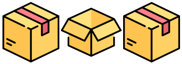
BDP 509: Applied Game Theory



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Lecture Nine: Games with uncertainty

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Today's Tasks

1. Illustrative example
2. Bayesian Nash equilibrium
3. Extensive games with imperfect information
4. What's left?

Returning to our neighbours

Suppose Alice values the streetlight at \$60 but is unsure what Bob's value is. In particular, she believes Bob has either a low valuation at \$40 or a high valuation at \$60. She then considers the following two games:

Bob low

Alice		Buy	Don't Buy
	Buy	$10, -10$	$10, 40$
	Don't Buy	$60, -10$	$0, 0$

Bob high

Alice		Buy	Don't Buy
	Buy	$10, 10$	$10, 60$
	Don't Buy	$60, 10$	$0, 0$

If Bob doesn't buy when he's low but buys when he's high, Alice would like to buy when Bob is low and not to buy when Bob is high. This however would require Alice to chose her action contingent on Bob's valuation, but she can't do that as she is uncertain of Bob's type!

Bayesian Nash equilibrium

A Bayes Nash equilibrium is a strategy profile and a set of beliefs such that:

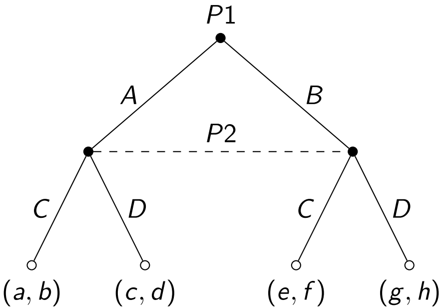
1. each players strategies are optimal given their beliefs and the other players strategies, and
2. each players beliefs are consistent with the equilibrium strategy profile and their prior beliefs.

Beliefs are probability judgements the players make about the types and strategies of their opponents. We require these beliefs are **properly** updated given the the equilibrium strategy profile and the role of Nature. Importantly, this requires our players to use Bayes rule where possible:

$$Pr(hypothesis|evidence) = \frac{Pr(evidence|hypothesis) \cdot Pr(hypothesis)}{Pr(evidence)}$$

Extensive games with imperfect information

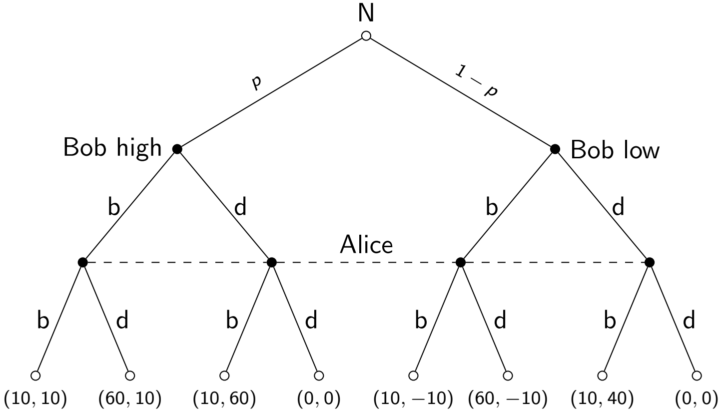
In extensive form games, we can represent uncertainty with information sets.



Notice here, the information set represents player 2's uncertainty about which of player 1's actions was played. As such, this game tree represents a usual 2×2 simultanious strategic game!

Including Nature

We can also use information sets to represent uncertainty about which type of player you are facing. We do this by introducing an initial player called Nature who selects this with some probability distribution:



What's left?

1. Incomplete and imperfect information; see *perfect Bayesian equilibrium* and *epistemics*
2. Collective actions; see *cooperative game theory* and *coalitional games*
3. Designing games and aligning incentives; see *mechanism design* and *market design*
4. Learning and gathering information; see *information acquisition*, *learning* and *search*
5. Testing and relaxing our rationality assumptions; see *behavioural economics* e.g. *ambiguity aversion*
6. Complexity in games; see *computer science* and *machine learning*