: classifiers built using customer data from two different towns *A* and *B*
    - To estimate performance of classifier from town *A* in completely new town, test it on data from *B*

Note on parameter tuning

- It is important that the test data is not used *in any way* to create the classifier
- Some learning schemes operate in two stages:
  - Stage 1: build the basic structure
  - Stage 2: optimize parameter settings
- The test data can't be used for parameter tuning!
- Proper procedure uses *three* sets: *training data*, *validation data*, and *test data*
  - Validation data is used to optimize parameters

Making the most of the data

- Once evaluation is complete, *all the data* can be used to build the final classifier
- Generally, the larger the training data the better the classifier (but returns diminish)
- The larger the test data the more accurate the error estimate
- *Holdout* procedure: method of splitting original data into training and test set
  - Dilemma: ideally both training set *and* test set should be large!

Predicting performance

- Assume the estimated error rate is 25%. How close is this to the true error rate?
  - Depends on the amount of test data
- Prediction is just like tossing a (biased!) coin
  - "Head" is a "success", "tail" is an "error"
- In statistics, a succession of independent events like this is called a *Bernoulli process*
  - Statistical theory provides us with confidence intervals for the true underlying proportion

Confidence intervals

- We can say: *p* lies within a certain specified interval with a certain specified confidence
- Example: *S*=750 successes in *N*=1000 trials
  - Estimated success rate: 75%
  - How close is this to true success rate *p*?
    - Answer: with 80% confidence *p* in [73.2,76.7]
- Another example: *S*=75 and *N*=100
  - Estimated success rate: 75%
  - With 80% confidence *p* in [69.1,80.1]

Mean and variance

- Mean and variance for a Bernoulli trial: p, p $(1–p)$
- Expected success rate $f=S/N$
- Mean and variance for f : p, p $(1–p)/N$
- For large enough N, f follows a Normal distribution
- c% confidence interval $[–z \leq X \leq z]$ for random variable with 0 mean is given by:

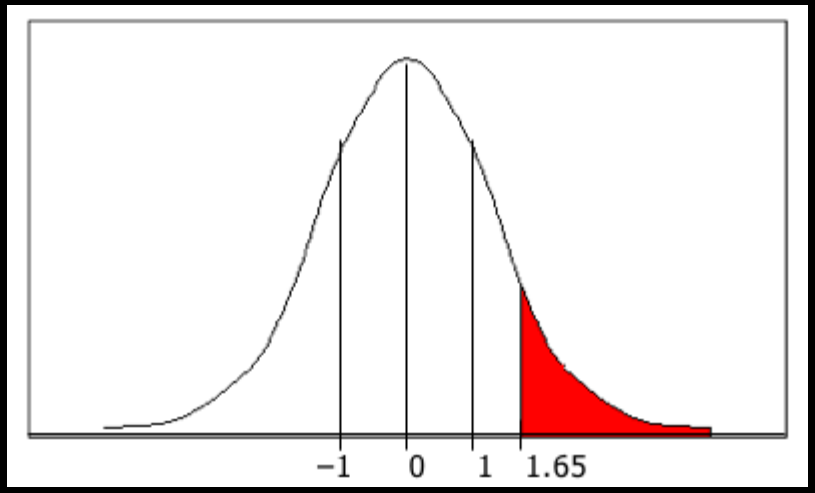
$$Pr[-z \leq X \leq z] = c$$

- With a symmetric distribution:

$$Pr[-z \leq X \leq z] = 1 - 2 \times Pr[x \geq z]$$

Confidence limits

- Confidence limits for the normal distribution with 0 mean and a variance of 1:



Pr[$X \geq z$]	z
0.1%	3.09
0.5%	2.58
1%	2.33
5%	1.65
10%	1.28
20%	0.84
40%	0.25

- Thus:

$$Pr[-1.65 \leq X \leq 1.65] = 90\%$$

- To use this we have to reduce our random variable f to have 0 mean and unit variance

Transforming *f*

- Transformed value for *f* : $\frac{f-p}{\sqrt{p(1-p)/N}}$

  (i.e. subtract the mean and divide by the *standard deviation*)

- Resulting equation: $Pr[-z \le \frac{f-p}{\sqrt{p(1-p)/N}} \le z] = c$

- Solving for *p* :

$$p = (f + \frac{z^2}{2N} \mp z\sqrt{\frac{f}{N} - \frac{f^2}{N} + \frac{z^2}{4N^2}})/(1 + \frac{z^2}{N})$$

Examples

- $f = 75\%$, $N = 1000$, $c = 80\%$ (so that $z = 1.28$):

$$p \in [0.732, 0.767]$$

- $f = 75\%$, $N = 100$, $c = 80\%$ (so that $z = 1.28$):

$$p \in [0.691, 0.801]$$

- Note that normal distribution assumption is only valid for large N (i.e. $N > 100$)
- $f = 75\%$, $N = 10$, $c = 80\%$ (so that $z = 1.28$):

$$p \in [0.549, 0.881]$$

(should be taken with a grain of salt)

Holdout estimation

- What to do if the amount of data is limited?
- The *holdout* method reserves a certain amount for testing and uses the remainder for training
  - Usually: one third for testing, the rest for training
- Problem: the samples might not be representative
  - Example: a class might be missing in the test data
- Advanced version uses *stratification*
  - Ensures that each class is represented with approximately equal proportions in both subsets

Repeated holdout method

- Holdout estimate can be made more reliable by repeating the process with different subsamples
  - In each iteration, a certain proportion is randomly selected for training (possibly with stratificiation)
  - The error rates on the different iterations are averaged to yield an overall error rate
- This is called the *repeated holdout* method
- Still not optimum: the different test sets overlap
  - Can we prevent overlapping?

Cross-validation

- *Cross-validation* avoids overlapping test sets
  - First step: split data into *k* subsets of equal size
  - Second step: use each subset in turn for testing, the remainder for training
- Called *k-fold cross-validation*
- Often the subsets are stratified before the cross-validation is performed
- The error estimates are averaged to yield an overall error estimate

More on cross-validation

- Standard method for evaluation: stratified ten-fold cross-validation
- Why ten?
  - Extensive experiments have shown that this is the best choice to get an accurate estimate
  - There is also some theoretical evidence for this
- Stratification reduces the estimate's variance
- Even better: repeated stratified cross-validation
  - E.g. ten-fold cross-validation is repeated ten times and results are averaged (reduces the variance)

Leave-One-Out cross-validation

- Leave-One-Out:
  a particular form of cross-validation:
  - Set number of folds to number of training instances
  - I.e., for *n* training instances, build classifier *n* times
- Makes best use of the data
- Involves no random subsampling
- Very computationally expensive
  - (exception: NN)

Leave-One-Out-CV and stratification

- Disadvantage of Leave-One-Out-CV: stratification is not possible
  - It *guarantees* a non-stratified sample because there is only one instance in the test set!
- Extreme example: random dataset split equally into two classes
  - Best inducer predicts majority class
  - 50% accuracy on fresh data
  - Leave-One-Out-CV estimate is 100% error!

The bootstrap

- CV uses sampling *without replacement*
  - The same instance, once selected, can not be selected again for a particular training/test set
- The *bootstrap* uses sampling *with replacement* to form the training set
  - Sample a dataset of *n* instances *n* times *with replacement* to form a new dataset of *n* instances
  - Use this data as the training set
  - Use the instances from the original dataset that don't occur in the new training set for testing

The 0.632 bootstrap

- Also called the *0.632 bootstrap*
  - A particular instance has a probability of 1–1/*n* of *not* being picked
  - Thus its probability of ending up in the test data is:

$$(1-\frac{1}{n})^n \approx e^{-1} \approx 0.368$$

  - This means the training data will contain approximately 63.2% of the instances

Estimating error with the bootstrap

- The error estimate on the test data will be very pessimistic
  - Trained on just ~63% of the instances
- Therefore, combine it with the resubstitution error:

$$err = 0.632 \times e_{\text{test instances}} + 0.368 \times e_{\text{training\_instances}}$$

- The resubstitution error gets less weight than the error on the test data
- Repeat process several times with different replacement samples; average the results

More on the bootstrap

- Probably the best way of estimating performance for very small datasets
- However, it has some problems
  - Consider the random dataset from above
  - A perfect memorizer will achieve
    0% resubstitution error and
    ~50% error on test data
  - Bootstrap estimate for this classifier:
    $err = 0.632 \times 50\% + 0.368 \times 0\% = 31.6\%$
  - True expected error: 50%

Comparing data mining schemes

- Frequent question: which of two learning schemes performs better?
- Note: this is domain dependent!
- Obvious way: compare 10-fold CV estimates
- Generally sufficient in applications (we don't loose if the chosen method is not truly better)
- However, what about machine learning research?
  - Need to show convincingly that a particular method works better

Comparing schemes II

- Want to show that scheme A is better than scheme B in a particular domain
  - For a given amount of training data
  - On average, across all possible training sets
- Let's assume we have an infinite amount of data from the domain:
  - Sample infinitely many dataset of specified size
  - Obtain cross-validation estimate on each dataset for each scheme
  - Check if mean accuracy for scheme A is better than mean accuracy for scheme B

Paired t-test

- In practice we have limited data and a limited number of estimates for computing the mean
- *Student's t-test* tells whether the means of two samples are significantly different
- In our case the samples are cross-validation estimates for different datasets from the domain
- Use a *paired* t-test because the individual samples are paired
  - The same CV is applied twice

Distribution of the means

- $x_1\ x_2\ \ldots\ x_k$ and $y_1\ y_2\ \ldots\ y_k$ are the $2k$ samples for the k different datasets
- m_x and m_y are the means
- With enough samples, the mean of a set of independent samples is normally distributed

- Estimated variances of the means are σ_x^2/k and σ_y^2/k

- If μ_x and μ_y are the true means then $\frac{m_x-\mu_x}{\sqrt{\sigma_x^2/k}}\ \frac{m_y-\mu_y}{\sqrt{\sigma_y^2/k}}$

  are *approximately* normally distributed with mean 0, variance 1

Student's distribution

- With small samples ($k < 100$) the mean follows *Student's distribution with k–1 degrees of freedom*
- Confidence limits:

Assuming we have 10 estimates

9 degrees of freedom distribution

Pr[$X \geq z$]	z
0.1%	4.30
0.5%	3.25
1%	2.82
5%	1.83
10%	1.38
20%	0.88

normal

Pr[$X \geq z$]	z
0.1%	3.09
0.5%	2.58
1%	2.33
5%	1.65
10%	1.28
20%	0.84

Distribution of the differences

- Let $m_d = m_x - m_y$
- The difference of the means (m_d) also has a Student's distribution with k–1 degrees of freedom
- Let σ_d^2 be the variance of the difference
- The standardized version of m_d is called the t-statistic: $$t = \frac{m_d}{\sqrt{\sigma_d^2 / k}}$$

- We use t to perform the t-test

Performing the test

- Fix a significance level
  - If a difference is significant at the α% level, there is a (100-α)% chance that the true means differ
- Divide the significance level by two because the test is two-tailed
  - I.e. the true difference can be +ve or – ve
- Look up the value for z that corresponds to $\alpha/2$
- If $t \leq -z$ or $t \geq z$ then the difference is significant
  - I.e. the *null hypothesis* (that the difference is zero) can be rejected

Unpaired observations

- If the CV estimates are from different datasets, they are no longer paired (or maybe we have k estimates for one scheme, and j estimates for the other one)
- Then we have to use an *un* paired t-test with $\min(k , j) - 1$ degrees of freedom
- The estimate of the variance of the difference of the means becomes:

$$\frac{\sigma_x^2}{k} + \frac{\sigma_y^2}{j}$$

Dependent estimates

- We assumed that we have enough data to create several datasets of the desired size
- Need to re-use data if that's not the case
  - E.g. running cross-validations with different randomizations on the same data
- Samples become dependent ⇒ insignificant differences can become significant
- A heuristic test is the *corrected resampled t-test*:
  - Assume we use the repeated hold-out method, with n_1 instances for training and n_2 for testing
  - New test statistic is:

$$t = \frac{m_d}{\sqrt{\left(\frac{1}{k} + \frac{n_2}{n_1}\right)\sigma_d^2}}$$

Predicting probabilities

- Performance measure so far: success rate
- Also called *0-1 loss function*:

$$\sum_i \begin{cases} 0 \text{ if prediction is correct} \\ 1 \text{ if prediction is incorrect} \end{cases}$$

- Most classifiers produce class probabilities
- Depending on the application, we might want to check the accuracy of the probability estimates
- 0-1 loss is not the right thing to use in those cases

Quadratic loss function

- $p_1 \ldots p_k$ are probability estimates for an instance
- c is the index of the instance's actual class
- $a_1 \ldots a_k = 0$, except for a_c which is 1

$$\sum_j (p_j - a_j)^2 = \sum_{j!=c} p_j^2 + (a - p_c)^2$$

- *Quadratic loss* is:

$$E[\sum_j (p_j - a_j)^2]$$

- Want to minimize
- Can show that this is minimized when $p_j = p_j^*$, the true probabilities

Informational loss function

- The informational loss function is $-\log(p_c)$, where c is the index of the instance's actual class
- Number of bits required to communicate the actual class
- Let p_1^* … p_k^* be the true class probabilities
- Then the expected value for the loss function is:

$$-p_1^* \log_2 p_1 - \ldots - p_k^* \log_2 p_k$$

- Justification: minimized when $p_j = p_j^*$
- Difficulty: *zero-frequency problem*

Discussion

- Which loss function to choose?
  - Both encourage honesty
  - Quadratic loss function takes into account all class probability estimates for an instance
  - Informational loss focuses only on the probability estimate for the actual class
  - Quadratic loss is bounded: $1+\sum_j p_j^2$
    it can never exceed 2
  - Informational loss can be infinite

Informational loss is related to *MDL principle* [later]

Counting the cost

- In practice, different types of classification errors often incur different costs
- Examples:
  - Spam filtering
  - Loan decisions
  - Oil-slick detection
  - Fault diagnosis

Counting the cost

- The *confusion matrix*:

		Predicted class	
		Yes	No
Actual class	Yes	True positive	False negative
	No	False positive	True negative

There are many other types of cost!

- E.g.: cost of collecting training data

Aside: the kappa statistic

- Two confusion matrices for a 3-class problem: actual predictor (left) vs. random predictor (right)

		Predicted class			
		a	*b*	*c*	*total*
Actual class	*a*	88	10	2	*100*
	b	14	40	6	*60*
	c	18	10	12	*40*
	total	*120*	*60*	*20*	

		Predicted class			
		a	*b*	*c*	*total*
Actual class	*a*	60	30	10	*100*
	b	36	18	6	*60*
	c	24	12	4	*40*
	total	*120*	*60*	*20*	

- Number of successes: sum of entries in diagonal (D)
- *Kappa* statistic: $\frac{D_{observed} - D_{random}}{D_{perfect} - D_{random}}$

measures relative improvement over random predictor

Classification with costs

- Two cost matrices:

		Predicted class	
		yes	*no*
Actual class	*yes*	0	1
	no	1	0

		Predicted class		
		a	*b*	*c*
Actual class	*a*	0	1	1
	b	1	0	1
	c	1	1	0

- Success rate is replaced by average cost per prediction
  - Cost is given by appropriate entry in the cost matrix

Cost-sensitive classification

- Can take costs into account when making predictions
  - Basic idea: only predict high-cost class when very confident about prediction
- Given: predicted class probabilities
  - Normally we just predict the most likely class
  - Here, we should make the prediction that minimizes the expected cost
    - Expected cost: dot product of vector of class probabilities and appropriate column in cost matrix
    - Choose column (class) that minimizes expected cost

Cost-sensitive learning

- So far we haven't taken costs into account at training time
- Most learning schemes do not perform cost-sensitive learning
  - They generate the same classifier no matter what costs are assigned to the different classes
  - Example: standard decision tree learner
- Simple methods for cost-sensitive learning:
  - Resampling of instances according to costs
  - Weighting of instances according to costs
- Some schemes can take costs into account by varying a parameter, e.g. naïve Bayes

Lift charts

- In practice, costs are rarely known
- Decisions are usually made by comparing possible scenarios
- Example: promotional mailout to 1,000,000 households
  - Mail to all; 0.1% respond (1000)
  - Data mining tool identifies subset of 100,000 most promising, 0.4% of these respond (400)
    40% of responses for 10% of cost may pay off
  - Identify subset of 400,000 most promising, 0.2% respond (800)
- A *lift chart* allows a visual comparison

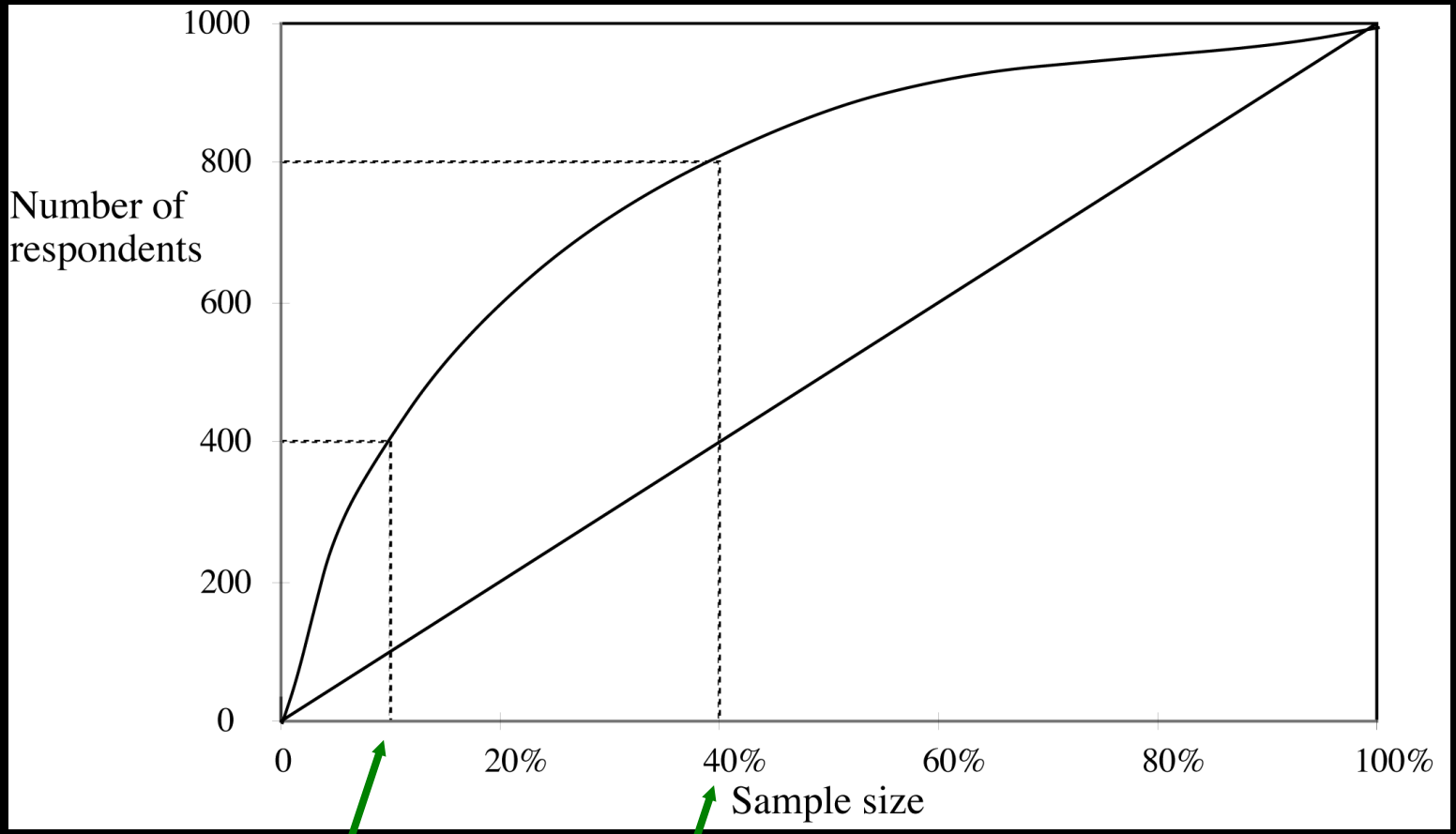
Generating a lift chart

- Sort instances according to predicted probability of being positive:

	Predicted probability	Actual class
1	0.95	Yes
2	0.93	Yes
3	0.93	No
4	0.88	Yes
...	...	...

- *x* axis is sample size
  y axis is number of true positives

A hypothetical lift chart



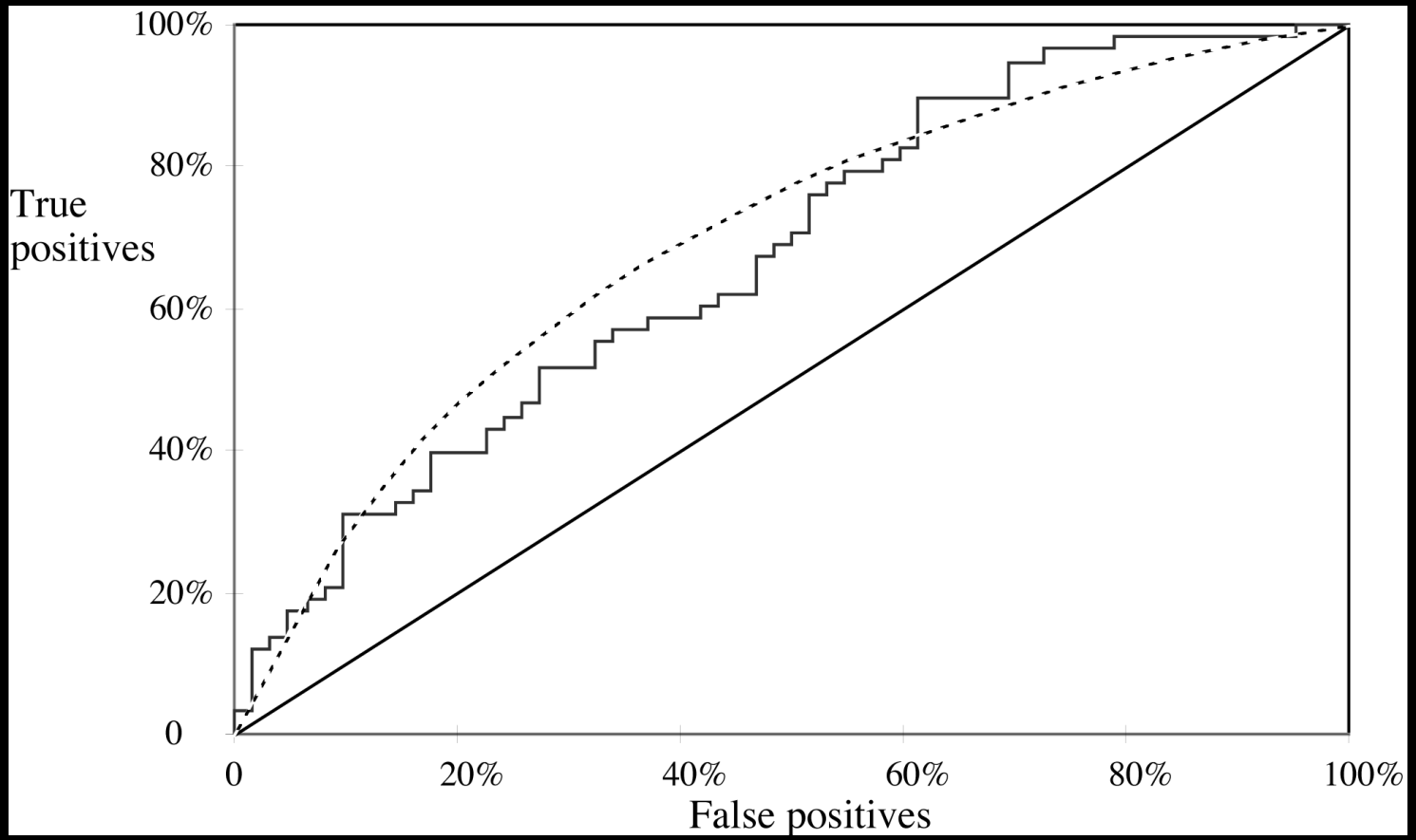
40% of responses for 10% of cost

80% of responses for 40% of cost

ROC curves

- *ROC curves* are similar to lift charts
  - Stands for "receiver operating characteristic"
  - Used in signal detection to show tradeoff between hit rate and false alarm rate over noisy channel
- Differences to lift chart:
  - *y* axis shows percentage of true positives in sample *rather than absolute number*
  - *x* axis shows percentage of false positives in sample *rather than sample size*

A sample ROC curve

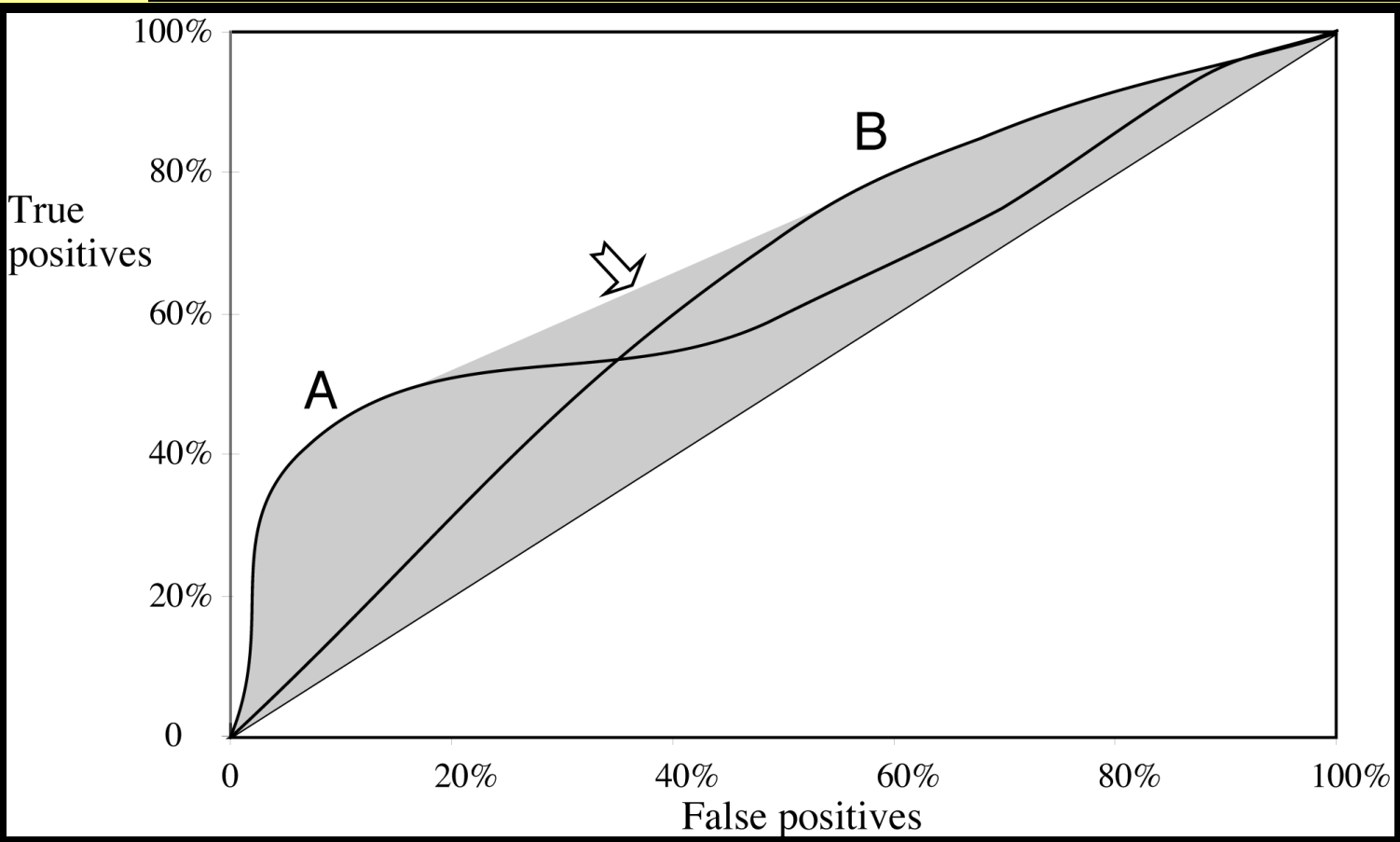


- Jagged curve—one set of test data
- Smooth curve—use cross-validation

Cross-validation and ROC curves

- Simple method of getting a ROC curve using cross-validation:
  - Collect probabilities for instances in test folds
  - Sort instances according to probabilities
- This method is implemented in WEKA
- However, this is just one possibility
  - Another possibility is to generate an ROC curve for each fold and average them

ROC curves for two schemes



- For a small, focused sample, use method A
- For a larger one, use method B
- In between, choose between A and B with appropriate probabilities

The convex hull

- Given two learning schemes we can achieve any point on the convex hull!
- TP and FP rates for scheme 1: t_1 and f_1
- TP and FP rates for scheme 2: t_2 and f_2
- If scheme 1 is used to predict $100 \times q$ % of the cases and scheme 2 for the rest, then
  - TP rate for combined scheme: $q \times t_1 + (1\text{-}q) \times t_2$
  - FP rate for combined scheme: $q \times f_1 + (1\text{-}q) \times f_2$

More measures...

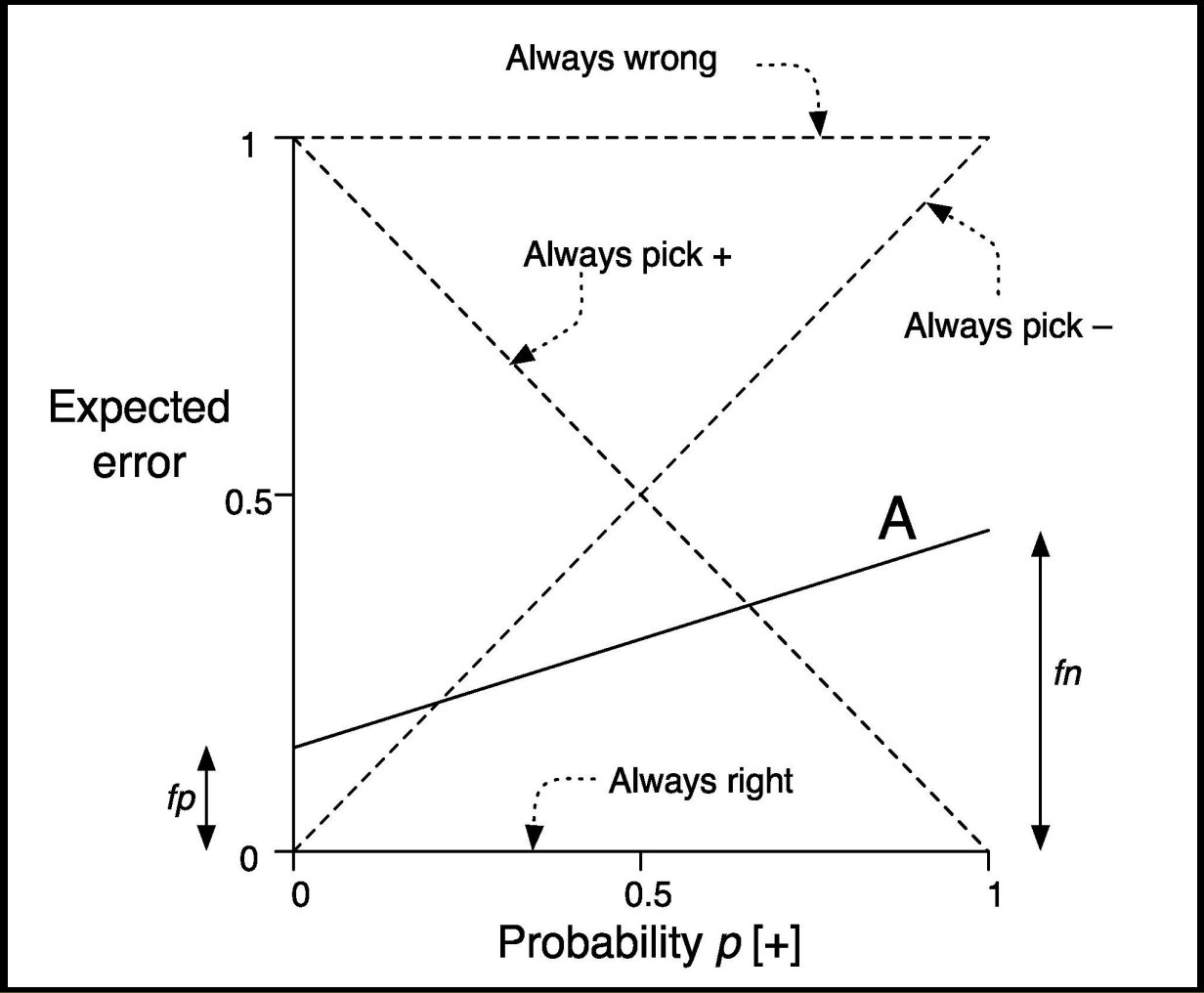
- Percentage of retrieved documents that are relevant: *precision*=TP/(TP+FP)
- Percentage of relevant documents that are returned: *recall* =TP/(TP+FN)
- Precision/recall curves have hyperbolic shape
- Summary measures: average precision at 20%, 50% and 80% recall (*three-point average recall*)
- *F-measure*=(2 × recall × precision)/(recall+precision)
- *sensitivity* × *specificity* = (TP / (TP + FN)) × (TN / (FP + TN))
- Area under the ROC curve (*AUC*):
  probability that randomly chosen positive instance is ranked above randomly chosen negative one

Summary of some measures

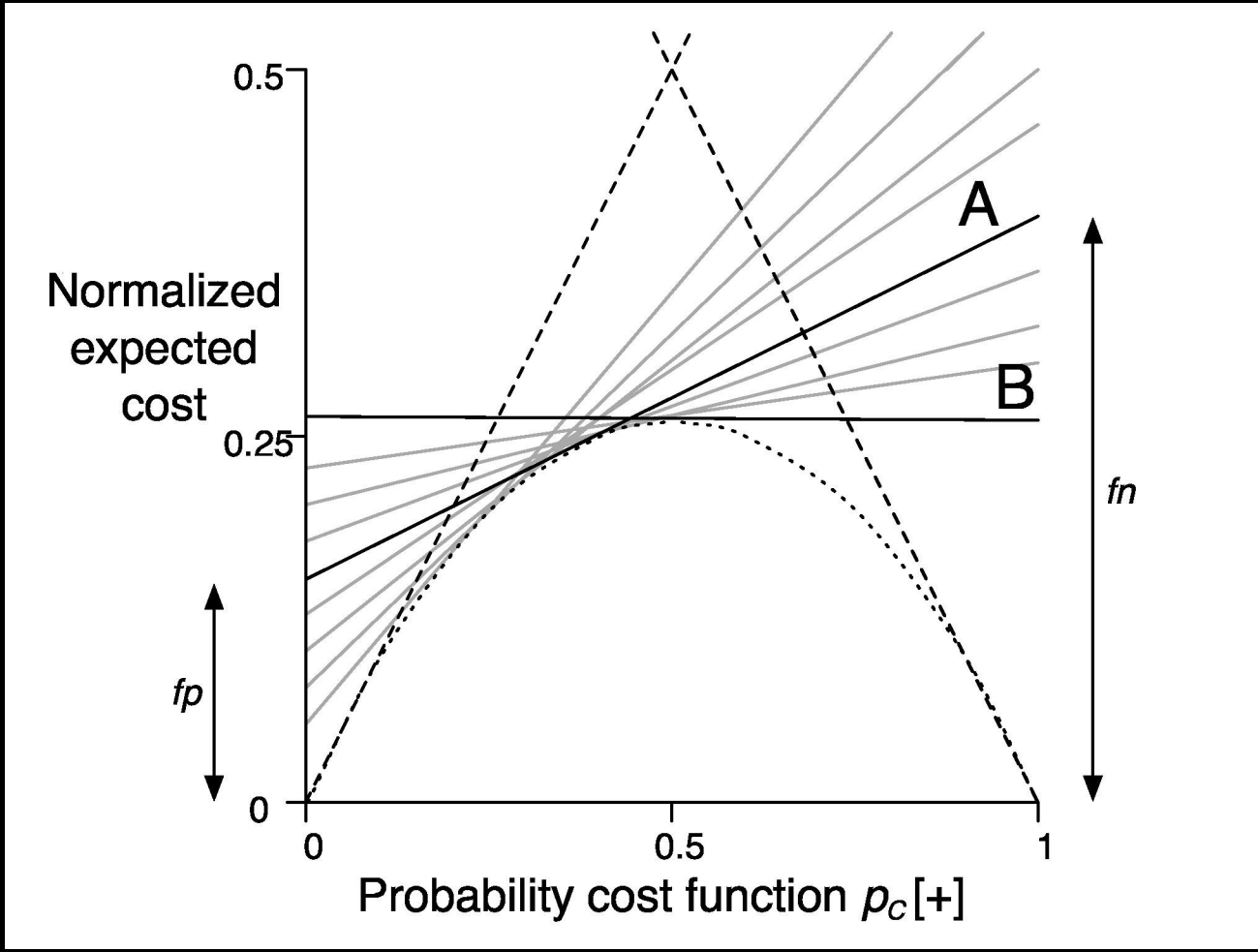
	Domain	Plot	Explanation
Lift chart	Marketing	TP Subset size	TP (TP+FP)/(TP+FP+TN+FN)
ROC curve	Communications	TP rate FP rate	TP/(TP+FN) FP/(FP+TN)
Recall-precision curve	Information retrieval	Recall Precision	TP/(TP+FN) TP/(TP+FP)

Cost curves

- *Cost curves* plot expected costs directly
- Example for case with uniform costs (i.e. error):



Cost curves: example with costs



$$\text{Probability cost function } p_c[+] = \frac{p[+]C[+|-]}{p[+]C[+|-] + p[-]C[-|+]}$$

$$\text{Normalized expected cost} = \text{fn} \times p_c[+] + \text{fp} \times (1 - p_c[+])$$

Evaluating numeric prediction

- Same strategies: independent test set, cross-validation, significance tests, etc.
- Difference: error measures
- Actual target values: $a_1\ a_2\ \ldots a_n$
- Predicted target values: $p_1\ p_2\ \ldots\ p_n$
- Most popular measure: *mean-squared error*

$$\frac{(p_1-a_1)^2+\ldots+(p_n-a_n)^2}{n}$$

  - Easy to manipulate mathematically

Other measures

- The *root mean-squared error* :

$$\sqrt{\frac{(p_1-a_1)^2+\ldots+(p_n-a_n)^2}{n}}$$

- The *mean absolute error* is less sensitive to outliers than the mean-squared error:

$$\frac{|p_1-a_1|+\ldots+|p_n-a_n|}{n}$$

- Sometimes *relative* error values are more appropriate (e.g. 10% for an error of 50 when predicting 500)

Improvement on the mean

- How much does the scheme improve on simply predicting the average?
- The *relative squared error* is:

$$\frac{(p_1-a_1)^2+\ldots+(p_n-a_n)^2}{(\bar{a}-a_1)^2+\ldots+(\bar{a}-a_n)^2}$$

- The *relative absolute error* is:

$$\frac{|p_1-a_1|+\ldots+|p_n-a_n|}{|\bar{a}-a_1|+\ldots+|\bar{a}-a_n|}$$

Correlation coefficient

- Measures the *statistical correlation* between the predicted values and the actual values

$$\frac{S_{PA}}{\sqrt{S_P S_A}}$$

$$S_{PA} = \frac{\sum_i (p_i - \bar{p})(a_i - \bar{a})}{n-1} \qquad S_P = \frac{\sum_i (p_i - \bar{p})^2}{n-1} \qquad S_A = \frac{\sum_i (a_i - \bar{a})^2}{n-1}$$

- Scale independent, between –1 and +1
- Good performance leads to large values!

Which measure?

- Best to look at all of them
- Often it doesn't matter
- Example:

	A	B	C	D
Root mean-squared error	67.8	91.7	63.3	57.4
Mean absolute error	41.3	38.5	33.4	29.2
Root rel squared error	42.2%	57.2%	39.4%	35.8%
Relative absolute error	43.1%	40.1%	34.8%	30.4%
Correlation coefficient	0.88	0.88	0.89	0.91

- D best
- C second-best
- A, B arguable

The MDL principle

- MDL stands for *minimum description length*
- The description length is defined as:

  space required to describe a theory

  +

  space required to describe the theory's mistakes
- In our case the theory is the classifier and the mistakes are the errors on the training data
- Aim: we seek a classifier with minimal DL
- MDL principle is a *model selection criterion*

Model selection criteria

- Model selection criteria attempt to find a good compromise between:
  - The complexity of a model
  - Its prediction accuracy on the training data
- Reasoning: a good model is a simple model that achieves high accuracy on the given data
- Also known as *Occam's Razor* :
  the best theory is the smallest one
  that describes all the facts

Elegance vs. errors

- Theory 1: very simple, elegant theory that explains the data almost perfectly
- Theory 2: significantly more complex theory that reproduces the data without mistakes
- Theory 1 is probably preferable
- Classical example: Kepler's three laws on planetary motion
  - Less accurate than Copernicus's latest refinement of the Ptolemaic theory of epicycles

MDL and compression

- MDL principle relates to data compression:
  - The best theory is the one that compresses the data the most
  - I.e. to compress a dataset we generate a model and then store the model and its mistakes
- We need to compute
  (a) size of the model, and
  (b) space needed to encode the errors
- (b) easy: use the informational loss function
- (a) need a method to encode the model

MDL and Bayes's theorem

- L[T]="length" of the theory
- L[E | T]=training set encoded wrt the theory
- Description length= L[T] + L[E | T]
- Bayes's theorem gives *a posteriori* probability of a theory given the data:

$$Pr[\mathrm{T}|\mathrm{E}]=\frac{Pr[\mathrm{E}|\mathrm{T}]Pr[T]}{Pr[E]}$$

- Equivalent to:

$$-\log Pr[\mathrm{T}|\mathrm{E}]=-\log Pr[\mathrm{E}|\mathrm{T}]-\log Pr[T]+\underbrace{\log Pr[E]}_{\textit{constant}}$$

MDL and MAP

- MAP stands for *maximum a posteriori probability*
- Finding the MAP theory corresponds to finding the MDL theory
- Difficult bit in applying the MAP principle: determining the prior probability Pr[T] of the theory
- Corresponds to difficult part in applying the MDL principle: coding scheme for the theory
- I.e. if we know a priori that a particular theory is more likely we need fewer bits to encode it

Discussion of MDL principle

- Advantage: makes full use of the training data when selecting a model
- Disadvantage 1: appropriate coding scheme/prior probabilities for theories are crucial
- Disadvantage 2: no guarantee that the MDL theory is the one which minimizes the expected error
- Note: Occam's Razor is an axiom!
- Epicurus's *principle of multiple explanations*: keep all theories that are consistent with the data

MDL and clustering

- Description length of theory:
  bits needed to encode the clusters
  - e.g. cluster centers
- Description length of data given theory:
  encode cluster membership and position relative to cluster
  - e.g. distance to cluster center
- Works if coding scheme uses less code space for small numbers than for large ones
- With nominal attributes, must communicate probability distributions for each cluster