BDP 509: Applied Game Theory

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Lecture Two: Equilibria and Benchmarks

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Today's Tasks

1. Quiz notification
2. Lecture one review and George's game
3. Best responses and dominant strategies
4. Nash equilibria
5. Pareto efficiency
6. Lessons from canonical games
   6.1 Prisoners dilemma
   6.2 Stag hunt
   6.3 Matching pennies
7. Extension: three player games

Review

Game theory is a set of tool for studying strategic behavior between agents who mutually recognize the interdependence of their actions.

Payoff representation: any agent who has complete and transitive preferences can have those preferences represented by a payoff function.

A normal form game is characterised by three objects: a list of **players**, their corresponding **strategies** and the resulting **payoffs**.

Payoff matrix:

		Column		
		left	*middle*	*right*
Row	*up*	$U_R(u,l), U_C(u,l)$	$U_R(u,m), U_C(u,m)$	$U_R(u,r), U_C(u,r)$
	center	$U_R(c,l), U_C(c,l)$	$U_R(c,m), U_C(c,m)$	$U_R(c,r), U_C(c,r)$
	down	$U_R(d,l), U_C(d,l)$	$U_R(d,m), U_C(d,m)$	$U_R(d,r), U_C(d,r)$

George's game

Description: George is unsure whether he has been offered a job by Mr Tuttle or not. He explains to his friend Jerry that if he shows up and he has been offered the job "I'm, fine!", but if he hasn't been offered he's confident in his ability to blend in in the company regardless. While Jerry worries that being fired after not being offered the job in the first place might be embarrassing, George doesn't seem to care; "yeah, so?". Jerry seemed to forget it was George they were talking about!

Representation:

		Mr Tuttle	
		hire	*fire*
George	*show up*	$3, 1$	$1, -1$
	stay home	$0, -1$	$0, 0$

Best responses

The strategy that gives a player the highest payoff conditional on what all other players play, is called that players best response to that particular set of strategies.

For example, George's best response to Mr Tuttle playing *hire*, is *show up*. Further his best response Mr Tuttle playing fire is also *show up*.

		Mr Tuttle: *hire*	Mr Tuttle: *fire*
George	*show up*	$3, 1$	$1, -1$
	stay home	$0, -1$	$0, 0$

What are Mr Tuttle's best responses? Note that given our payoff representation a player always has a best response and their best response need not be singular/unique.

Dominant strategies

When a player has a strategy that is **always** a best response, regardless what strategy the other players play, we call this strategy a dominant strategy.

For example, George has a dominant strategy of *show up* where Mr Tuttle does not have a dominant strategy as his best response depends on what George does.

		Mr Tuttle	
		hire	*fire*
George	*show up*	$3, 1$	$1, -1$
	stay home	$0, -1$	$0, 0$

If the payoff from playing a dominant strategy is always strictly higher than from playing any other strategy, we call this a strictly dominant strategy. Otherwise, if for some strategy of the other players the dominant strategy yields the same payoff as another strategy, we say that this strategy is only a weakly dominant strategy

Dominated strategies

When there is a strategy that always yields a lower payoff than another, we say this is a dominated strategy, and it is dominated by that other strategy.

For example, *stay home* is dominated by *show up* for George.

		Mr Tuttle *hire*	Mr Tuttle *fire*
George	*show up*	$3, 1$	$1, -1$
George	*stay home*	$0, -1$	$0, 0$

Note that just because a strategy isn't a dominant strategy, doesn't mean that it is itself dominated. For example, Mr Tuttle **neither** has a dominant or a dominated strategy.

Nash equilibria

Equilibrium in the social and physical sciences is a notion of stability. To this end, **John Nash** contributed a very attractive notion of equilibria in games in 1950:

A Nash equilibrium is a profile of strategies such that each player is playing a best response to the other strategies in the profile.

By a strategy profile we mean a strategy assignment for each player in our game.

You may also see this written as: a Nash equilibrium is a profile of mutually best responses. It represents the notion of equilibrium because given we are playing this profile, no player has a **strict** incentive to alter their strategy.

Equilibrium in George's game

		Mr Tuttle	
		hire	*fire*
George	*show up*	$3, 1$	$1, -1$
	stay home	$0, -1$	$0, 0$

If we underline the payoff of each players best response in matrix, the intersection of these strategies becomes apparent and as such, so does our Nash Equilibrium:

		Mr Tuttle	
		hire	*fire*
George	*show up*	$\underline{3}, \underline{1}$	$\underline{1}, -1$
	stay home	$0, -1$	$0, \underline{0}$

The Nash equilibrium in George's game is: (*show up*, *hire*)

Pareto efficiency

While Nash equilribria is an attractive notion of what **will** likely happen in a game, it doesn't necessarily reflect what ideally **should** happen. A simple question we may ask about a particular equilibria is whether all players would unanimously agree to shift to a different outcome:

An outcome is a said to be Pareto efficient if no player can be made better off without making another player worse off. If one outcome gives a higher payoff for all players than another, we say a change to this outcome represents a Pareto improvement.

In George's game, not only is our Nash equilibrium Pareto Efficient but there are no other Pareto Efficient outcomes.

		Mr Tuttle	
		hire	*fire*
George	*show up*	$\underline{3}, \underline{1}$	$\underline{1}, -1$
	stay home	$0, -1$	$0, \underline{0}$

Not all Nash equilibria are Pareto efficient ...

Prisoners dilemma

Description: two members of a criminal gang, Alice and Bob, are arrested and imprisoned. Each prisoner is in solitary confinement with no means of speaking to or exchanging messages with the other. The police admit they don't have enough evidence to convict the pair on the principal charge. They plan to sentence both to a year in prison on a lesser charge. Simultaneously, the police offer each prisoner a bargain: confess to the crime and we'll let you go free and sentence the other criminal three years. If both of you confess however we'll sentence you both to two years.

Representation:

		Bob: *deny*	Bob: *confess*
Alice	*deny*	$-1, -1$	$-3, 0$
	confess	$0, -3$	$-2, -2$

Lesson 1

		Bob	
		deny	*confess*
Alice	*deny*	$-1, -1$	$-3, \underline{0}$
	confess	$\underline{0}, -3$	$\underline{-2}, \underline{-2}$

There is a unique Nash equilibria: (*confess*, *confess*)

Lessons:

- This game is often used to demonstrate that despite the existence of a more favourable (Pareto efficient) outcome, individual incentives can drive players to coordinating on the worst outcome. In this sense, it is often compared to the opposite finding in economics that competition improves social welfare (**Adam Smith**).
- This is a game that has both players playing strictly dominant strategies, so despite the dire outcome, any other prediction seems implausible as there is always an incentive for either player to unilaterally deviate and be better for it.

Stag hunt

Description: Two hunters are combing the land for their next meal. If they chose to cooperate and hunt a large stag they can increase their likelihood of catching the animal and can expect to get a large amount to keep for themselves: from an expected single meal to four meals each. Hunting hares on the other hand is a solo and competitive task; if only one hunter searches for hares they are very likely to catch some and earn three meals, but if both hunt hares it becomes more difficult and they can only expect two meals.

Representation:

		Hunter 2	
		stag	*hare*
Hunter 1	*stag*	$4, 4$	$1, 3$
	hare	$3, 1$	$2, 2$

Lesson 2

		Hunter 2	
		stag	*hare*
Hunter 1	*stag*	$\underline{4}, \underline{4}$	$1, 3$
	hare	$3, 1$	$\underline{2}, \underline{2}$

There are two Nash equilibria: (*stag*, *stag*) and (*hare*, *hare*)

Lessons:

- This game is often used to demonstrate **coordination failure** in social settings. Even if players have an explicit incentive to co-ordinate, it doesn't guarantee that they will end up on the ideal (Pareto efficient) outcome.
- This game also illustrates that our notion of equilibria may not give us a unique prediction of how players behave. In experiments it's observed that most players chose to co-ordinate on hunting the stag. What's missing in our notion of equilibria?

Matching pennies

Description: Australia is through to the FIFA World Cup final against the USA and at the end of the match the scores are drawn. All penalty kicks have been missed by both teams and the final penalty kick for Australia is about to take place.

Representation:

		US Goalie	
		left	*right*
Aus. Striker	*left*	$-1, 1$	$1, -1$
	right	$1, -1$	$-1, 1$

Lesson 3

		US Goalie	
		left	*right*
Aus. Striker	*left*	$-1, \underline{1}$	$\underline{1}, -1$
	right	$\underline{1}, -1$	$-1, \underline{1}$

There are no (pure strategy) Nash equilibria!

- Here we have another issue with our notion: it doesn't provide a prediction at all! John Nash actually proved that there is always a Nash equilibrium in these games, however they may involve players playing randomised strategies. More on this next lecture ...
- This game is also an example of a **zero sum** game, that is a game who's payoffs always sum to zero. This does not directly imply that there are no pure strategy Nash equilibrium (consider a game will payoffs that are all zeros!) but does describe the most pure form of a competitive game where one players gain is another players loss. This was worked on extensively before John Nash, most notably by: **John Von Nueman** and **Oskar Morgenstern**.

3 player games

Including a third player in our simultaneous games is straightforward if we introduce another matrix for each additional strategy the third player has and listing their payoff third. Here's a randomly generated example:

Player 3 plays *top*,

		Player 2	
		left	*right*
Player 1	*up*	$10, 1, 2$	$9, 3, 10$
	down	$10, 5, 7$	$2, 5, 8$

Player 3 plays *bottom*,

		Player 2	
		left	*right*
Player 1	*up*	$9, 5, 9$	$6, 1, 3$
	down	$6, 8, 9$	$6, 4, 2$

Equilibria

Player 3 plays *top*,

		Player 2	
		left	*right*
Player 1	*up*	$\underline{10}, 1, 2$	$\underline{9}, \underline{3}, \underline{10}$
	down	$\underline{10}, \underline{5}, 7$	$2, \underline{5}, \underline{8}$

Player 3 plays *bottom*,

		Player 2	
		left	*right*
Player 1	*up*	$\underline{9}, \underline{5}, \underline{9}$	$\underline{6}, 1, 3$
	down	$6, \underline{8}, \underline{9}$	$\underline{6}, 4, 2$

There are two NE: (*up*, *left*, *bottom*) and (*up*, *right*, *top*)