Distributed Algorithms for Connectivity and MST in Large Graphs with Efficient Local Computation

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    - assume $n \gg k$ (e.g. $k = \mathcal{O}(n^{\varepsilon})$)

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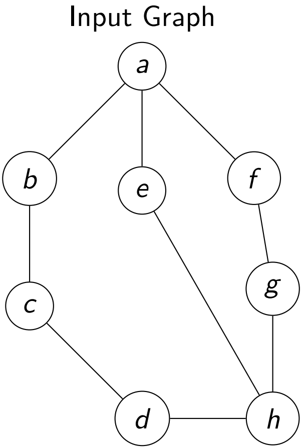

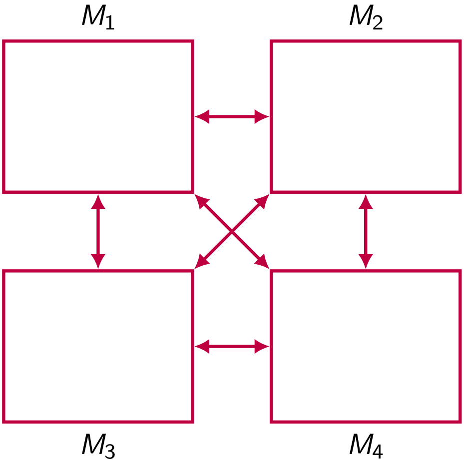

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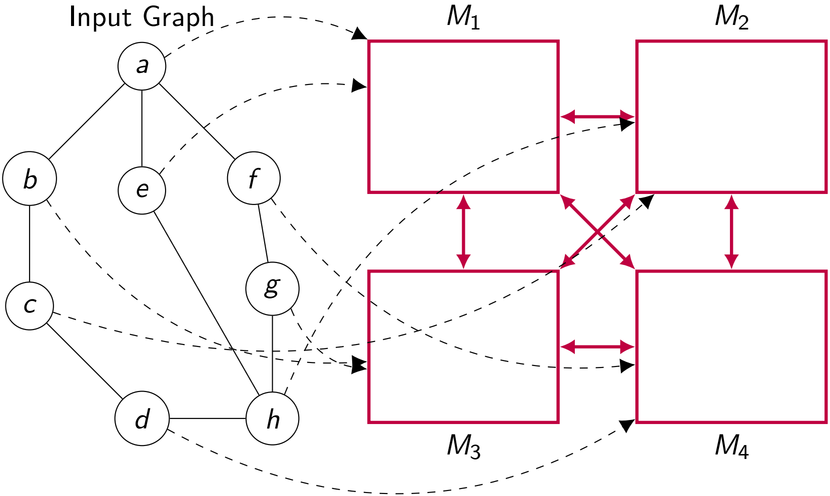

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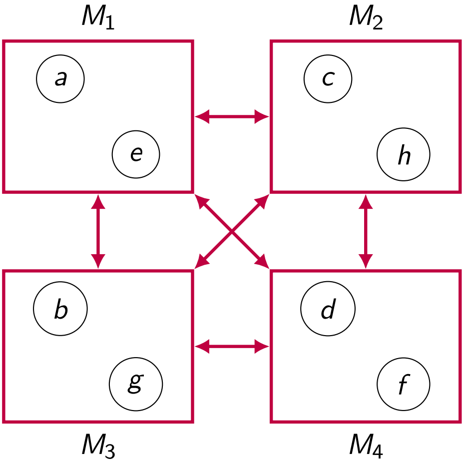

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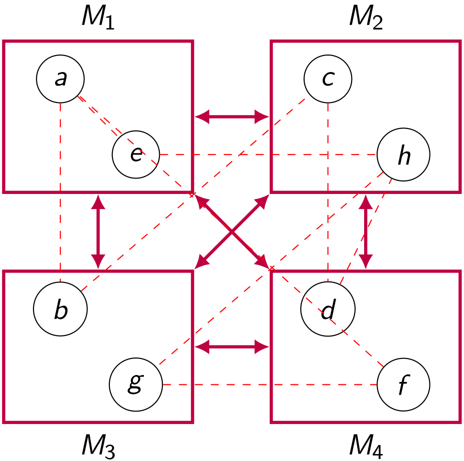

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- in practical implementations of k-machine model algorithms, often see suboptimal speedup
  - e.g. PageRank with message complexity $\mathcal{O}(n/k)$ but whose wall-clock time is approximately $n/k^{0.8}$

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- we analyze Connectivity and MST algorithms using this new complexity measure

Summary of Results

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Algorithm	Round complexity	Local runtime
Flooding	$\tilde{O}\left(\frac{n}{k} + D\right)$	$\tilde{O}\left(\frac{m}{k} + \Delta + k\right)$
Filtering	$\tilde{O}\left(\frac{n}{k}\right)$	$\tilde{O}\left(\frac{m}{k} + n\right)$
Borůvka-Style	$\tilde{O}\left(\frac{n}{k}\right)$	$\tilde{O}\left(\frac{m+n}{k} + \Delta + k\right)$
Randomized CC	$\tilde{O}\left(\frac{n}{k^2}\right)$	$\tilde{O}\left(\frac{m+n}{k} + \Delta + k\right)$

Some Useful Lemmas

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  - the Node Distribution Lemma
  - the Mapping Lemma

Node Distribution Lemma

Lemma (Node Distribution Lemma)

Consider a graph G of nodes $v_1, v_2, \ldots, v_n$ with associated non-negative real-valued "weights" $w(v)$ for each node v. Given a uniform, random distribution of the n nodes to k machines, as in the k-machine model, then, with probability at least $1 - 1/n^a$ for any $a > 0$, the total weight of nodes at every machine is bounded above by

$$\mathcal{O}(T_{avg} + \log n \cdot w_{max})$$

where $T_{avg} = \frac{1}{k}\sum_{i=1}^{n} w(v_i)$ and $w_{max} = max\{w(v_i)\}$.

Mapping Lemma

Lemma (Mapping Lemma)

Let an n-node, m-edge graph G be partitioned among the k machines as $N = \{p_1, \ldots, p_k\}$*. Then with probability at least* $1 - 1/n^{\alpha}$*, where* $\alpha > 1$ *is an arbitrary fixed constant, the following bounds hold:*

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1. *The number of vertices mapped to any machine is $\mathcal{O}(n/k)$.*
2. *The number of edges mapped to any machine is $\mathcal{O}(m/k + \Delta \log n)$.*
3. *The number of edges mapped to any link of the network is $\mathcal{O}(m/k^2 + n/k)$.*

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  - upon receiving a message (with an ID), if the new ID is greater than the current ID, the node updates its ID and floods

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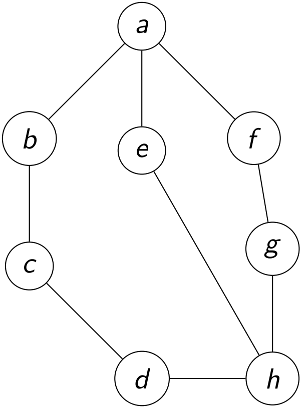

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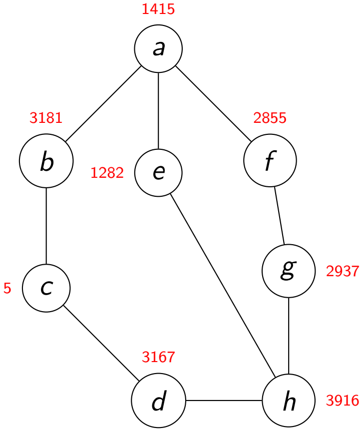

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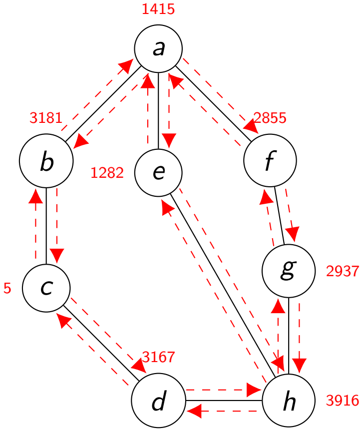

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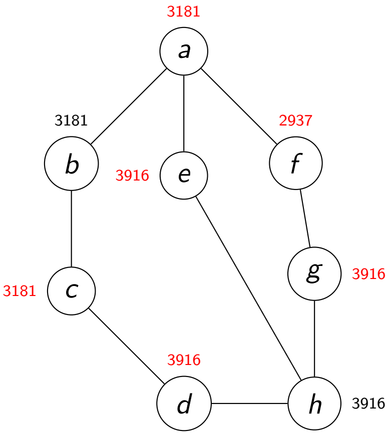

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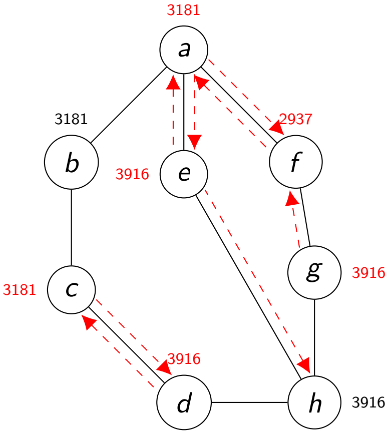

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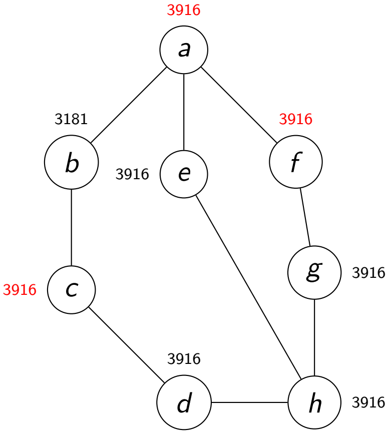

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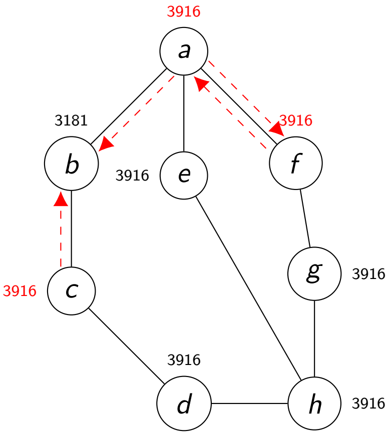

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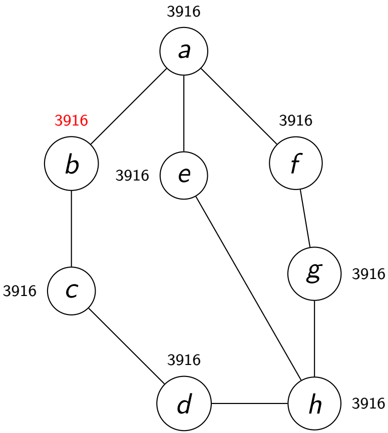

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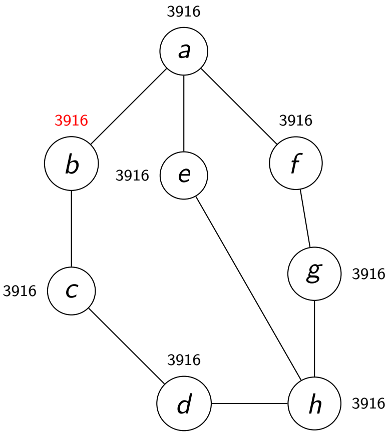

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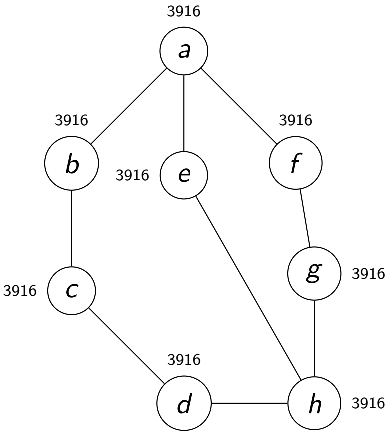

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  - after some $\mathcal{O}(n \log n/k + D)$ rounds, aggregate all IDs located on machine M and send them to machine M_1
- Finally, machine M_1 counts the number of distinct IDs (which is the number of connected components) and broadcasts it to all machines

Theorem

With high probability, the above algorithm correctly counts the number of connected components with

$$T_\ell = \mathcal{O}((m/k + \Delta \log n) \log n + k)$$
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- maximum runtime is therefore $\mathcal{O}(\Delta \log n)$.

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- apply node distribution lemma on the degree of nodes

$$\begin{aligned} T_\ell &= \mathcal{O}\left(\frac{1}{k}\left(\sum_{i=1}^{n}(d(v_i)\log n)\right) + \log n \cdot \Delta \log n\right) \\ &= \mathcal{O}\left(\left(\frac{m}{k} + \Delta \log n\right)\log n\right) \end{aligned}$$

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$$= \mathcal{O}\left(\left(\frac{m}{k} + \Delta \log n\right)\log n\right)$$

- in the worst case, a machine sends/receives messages from all other machines, totaling

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- thus, $\mathcal{O}(n \log n)$ broadcasts initiated with high probability

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  - when exactly one fragment remains, it forms an MST

The Algorithm

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- algorithm then continues on remaining subgraphs

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- the machine then merges fragments

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    - construct maximal matching (using e.g. Cole-Vishkin)
    - merge all matched edges and any edge where exactly one endpoint is matched

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- $T_\ell = \tilde{O}\left(\frac{m+n}{k} + \Delta + k\right)$
  - filtering requires $\tilde{O}(m/k + \Delta)$

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- relies on linear graph sketches to efficiently merge multiple components

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  - optimal, up to polylog factors!

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Conclusion

- Augmented the k-machine model with a metric to measure local computation

Algorithm	Round complexity	Local runtime
Flooding	$\tilde{O}\left(\frac{n}{k} + D\right)$	$\tilde{O}\left(\frac{m}{k} + \Delta + k\right)$
Filtering	$\tilde{O}\left(\frac{n}{k}\right)$	$\tilde{O}\left(\frac{m}{k} + n\right)$
Borůvka-Style	$\tilde{O}\left(\frac{n}{k}\right)$	$\tilde{O}\left(\frac{m+n}{k} + \Delta + k\right)$
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Conclusion

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    - Many "optimal" algorithms are only optimal with respect to T_c
- Analyzed well-known MIS/CC algorithms with respect to T_ℓ and T_c

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- Analyze other graph problems in this model using
- Implementing algorithms and comparing wall-clock time with T_ℓ and T_c
  - e.g. are there constants α and β (for a given set of hardware) such that wall-clock time is $\alpha T_\ell + \beta T_c$?