# COSC 6364 Adv. Numerical Analysis

## Quiz 1 Prep

K. Hourani[1]

[1]Computer Science
University of Houston

Spring 2020

# Table of Contents

# Orthogonal Projector



- What is an orthogonal projector?

# Orthogonal Projector



- What is an orthogonal projector?
  - An orthogonal projector is a matrix $P$ such that $P^2 = P$ and $P^* = P$.

# Orthogonal Projector



- What is an orthogonal projector?
  - An orthogonal projector is a matrix $P$ such that $P^2 = P$ and $P^* = P$.
- What is an orthogonal matrix?

# Orthogonal Projector



- What is an orthogonal projector?
  - An orthogonal projector is a matrix $P$ such that $P^2 = P$ and $P^* = P$.
- What is an orthogonal matrix?
  - A real matrix $Q$ is orthogonal if $QQ^{\mathrm{T}} = Q^{\mathrm{T}}Q = I$.

# Orthogonal Projector



- What is an orthogonal projector?
  - An orthogonal projector is a matrix $P$ such that $P^2 = P$ and $P^* = P$.
- What is an orthogonal matrix?
  - A real matrix $Q$ is orthogonal if $QQ^{\mathrm{T}} = Q^{\mathrm{T}}Q = I$.
- Suppose we have a unit vector $\boldsymbol{v}$. What is the orthogonal projector that projects to $\mathrm{range}(\boldsymbol{v})$?

# Orthogonal Projector



- What is an orthogonal projector?
  - An orthogonal projector is a matrix $P$ such that $P^2 = P$ and $P^* = P$.
- What is an orthogonal matrix?
  - A real matrix $Q$ is orthogonal if $QQ^{\mathrm{T}} = Q^{\mathrm{T}}Q = I$.
- Suppose we have a unit vector $\boldsymbol{v}$. What is the orthogonal projector that projects to $\mathrm{range}(\boldsymbol{v})$?
  - The matrix $\boldsymbol{v}\boldsymbol{v}^{\mathrm{T}}$ projects onto $\mathrm{range}(\boldsymbol{v})$, since

$$\begin{aligned} \boldsymbol{v}\boldsymbol{v}^{\mathrm{T}}\boldsymbol{x} &= \boldsymbol{v}\left(\boldsymbol{v}^{\mathrm{T}}\boldsymbol{x}\right) \\ &= \langle \boldsymbol{v}, \boldsymbol{x} \rangle \boldsymbol{v} \end{aligned}$$

# Orthogonal Projector



- What is an orthogonal projector?
  - An orthogonal projector is a matrix $P$ such that $P^2 = P$ and $P^* = P$.
- What is an orthogonal matrix?
  - A real matrix $Q$ is orthogonal if $QQ^\mathrm{T} = Q^\mathrm{T}Q = I$.
- Suppose we have a unit vector $\boldsymbol{v}$. What is the orthogonal projector that projects to $\mathrm{range}(\boldsymbol{v})$?
  - The matrix $\boldsymbol{v}\boldsymbol{v}^\mathrm{T}$ projects onto $\mathrm{range}(\boldsymbol{v})$, since

$$\begin{aligned}\boldsymbol{v}\boldsymbol{v}^\mathrm{T}\boldsymbol{x} &= \boldsymbol{v}\left(\boldsymbol{v}^\mathrm{T}\boldsymbol{x}\right)\\ &= \langle \boldsymbol{v}, \boldsymbol{x}\rangle \boldsymbol{v}\end{aligned}$$

- Suppose we have an ortho-normal matrix $Q$ (with orthogonal and normalized columns). What is the orthogonal projector that projects to $\mathrm{range}(Q)$?

# Orthogonal Projector



- What is an orthogonal projector?
  - An orthogonal projector is a matrix $P$ such that $P^2 = P$ and $P^* = P$.
- What is an orthogonal matrix?
  - A real matrix $Q$ is orthogonal if $QQ^{\mathrm{T}} = Q^{\mathrm{T}}Q = I$.
- Suppose we have a unit vector $\boldsymbol{v}$. What is the orthogonal projector that projects to $\mathrm{range}(\boldsymbol{v})$?
  - The matrix $\boldsymbol{v}\boldsymbol{v}^{\mathrm{T}}$ projects onto $\mathrm{range}(\boldsymbol{v})$, since

$$\begin{aligned}\boldsymbol{v}\boldsymbol{v}^{\mathrm{T}}\boldsymbol{x} &= \boldsymbol{v}\left(\boldsymbol{v}^{\mathrm{T}}\boldsymbol{x}\right)\\ &= \langle \boldsymbol{v}, \boldsymbol{x}\rangle \boldsymbol{v}\end{aligned}$$

- Suppose we have an ortho-normal matrix $Q$ (with orthogonal and normalized columns). What is the orthogonal projector that projects to $\mathrm{range}(Q)$?
  - The matrix $QQ^{\mathrm{T}}$ projects onto $\mathrm{range}(Q)$.

# Orthogonal Projector



- What is an orthogonal projector?
  - An orthogonal projector is a matrix $P$ such that $P^2 = P$ and $P^* = P$.
- What is an orthogonal matrix?
  - A real matrix $Q$ is orthogonal if $QQ^{\mathrm{T}} = Q^{\mathrm{T}}Q = I$.
- Suppose we have a unit vector $\boldsymbol{v}$. What is the orthogonal projector that projects to $\operatorname{range}(\boldsymbol{v})$?
  - The matrix $\boldsymbol{v}\boldsymbol{v}^{\mathrm{T}}$ projects onto $\operatorname{range}(\boldsymbol{v})$, since

$$\begin{aligned} \boldsymbol{v}\boldsymbol{v}^{\mathrm{T}}\boldsymbol{x} &= \boldsymbol{v}\left(\boldsymbol{v}^{\mathrm{T}}\boldsymbol{x}\right) \\ &= \langle \boldsymbol{v}, \boldsymbol{x} \rangle \boldsymbol{v} \end{aligned}$$

- Suppose we have an ortho-normal matrix $Q$ (with orthogonal and normalized columns). What is the orthogonal projector that projects to $\operatorname{range}(Q)$?
  - The matrix $QQ^{\mathrm{T}}$ projects onto $\operatorname{range}(Q)$.
- Suppose we have a matrix $A$. What is the orthogonal projector that projects to $\operatorname{range}(A)$?

# Orthogonal Projector



- What is an orthogonal projector?
  - An orthogonal projector is a matrix $P$ such that $P^2 = P$ and $P^* = P$.
- What is an orthogonal matrix?
  - A real matrix $Q$ is orthogonal if $QQ^{\mathrm{T}} = Q^{\mathrm{T}}Q = I$.
- Suppose we have a unit vector $\boldsymbol{v}$. What is the orthogonal projector that projects to $\operatorname{range}(\boldsymbol{v})$?
  - The matrix $\boldsymbol{v}\boldsymbol{v}^{\mathrm{T}}$ projects onto $\operatorname{range}(\boldsymbol{v})$, since

$$\begin{aligned} \boldsymbol{v}\boldsymbol{v}^{\mathrm{T}}\boldsymbol{x} &= \boldsymbol{v}\left(\boldsymbol{v}^{\mathrm{T}}\boldsymbol{x}\right) \\ &= \langle \boldsymbol{v}, \boldsymbol{x} \rangle \boldsymbol{v} \end{aligned}$$

- Suppose we have an ortho-normal matrix $Q$ (with orthogonal and normalized columns). What is the orthogonal projector that projects to $\operatorname{range}(Q)$?
  - The matrix $QQ^{\mathrm{T}}$ projects onto $\operatorname{range}(Q)$.
- Suppose we have a matrix $A$. What is the orthogonal projector that projects to $\operatorname{range}(A)$?
  - The matrix $A(A^{\mathrm{T}}A)^{-1}A^{\mathrm{T}}$ projects onto $\operatorname{range}(A)$.

# Householder Reflector and QR factorization

- How does Householder reflector accomplish QR factorization?

# Householder Reflector and QR factorization

- How does Householder reflector accomplish QR factorization?
  - The Householder reflector introduces zeros below the diagonal in the $k^{\text{th}}$ column while preserving all the zeroes previously introduced.

$$\underset{A}{\begin{bmatrix} \times & \times & \times \\ \times & \times & \times \\ \times & \times & \times \\ \times & \times & \times \\ \times & \times & \times \end{bmatrix}} \xrightarrow{Q_1} \underset{Q_1 A}{\begin{bmatrix} \times & \times & \times \\ 0 & \times & \times \\ 0 & \times & \times \\ 0 & \times & \times \\ 0 & \times & \times \end{bmatrix}} \xrightarrow{Q_2} \underset{Q_2 Q_1 A}{\begin{bmatrix} \times & \times & \times \\ 0 & \times & \times \\ 0 & 0 & \times \\ 0 & 0 & \times \\ 0 & 0 & \times \end{bmatrix}} \xrightarrow{Q_3} \underset{Q_3 Q_2 Q_1 A}{\begin{bmatrix} \times & \times & \times \\ 0 & \times & \times \\ 0 & 0 & \times \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}}$$

Then

$$Q = Q_1 Q_2 \ldots Q_n$$

# Householder Reflector and QR factorization



- Given a vector $x$, what is the Householder reflector that maps $x$ to the first axis?

# Householder Reflector and QR factorization



- Given a vector $\boldsymbol{x}$, what is the Householder reflector that maps $\boldsymbol{x}$ to the first axis?
  - Write $\boldsymbol{v} = \boldsymbol{x} + \|\boldsymbol{x}\|\boldsymbol{e}_1$. Then the Householder reflector is

$$I - 2\frac{\boldsymbol{v}\boldsymbol{v}^{\mathrm{T}}}{\boldsymbol{v}^{\mathrm{T}}\boldsymbol{v}}$$

Equivalently, write $\boldsymbol{u} = \frac{\boldsymbol{v}}{\|\boldsymbol{v}\|}$ and our reflector is

$$I - 2\boldsymbol{u}\boldsymbol{u}^{\mathrm{T}}$$

# Householder Reflector and QR factorization



- Given a vector $\boldsymbol{x}$, what is the Householder reflector that maps $\boldsymbol{x}$ to the first axis?
  - Write $\boldsymbol{v} = \boldsymbol{x} + \|\boldsymbol{x}\|\boldsymbol{e}_1$. Then the Householder reflector is

$$I - 2\frac{\boldsymbol{v}\boldsymbol{v}^{\mathrm{T}}}{\boldsymbol{v}^{\mathrm{T}}\boldsymbol{v}}$$

    Equivalently, write $\boldsymbol{u} = \frac{\boldsymbol{v}}{\|\boldsymbol{v}\|}$ and our reflector is

$$I - 2\boldsymbol{u}\boldsymbol{u}^{\mathrm{T}}$$

- What's the FLOP count of Householder QR factorization?

# Householder Reflector and QR factorization



- Given a vector $\boldsymbol{x}$, what is the Householder reflector that maps $\boldsymbol{x}$ to the first axis?
  - Write $\boldsymbol{v} = \boldsymbol{x} + \|\boldsymbol{x}\|\boldsymbol{e}_1$. Then the Householder reflector is

$$I - 2\frac{\boldsymbol{v}\boldsymbol{v}^{\mathrm{T}}}{\boldsymbol{v}^{\mathrm{T}}\boldsymbol{v}}$$

    Equivalently, write $\boldsymbol{u} = \frac{\boldsymbol{v}}{\|\boldsymbol{v}\|}$ and our reflector is

$$I - 2\boldsymbol{u}\boldsymbol{u}^{\mathrm{T}}$$

- What's the FLOP count of Householder QR factorization?
  - $2mn^2 - \frac{2}{3}n^3$

# Conditioning and Backward Stability



- What's conditioning & backward stability?

# Conditioning and Backward Stability



- What's conditioning & backward stability?
  - Given a problem $f$ with input $\boldsymbol{x}$, write $\delta\boldsymbol{x} = (1+\delta)\boldsymbol{x}$ and $\delta f = f(\delta\boldsymbol{x}) - f(\boldsymbol{x})$. The absolute condition number of $f$ is given by
$$\lim_{\delta\to 0}\sup_{\delta}\frac{\|\delta f\|}{\|\delta\boldsymbol{x}\|}$$
The relative condition number is given by
$$\lim_{\delta\to 0}\sup_{\delta}\frac{\|\delta f\|/\|f\|}{\|\delta\boldsymbol{x}\|/\|\boldsymbol{x}\|}$$

# Conditioning and Backward Stability



- What's conditioning & backward stability?
  - Given a problem $f$ with input $\boldsymbol{x}$, write $\delta\boldsymbol{x} = (1+\delta)\boldsymbol{x}$ and $\delta f = f(\delta\boldsymbol{x}) - f(\boldsymbol{x})$. The absolute condition number of $f$ is given by
  $$\lim_{\delta\to 0}\sup_{\delta}\frac{\|\delta f\|}{\|\delta\boldsymbol{x}\|}$$
  The relative condition number is given by
  $$\lim_{\delta\to 0}\sup_{\delta}\frac{\|\delta f\|/\|f\|}{\|\delta\boldsymbol{x}\|/\|\boldsymbol{x}\|}$$
  - An algorithm is backward stable if, for every $\boldsymbol{x}$, there exists an $\tilde{\boldsymbol{x}}$ with
  $$\frac{\|\tilde{\boldsymbol{x}} - \boldsymbol{x}\|}{\|\boldsymbol{x}\|} = \mathcal{O}(\epsilon_{\text{machine}})$$
  such that
  $$\tilde{f}(\boldsymbol{x}) = f(\tilde{\boldsymbol{x}})$$

# Conditioning and Backward Stability



- Why is forward error bounded by backward error times $\kappa$?

# Conditioning and Backward Stability



- Why is forward error bounded by backward error times $\kappa$?
  - This follows from the definitions of forward error and $\kappa$:

$$\frac{\left\|\tilde{f}(\boldsymbol{x}) - f(\tilde{\boldsymbol{x}})\right\|}{\left\|f(\tilde{\boldsymbol{x}})\right\|} = \frac{\left\|\tilde{f}(\boldsymbol{x}) - f(\tilde{\boldsymbol{x}})\right\| / \|f(\tilde{\boldsymbol{x}})\|}{\|\tilde{\boldsymbol{x}} - \boldsymbol{x}\| / \|\boldsymbol{x}\|} \cdot \frac{\|\tilde{\boldsymbol{x}} - \boldsymbol{x}\|}{\|\boldsymbol{x}\|}$$

$$\leq \kappa \frac{\|\tilde{\boldsymbol{x}} - \boldsymbol{x}\|}{\|\boldsymbol{x}\|}$$

# Conditioning and Backward Stability



- Why is forward error bounded by backward error times $\kappa$?
  - This follows from the definitions of forward error and $\kappa$:

$$\frac{\left\|\tilde{f}(\boldsymbol{x}) - f(\tilde{\boldsymbol{x}})\right\|}{\left\|f(\tilde{\boldsymbol{x}})\right\|} = \frac{\left\|\tilde{f}(\boldsymbol{x}) - f(\tilde{\boldsymbol{x}})\right\| / \|f(\tilde{\boldsymbol{x}})\|}{\|\tilde{\boldsymbol{x}} - \boldsymbol{x}\| / \|\boldsymbol{x}\|} \cdot \frac{\|\tilde{\boldsymbol{x}} - \boldsymbol{x}\|}{\|\boldsymbol{x}\|}$$

$$\leq \kappa \frac{\|\tilde{\boldsymbol{x}} - \boldsymbol{x}\|}{\|\boldsymbol{x}\|}$$

- Is the simple running sum algorithm for computing inner product of two vectors $\boldsymbol{x}^{\mathrm{T}}\boldsymbol{y} = \sum_{i=1}^{n} x_i y_i$ backward stable?

# Conditioning and Backward Stability

- Why is forward error bounded by backward error times $\kappa$?
  - This follows from the definitions of forward error and $\kappa$:

$$\frac{\left\|\tilde{f}(\boldsymbol{x}) - f(\tilde{\boldsymbol{x}})\right\|}{\left\|f(\tilde{\boldsymbol{x}})\right\|} = \frac{\left\|\tilde{f}(\boldsymbol{x}) - f(\tilde{\boldsymbol{x}})\right\| / \|f(\tilde{\boldsymbol{x}})\|}{\|\tilde{\boldsymbol{x}} - \boldsymbol{x}\| / \|\boldsymbol{x}\|} \cdot \frac{\|\tilde{\boldsymbol{x}} - \boldsymbol{x}\|}{\|\boldsymbol{x}\|}$$

$$\leq \kappa \frac{\|\tilde{\boldsymbol{x}} - \boldsymbol{x}\|}{\|\boldsymbol{x}\|}$$

- Is the simple running sum algorithm for computing inner product of two vectors $\boldsymbol{x}^{\mathrm{T}}\boldsymbol{y} = \sum_{i=1}^{n} x_i y_i$ backward stable?
  - Yes. Let $f$ denote the inner product and $\tilde{f}$ its computation by running sum. When $n = 2$, we have

$$\begin{aligned}
\tilde{f}(\boldsymbol{x}, \boldsymbol{y}) &= x_1 \otimes y_1 \oplus x_2 \otimes y_2 \\
&= (x_1 y_1)(1 + \epsilon_1) \oplus (x_2 y_2)(1 + \epsilon_2) \\
&= [(x_1 y_1)(1 + \epsilon_1) + (x_2 y_2)(1 + \epsilon_2)](1 + \epsilon_3) \\
&= (x_1 y_1 + x_2 y_2 + x_1 y_1 \epsilon_1 + x_2 y_2 \epsilon_2)(1 + \epsilon_3) \\
&= x_1 y_1 + x_2 y_2 + x_1 y_1 \epsilon_1 + x_2 y_2 \epsilon_2 + x_1 y_1 \epsilon_3 + x_2 y_2 \epsilon_3 + x_1 y_1 \\
&= \text{this is unfinished because these proofs are the worst}
\end{aligned}$$

# LU factorization

- How to use LU factorization to solve a linear system?

# LU factorization



- How to use LU factorization to solve a linear system?
  - To solve $A\boldsymbol{x} = \boldsymbol{b}$, solve $L\boldsymbol{y} = \boldsymbol{b}$ for $\boldsymbol{y}$ then solve $U\boldsymbol{x} = \boldsymbol{y}$ for $\boldsymbol{x}$.

# LU factorization



- How to use LU factorization to solve a linear system?
  - To solve $A\boldsymbol{x} = \boldsymbol{b}$, solve $L\boldsymbol{y} = \boldsymbol{b}$ for $\boldsymbol{y}$ then solve $U\boldsymbol{x} = \boldsymbol{y}$ for $\boldsymbol{x}$.
  - A better solution is to write $PA = LU$ for a permutation matrix $P$. Now, solve $L\boldsymbol{y} = P\boldsymbol{b}$ for $\boldsymbol{y}$ then solve $U\boldsymbol{x} = \boldsymbol{y}$ for $\boldsymbol{x}$.

# LU factorization



- How to use LU factorization to solve a linear system?
  - To solve $A\boldsymbol{x} = \boldsymbol{b}$, solve $L\boldsymbol{y} = \boldsymbol{b}$ for $\boldsymbol{y}$ then solve $U\boldsymbol{x} = \boldsymbol{y}$ for $\boldsymbol{x}$.
  - A better solution is to write $PA = LU$ for a permutation matrix $P$. Now, solve $L\boldsymbol{y} = P\boldsymbol{b}$ for $\boldsymbol{y}$ then solve $U\boldsymbol{x} = \boldsymbol{y}$ for $\boldsymbol{x}$.
- How to use LU factorization to invert a square non-singular matrix A?

# LU factorization



- How to use LU factorization to solve a linear system?
  - To solve $A\boldsymbol{x} = \boldsymbol{b}$, solve $L\boldsymbol{y} = \boldsymbol{b}$ for $\boldsymbol{y}$ then solve $U\boldsymbol{x} = \boldsymbol{y}$ for $\boldsymbol{x}$.
  - A better solution is to write $PA = LU$ for a permutation matrix $P$. Now, solve $L\boldsymbol{y} = P\boldsymbol{b}$ for $\boldsymbol{y}$ then solve $U\boldsymbol{x} = \boldsymbol{y}$ for $\boldsymbol{x}$.
- How to use LU factorization to invert a square non-singular matrix A?
  - Solve $LU\boldsymbol{x}_i = \boldsymbol{e}_i$ for all $i$. Then $A^{-1} = \begin{bmatrix} \boldsymbol{x}_1 & \boldsymbol{x}_2 & \dots & \boldsymbol{x}_n \end{bmatrix}$.

# LU factorization



- How to use LU factorization to solve a linear system?
  - To solve $A\boldsymbol{x} = \boldsymbol{b}$, solve $L\boldsymbol{y} = \boldsymbol{b}$ for $\boldsymbol{y}$ then solve $U\boldsymbol{x} = \boldsymbol{y}$ for $\boldsymbol{x}$.
  - A better solution is to write $PA = LU$ for a permutation matrix $P$. Now, solve $L\boldsymbol{y} = P\boldsymbol{b}$ for $\boldsymbol{y}$ then solve $U\boldsymbol{x} = \boldsymbol{y}$ for $\boldsymbol{x}$.
- How to use LU factorization to invert a square non-singular matrix A?
  - Solve $LU\boldsymbol{x}_i = \boldsymbol{e}_i$ for all $i$. Then $A^{-1} = \begin{bmatrix} \boldsymbol{x}_1 & \boldsymbol{x}_2 & \dots & \boldsymbol{x}_n \end{bmatrix}$.
- What's the cost of LU factorization? How does it compare to Householder QR factorization on the same matrix?

# QR Algorithm for Eigenvalue Decomposition



- How does QR algorithm work?

# QR Algorithm for Eigenvalue Decomposition



- How does QR algorithm work?
  - Begin with matrix $A^{(0)} = A$ and, for $k = 1, 2, \ldots$, write

$$\begin{aligned} Q^{(k)} R^{(k)} &= A^{(k-1)} \\ A^{(k)} &= R^{(k)} Q^{(k)} \end{aligned}$$

This converges to an upper triangular matrix with eigenvalues along the diagonal.

# QR Algorithm for Eigenvalue Decomposition



- How does QR algorithm work?
  - Begin with matrix $A^{(0)} = A$ and, for $k = 1, 2, \ldots$, write

$$\begin{aligned} Q^{(k)} R^{(k)} &= A^{(k-1)} \\ A^{(k)} &= R^{(k)} Q^{(k)} \end{aligned}$$

    This converges to an upper triangular matrix with eigenvalues along the diagonal.

- What's the cost of one iteration in QR algorithm?

# QR Algorithm for Eigenvalue Decomposition



- How does QR algorithm work?
  - Begin with matrix $A^{(0)} = A$ and, for $k = 1, 2, \ldots$, write

$$\begin{aligned} Q^{(k)} R^{(k)} &= A^{(k-1)} \\ A^{(k)} &= R^{(k)} Q^{(k)} \end{aligned}$$

    This converges to an upper triangular matrix with eigenvalues along the diagonal.
- What's the cost of one iteration in QR algorithm?
  - If we run it on $A$ directly, it is $\mathcal{O}(m^3)$ flops. If instead we perform it on $H$, the reduced Hessenberg form of $A$ ($\frac{10}{3}m^3$ flops), each phase is $\mathcal{O}(m^2)$ flops.

# QR Algorithm for Eigenvalue Decomposition



- How does QR algorithm work?
  - Begin with matrix $A^{(0)} = A$ and, for $k = 1, 2, \ldots$, write

$$\begin{aligned} Q^{(k)} R^{(k)} &= A^{(k-1)} \\ A^{(k)} &= R^{(k)} Q^{(k)} \end{aligned}$$

    This converges to an upper triangular matrix with eigenvalues along the diagonal.
- What's the cost of one iteration in QR algorithm?
  - If we run it on $A$ directly, it is $\mathcal{O}(m^3)$ flops. If instead we perform it on $H$, the reduced Hessenberg form of $A$ ($\frac{10}{3} m^3$ flops), each phase is $\mathcal{O}(m^2)$ flops.
- What's the convergence rate of QR algorithm?

# QR Algorithm for Eigenvalue Decomposition



- How does QR algorithm work?
  - Begin with matrix $A^{(0)} = A$ and, for $k = 1, 2, \ldots$, write

$$\begin{aligned} Q^{(k)}R^{(k)} &= A^{(k-1)} \\ A^{(k)} &= R^{(k)}Q^{(k)} \end{aligned}$$

    This converges to an upper triangular matrix with eigenvalues along the diagonal.
- What's the cost of one iteration in QR algorithm?
  - If we run it on $A$ directly, it is $\mathcal{O}(m^3)$ flops. If instead we perform it on $H$, the reduced Hessenberg form of $A$ ($\frac{10}{3}m^3$ flops), each phase is $\mathcal{O}(m^2)$ flops.
- What's the convergence rate of QR algorithm?
  - Without shifting, the QR algorithm exhibits linear convergence. With shifting, it exhibits cubic convergence.