Trigonometric Equations

Find the value of x:

1) $Sinx = -0.5$ $[0 \leq x \leq 360]$

Calculator

$Sin^{-1}(-0.5) = -30$

$x = -30$ (Out of domain)

$x = -30 + 360 = 330$ (In the domain)

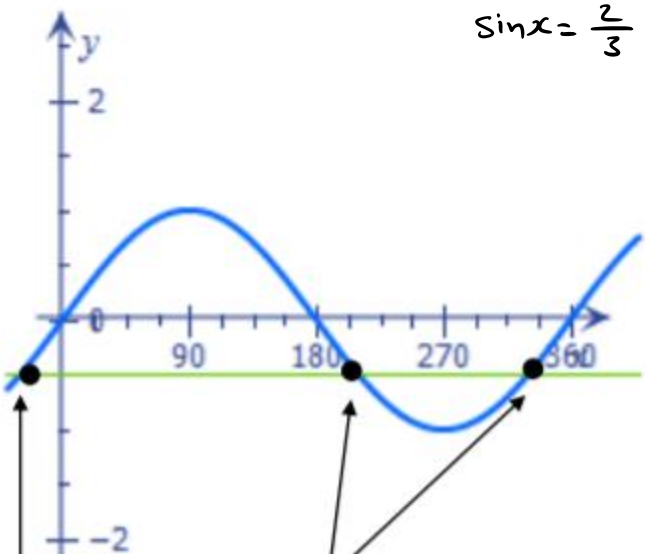
180 −

$180 - (-30) = 210$

$Cosec x = \frac{3}{2}$

$\sin x = \frac{2}{3}$

$\cos x = \frac{1}{2}$



This is a solution too at $X = -30$, but its out of the domain so we don't state it.

$Sin\ x = -0.5$ at $X = 210°, 330°$

Which are inside the domain.

2) Jun 09 #8

(a) Write down $\sin 2x$ in terms of $\sin x$ and $\cos x$. $\rightarrow$ $\sin 2x = 2\sin x \cos x$

(1)

(b) Find, for $0 < x < \pi$, all the solutions of the equation

$$\operatorname{cosec} x - 8\cos x = 0$$

giving your answers to 2 decimal places.

(5)

$$\frac{1}{\sin x} - 8\cos x = 0$$

$$\frac{1}{\sin x} = 8\cos x$$

$$1 = 8\sin x \cos x$$

$$1 = 4 \times 2\sin x\cos x$$

$$1 = 4\sin 2x$$

$$\boxed{\sin 2x = \frac{1}{4}}$$

Calc. $\qquad\qquad$ $\pi -$

$2x = 0.2526$ $\qquad\qquad$ $2x = 2.89$

$\boxed{x = 0.13}$ $\qquad\qquad$ $\boxed{x = 1.45}$

$$\frac{\sin^2\theta}{\sin^2\theta}+\frac{\cos^2\theta}{\sin^2\theta}=\frac{1}{\sin^2\theta} \quad \div \sin^2\theta$$

$$1+\cot^2\theta=\operatorname{cosec}^2\theta$$

3) Jun 06 #6

(a) Using $\sin^2\theta + \cos^2\theta \equiv 1$, show that $\operatorname{cosec}^2\theta - \cot^2\theta \equiv 1$.

(2)

(b) Hence, or otherwise, prove that

$$\operatorname{cosec}^4\theta - \cot^4\theta \equiv \operatorname{cosec}^2\theta + \cot^2\theta.$$

$$\underbrace{(\operatorname{cosec}^2\theta - \cot^2\theta)}_{1}(\operatorname{cosec}^2\theta + \cot^2\theta)$$

(2)

(c) Solve, for $90° < \theta < 180°$,

$$\operatorname{cosec}^4\theta - \cot^4\theta = 2 - \cot\theta.$$

(6)

$$\operatorname{cosec}^2\theta + \cot^2\theta = 2 - \cot\theta$$

$$\cot^2\theta + 1 = \operatorname{cosec}^2\theta$$

$$\cot^2\theta + 1 + \cot^2\theta + \cot\theta - 2 = 0$$

$$2\cot^2\theta + \cot\theta - 1 = 0$$

$$\cot\theta = \frac{1}{2} \qquad \cot\theta = -1$$

$$\boxed{\tan\theta = 2} \qquad \boxed{\tan\theta = -1}$$

$\downarrow$

AS Solving

8) Jun 08 #5

$\div \sin^2\theta$

(*a*) Given that $\sin^2\theta + \cos^2\theta \equiv 1$, show that $1 + \cot^2\theta \equiv \operatorname{cosec}^2\theta$.

(2)

(*b*) Solve, for $0 \leq \theta < 180°$, the equation

$$2\cot^2\theta - 9\operatorname{cosec}\theta = 3,$$

giving your answers to 1 decimal place.

(6)

$\cot^2\theta + 1 = \operatorname{cosec}^2\theta$

$\cot^2\theta = \operatorname{cosec}^2\theta - 1$

$2[\operatorname{cosec}^2\theta - 1] - 9\operatorname{cosec}\theta - 3 = 0$

$2\operatorname{cosec}^2\theta - 9\operatorname{cosec}\theta - 5 = 0$

$\operatorname{cosec}\theta = 5$ $\qquad$ $\operatorname{cosec}\theta = -\frac{1}{2}$

$\boxed{\sin\theta = \frac{1}{5}}$ $\qquad$ $\boxed{\sin\theta = -2}$

As Solving $\qquad$ Rej

Q.7[JUNE 2017, C3, NO.9]:

9. (a) Prove that

$$\sin 2x - \tan x \equiv \tan x \cos 2x, \qquad x \neq (2n+1)90°, \qquad n \in \mathbb{Z}$$

(4)

(b) Given that $x \neq 90°$ and $x \neq 270°$, solve, for $0 \leqslant x < 360°$,

$$\sin 2x - \tan x = 3 \tan x \sin x$$

Give your answers in degrees to one decimal place where appropriate.

(*Solutions based entirely on graphical or numerical methods are not acceptable.*)

(5)

(a) $\frac{\cos x \times}{\cos x \times} \frac{2 \sin x \cos x}{1} - \frac{\sin x}{\cos x}$

$$\frac{2 \sin x \cos^2 x - \sin x}{\cos x} = \frac{\sin x (2\cos^2 x - 1)}{\cos x}$$

$$= \tan x \cos 2x$$

(b) $\tan x \cos 2x = 3 \tan x \sin x$

$3x\cancel{^2} = 2\cancel{x}$

$3x = 2$

$x = \frac{2}{3}$

$3x^2 - 2x = 0$

$x(3x-2) = 0$

$x = 0 \qquad x = \frac{2}{3}$

$\tan x \cos 2x - 3 \tan x \sin x = 0$

$\tan x (\cos 2x - 3 \sin x) = 0$

$\boxed{\tan x = 0}$

Calc. $\rightarrow \boxed{x = 0}$

180+ $\rightarrow \boxed{x = 180}$

$\cos 2x - 3 \sin x = 0$

$1 - 2\sin^2 x - 3 \sin x = 0$

$2 \sin^2 x + 3 \sin x - 1 = 0$

$$\boxed{\sin x = \frac{-3 \pm \sqrt{17}}{4}}$$

$$\boxed{\sin x = \frac{-3 \pm \sqrt{17}}{4}}$$

$$\boxed{\sin x = \frac{-3 + \sqrt{17}}{4}}$$

calc.

$\boxed{x = 16.3}$

180 −

$\boxed{x = 163.7}$

Sin	$\Rightarrow$	180 −
Cos	$\Rightarrow$	360 −
tan	$\Rightarrow$	180 +

Q.20[JAN 2021, P3, NO.7]:

In this question you must show all stages of your working.

Solutions relying entirely on calculator technology are not acceptable.

7. (a) Prove that

$$\frac{\sin x}{\sin x} \times \frac{\sin 2x}{\cos x} + \frac{\cos 2x}{\sin x}\frac{\cos x}{\cos x} \equiv \operatorname{cosec} x \qquad x \neq \frac{n\pi}{2} \quad n \in \mathbb{Z}$$

(3)

(b) Hence solve, for $-\frac{\pi}{2} < \theta < \frac{\pi}{2}$

$$7 + \frac{\sin 4\theta}{\cos 2\theta} + \frac{\cos 4\theta}{\sin 2\theta} = 3\cot^2 2\theta$$

giving your answers in radians to 3 significant figures where appropriate.

(6)

$$\frac{\sin x \sin 2x + \cos 2x \cos x}{\sin x \cos x}$$

$$\frac{\cos 2x \cos x + \sin 2x \sin x}{\sin x \cos x}$$

$$\frac{\cos(2x - x)}{\sin x \cos x} = \frac{\cancel{\cos x}}{\sin x \cancel{\cos x}} = \frac{1}{\sin x} = \operatorname{cosec} x$$

Q.24[JAN 2021, P3, NO.9]:

9. **In this question you must show detailed reasoning.**

Solutions relying entirely on calculator technology are not acceptable.

(i) Solve, for $0 < x \leqslant \pi$, the equation

$$2\sec^2 x - 3\tan x = 2$$

giving the answers, as appropriate, to 3 significant figures.

(4)

(ii) Prove that

$$\frac{\sin 3\theta}{\sin\theta} - \frac{\cos 3\theta}{\cos\theta} \equiv 2$$

(4)

(i) $2\left[\tan^2 x + 1\right] - 3\tan x - 2 = 0$

$2\tan^2 x - 3\tan x = 0$

$\tan x(2\tan x - 3) = 0$

$\boxed{\tan x = 0}$ $\boxed{\tan x = \frac{3}{2}}$

As Solving