

Optimal Allocation with Noisy Inspection

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Inspection

A core economic activity

- employers **interview** potential employees
- public funds **assess** grant applications
- venture capitalists **evaluate** investment opportunities



Why **inspect**?

1. discovery or *information acquisition*
2. verification or *screening*

A class of problems

A **principal** receives an unknown reward from allocating to an **agent**.

The agent has **imperfect private information** about this unknown reward; they receive a unit reward from being allocated to.

The principal may elicit a report from the agent, as well as **inspect** the reward at a **cost**.

The principal can commit to a mechanism, but must do so **without transfers**.

How should the principal design the inspection and allocation mechanism to maximize their ex ante expected return?

Applications

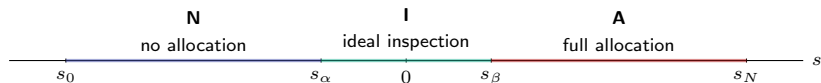
Mechanism design problems with **noisy information**, **costly inspection**, and **limited transfers** are widespread.

1. **Job hiring:** a firm seeks to fill an open position in their operation with a potential employee.
2. **Grant approval:** a public fund is tasked with assessing a grant application.
3. **Impact investment:** a venture capitalist sets the mechanism by which it reviews and invests in startups.

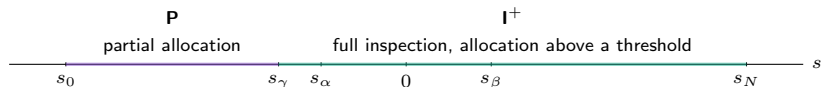
A simple solution

Let r be the principal's **reward**, and s be the agent's **type**, sorted and labelled by the expected value of the reward.

Symmetric information benchmark:



Optimal separating mechanism:

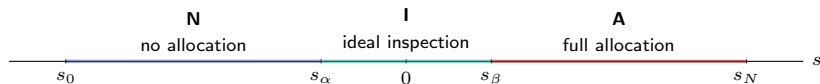


Losses

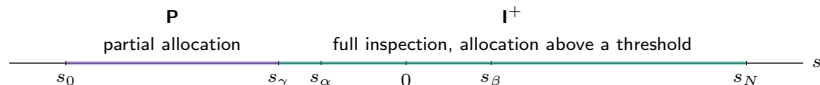
Three types of losses from private information:

1. over-allocation at the bottom,
2. over-inspection at the top and bottom, and
3. under-allocation post-inspection.

Symmetric information benchmark:

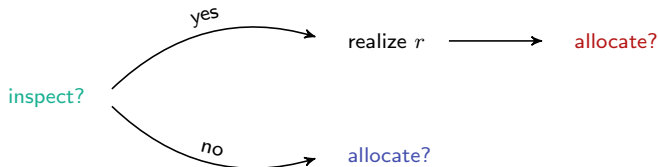


Optimal (separating) mechanism:



Mechanism

After the agent reports to the principal, what can the principal do?



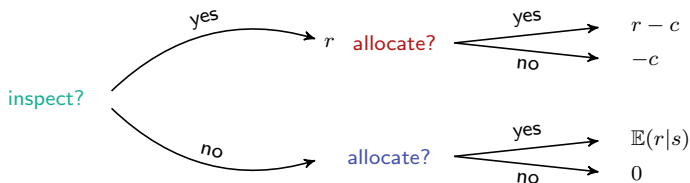
Then, a **mechanism** specifies for each type s ,

- an inspection rule,
- a pre-inspection allocation, and
- a post-inspection allocation for each r .

These are potentially probabilistic choices, so are bounded between 0 and 1.

Optimal allocation

Principal's objective:



Agent's incentives: 1 if allocated to, 0 otherwise.

An **optimal allocation** is a mechanism that maximizes the ex ante expected objective subject to *incentive compatibility* (IC) for each type s :

$$u(s|s) \geq u(\hat{s}|s) \quad \forall \hat{s}$$

A solution recipe

Consider a **relaxation** of the principal's problem that only requires the upward local IC constraints to be satisfied.

Claim 1: Optimal post-inspection rules are threshold rules. That is, for each s_n there exists some τ_n such that allocation only occurs post-inspection if $r > \tau_n$.

Claim 2: Each upward local incentive compatibility constraint binds. That is, for each s_n , $u(s_n | s_n) = u(s_{n+1} | s_n)$.

Claim 3: Optimal inspection rules are themselves threshold rules. That is, there exists γ such that the agent is only inspected if $s_n > s_\gamma$.

\Rightarrow Optimal post-inspection thresholds are constant: $\tau_n = \tau \forall n$.

Optimal separating policy

Given **Claims 1-3**, we are only left to optimize by selecting:

- γ : the first type to inspect, and
- τ : the threshold for passing those who are inspected.

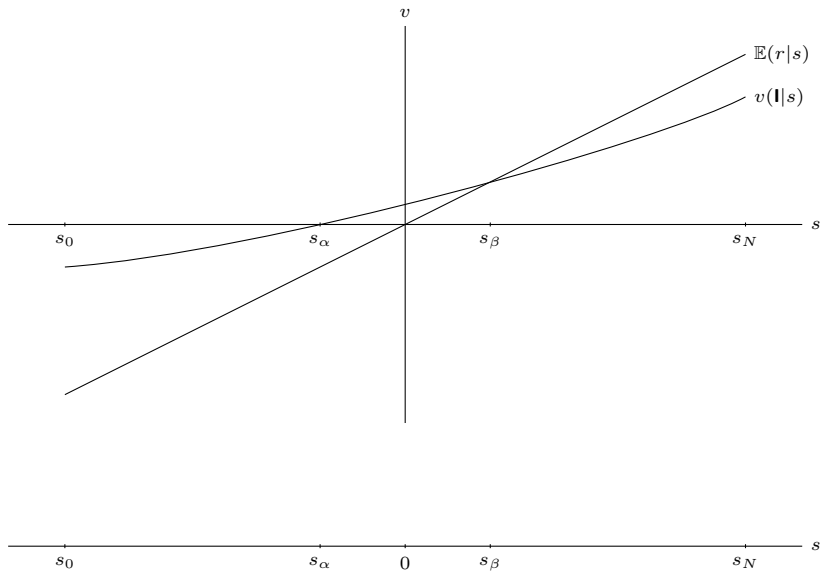
This is given by:

- the value of those high signals that we inspect with threshold τ , and
- the value of those low signals that we partially allocate to.

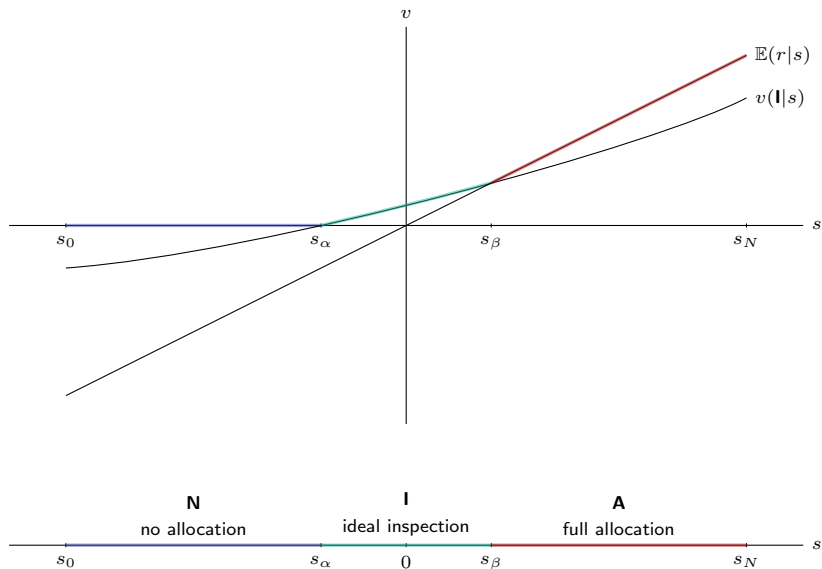
$$\max_{\gamma, \tau} v(\mathbf{I}(\tau)|s > s_\gamma) \cdot Pr(s > s_\gamma) + Pr(r > \tau|s_\gamma)\mathbb{E}(r|s \leq s_\gamma) \cdot Pr(s \leq s_\gamma)$$

This satisfies the **global** IC constraints for all γ and τ , and thus must be a solution to the original problem.

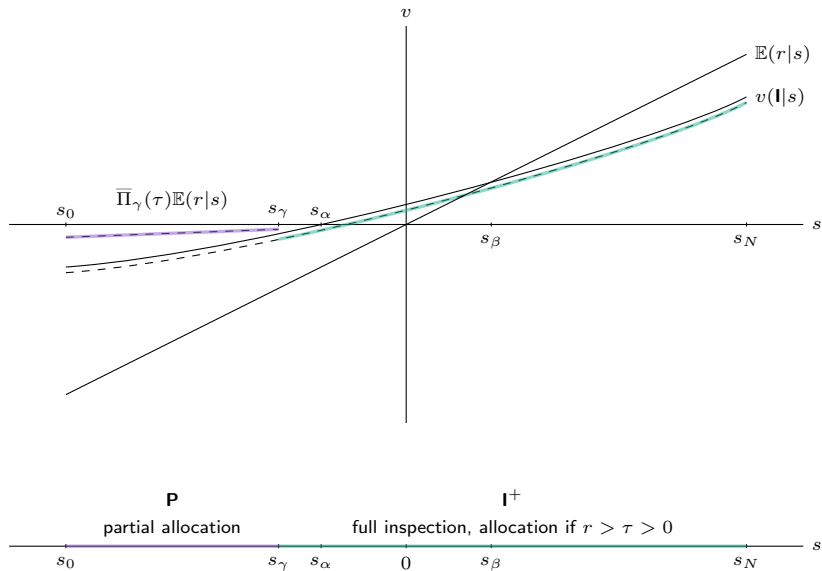
A visual representation



First best policy



Second best policy



Literature

Perfect information: Green and Laffont (1986), [Ben-Porath, Dekel and Lipman \(2014\)](#), Mylovanov and Zapechelnyuk (2017), Epitropou and Vohra (2019).

Transfers: Townsend (1979), Border and Sobel (1987), Mookherjee and Png (1989), Alaei et al. (2020).

Limited transfers: Mylovanov and Zapechelnyuk (2017), [Silva \(2019b\)](#), Li (2021).

Efficient mechanisms: [Ball and Kattwinkel \(2019\)](#), Silva (2019a), Siegel and Strulovici (2021), [Pereyra and Silva \(2021\)](#), Erlanson and Kleiner (2020).

Scoring rules: McCarthy (1956), Savage (1971), Gneiting and Raftery (2007).

Noisy inspection

Optimal inspection balances *discovery* and *verification*.

When agents have **noisy private information**, the principal:

- **over-inspects** high and low types,
- **under-allocates** to agents who are inspected, and
- **over-allocates** to agents who are not inspected.

Weakening commitment magnifies the losses from over-allocating to agents who aren't inspected.

For **separating to be optimal**, signals need to be sufficiently accurate, costs sufficiently small and information sufficiently valuable.

Outstanding questions?

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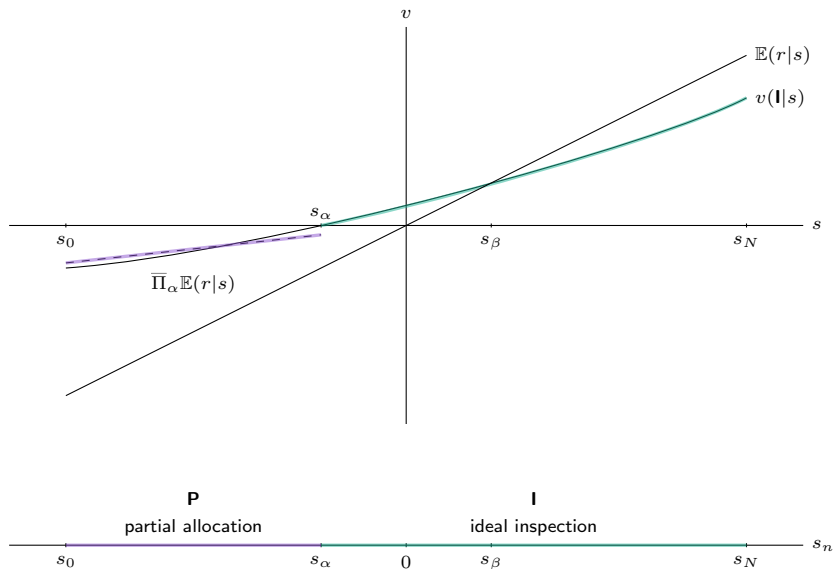
Relaxing commitment

There are three natural relaxations to the commitment assumption:

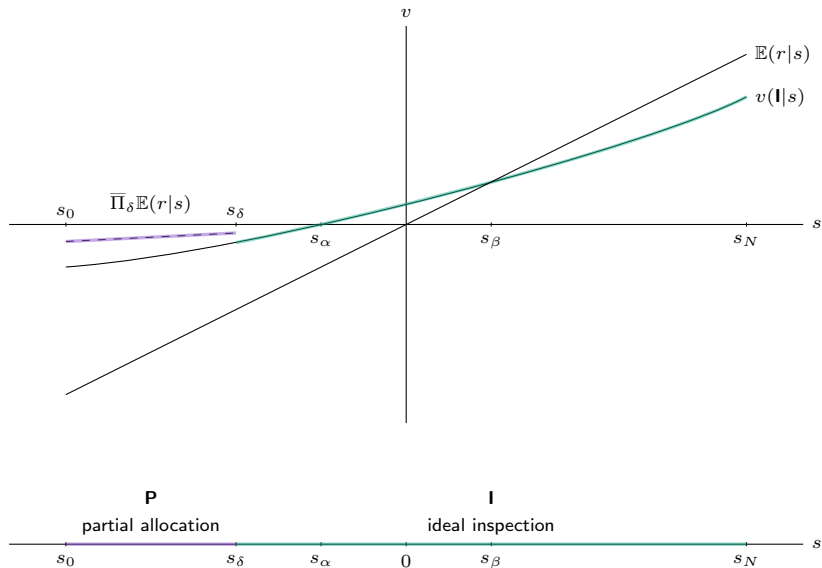
1. **pre-inspection commitment**: the principal can commit to pre-inspection allocations and an inspection rule but cannot commit to post-inspection allocations,
2. **pre-assessment commitment**: the principal cannot commit to either an inspection rule or post-inspection allocations, but can commit to pre-inspection allocations, and
3. **no commitment**: the principal cannot commit to allocations or an inspection rule.

For **no commitment**, the principal can only choose between the pooling mechanisms and reports convey no information. We know what this looks like, so let's turn to the first two relaxations.

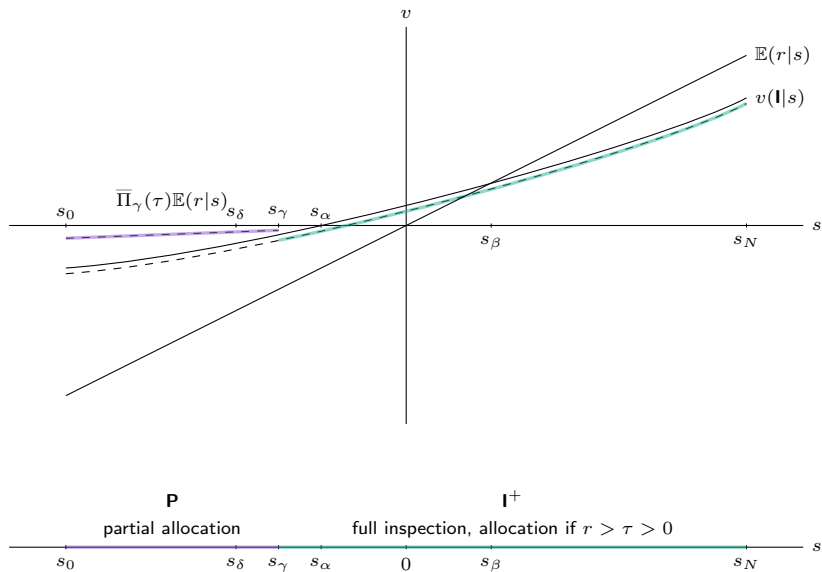
Pre-assessment commitment



Pre-inspection commitment



Full commitment



Gaussian environment

Suppose the prior over the rewards is given by: $r \sim N(\mu, 1)$, and the agent receives a signal of this reward, $\hat{s} = r + \varepsilon$, where $\varepsilon \sim N(0, \sigma^2)$.

Relabelling the signal by the expected reward given the signal, the posterior distribution of rewards, Π_s , is given by: $r \mid s \sim N(s, \hat{\sigma}^2)$ where:

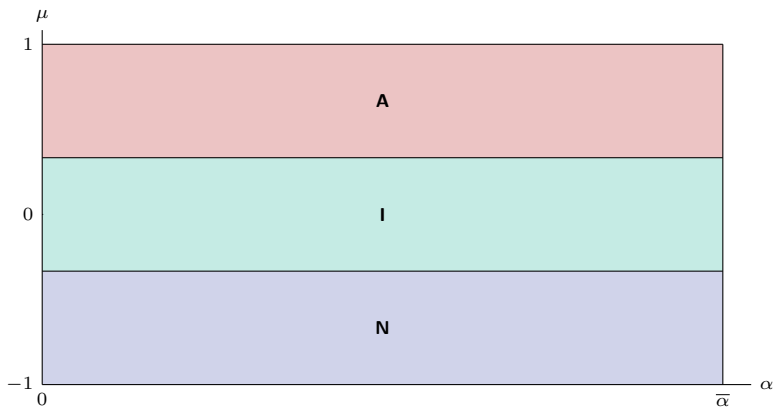
$$s = \frac{\sigma^2}{\sigma^2 + 1} \left[\mu + \frac{\hat{s}}{\sigma^2} \right] \quad \text{and} \quad \hat{\sigma}^2 = \frac{\sigma^2}{\sigma^2 + 1}$$

The induced distribution of signals, P , is then given by: $s \sim N(\mu, \frac{1}{\sigma^2 + 1})$.

This defines the environment by a triple:

- μ , the ex-ante expected reward of allocating to an agent,
- $\alpha := 1/\sigma^2$, the precision of the agent's signal of the reward, and
- c , the inspection cost to the principal.

Pooling equilibria



Comparative statics

