

COSC6364 Adv. Numerical Analysis

Wu, Panrı

What is this course about

Tentative To

Numerical Linear

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Convex Optimizat

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/ IIBCDIU

Optimization

C. b. L.

Can't I just use a library?

COSC6364 Adv. Numerical Analysis

P. Wu¹

¹Computer Science University of Houston

Spring 2020



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Basically two major parts:

Numerical linear algebra (matrix computations)



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Why study a these?

- Basically two major parts:
 - Numerical linear algebra (matrix computations)
 - (Convex) optimization



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Why study a

Can't I just use a

- Basically two major parts:
 - Numerical linear algebra (matrix computations)
 - (Convex) optimization
- The topics are primarily geared towards (large scale) statistical learning (learning from data) and to scientific/engineering computing (technical computing).



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Can't I just use a

■ Matrix multiplication, norms, orthogonality



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Why study a these?

- Matrix multiplication, norms, orthogonality
- Singular value decomposition (SVD)



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Why study a these?

- Matrix multiplication, norms, orthogonality
- Singular value decomposition (SVD)
- QR factorization and least square problems



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Can't I just use a

- Matrix multiplication, norms, orthogonality
- Singular value decomposition (SVD)
- QR factorization and least square problems
- LU/Cholesky factorization for linear systems



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- QR algorithm and Eigenvalue Decomposition (EVD)



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- QR algorithm and Eigenvalue Decomposition (EVD)
- Iterative methods for solving linear equation and eigen/singular decomposition



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- QR factorization and least square problems
- LU/Cholesky factorization for linear systems
- QR algorithm and Eigenvalue Decomposition (EVD)
- Iterative methods for solving linear equation and eigen/singular decomposition
- Fast implementation, parallelization, randomization, etc...



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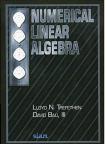
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Why study

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 (Highly recommend; main reference)
 Numerical Linear Algebra
 (Lloyd N. Trefethen, David Bau)





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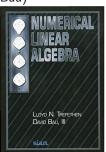
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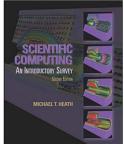
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Why study a these?

- Convexity
- Gradient/subgradient descent, proximal GD, stochastic GD



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Can't I just use a

- Convexity
- Gradient/subgradient descent, proximal GD, stochastic GD
- Duality, KKT conditions



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Why study a

- Convexity
- Gradient/subgradient descent, proximal GD, stochastic GD
- Duality, KKT conditions
- (Quasi) Newton methods, Interior point methods



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Why study al these?

- Convexity
- Gradient/subgradient descent, proximal GD, stochastic GD
- Duality, KKT conditions
- (Quasi) Newton methods, Interior point methods
- Coordinate descent, dual ascent, Alternating direction method of multipliers (ADMM), etc...



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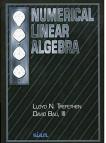
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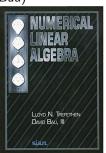




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Why study

Can't I just use a library? Upon successful completion of this course, the students should be able to

Understand basic matrix computation concepts and algorithms;



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Can't I just use a

- Understand basic matrix computation concepts and algorithms;
- Know how to compute matrix factorization/inversion, linear systems, least square problems, eigen/singular value decomposition, ...



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Can't I just use

- Understand basic matrix computation concepts and algorithms;
- Know how to compute matrix factorization/inversion, linear systems, least square problems, eigen/singular value decomposition, ...
- Analyze the computational cost of numerical algorithms



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Why study a these?

Can't I just use library?

- Understand basic matrix computation concepts and algorithms;
- Know how to compute matrix factorization/inversion, linear systems, least square problems, eigen/singular value decomposition, ...
- Analyze the computational cost of numerical algorithms
- Apply different algorithms for different situations; able to diagnose numerical stability and errors



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Upon successful completion of this course, the students should be able to

Recognize convex optimizations



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Can't I just use a

- Recognize convex optimizations
- Be able to formulate and transform convex optimization problem



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Can't I just use a library?

- Recognize convex optimizations
- Be able to formulate and transform convex optimization problem
- Understand how different optimization algorithms work; under what conditions do they work; the convergence rate; the cost of each iteration.



Objectives for Optimization

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Why study all these?

Upon successful completion of this course, the students should be able to

- Recognize convex optimizations
- Be able to formulate and transform convex optimization problem
- Understand how different optimization algorithms work; under what conditions do they work; the convergence rate; the cost of each iteration.
- Use KKT conditions to solve/characterize optimization problem



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- Recognize convex optimizations
- Be able to formulate and transform convex optimization problem
- Understand how different optimization algorithms work; under what conditions do they work; the convergence rate; the cost of each iteration.
- Use KKT conditions to solve/characterize optimization problem
- Understand the role of optimization in statistical learning



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Why study all these?

■ Data science is largely about manipulating matrices/vectors.



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Why study all these?

Can't I just use a library?

- Data science is largely about manipulating matrices/vectors.
- There are four aspects that are important for manipulating large matrices (which are not covered in typical college linear algebra courses):



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Why study all these?

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- Data science is largely about manipulating matrices/vectors.
- There are four aspects that are important for manipulating large matrices (which are not covered in typical college linear algebra courses):
 - Speed: big matrix can be very slow to compute.



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Why study all

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- Data science is largely about manipulating matrices/vectors.
- There are four aspects that are important for manipulating large matrices (which are not covered in typical college linear algebra courses):
 - Speed: big matrix can be very slow to compute.
 - Doing pretty much anything interesting on dense matrix cost about $\mathcal{O}(n^3)$ floating point operations; on a large matrix where $n\approx 10^5,\ n^3\approx 10^{15}$ operations ≈ 280 hours assuming 10^9 operations/s.



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 - Accuracy: digital computers do finite precision arithmetic, meaning that each arithmetic (usually) incurs some small error (rounding error). If you do billions of operations does the errors accumulate or amplify?



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 - Some algorithms are more stable than the others;



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 - Accuracy: digital computers do finite precision arithmetic, meaning that each arithmetic (usually) incurs some small error (rounding error). If you do billions of operations does the errors accumulate or amplify?
 - Some algorithms are more stable than the others;
 - Memory: the storage is usually $\mathcal{O}(n^2)$; again grows fast with size of matrix.



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 - Memory: the storage is usually $\mathcal{O}(n^2)$; again grows fast with size of matrix.
 - Scalability: you don't have enough memory space or computing power so you wantto use parallel computers. Does more nodes mean faster execution time?



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 - Some algorithms are more stable than the others;
 - Memory: the storage is usually $\mathcal{O}(n^2)$; again grows fast with size of matrix.
 - Scalability: you don't have enough memory space or computing power so you wantto use parallel computers. Does more nodes mean faster execution time?
- Because the world is harsh, we'll frequently need to tradeoff between them: but first you need to know and understand them



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...like Sci-kit learn / [put your favorite framework here] Probably, but what if



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■ Too slow



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- Too slow
- Low quality result. What's wrong?



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- Too slow
- Low quality result. What's wrong?
- Variants not implemented (⊕)?



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- Doing cutting edge research/development?



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- Doing cutting edge research/development?

It's worth knowing what's going on under the hood. Plus, you get to study and play with really cool stuff that I found fascinating.