Optimal Allocation with Noisy Inspection

Nawaaz Khalfan

November 13, 2022

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A core economic activity

- employers **interview** potential employees
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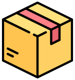

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2. verification or *screening*

A class of problems

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How should the principal design the inspection and allocation mechanism to maximize their ex ante expected return?

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Mechanism design problems with noisy information, costly inspection, and limited transfers are widespread.

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1. **Job hiring**: a firm seeks to fill an open position in their operation with a potential employee.

2. **Grant approval**: a public fund is tasked with assessing a grant application.

3. **Impact investment**: a venture capitalist sets the mechanism by which it reviews and invests in startups.

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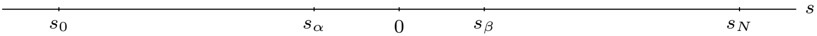

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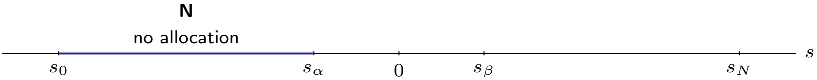

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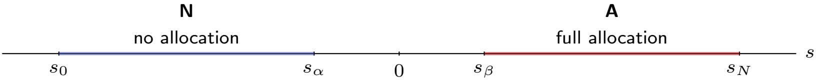

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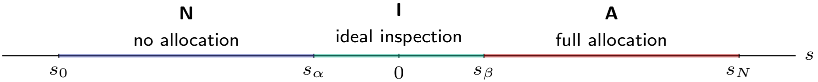

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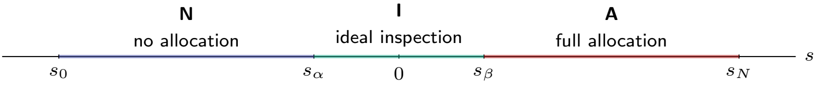

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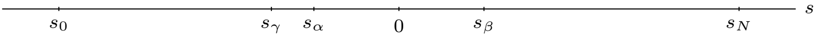

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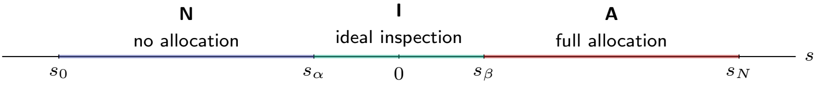

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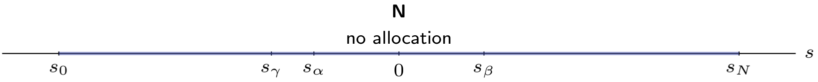

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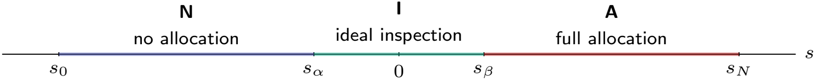

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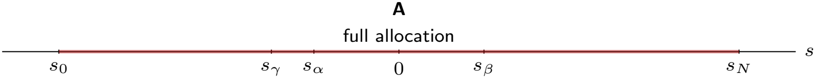

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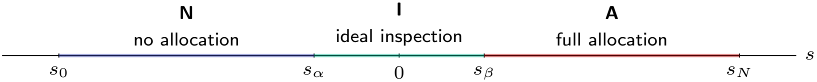

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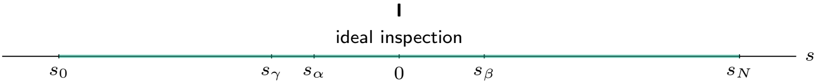

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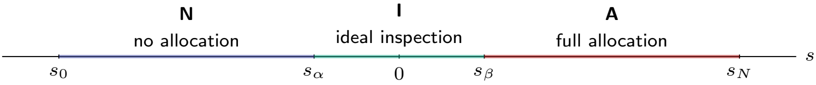

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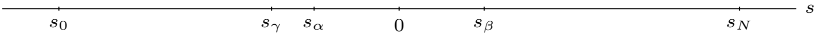

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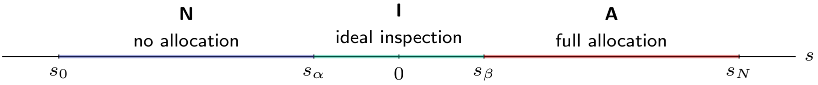

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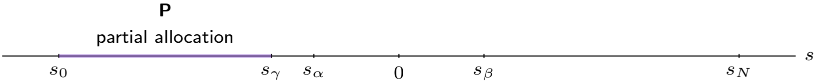

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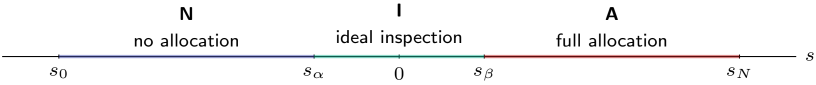

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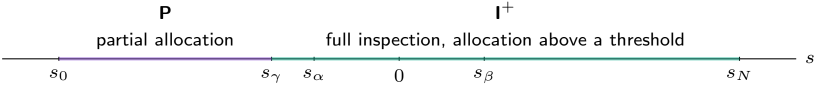

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Three types of losses from private information:

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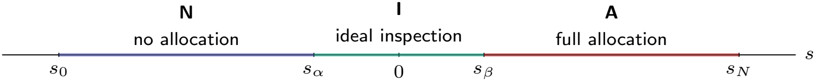

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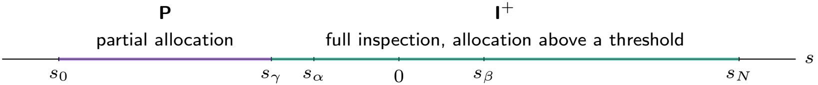

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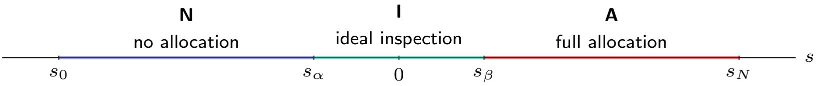

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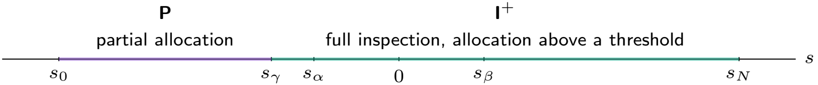

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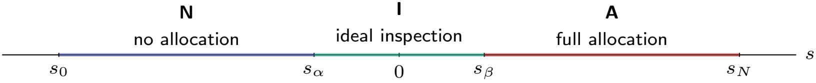

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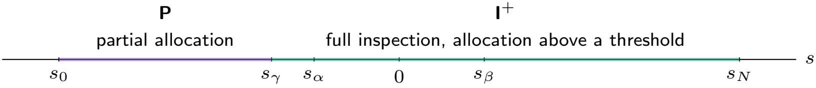

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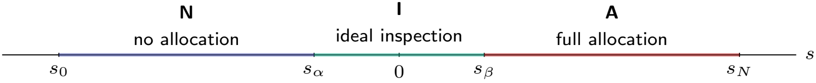

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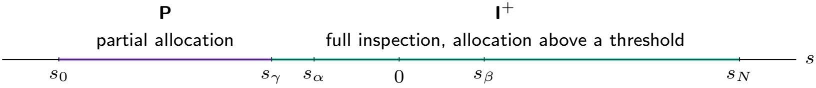

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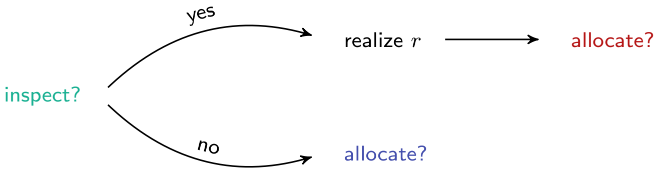

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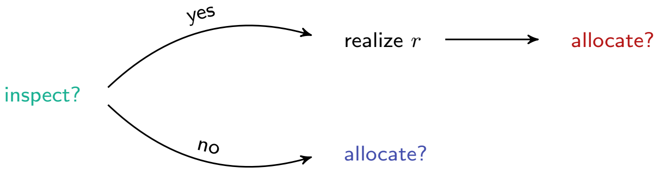


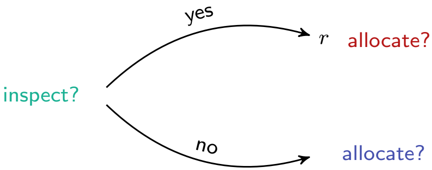
Then, a **mechanism** specifies for each type s,

- an inspection rule,
- a pre-inspection allocation, and
- a post-inspection allocation for each r.

These are potentially probabilistic choices, so are bounded between 0 and 1.

Optimal allocation

Principal's objective:



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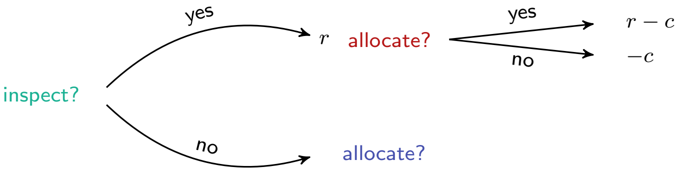

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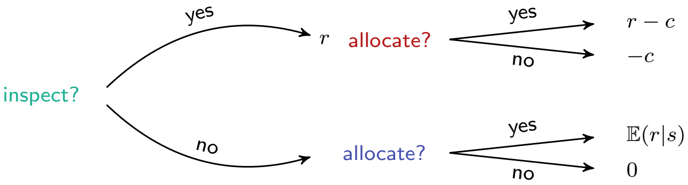

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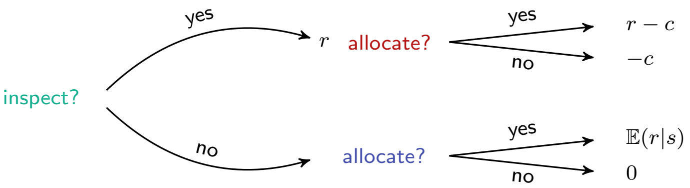

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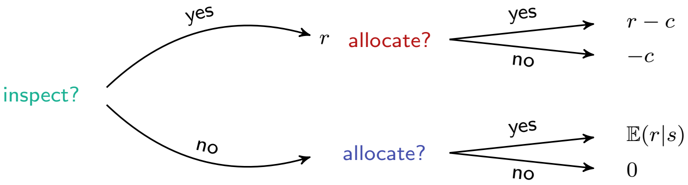

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An **optimal allocation** is a mechanism that maximizes the ex ante expected objective subject to *incentive compatibility* (IC) for each type s:

$$u(s|s) \geq u(\hat{s}|s) \quad \forall \hat{s}$$

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$\Rightarrow$ Optimal post-inspection thresholds are constant: $\tau_n = \tau \ \forall n$.

Optimal separating policy

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- the value of those high signals that we inspect with threshold τ, and
- the value of those low signals that we partially allocate to.

$$\max_{\gamma,\tau} \; v(\mathbf{I}(\tau)|s > s_\gamma) \cdot Pr(s > s_\gamma) + Pr(r > \tau|s_\gamma)\mathbb{E}(r|s \leq s_\gamma) \cdot Pr(s \leq s_\gamma)$$

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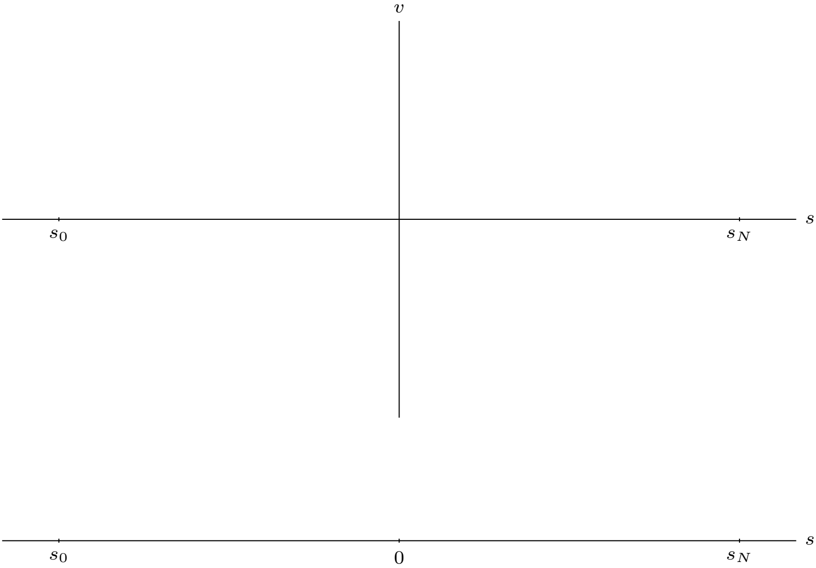
This satisfies the **global** IC constraints for all γ and τ, and thus must be a solution to the original problem.

A visual representation



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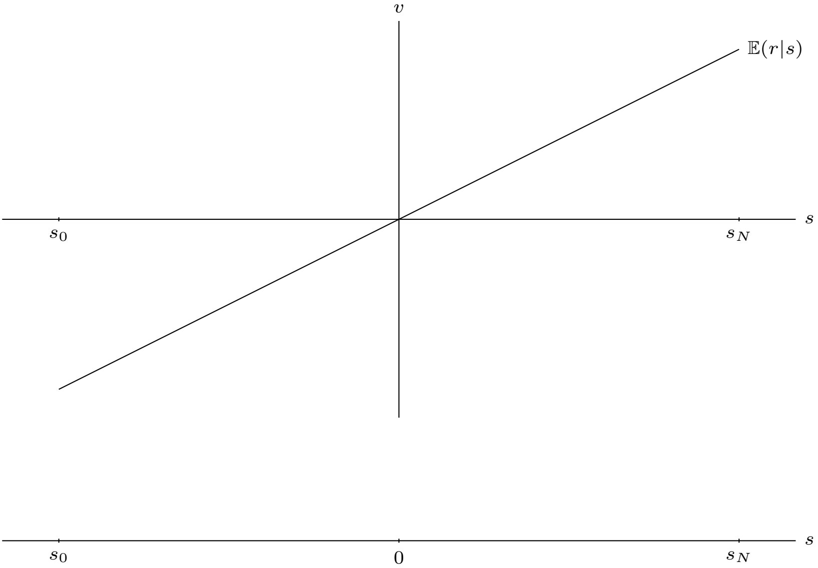

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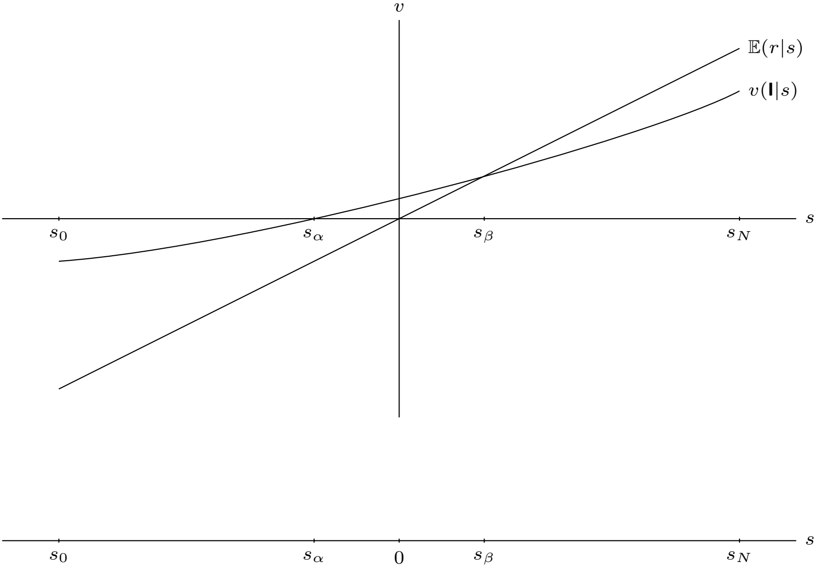

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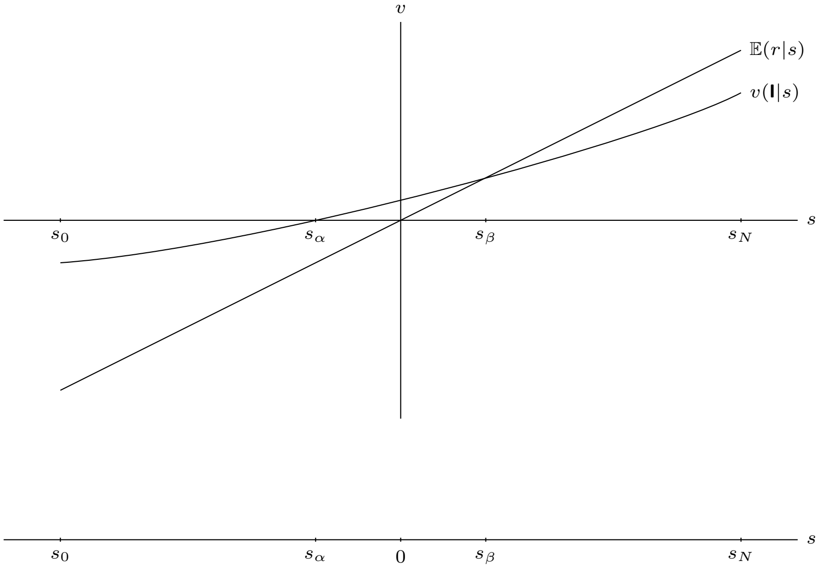

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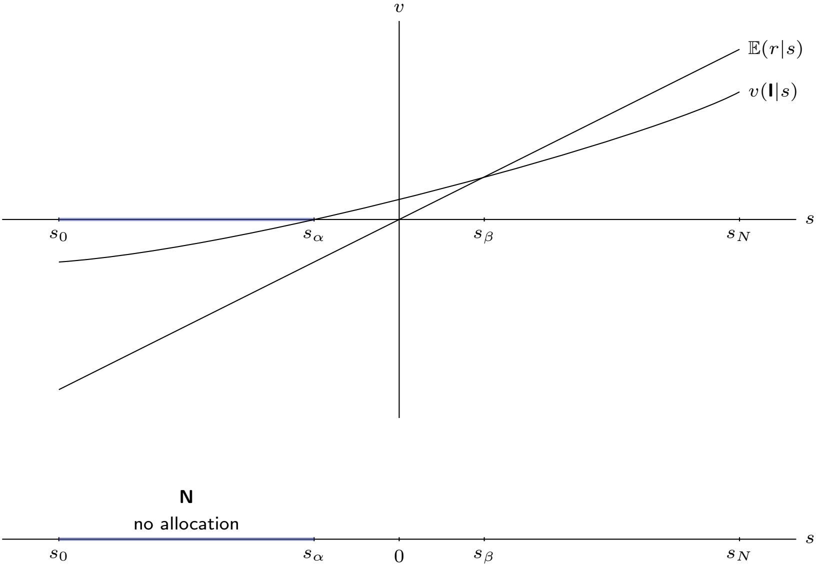

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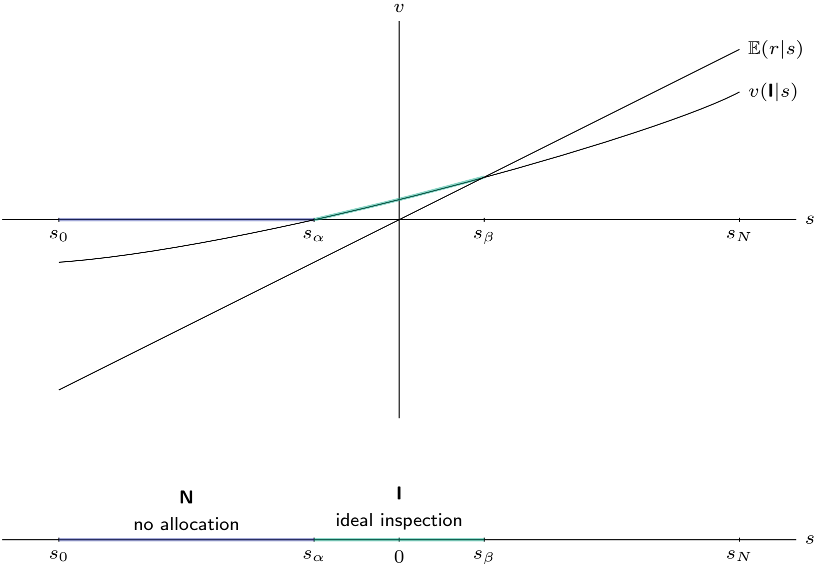

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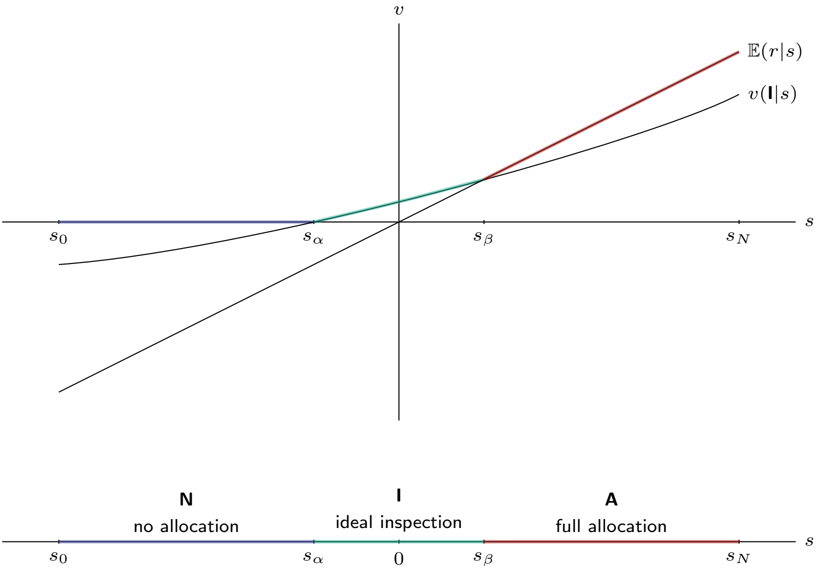

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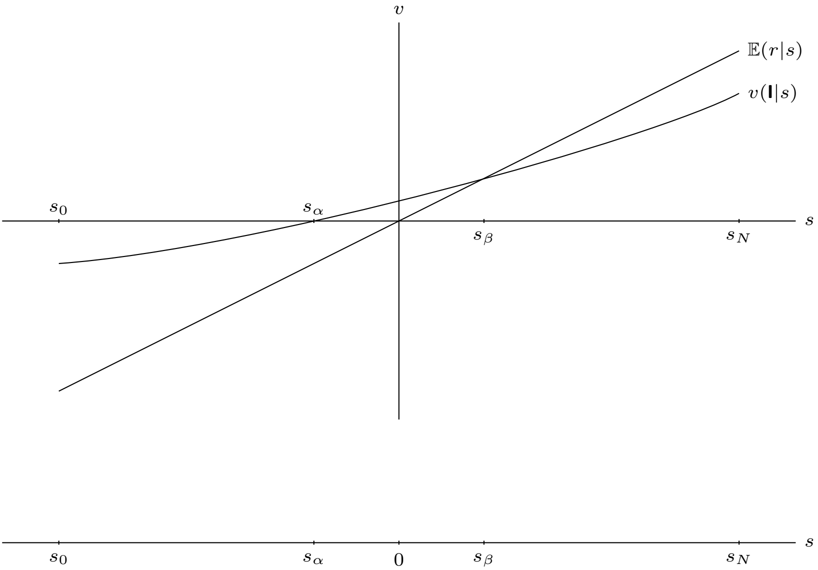

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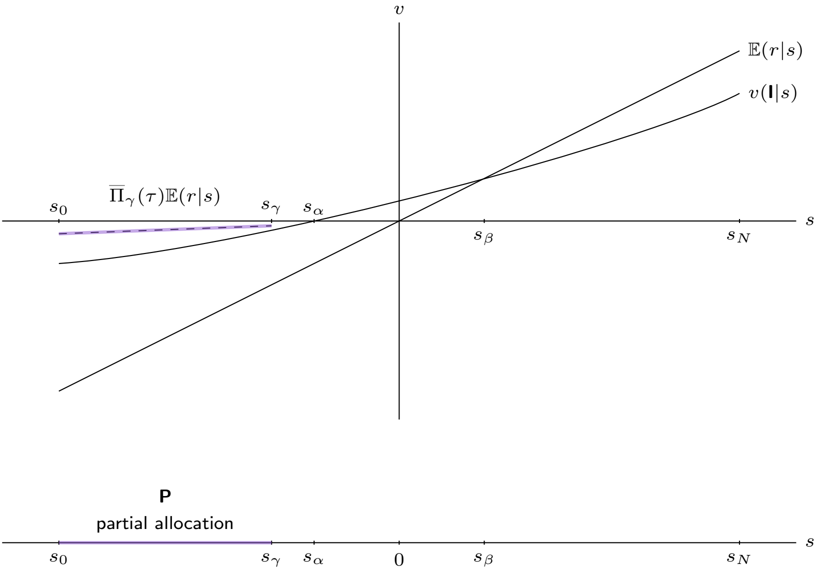

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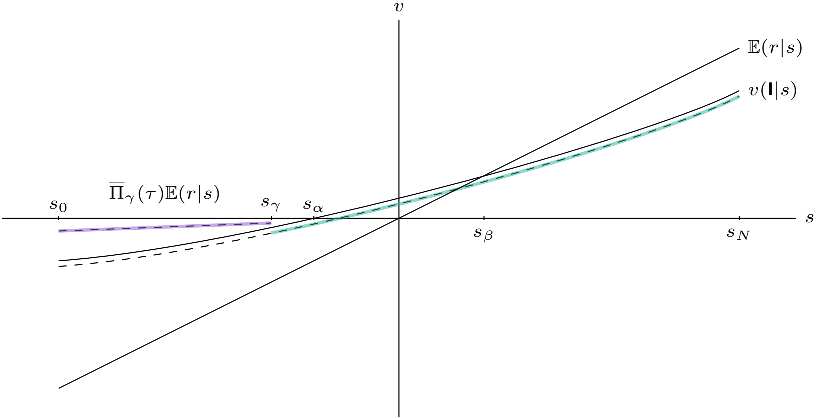

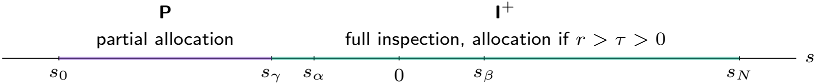

Literature

Perfect information: Green and Laffont (1986), Ben-Porath, Dekel and Lipman (2014), Mylovanov and Zapechelnyuk (2017), Epitropou and Vohra (2019).

Transfers: Townsend (1979), Border and Sobel (1987), Mookherjee and Png (1989), Alaei et al. (2020).

Limited transfers: Mylovanov and Zapechelnyuk (2017), Silva (2019*b*), Li (2021).

Efficient mechanisms: Ball and Kattwinkel (2019), Silva (2019*a*), Siegel and Strulovici (2021), Pereyra and Silva (2021), Erlanson and Kleiner (2020).

Scoring rules: McCarthy (1956), Savage (1971), Gneiting and Raftery (2007).

Noisy inspection

Optimal inspection balances *discovery* and *verification*.

When agents have **noisy private information**, the principal:

- over-inspects high and low types,
- under-allocates to agents who are inspected, and
- over-allocates to agents who are not inspected.

Weakening commitment magnifies the losses from over-allocating to agents who aren't inspected.

For separating to be optimal, signals need to be sufficiently accurate, costs sufficiently small and information sufficiently valuable.

Outstanding questions?

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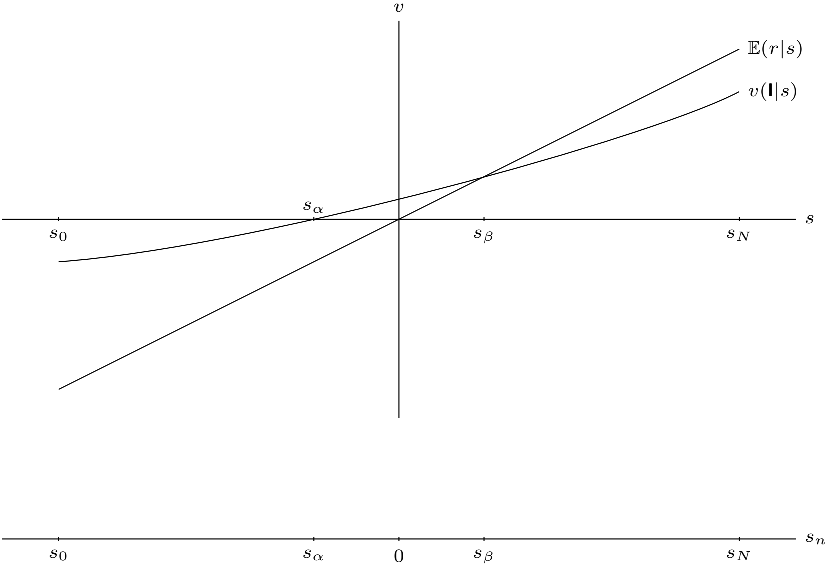
For **no commitment**, the principal can only choose between the pooling mechanisms and reports convey no information. We know what this looks like, so let's turn to the first two relaxations.

Pre-assessment commitment



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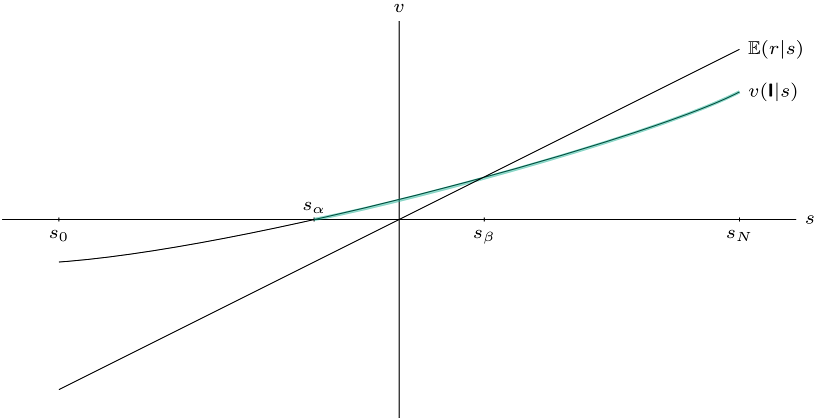

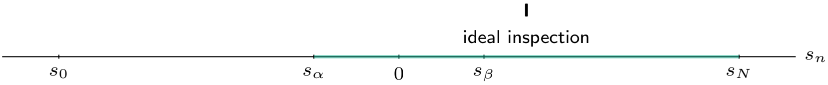

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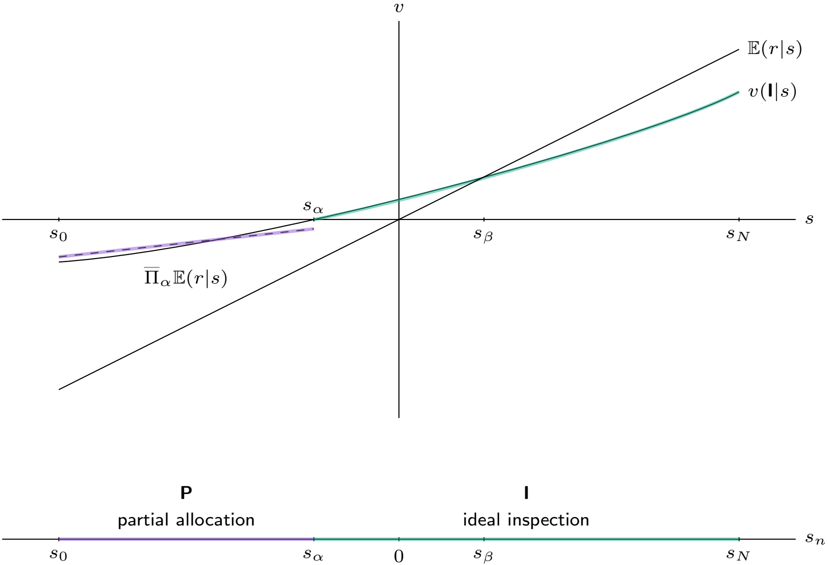

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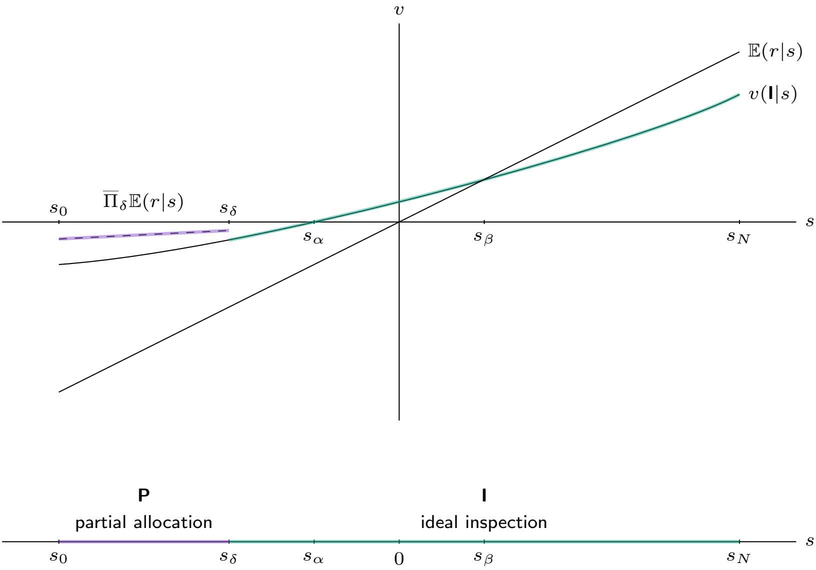
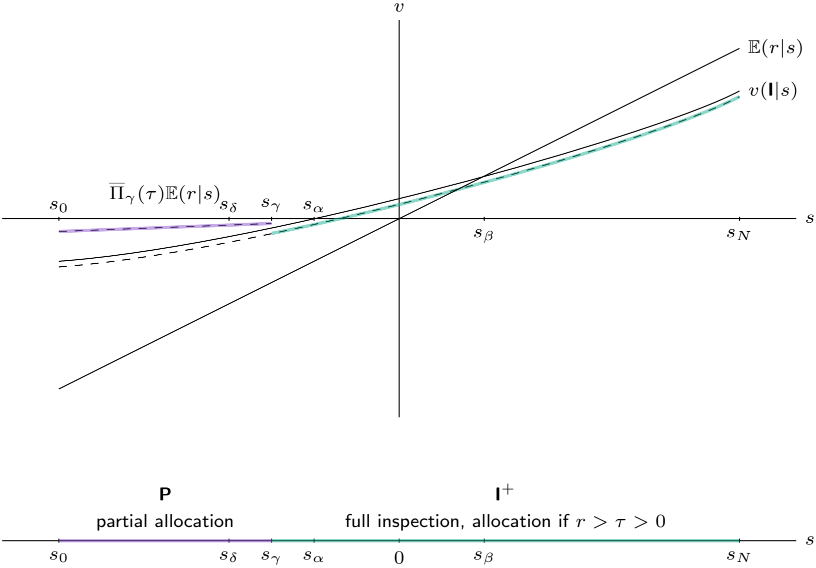


Full commitment



Gaussian environment

Suppose the prior over the rewards is given by: $r \sim N(\mu, 1)$, and the agent receives a signal of this reward, $\hat{s} = r + \varepsilon$, where $\varepsilon \sim N(0, \sigma^2)$.

Relabelling the signal by the expected reward given the signal, the posterior distribution of rewards, Π_s, is given by: $r \mid s \sim N(s, \hat{\sigma}^2)$ where:

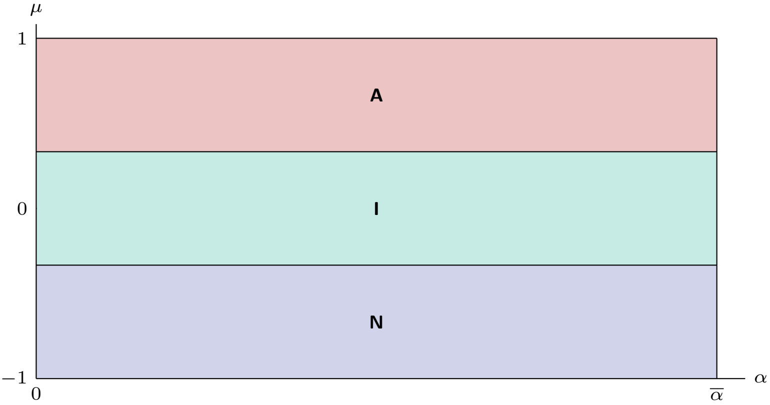
$$s = \frac{\sigma^2}{\sigma^2 + 1}\left[\mu + \frac{\hat{s}}{\sigma^2}\right] \quad \text{and} \quad \hat{\sigma}^2 = \frac{\sigma^2}{\sigma^2 + 1}$$

The induced distribution of signals, P, is then given by: $s \sim N(\mu, \frac{1}{\sigma^2+1})$.

This defines the environment by a triple:

- μ, the ex-ante expected reward of allocating to an agent,
- $\alpha := 1/\sigma^2$, the precision of the agent's signal of the reward, and
- c, the inspection cost to the principal.

Pooling equilibria



Comparative statics