

## Instance Based Learning

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[Read Ch. 8]

- $k$ -Nearest Neighbor
- Locally weighted regression
- Radial basis functions
- Case-based reasoning
- Lazy and eager learning

## When To Consider Nearest Neighbor

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- Instances map to points in  $\mathcal{R}^n$
- Less than 20 attributes per instance
- Lots of training data

Advantages:

- Training is very fast
- Learn complex target functions
- Don't lose information

Disadvantages:

- Slow at query time
- Easily fooled by irrelevant attributes

## Instance-Based Learning

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Key idea: just store all training examples  $\langle x_i, f(x_i) \rangle$

Nearest neighbor:

- Given query instance  $x_q$ , first locate nearest training example  $x_n$ , then estimate  $\hat{f}(x_q) \leftarrow f(x_n)$

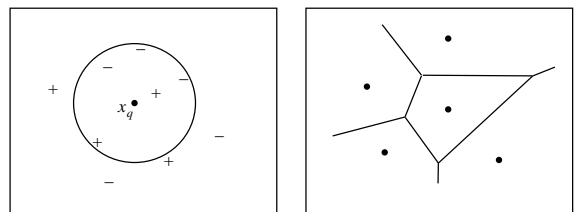
$k$ -Nearest neighbor:

- Given  $x_q$ , take vote among its  $k$  nearest nbrs (if discrete-valued target function)
- take mean of  $f$  values of  $k$  nearest nbrs (if real-valued)

$$\hat{f}(x_q) \leftarrow \frac{\sum_{i=1}^k f(x_i)}{k}$$

## Voronoi Diagram

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## Behavior in the Limit

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Consider  $p(x)$  defines probability that instance  $x$  will be labeled 1 (positive) versus 0 (negative).

Nearest neighbor:

- As number of training examples  $\rightarrow \infty$ , approaches Gibbs Algorithm
- Gibbs: with probability  $p(x)$  predict 1, else 0

$k$ -Nearest neighbor:

- As number of training examples  $\rightarrow \infty$  and  $k$  gets large, approaches Bayes optimal
- Bayes optimal: if  $p(x) > .5$  then predict 1, else 0

Note Gibbs has at most twice the expected error of Bayes optimal

## Curse of Dimensionality

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Imagine instances described by 20 attributes, but only 2 are relevant to target function

*Curse of dimensionality*: nearest nbr is easily misled when high-dimensional  $X$

One approach:

- Stretch  $j$ th axis by weight  $z_j$ , where  $z_1, \dots, z_n$  chosen to minimize prediction error
- Use cross-validation to automatically choose weights  $z_1, \dots, z_n$
- Note setting  $z_j$  to zero eliminates this dimension altogether

see [Moore and Lee, 1994]

## Distance-Weighted $k$ NN

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Might want weight nearer neighbors more heavily...

$$\hat{f}(x_q) \leftarrow \frac{\sum_{i=1}^k w_i f(x_i)}{\sum_{i=1}^k w_i}$$

where

$$w_i \equiv \frac{1}{d(x_q, x_i)^2}$$

and  $d(x_q, x_i)$  is distance between  $x_q$  and  $x_i$

Note now it makes sense to use *all* training examples instead of just  $k$

$\rightarrow$  Shepard's method

## Locally Weighted Regression

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Note  $k$ NN forms local approximation to  $f$  for each query point  $x_q$

Why not form an explicit approximation  $\hat{f}(x)$  for region surrounding  $x_q$

- Fit linear function to  $k$  nearest neighbors
- Fit quadratic, ...
- Produces "piecewise approximation" to  $f$

Several choices of error to minimize:

- Squared error over  $k$  nearest neighbors

$$E_1(x_q) \equiv \frac{1}{2} \sum_{x \in k \text{ nearest nbrs of } x_q} (f(x) - \hat{f}(x))^2$$

- Distance-weighted squared error over all nbrs

$$E_2(x_q) \equiv \frac{1}{2} \sum_{x \in D} (f(x) - \hat{f}(x))^2 K(d(x_q, x))$$

- ...

## Radial Basis Function Networks

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- Global approximation to target function, in terms of linear combination of local approximations
- Used, e.g., for image classification
- A different kind of neural network
- Closely related to distance-weighted regression, but “eager” instead of “lazy”

## Training Radial Basis Function Networks

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Q1: What  $x_u$  to use for each kernel function  $K_u(d(x_u, x))$

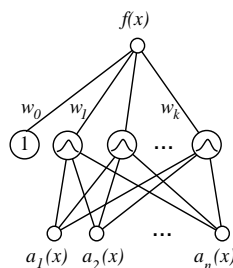
- Scatter uniformly throughout instance space
- Or use training instances (reflects instance distribution)

Q2: How to train weights (assume here Gaussian  $K_u$ )

- First choose variance (and perhaps mean) for each  $K_u$ 
  - e.g., use EM
- Then hold  $K_u$  fixed, and train linear output layer
  - efficient methods to fit linear function

## Radial Basis Function Networks

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where  $a_i(x)$  are the attributes describing instance  $x$ , and

$$f(x) = w_0 + \sum_{u=1}^k w_u K_u(d(x_u, x))$$

One common choice for  $K_u(d(x_u, x))$  is

$$K_u(d(x_u, x)) = e^{-\frac{1}{2\sigma_u^2} d^2(x_u, x)}$$

## Case-Based Reasoning

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Can apply instance-based learning even when  $X \neq \mathbb{R}^n$

→ need different “distance” metric

Case-Based Reasoning is instance-based learning applied to instances with symbolic logic descriptions

```
((user-complaint error53-on-shutdown)
 (cpu-model PowerPC)
 (operating-system Windows)
 (network-connection PCIA)
 (memory 48meg)
 (installed-applications Excel Netscape VirusScan)
 (disk 1gig)
 (likely-cause ???))
```

## Case-Based Reasoning in CADET

CADET: 75 stored examples of mechanical devices

- each training example:  $\langle$  qualitative function, mechanical structure  $\rangle$
- new query: desired function,
- target value: mechanical structure for this function

Distance metric: match qualitative function descriptions

## Case-Based Reasoning in CADET

- Instances represented by rich structural descriptions
- Multiple cases retrieved (and combined) to form solution to new problem
- Tight coupling between case retrieval and problem solving

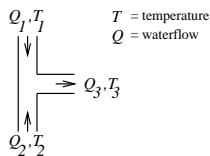
Bottom line:

- Simple matching of cases useful for tasks such as answering help-desk queries
- Area of ongoing research

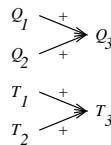
## Case-Based Reasoning in CADET

**A stored case:** T-junction pipe

Structure:



Function:

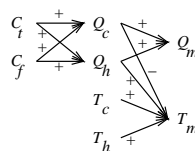


**A problem specification:** Water faucet

Structure:

?

Function:



## Lazy and Eager Learning

Lazy: wait for query before generalizing

- $k$ -NEAREST NEIGHBOR, Case based reasoning

Eager: generalize before seeing query

- Radial basis function networks, ID3, Backpropagation, NaiveBayes, ...

Does it matter?

- Eager learner must create global approximation
- Lazy learner can create many local approximations
- if they use same  $H$ , lazy can represent more complex fns (e.g., consider  $H$  = linear functions)