Chapter 8

Signal Averaging

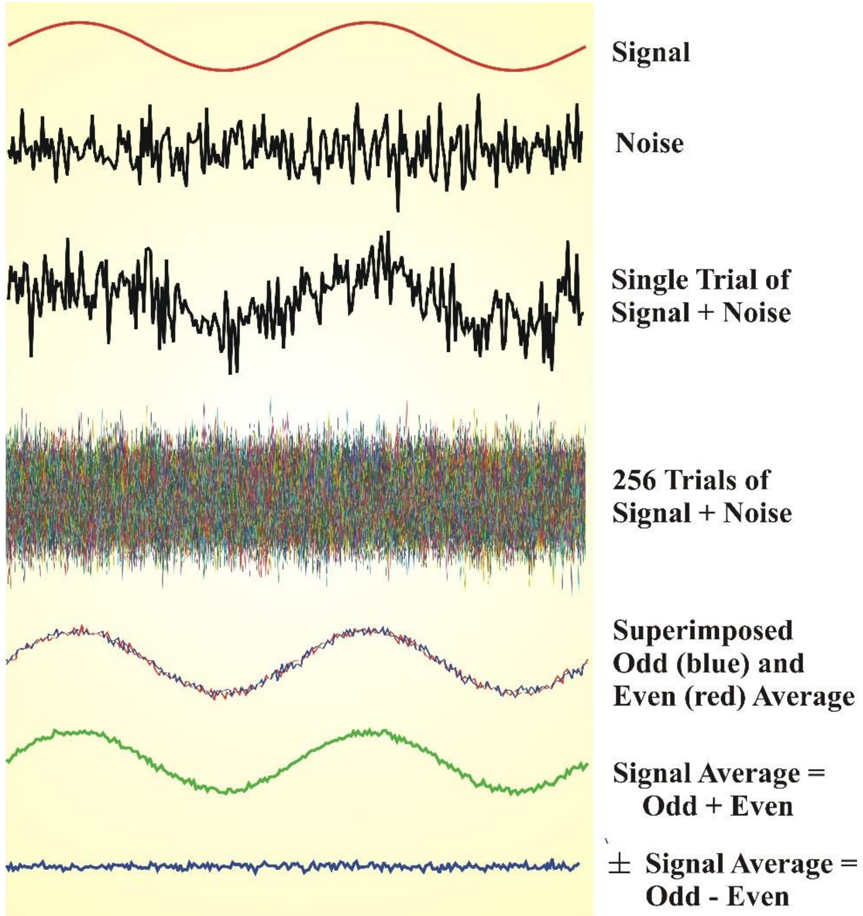
Introduction

Data analysis techniques are commonly subdivided into operations in the **time domain** (or spatial domain) and **frequency domain**. In this chapter we discuss processing techniques applied in the time (spatial) domain with a strong emphasis on signal averaging. Signal averaging is an important technique that allows estimation of low-amplitude signals that are buried in noise. The technique usually assumes that:

1. **signal and noise are uncorrelated**.
2. **The timing of the signal is known**.
3. **Signal component is consistent when performing repeated measurements.**
4. **The noise is truly random with zero mean.**

In the real world all these assumptions may be **violated** to some degree; however, the averaging technique has proven sufficiently robust to survive **minor violations** of these four basic assumptions.

TIME-LOCKED SIGNALS …



Signal averaging of a signal buried in noise (Signal + Noise). This example shows 256 superimposed trials of such a measurement and the average thereof. The average results of the odd and even trials are shown separately. The sum of all trials (Signal Average) shows the signal with a small component of residual noise. A signal average is shown as an estimate of residual noise.

Figure 4.2 Signal processing for Neuroscientist, second edition by Wim van Drongelen

NOISE ESTIMATES

The ultimate reason to perform signal averaging is to increase the signal-to-noise ratio .The estimate of residual noise can easily be established in theory where all the components are known. In real measurements the noise and signal components are unknown and the averaged result is certain to contain both signal and residual noise . In practical applications, there is a number of techniques one might use to estimate the residual noise in the averaged result.

Prestimulus Noise
Bootstrap
+ - Average

Prestimulus Noise

One might estimate the residual noise by using the prestimulus epoch if there is a reason to assume that **the time-locked signal only occurs after the trigger for the signal average**, such as an evoked potential. Without a clear indication of a post stimulus response, such as activity surrounding the occurrence of a spike in a spike-triggered average , this approach obviously won't work**. It should also be noted that the prestimulus epoch is not necessarily reliable in the case of a stimulus-evoked potential**. Since this type of average is obtained by repetitive stimulation, the late component of the response to a stimulus can still be ongoing in the prestimulus epoch of the next stimulus. In this case, the noise estimate of the prestimulus average will include the effect of the late component of the response. One could attempt to mitigate this problem by using **larger inter stimulus intervals**, **removing the late/slow components by high-pass filtering** , or using an alternative noise estimator.

NOISE ESTIMATES …

Prestimulus Noise
Bootstrap
+ - Average

Bootstrapping is another method that works well, especially in **off-line average procedures** where the signal average is produced with signals that were previously recorded and stored. In this case, **a control-average** can be produced by picking random triggers rather than the triggers used for producing the **“true” average**. By picking the trigger randomly, **the time-locked aspect of the epochs producing the average is destroyed**. This procedure will produce a control-average that still includes the effects of the time-locked signal. However, since the averages are not aligned with the real trigger, the time-locked signal will not be enhanced in this control average. **Obviously, this bootstrap method will work well if the power of the time-locked component is small with respect to the noise that embeds it.** Fortunately, this is usually the case because it is a principal motivation to employ signal averaging in the first place! One can produce a series of control-averages by using the bootstrap multiple times. In this case, you can compare the statistics of the control averages with the “true” average and use this comparison to validate the “true” average result.

NOISE ESTIMATES …

Prestimulus Noise
Bootstrap
+ - Average

+ - Average

One efficient way of establishing the amount of residual noise using the same epochs as those in the “true” average is by using so-called averaging. This is a procedure in which measurements from every other trial are inverted prior to creating the averaged result. This technique removes any consistent signal component by alternating addition and subtraction. However, the residual noise is maintained in the end result . The *rms* value of the noise component estimated from the average is the same as that produced by the standard average because random noise samples from the inverted traces have the same distribution as those from noninverted trials.

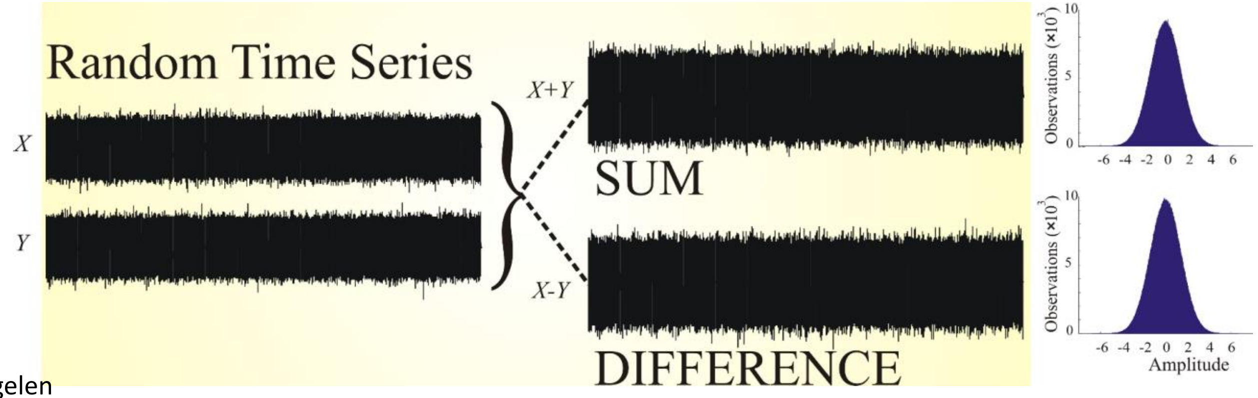
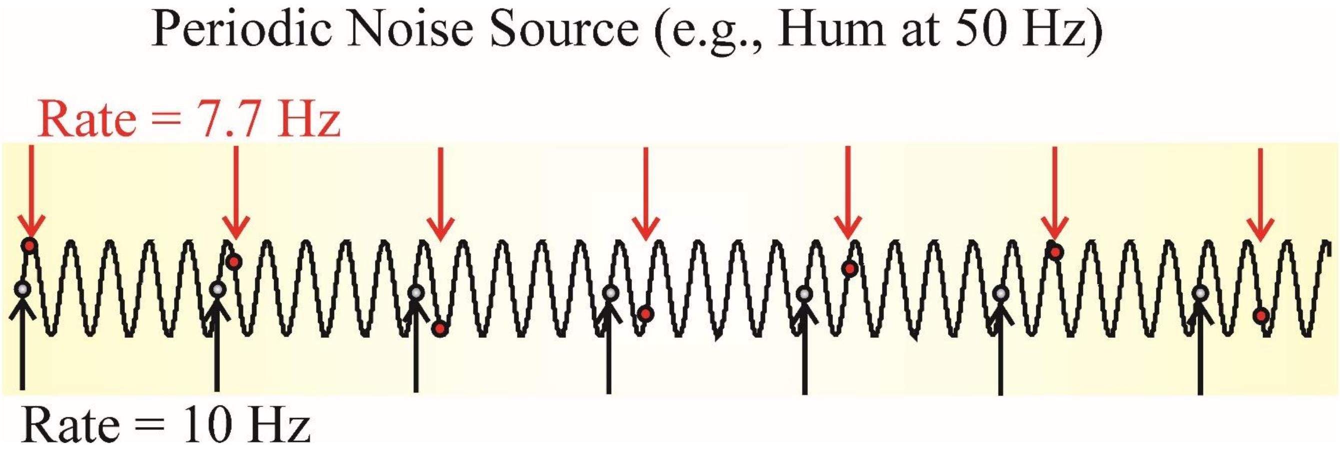


Figure 4.3 Signal processing for Neuroscientist, second edition by Wim van Drongelen

SIGNAL AVERAGING AND NONRANDOM NOISE

The result in the previous section depends heavily on a noise component being random, having **zero mean, and being unrelated** to the signal. A special case occurs when the noise is not random. This situation may affect the performance of the average and even make it impossible to apply the technique without a few critical adaptations.

This problem is often avoided by either **randomizing the stimulus interval** or by using a **noninteger stimulus rate**.



The stimulus-rate and a periodic component (e.g., a 50-Hz or 60-Hz hum artifact) in the unaveraged signal can produce an undesired effect in the average. An average produced with a 10-Hz rate will contain a large 50-Hz signal. In contrast, an average produced at a 7.7-Hz rate will not contain such a strong 50-Hz artifact. This difference is due to the fact that a rate of 10-Hz results in a stimulus onset that coincides with the same phase in the 50-Hz sinusoidal noise source (black dots), whereas the noninteger rate of 7.7 Hz produces a train of stimuli for which the relative phase of the noise source changes with each stimulus (red dots).

Figure 4.5 Signal processing for Neuroscientist, second edition by Wim van Drongelen

Characteristics of Background EEG

- EEG frequency range: 0.01 to 100 Hz
- EEG amplitudes: typically around 100 microvolts
- Power spectral density follows a power law
- EEG divided into five bands: delta, theta, alpha, beta, gamma
- EEG considered stochastic due to limited genuine measurements
- Long-term EEG signals are non-stationary time series
- Short-term EEG signals can be approximately stationary
- Stationary time window lengths vary, typically several seconds to minutes

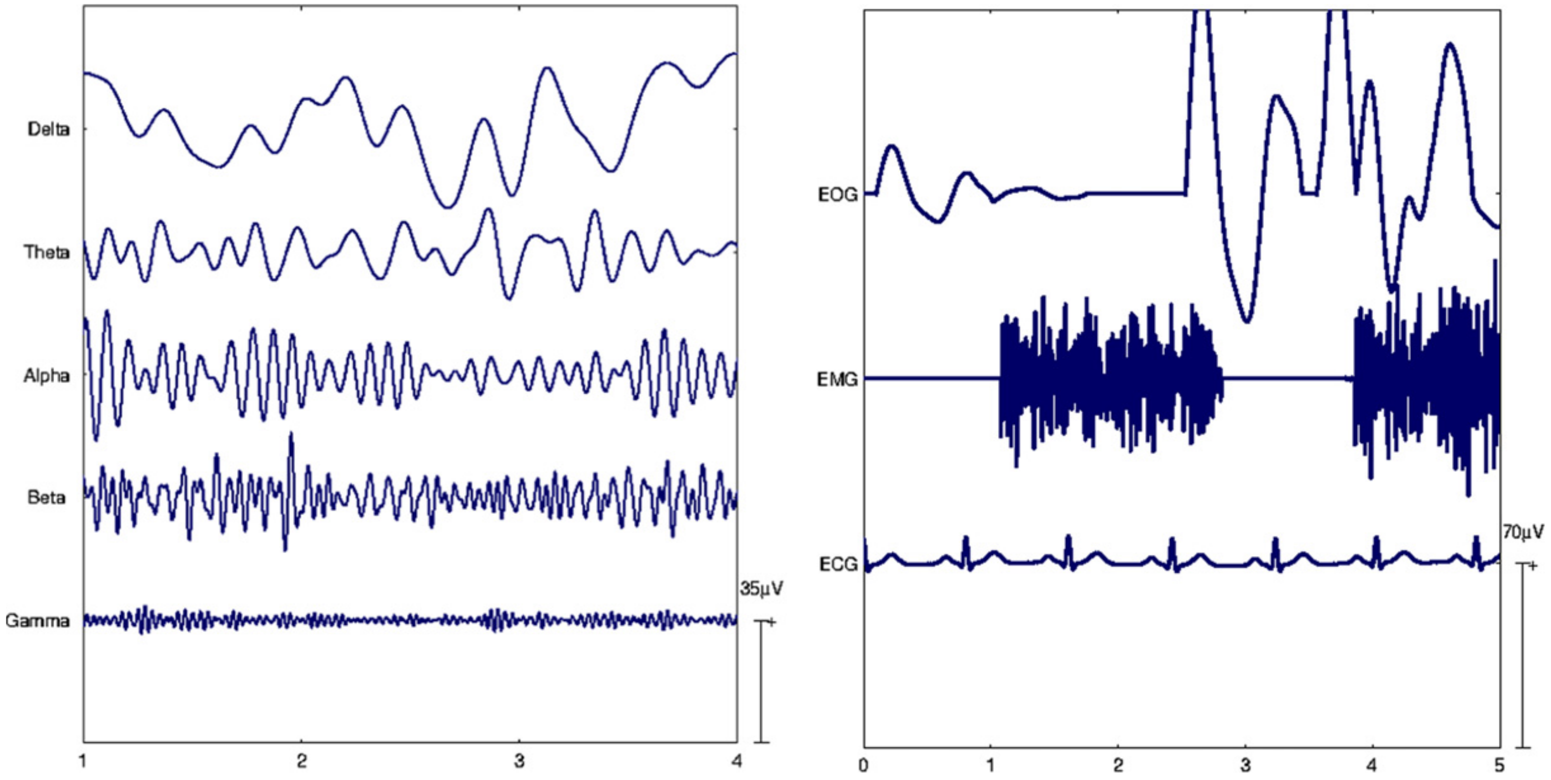


Figure 1. (a) Five normal brain rhythms, from low to high frequencies. Delta, theta, alpha, beta and gamma rhythms comprise the background EEG spectrum. (b) Three different kinds of artifacts. Ocular, muscular and cardiac artifacts are the most frequent physiological contaminants in the literature on EEG artifact removal.
Figure 1, J.A. Urigüen 2015

Power Line Noise Removal

- Notch filtering used to eliminate line noise at 50 or 60 Hz
- Implemented with a certain frequency width (e.g., 10 Hz)
- Successful in removing line noise but may distort signal components between 50 and 70 Hz
- Notch filter can generate transient oscillation in baseline activity, impacting data interpretation
- Follow-up low-pass filtering below 50 Hz may address issues but can alter EEG temporal structures or cause spurious interactions between channels

Example:

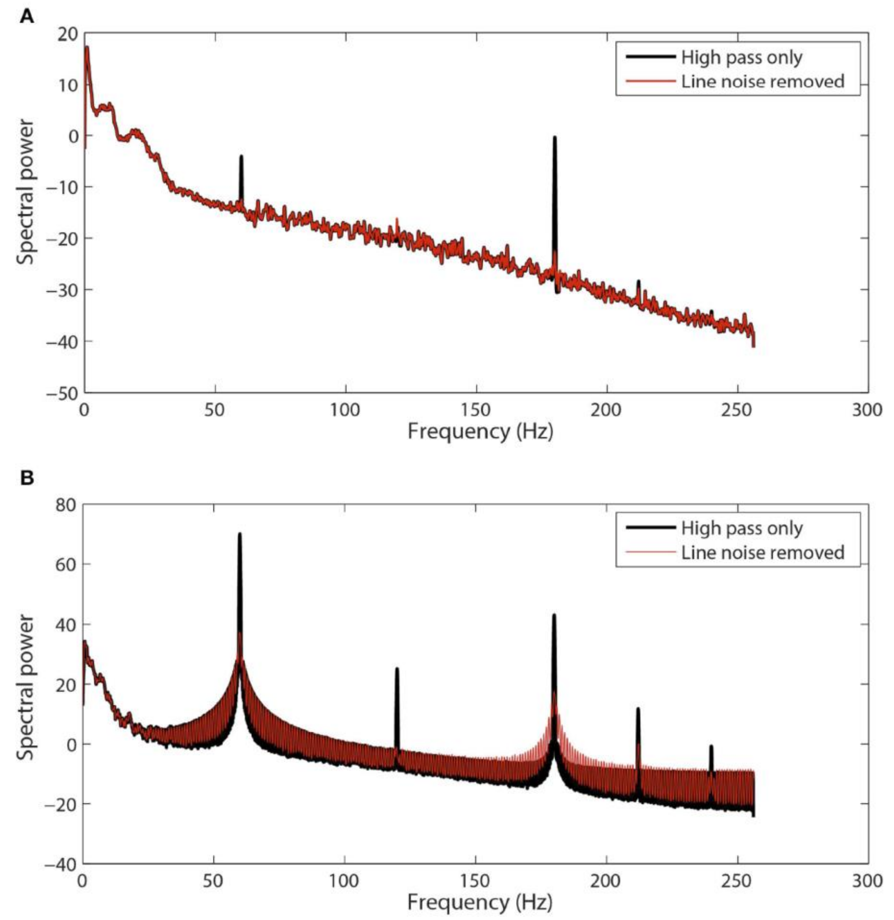
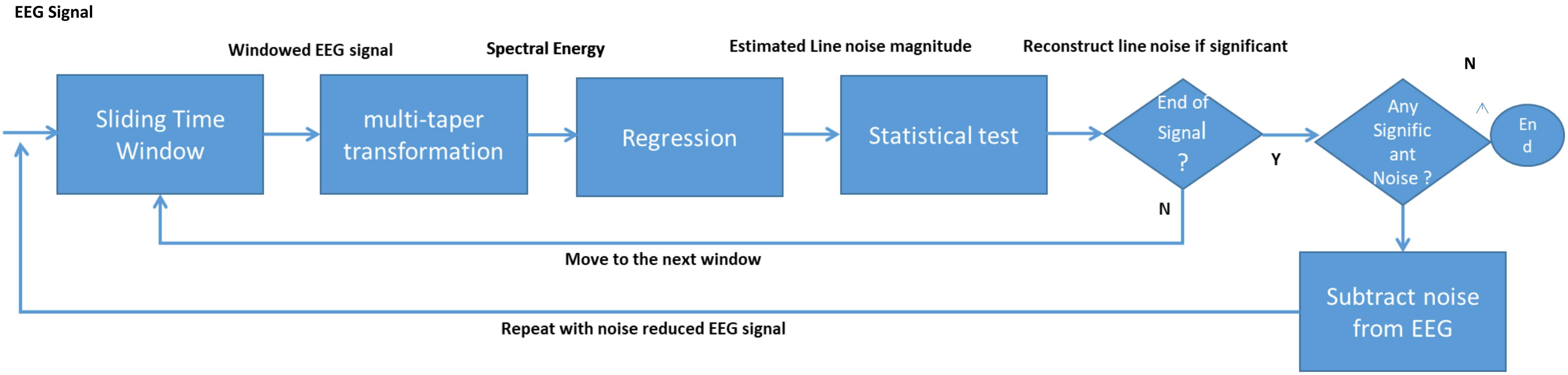


FIGURE 1 | Selected channel spectra from 64-channel Biosemi EEG before and after line noise removal. Spectral power is in units of $10\log_{10}\left(\mu V^2\right)$. Data was high-pass filtered at 1 Hz prior to line noise removal. **(A)** A channel with sharp peaks at 60 and 180 Hz. **(B)** A channel contaminated by non-stationary transformer noise. This channel is interpolated at a later stage in the pipeline.

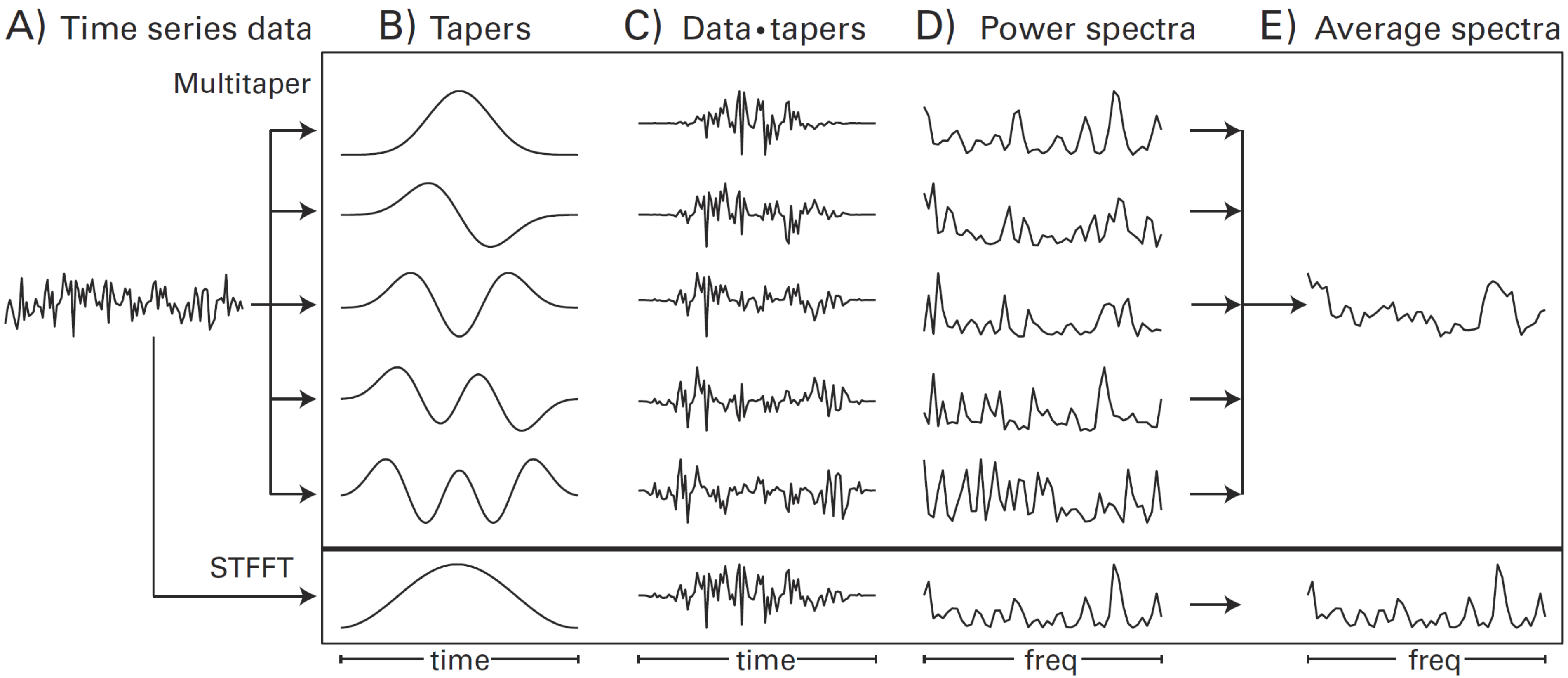
Bigdely-Shamlo et al. 2015

multi-taper decomposition

Estimate and subtract line noise in EEG signals using multi-taper decomposition, regression modelling, and significance testing for effective removal without damaging background spectral components.



Multitapper tranfromation



- Multitaper method is useful for low signal-to-noise ratio situations.
- Particularly effective for higher-frequency activity or single-trial estimates of power.
- Frequencies lower than around 30 Hz may make multitaper less appropriate.
- Signal-to-noise ratio is already relatively high at lower frequencies.
- Spectral smoothing from multitaper may impede frequency isolation.
- Activities from multiple frequency bands can become averaged together.

Referencing

- Standard practice: Subtract reference signal with the same time resolution from original EEG signal at each channel.

- Common reference choices:

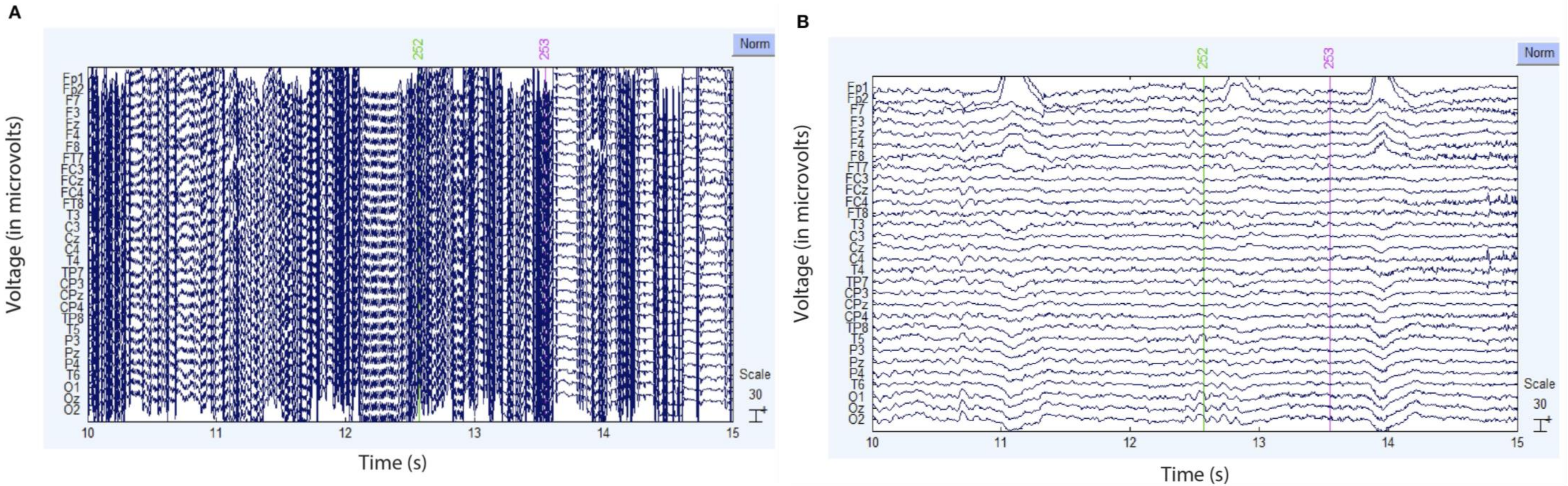
  mastoid channel, specific EEG channel, average of two mastoid signals, or average of all EEG channels.

- Importance: Chosen reference should remain unchanged relative to EEG signals, ensuring effective representation of brain activity.

- Caution: Carefully inspect chosen reference signal for comparable amplitude levels and no correlation with task-induced brain activity.

- Common Average Reference (CAR): Reduces single-point failure impact but may suffer from outlier channels.
  - Solution: Detect and remove bad channels before using CAR.

Example:



Comparison of theordinary and robustaverage references

(A) Example signal after ordinary average referencing.

(B) Same signal using robust average referencing.

Bigdely-Shamlo 2015, frontrier, Figure 4.

Bad Channel Detection

1. **Detecting Noisy or Bad Channels:**
    - Identify channels with excessively large amplitudes.
    - Use robust z-score to detect extreme amplitudes.
    - Bad channel determined if robust z-score exceeds a threshold.
2. **Correlation-based Detection:**
    - Investigate correlation of a single channel with others.
    - Normal EEG shows low-frequency correlations across channels.
    - Detect bad channels by correlating one channel after low-pass filtering.
    - Attempt prediction if two bad channels are correlated.
3. **Frequency-based Detection:**
    - Measure ratio of high-frequency power to low-frequency power.
    - Detect bad channels with a ratio higher than a threshold.

Bad Channel Detection

4. Replacement of Bad Channels:

- Replace bad channels with virtual healthy channels.
- Reconstruction of global brain responses.

5. Interpolation Schemes:

- Various interpolation schemes for channel reconstruction:
  - Spherical splines
  - Higher-order polynomials
  - Nearest-neighbor averaging
  - Radial basis function

Advantages of Interpolation Methods:

- Spherical splines provide accurate scalp potential estimation with dense electrode mapping.
- Statistical methods like radial basis functions offer cost-effectiveness with lower computational loads.