COSC6364 Adv. Numerical Analysis

Introduction

P. Wu[1]

[1]Computer Science
University of Houston

Spring 2020

Table of Contents

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  - Numerical linear algebra (matrix computations)
  - (Convex) optimization
- The topics are primarily geared towards (large scale) statistical learning (learning from data) and to scientific/engineering computing (technical computing).

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- Iterative methods for solving linear equation and eigen/singular decomposition
- Fast implementation, parallelization, randomization, etc...

Books

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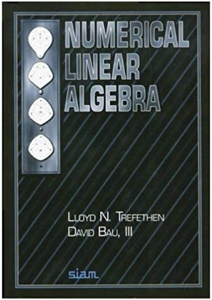
- (Highly recommend; main reference)
  Numerical Linear Algebra (Lloyd N. Trefethen, David Bau)



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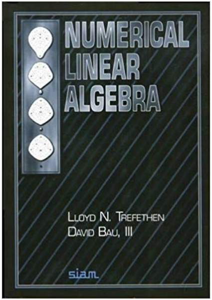

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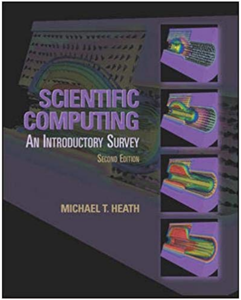

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- Coordinate descent, dual ascent, Alternating direction method of multipliers (ADMM), etc...

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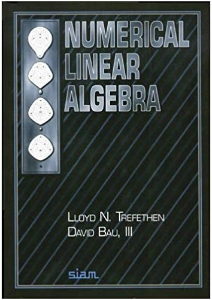

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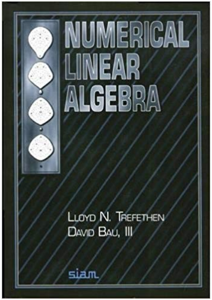

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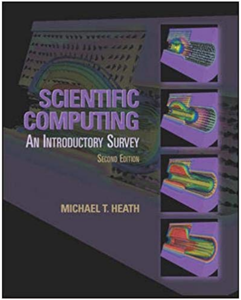

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- Apply different algorithms for different situations; able to diagnose numerical stability and errors

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- Use KKT conditions to solve/characterize optimization problem
- Understand the role of optimization in statistical learning

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  - Scalability: you don't have enough memory space or computing power so you wantto use parallel computers. Does more nodes mean faster execution time?
- Because the world is harsh, we'll frequently need to tradeoff between them: but first you need to know and understand them

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It's worth knowing what's going on under the hood. Plus, you get to study and play with really cool stuff that I found fascinating.