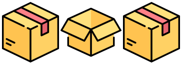
BDP 509: Applied Game Theory



Instructor: Nawaaz Khalfan

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Lecture One: What is Game Theory?

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Today's Tasks

1. Introduction, canvas and syllabus
2. What is Game theory?
3. Payoff representation
4. Normal form games
    4.1 Prisoners dilemma
    4.2 Stag hunt
    4.3 Matching pennies
5. What's missing?

What is Game Theory?

Game theory is a set of tool for studying strategic behavior between agents who mutually recognize the interdependence of their actions.

By strategic we mean that the consequences for each agent depends not just on the actions they chose but what others chose as well

and by agents we may be referring to individual decision makers (e.g. voters in an election) or whole groups of decision makers (e.g. company's choosing a price for their goods).

Game theory is just a set of tools, helpful in (scientifically) studying strategic scenarios. It is not a separate philosophy or ideology. Think of it like calculus, statistics, a recipe book . . .

Aside: what is Economics?

Economics is the study of choices. As such economics uses game theory to expand their understanding of complex decision making. This is more and more true of all the social sciences (as it should be!).

Just like game theory, economics is just a set of tools, helpful in studying choices. It is not a separate philosophy or ideology.

Payoff representation

In order to have a consistent theory of decision making, it is important to make several important assumptions on the behavior of our agents. For now, we will assume that the agents preferences are rational, meaning their preferences over consequences are:

- Complete: The agent can rank any two consequences. That is, for the agent either: consequence **A** is preferred to **B**, **B** is preferred to **A** or they are indifferent between **A** and **B**.
- Transitive: The agent's rankings are consistent. That is if consequence **A** is preferred to **B**, and **B** is preferred to **C**, it must be true that consequence **A** is preferred to **C**.

Payoffs are numbers we assign to consequences that represent the preferences of our agents. This allows us to rank alternatives in the eyes of our players and subsequently analyse games they are involved in.

Theorem: any agent who has complete and transitive preferences can have those preferences represented by a payoff function, U.

Normal form games

A normal form game consists of three things:

1. Players: A list of people which agents are playing the game.
2. Strategies: A complete list of possible actions each player can choose.
3. Payoffs: A reward given to each player as a function of what strategy all players have chosen.

As there are only a few things we need to completely describe a game, we can represent a (two or three player) game with a payoff matrix:

		Column	
		left	*right*
Row	*up*	$U_R(u,l), U_C(u,l)$	$U_R(u,r), U_C(u,r)$
	down	$U_R(d,l), U_C(d,l)$	$U_R(d,r), U_C(d,r)$

Prisoners dilemma

Description: two members of a criminal gang, Alice and Bob, are arrested and imprisoned. Each prisoner is in solitary confinement with no means of speaking to or exchanging messages with the other. The police admit they don't have enough evidence to convict the pair on the principal charge. They plan to sentence both to a year in prison on a lesser charge. Simultaneously, the police offer each prisoner a bargain: confess to the crime and we'll let you go free and sentence the other criminal three years. If both of you confess however we'll sentence you both to two years.

Representation:

		Bob	
		deny	*confess*
Alice	*deny*	$-1, -1$	$-3, 0$
	confess	$0, -3$	$-2, -2$

Stag hunt

Description: Two hunters are combing the land for their next meal. If they chose to cooperate and hunt a large stag they can increase their likelihood of catching the animal and can expect to get a large amount to keep for themselves: from an expected single meal to four meals each. Hunting hares on the other hand is a solo and competitive task; if only one hunter searches for hares they are very likely to catch some and earn three meals, but if both hunt hares it becomes more difficult and they can only expect two meals.

Representation:

		Hunter 2	
		stag	*hare*
Hunter 1	*stag*	$4, 4$	$1, 3$
	hare	$3, 1$	$2, 2$

Matching pennies

Description: Australia is through to the FIFA World Cup final against the USA and at the end of the match the scores are drawn. All penalty kicks have been missed by both teams and the final penalty kick for Australia is about to take place.

Representation:

		US Goalie	
		left	*right*
Aus. Striker	*left*	$-1, 1$	$1, -1$
	right	$1, -1$	$-1, 1$

What's missing?

Perhaps we're interested in a strategic scenario (a game) that has some more nuanced features like:

- *time*: there may be an order in which the players make an observable action
- *uncertainty*: payoffs from playing a particular action may be unknown to the agent, or the agent may not be fully aware who they are playing against
- *communication*: there may be some scope for our players to talk to one another or deliver a signal
- *complexity*: some action sets and payoff correspondences may be too large for the player to completely consider

More on this going forward ...