BDP 509: Applied Game Theory

Assignment One

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Summer Session II, 2022

Abstract

In class, we have explored core definitions and results in the following topics. In this assignment we will extend the lessons we have learned to more complicated games. Feel free to discuss the questions on Canvas, but do not post your answers - try to ensure the discussion is about the theory behind the query. In each question there is an extension part, which are optional but the ultimate point of the exercise. They will not count towards your grade. Submit your answers as a PDF on Canvas by **Wednesday 27th of July by 12pm EST**. It's suggested that this is a hand written assignment and scanned into the computer, but you may do them on the computer if that's convenient.

1 Pure strategy Nash Equilibrium

Firms are often said to be "profit driven". What this means in our context is that their payoff from a particular outcome is their total revenue they receive from selling their product minus the cost to produce the product.

In a town called Alice Springs there are two profit driven pubs, The Alice Hotel (**A**) and Billabong Brewery (**B**). On any given day there are 100 people looking to buy a beer, each of whom will only buy if the price is equal or below \$10. Further, they are indifferent between the pubs and if they do chose to buy a beer they will do so from the pub with the lowest price. If both pubs set the same price, they will split up equally among the pubs. It costs both pubs \$5 to produce each beer. We are going to investigate what the equilibrium price should be in this competitive environment.

First suppose Alice Springs has some unusual liquor laws that specify the pubs can only choose a price that is a multiple of 5 (e.g. \$0, \$5, \$10, etc.).

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a) Represent this situation as a strategic form game with profits as the payoffs and prices as the action. (Hint: you don't have to list every possible price as there are infinitely many prices to choose from. It should be clear which prices the pubs are considering and include one more to be safe.)

b) Find and describe the best responses for both pubs.

c) What are the pure strategy Nash equilibrium of this game and which one is Pareto efficient (with respect to our players)?

Now suppose Alice Springs allows the publicans to advertise and grant a $1 discount to their patrons.

d) Which Nash equilibrium would not survive under the new policy and why?

Now suppose Alice Springs abolishes all price regulations and lets the pubs set whatever price they wish.

e)[Extension] Without drawing a payoff matrix, what will the Nash equilibrium be and why?

2 Mixed strategy Nash equilibrium

There are two independent boats in the small coastal town of Ku-ring-gai. They are profit driven and there are two economic activities that they find most productive: fishing and touring.

Fishing is competitive in two ways: 1) when there are multiple boats fishing at the same time, it is more difficult to catch fish, and 2) when there are multiple merchants selling at the same time, it is more difficult to make a profit on the catch (see Question 1). However, due to a recent tourist boom in Ku-ring-gai, touring isn't competitive at all: no matter how many boats offer tours, they can always find a group to take out and show them their favourite spots along the coast.

To operate the boats it costs $20 per day in fuel regardless of the activity they choose and an additional $10 for the bait if they decide to fish. On the revenue side, fishing brings in $200 if they are the only boat to go fishing that day, but only $40 if both boats decide to fish. On the other hand, tourists will pay $70 to go on a day tour regardless of how many boats are offering to tour the coast.

a) Represent this situation as a strategic form game with profits as the payoffs.

b) Find all Nash equilibrium of this game.

Now suppose a new boat from neighbouring Awabakal decides to move permanently to Ku-ring-gai, so there are now three boats. For simplicity assume the payoffs remain the same. In particular, fishing brings in $200 if they are the only boat to go fishing that day, but only $40 if more than one boat decides to fish. On the other hand, tourists will always pay $70 regardless of how many boats are touring.

c) Represent this new situation as a strategic form game.

d) Show that each boat playing *fish* with probability 0.5 and *tour* with probability 0.5 is a mixed strategy Nash equilibrium of this game.

Now suppose that boats from all over the coast move a Ku-ring-gai and there are now $n+1$ total boats in Ku-ring-gai. For simplicity assume the payoffs are the same:

e) [Extension] What is the symmetric mixed strategy Nash equilibrium? That is, if every boat plays *fish* with probability p and *tour* with probability $1-p$, what is the equilibrium value of p?

3 Extensive form games[1]

There are two pirates, Old Blackbeard and Young Cook, who've stumbled upon a treasure chest containing 3 gold doubloons. According to the Code of the Pirate Brethren, whenever the pirates find treasure, the split is determined as follows: the oldest pirate suggests a split and all pirates present, including the oldest, takes a vote. If a strict majority vote in favour, the treasure is split as proposed. If not, the oldest pirate is thrown overboard and the process is repeated with the remaining pirates.

It is well known that traditional pirates love both gold and anarchy. In particular, when presented with an opportunity to increase their share of treasure a pirate will always do so. If presented with two equal shares, they choose to chose to break the tie in favour of throwing a fellow pirate overboard. Lastly, they value their own lives above all else, so getting thrown overboard is the worst outcome.

a) Represent this game for the two pirates in extensive form. You may assume that they cannot split a single doubloon and a pirate will always vote for their own proposal.

[1] This is an old riddle. It's a lot more fun and educational to work through and find the solutions yourself, so give it a go before turning to the internet.

b) What is the subgame Nash equilibrium of this game? Is there any way for Blackbeard to avoid being thrown overboard?

Suppose that Cook isn't a traditional pirate and he actually quite likes Blackbeard and doesn't wish to throw him overboard. He still however, prefers treasure above all else.

c) How does this change your answer to b).

Now suppose there was a third traditional pirate, Very Old Ahab, who was with them when they found the 3 doubloons.

d) Knowing what would happen if his proposal was rejected (that is, backwards induction), what split would Ahab propose?

Now suppose five traditional pirates (A, B, C, D and E) find a treasure chest with 100 gold doubloons. These pirates have a slightly different constitution however: if the vote is tied, the proposed split wins and is enacted.

e)[Extension] What is the equilibrium split in the subgame Nash equilibrium of this game? [Hint: use backwards induction from the the situation where D and E are the only ones who haven't been ejected. There is no need to draw the entire game tree.]

Updated: July 22, 2022