Assignment 2

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1)

a. The worst kind of input would be an array that is sorted from largest value to smallest value with no repeating values. This is due to the fact that the jth value will always be bigger than the j+1th value, this will cause a swap to always occur. So this will induce a maximum number of calls, as a consequence.

b.

$T_{cond} + T_{Assign} + T_{Comp} + T_{Arith}$	for i ← 1 to n – 1 do
$T_{cond} + T_{Assign} + T_{Comp} + 2T_{Arith}$	for j ← 0 to n – 1 – i do
$T_{cond} + 2T_{Index} + T_{Comp} + T_{Arith}$	if (A[j + 1] < A[j]) then
$T_{Assign} + T_{Index}$	tmp ← A[j]
$T_{Assign} + 2T_{Index} + T_{Arith}$	A[j] ← A[j + 1]
$T_{Assign} + T_{Index} + T_{Arith}$	A[j + 1] ← tmp
$T_{Assign} + T_{Arith}$	for j ← 0 to n – 1 – i do
$T_{Assign} + T_{Arith}$	for i ← 1 to n – 1 do

$$= 2T_{Cond} + T_{Assign} + 2T_{Comp} + T_{Arith} + \sum_{i=1}^{n-1}\left[T_{Cond} + 3T_{Assign} + T_{Comp} + 4T_{Arith} + \left(2T_{Cond} + 6T_{Index} + 3T_{Arith} + 2T_{Comp} + 3T_{Assign}\right)(n-i)\right]$$
$$= 6 + \sum_{i=1}^{n-1}[9 + 16(n-i)]$$
$$= 6 + \sum_{i=1}^{n-1}[9 + 16n - 16i]$$
$$= 6 + (n-1)(9+16n)\sum_{i=1}^{n-1}[-16i]$$
$$= 6 + 9n + 16n^2 - 9 - 16n - 16\sum_{i=1}^{n-1}[i]$$
$$= 16n^2 - 7n - 3 - \frac{16(n-1)\left((n-1)+1\right)}{2}$$
$$= 16n^2 - 7n - 3 - 8n^2 + 8n$$
$$T(n) = 8n^2 + n - 3$$

c. $T(n) \in O(n^2)$

2. RunningTime

a. $O(n)$
b. $O(n^2)$
c. $O(\log_2(n))$
d. $O(1)$

3. $4^n < n!$ for $n \geq 9$

Proof:

Base Case:

$4^9 < 9!$

$262{,}144 < 362{,}880$ ■

Assume $4^k < k!, for\ k\ \geq 9]$
$4^{k+1} < (k+1)!$
$(k+1)! = (k+1)(k)(k-1)(k-2)\ldots(2)(1) = (k+1)k!$
$(k+1)k!\ > (k+1)!\,4^k > 4 \cdot 4^k\ if\ k \geq 9$
$Since\ 4 < k+1\ for\ k \geq 9$

4)

$$T(n) = 3T(n-1) + 2 = T(n-1) = 3T\big((n-1)-1\big) + 2 = 3T(n-2) + 2$$
$$T(n) = 3(3T(n-2)+2) + 2 = 9T(n-2) + 6 + 2 = 9T\,(n-2) + 8$$
$$T(n-2) = 3T(n-3) + 2$$
$$T(n) = 9(3T(n-3)+2) + 8\ = 27T(n-3) + 18 + 8\ = 27T(n-3) + 26$$
$$T(n) = 3^kT(n-k) + 3^k - 1$$
$$n - k = 1 => k = n - 1$$

5) $25n + 5 \leq 25n + 5n = 30n = 30n\ = c * n$

$25n + 5\ is\ therefore\ O(n)$

6) $(n+10)^{2.5} + n^2 + 1 \leq (n+10n)^{2.5} + n^{2.5}\ \leq (13n)^{2.5} \leq 13^{2.5}n^{2.5}$ *for n>0*

Therefore, $(n+10)^{2.5}+n^2+1$ is $O(n^{2.5})$.

7) $n^2 + 2n + 1 \leq n^2 + 3n \leq n^2 + 3n^2 \leq 4n^2\ IS\ NOT \leq n$

$$Therefore;\ (n+1)^2\ is\ not\ O(n)$$

8) Imagine if there was only one person with a blue face, knowing that there is at least one person on the island with a blue face, yet having not seen one, he immediately realizes that he must be that person and kills himself on the first night.

Now, Imagine if there were two people with blue faces, each would see one person with a blue face, therefore they would both assume that the other will suicide by the first night . When they wake up and realize that they didn't suicide, they deduce they also must have a blue face and therefore they will suicide on the 2nd day.

Now, Imagine if there were three people with blue faces, each would see two people with a blue face, therefore they would both assume a similar scenario will concur as written in the last paragraph. When they wake up on the third day and realize that they didn't suicide, they deduce that they also must have a blue face and therefore they will commit suicide on that 3nd night.

Therefore, in this scenario, an individual with a blue face will see 9 people with blue faces. He does not know if he also has a blue face, therefore he will think that either the 9 others also see 9 faces (if he has a blue face) or they see 8 (if he does not). Given their high IQ, a similar assumption is shared amongst all inhabitants. When the 9th night happens and no one kills himself, then they (all 10 people) will know that they also have a blue face and they will go and commit the suicide on the 10th night.