Mechanics

→ Lec. 1 Force Vectors

mechanics → statics: equilibrium
mechanics → dynamics: no equilibrium

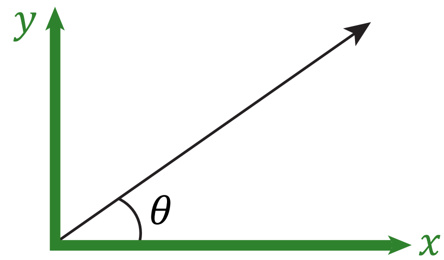
Bodies → fluid
Bodies → rigid (* our interest)
Bodies → flexible

* rigid bodies: no change in dimensions

has → mass and dimensions

particle: no dimensions (not important)

only magnitude ⟵ scalars ⟵ - - - **quantities** - - - → vectors ⟶ magnitude and direction



y and x should be positive if not given

θ : inclination angle from $+x$-axis counterclockwise

Forces ⟶ special case of vectors: which point it acts on

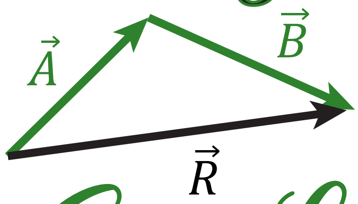
should have: magnitude - direction - line of action

Rigid bodies: F acts on ∞ points - - - - - - - - → Net Force with line of action

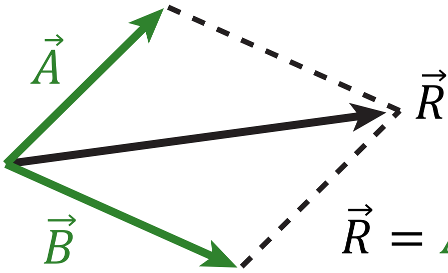
Particle: F acts on 1 point ⟶ **Vector Addition of Forces**

1 graphical

- - - → Parallelogram
- - - → Triangle



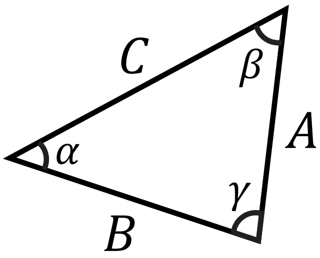
$$\vec{R} = \vec{A} + \vec{B}$$



$$\vec{R} = \vec{A} + \vec{B}$$

Sine law

$$\frac{A}{\sin\alpha} = \frac{B}{\sin\beta} = \frac{C}{\sin\gamma}$$



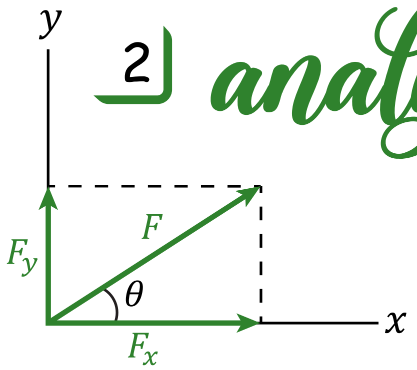
Cosine law

$$A = \sqrt{B^2 + C^2 - 2BC\cos\alpha}$$
$$B = \sqrt{A^2 + C^2 - 2AC\cos\beta}$$
$$C = \sqrt{A^2 + B^2 - 2AB\cos\gamma}$$

$\alpha > 90^\circ \rightarrow -2BC\cos\alpha \rightarrow \oplus$, $\alpha > 90^\circ \rightarrow -2BC\cos\alpha \rightarrow \ominus$

→ Lec. 2 Coplanar Forces

2 analytical



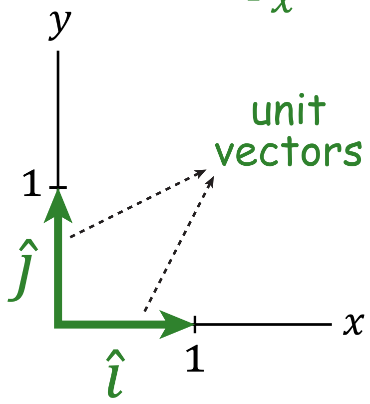
$$\begin{cases} F_x = F\cos\theta \\ F_y = F\sin\theta \end{cases} \quad F = \sqrt{F_x{}^2 + F_y{}^2} \quad \theta = \tan^{-1}\frac{F_y}{F_x}$$

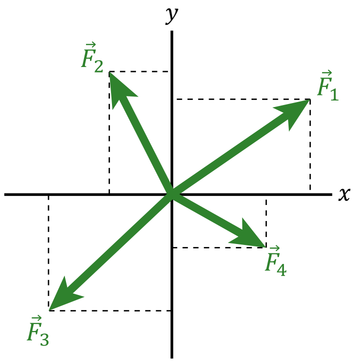
→ x and y components

$$\vec{F} = F_x\hat{\imath} + F_y\hat{\jmath}$$

$$\vec{F} = (F\cos\theta)\hat{\imath} + (F\sin\theta)\hat{\jmath}$$

→ cartesian vector representation → F_x , F_y → magnitude





$$\vec{F}_R = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \vec{F}_4 = (F_R)_x\hat{\imath} + (F_R)_y\hat{\jmath}$$

$$(F_R)_x = \Sigma F_x = F_{1x} - F_{2x} - F_{3x} + F_{4x}$$

$$(F_R)_y = \Sigma F_y = F_{1y} - F_{2y} - F_{3y} + F_{4y}$$

→ Lec. 3 Equilibrium of a particle

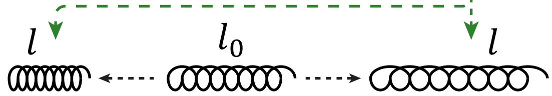
$$\Sigma F = 0 \rightarrow \Sigma F_x\hat{\imath} + \Sigma F_y\hat{\jmath} = 0 \rightarrow \begin{matrix} \Sigma F_x = 0 \\ \Sigma F_y = 0 \end{matrix}$$

supports and reactions included in a free-body diagram

① Springs

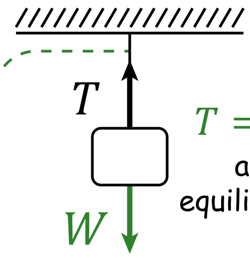
$$F = k\underbrace{s}$$

spring constant



$s = l_0 - l$ or $s = l - l_0$

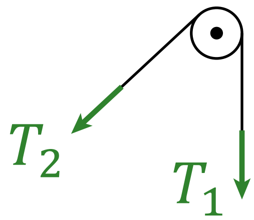
② Cables



$T = W$ at equilibrium

a cable of negligible weight that can't stretch

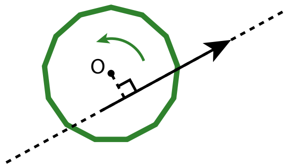
③ Pulleys



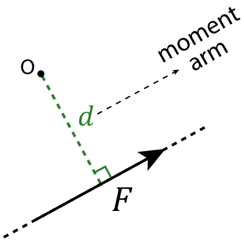
$T_1 = T_2$

force vectors have the same magnitude over a frictionless pulley

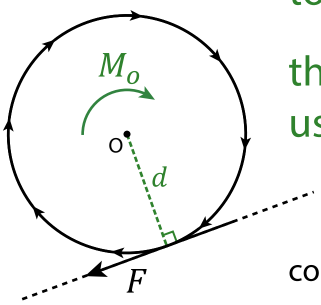
→Lec. 3 Force System Resultants



for a point on a body "point O", if a force is applied to the body with a line of action that doesn't pass the point, then it will produce $\underbrace{\text{a tendency to rotate}}$ about the point "O"



M $\boxed{\text{moment}}$ ←or— the moment of a force ←or— a torque

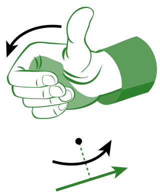


to find **magnitude** of M ⇢ $\boxed{M_o = Fd}$

- d → perpindicular distance
- → units of $(N \cdot m)$

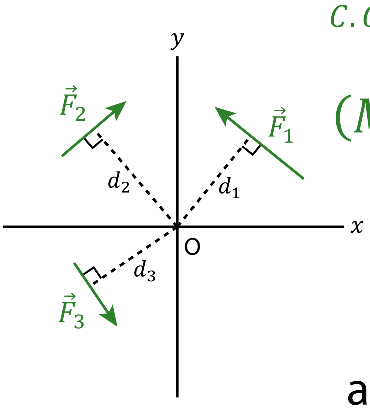
the **sense of direction** is determined using $\underbrace{\text{the right hand rule}}$

- counterclockwise $C.C.W$
- clockwise $C.W$



→ as a convention

for $C.C.W$ ⊕ positive, $C.W$ ⊖ negative



$$(M_R)_o = F_1 d_1 - F_2 d_2 + F_3 d_3$$

⇢ finding a resultant moment in 2d

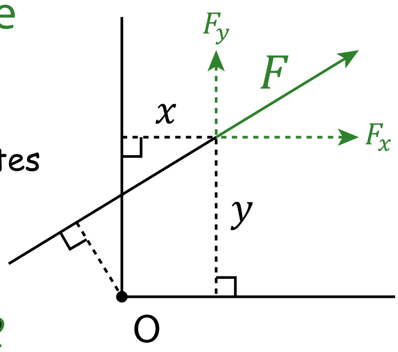
Principle of Moments

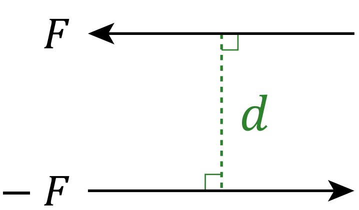
also referred to as Varingnon's theorem, it states that:

➳ the moment of a force about a point is equal to the sum of the moments of the components of the force about the point.

$$\boxed{M_o = F_y x - F_x y}$$

✻ the sign indicates the direction of the rotation



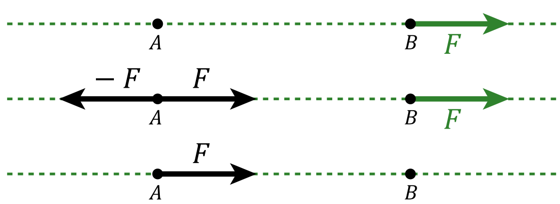


Moment of a Couple

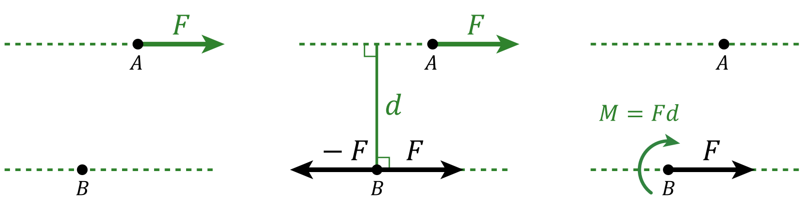
two parallel forces with the same magnitude but opposite directionse

$\boxed{M = Fd}$ ⇢ couple moment is a free vector which can act at any point

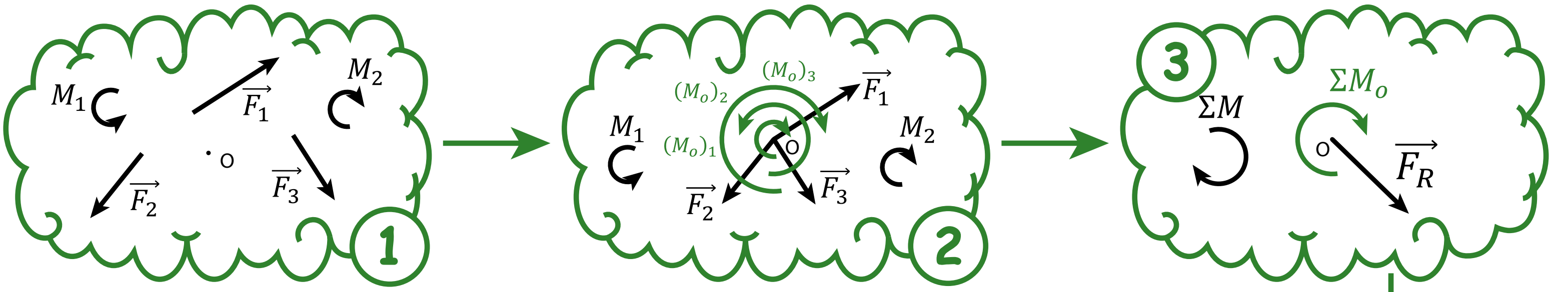
✻ force is a sliding vector which can act at any point along its line of action



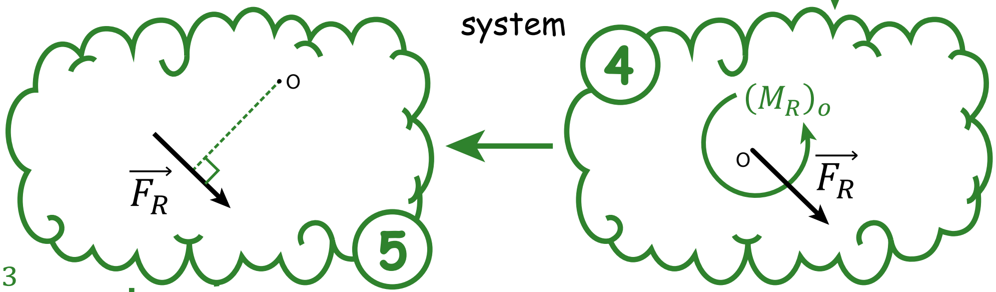
➳ how do you move a force to a point that is not on its line of action ?



Simplification of a Force and Couple System:



✻ Simplification of a: non-concurrent forces system



1. a system of couples and forces
2. move the forces to point o
3. find the sum of the forces[1], their couple moments about point o[2], and the other couple moments[3]

$$\vec{F_R} = \vec{F_4} + \vec{F_4} + \vec{F_4} + \vec{F_4}$$
$$\Sigma M_o = (M_o)_1 + (M_o)_2 - (M_o)_3$$
$$\Sigma M = M_1 - M_2$$

4. find the resultant couple moment

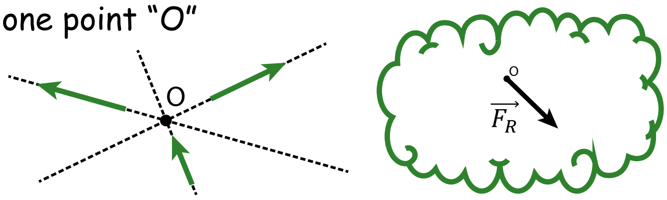
$$(M_R)_o = \Sigma M_o + \Sigma M$$

5. move the resultant force so that it produces the same resultant moment

$$\left(M_{F_R}\right)_o = (F_R)_x y + (F_R)_y x = (M_R)_o$$

if the system is a **concurrent forces system**

the lines of action of all the forces intersect at one point "O"

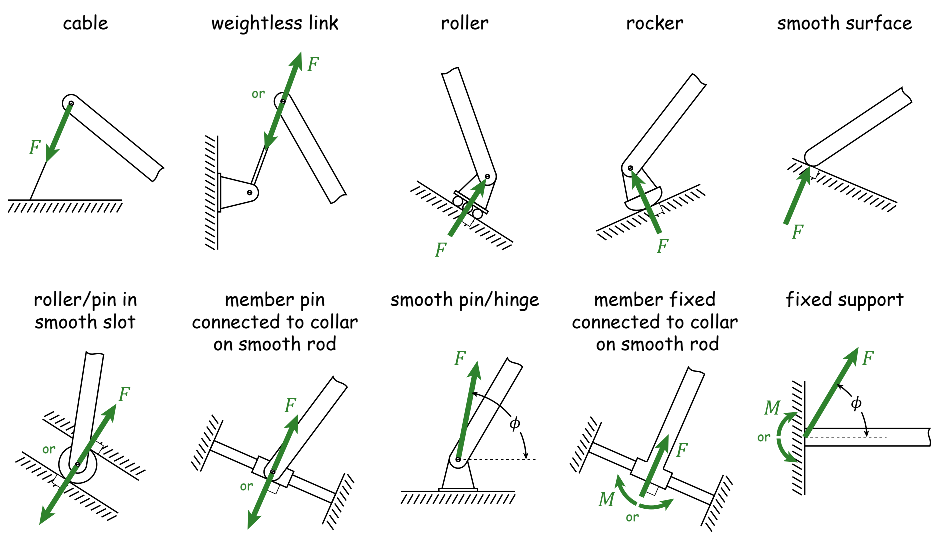


the resultant force acts on point "O" because the system produces no moment about point "O"

→ Lec. 5 Equilibrium of a Rigid Body

$$\Sigma F = 0 \quad \begin{cases} \Sigma F_x = 0 \\ \Sigma F_y = 0 \end{cases} \qquad \Sigma M_o = 0$$

types of reactions that occur at supports and points of contact between bodies that are subjected to coplanar force systems

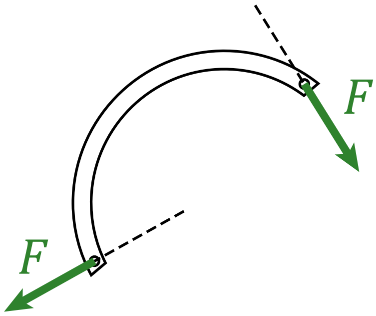


a couple moment is exerted
rotation
if a support prevents
traslation
a force is exrted in the direction of translation

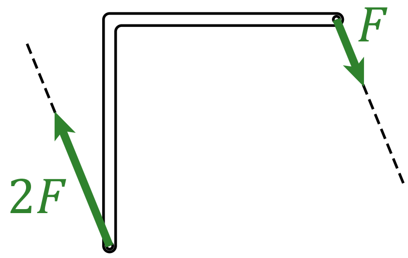
→Lec. 6 2- and 3-Force Members

1 two-force members

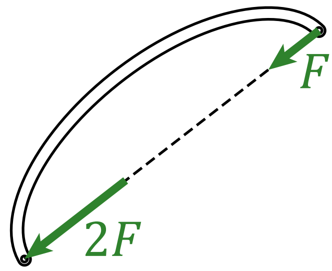
members subjected to two forces only



not equilibrium
① same magnitude

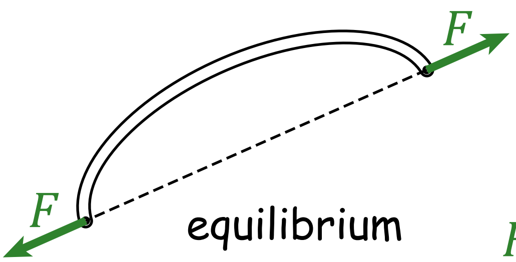


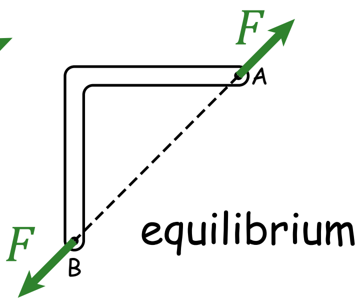
not equilibrium
② opposite directions



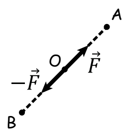
not equilibrium
③ collinear

∴ the two-force member is in equilibrium ← if 1,2,3 are satesfied





$$\Sigma\vec{F} = \vec{F} + (-\vec{F})$$

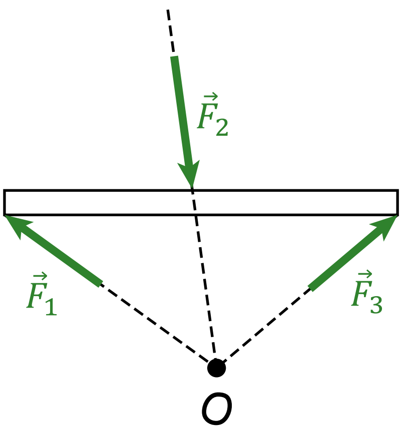


$$\Sigma\vec{F} = 0$$

$$\Sigma M_A = \Sigma M_B = \Sigma M_O = 0$$

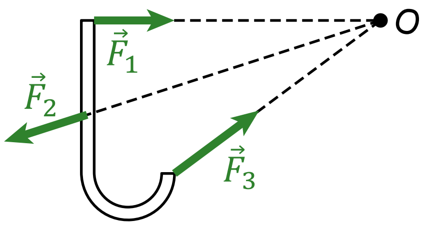
2 three-force members

members subjected to three forces only



$$\Sigma\vec{F} = 0$$
$$\Sigma M_O = 0$$

equilibrium
"concurrent forces system"

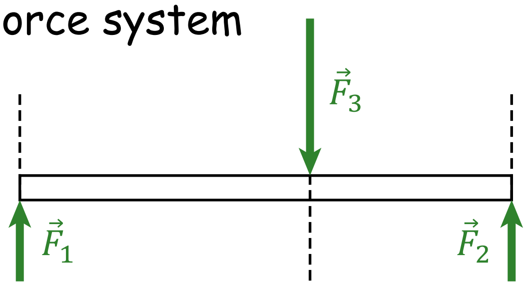


$$\Sigma\vec{F} = 0$$
$$\Sigma M_O = 0$$

equilibrium
"concurrent forces system"

intersection point O will approach infinity in a parallel force system

✳ for a three force member to be in equilibrium, the three forces have to form a concurrent or a parallel force system

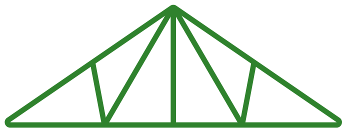


$$\Sigma\vec{F} = 0$$
$$\Sigma M_O = 0$$

equilibrium
"parallel forces system"

→ Lec. 7 8

Trusses

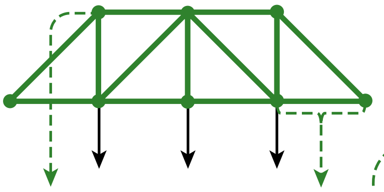


truss: a structure composed of selender members joined together at their end points.

members: wooden struts, metal bars

Assumptions for design:

1. All loadings are applied at the joints
2. The members are joined together by smooth pins



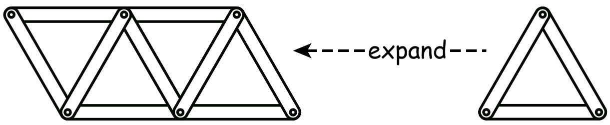
smooth pins

member (weight neglected) can be elongated or shorten → two-force member → tensile force T / compressive force C

two-force member: 2 forces directed along the axis of the member

simple truss: a truss constructed by expanding a basic triangular truss

triangular truss: three members pin connected at their ends

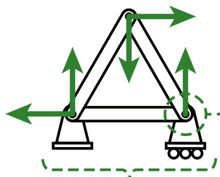


simple truss triangular truss

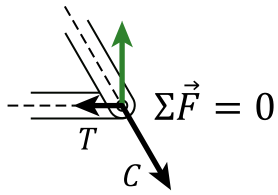
* to analyze a truss we can use two methods:

1. methods of joints
2. methods of sections

1 Method of Joints



for this joint



if the entire truss is at equilibrium:
each of its joints are also in equilibrium,
subjected to coplanar concurrent forces

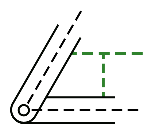
$\Sigma F_x = 0$ ←only→ $\Sigma F_y = 0$

the green arrows are external forces on the entire truss like reactions or other loads

T and C are forces from the members: not included when studying the entire truss

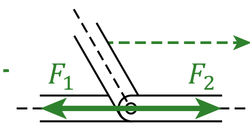
Zero-force Members: members that support no loading.

1. two non-collinear members forming a joint with no external forces/loads/support reactions applied to the joint



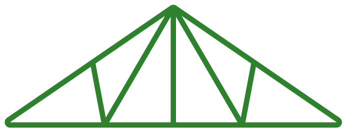
both members are zero-force members

2. three members forming a joint for which two members are collinear with no external loads/reactions applied to the joint



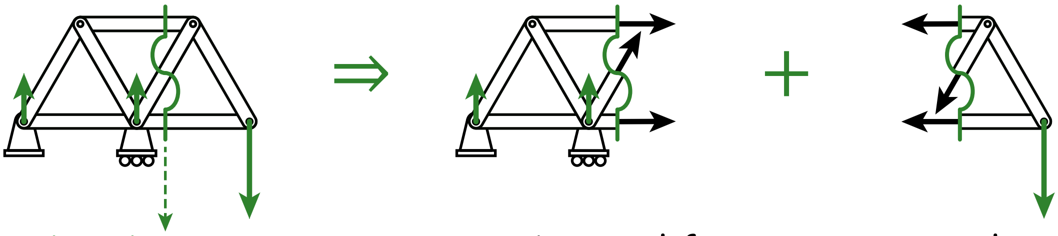
the third member is a zero-force member and $F_1 = F_2$

→ Lec. 8 Trusses II



2 Method of Sections

if the entire truss is in equilibrium then any segment of the truss is also in equilibrium:



imaginary section

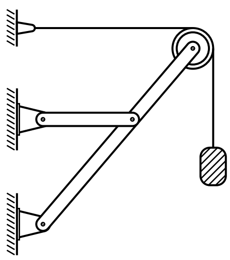
expose internal forces as external forces for each segment

* there should be only 3 unknowns

solve by

$$\Sigma F_x = \Sigma F_y = \Sigma M_O = 0$$

→ Lec. 9 Frames



the structure

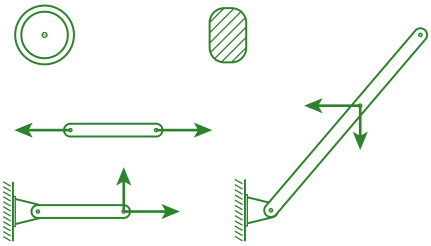
pin-connected multiforce members to support load

members subjected to more than two forces

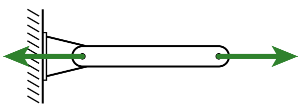
disassemble:

1. isolate each part

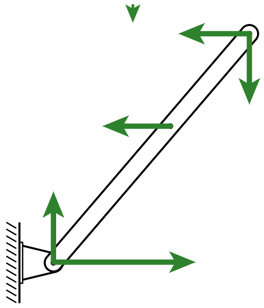
2. identify two-force members



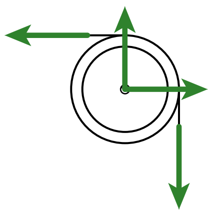
3. forces common to two contacting members act with equal magnitudes but opposite direction relative to each member



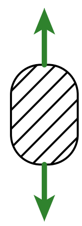
$\Sigma F_x = 0$
$\Sigma F_y = 0$
$\Sigma M_O = 0$



$\Sigma F_x = 0$
$\Sigma F_y = 0$
$\Sigma M_O = 0$



$\Sigma F_x = 0$
$\Sigma F_y = 0$
$\Sigma M_O = 0$



$\Sigma F_x = 0$
$\Sigma F_y = 0$
$\Sigma M_O = 0$

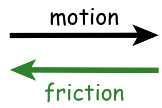
→ Lec. 10 8

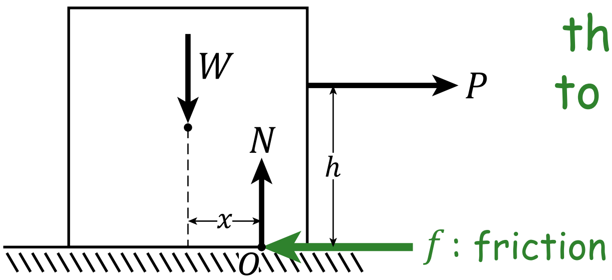
Friction Force

friction: a foce that resists the movement of two contacting surfaces that slide relative to one another.

tangent to the surface of contact

directed to oppose the possible or existing motion





the normal force N acts at a distance x to balance the tipping effect of force P

$$\Sigma M_O = 0 = Wx - Ph \quad \Rightarrow \quad x = Ph/W$$

$$\Sigma F_x = 0 = P - f \quad \Rightarrow \quad P = f$$

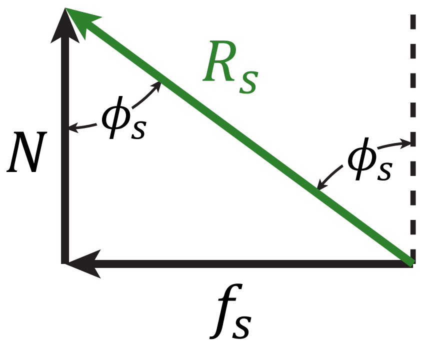
as P increases, f increases until it reaches a maximum value

$$\boxed{f_s = \mu_s N}$$

it is directly propotional to N with a constant of propotionality $= \mu_s$ → coefficient of static friction

limiting static frictional force

impending motion (the body is about to slip or is on the verge of sliding)



R_s : resultant reactive force

$$\tan\phi_s = \frac{f_s}{N} = \frac{\mu_s N}{N} = \mu_s$$

$$\tan^{-1}\tan\phi_s = \tan^{-1}\mu_s$$

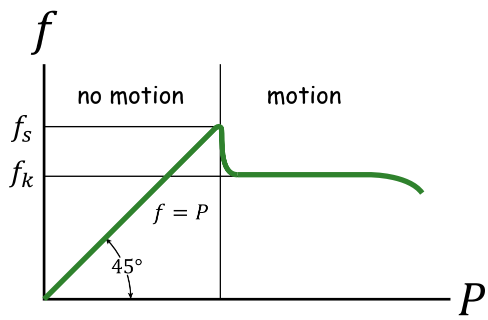
$$\boxed{\phi_s = \tan^{-1}\mu_s}$$

angle of static friction

⁎ if P increases even more, then the body starts moving and

$$f_k = \mu_k N$$

where $\underbrace{\mu_k}_{\text{coefficient of kinetic friction}} < \mu_s$



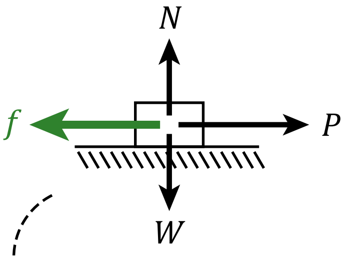
⁎ the coefficient of friction depends on the conditions of contacting surfaces

→ Lec. 11

Friction Force II

friction of a particle

no $\Sigma M_O = 0$



$$\Sigma F_y = 0 = N - W \Rightarrow N = W$$

$$\Sigma F_x = 0 = P - f \Rightarrow f = P$$

$$\underbrace{f = f_s = \mu_s N}_{\text{in case of impending motion}}$$

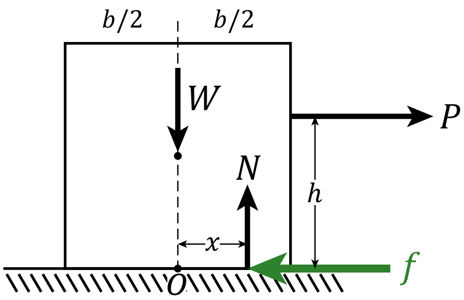
a particle can't tip because it has no dimensions

friction of a body

$$\Sigma F_x = 0 \qquad \Sigma F_y = 0 \qquad \Sigma M_O = 0$$

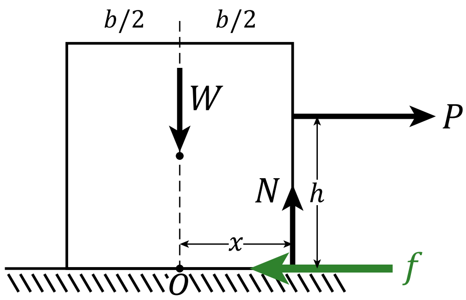
* the impending motion can be either slipping or tipping

slipping



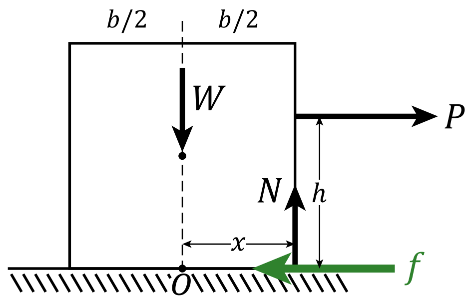
$$f = f_s = \mu_s N$$
$$x < b/2$$
$$\Sigma F_y = 0 = N - W$$
$$\Sigma F_x = 0 = P - \mu_s N$$
$$\Sigma M_O = 0 = Nx - Ph$$

tipping



$$f < \mu_s N$$
$$x = b/2$$
$$\Sigma F_y = 0 = N - W$$
$$\Sigma F_x = 0 = P - f$$
$$\Sigma M_O = 0 = Nb/2 - Ph$$

slipping + tipping



$$f = f_s = \mu_s N$$
$$x = b/2$$
$$\Sigma F_y = 0 = N - W$$
$$\Sigma F_x = 0 = P - \mu_s N$$
$$\Sigma M_O = 0 = Nb/2 - Ph$$

Note: in the case of no impending motion then the body isn't going to slip or tip and $f < \mu_s N$ and $x < b/2$