Distributed Algorithms for Connectivity and MST in Large Graphs with Efficient Local Computation

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January 4, 2022

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    - commonly called vertex centric model
  - assume $n \gg k$ (e.g. $k = \mathcal{O}(n^{\varepsilon})$)

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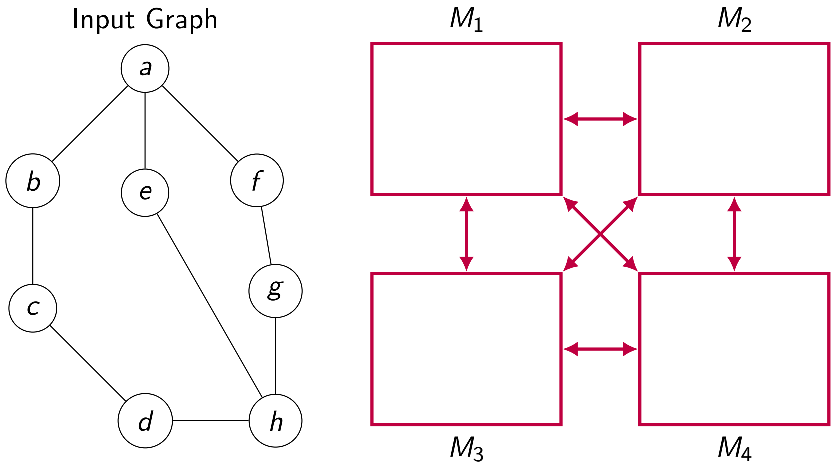

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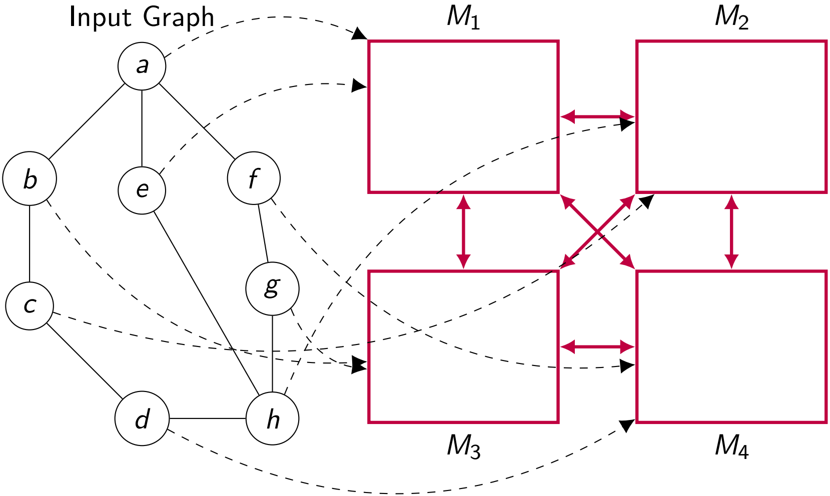

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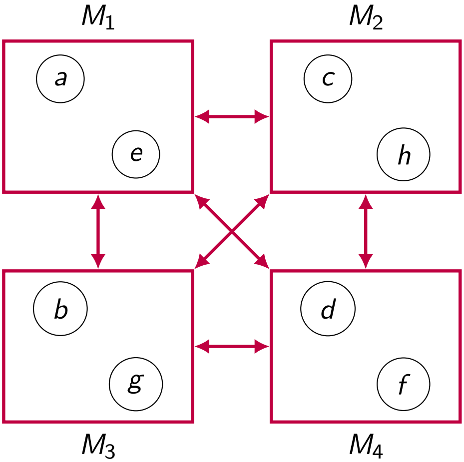

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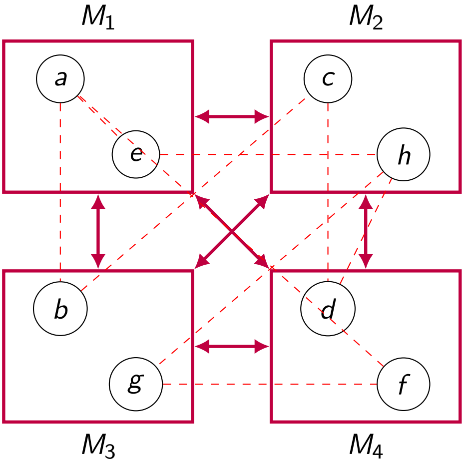

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- in practical implementations of k-machine model algorithms, often see suboptimal speedup
  - e.g. PageRank with message complexity $\mathcal{O}(n/k)$ but whose wall-clock time is approximately $n/k^{0.8}$

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- we analyze Connectivity and MST algorithms using this new complexity measure

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Algorithm	Round complexity	Local runtime
Flooding	$\tilde{O}\left(\frac{n}{k} + D\right)$	$\tilde{O}\left(\frac{m}{k} + \Delta + k\right)$
Filtering	$\tilde{O}\left(\frac{n}{k}\right)$	$\tilde{O}\left(\frac{m}{k} + n\right)$
Borůvka-Style	$\tilde{O}\left(\frac{n}{k}\right)$	$\tilde{O}\left(\frac{m+n}{k} + \Delta + k\right)$
Randomized CC	$\tilde{O}\left(\frac{n}{k^2}\right)$	$\tilde{O}\left(\frac{m+n}{k} + \Delta + k\right)$

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  - the Node Distribution Lemma
  - the Mapping Lemma

Node Distribution Lemma

Lemma (Node Distribution Lemma)

Consider a graph G of nodes $v_1, v_2, \ldots, v_n$ with associated non-negative real-valued "weights" $w(v)$ for each node v. Given a uniform, random distribution of the n nodes to k machines, as in the k-machine model, then, with probability at least $1 - 1/n^a$ for any $a > 0$, the total weight of nodes at every machine is bounded above by

$$\mathcal{O}(T_{avg} + \log n \cdot w_{max})$$

where $T_{avg} = \frac{1}{k}\sum_{i=1}^{n} w(v_i)$ and $w_{max} = max\{w(v_i)\}$.

Mapping Lemma

Lemma (Mapping Lemma)

Let an n-node, m-edge graph G be partitioned among the k machines as $N = \{p_1, \ldots, p_k\}$. *Then with probability at least* $1 - 1/n^{\alpha}$, *where* $\alpha > 1$ *is an arbitrary fixed constant, the following bounds hold:*

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Let an n-node, m-edge graph G be partitioned among the k machines as $N = \{p_1, \ldots, p_k\}$. Then with probability at least $1 - 1/n^{\alpha}$, where $\alpha > 1$ is an arbitrary fixed constant, the following bounds hold:

1. *The number of vertices mapped to any machine is $\mathcal{O}(n/k)$.*

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1. *The number of vertices mapped to any machine is $\mathcal{O}(n/k)$.*
2. *The number of edges mapped to any machine is $\mathcal{O}(m/k + \Delta \log n)$.*
3. *The number of edges mapped to any link of the network is $\mathcal{O}\left(m/k^2 + n/k\right)$.*

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  - each node chooses an ID uniformly at random in $[1, n^4]$
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  - upon receiving a message (with an ID), if the new ID is greater than the current ID, the node updates its ID and floods

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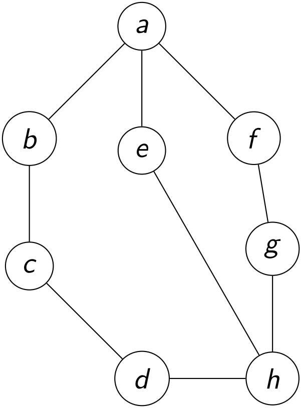

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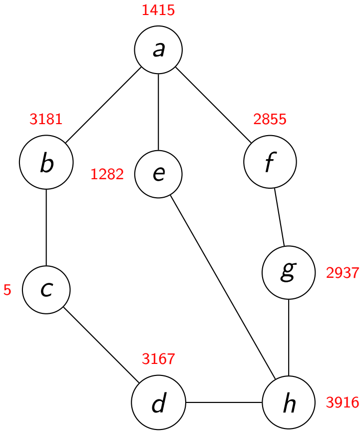

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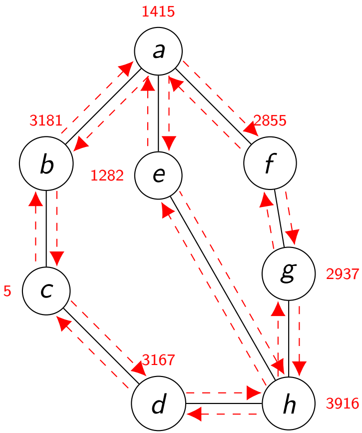

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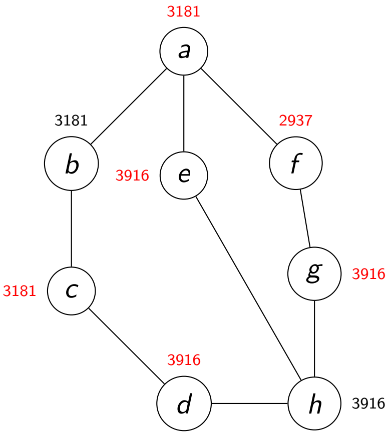

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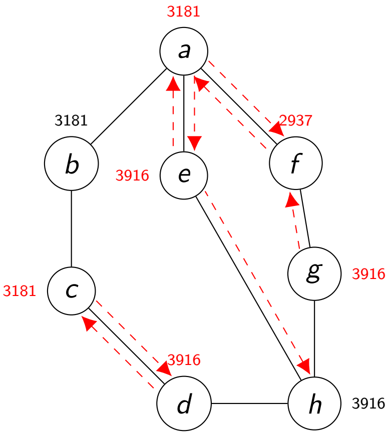

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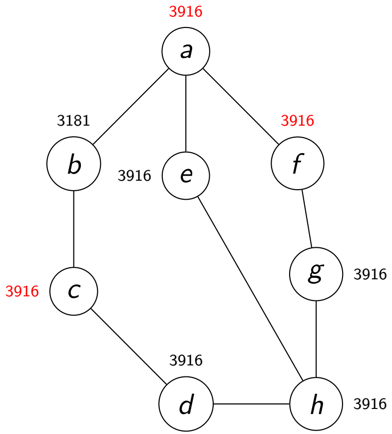

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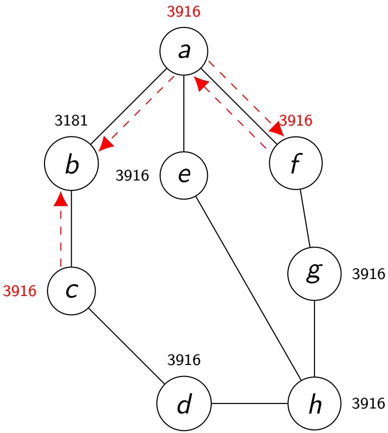

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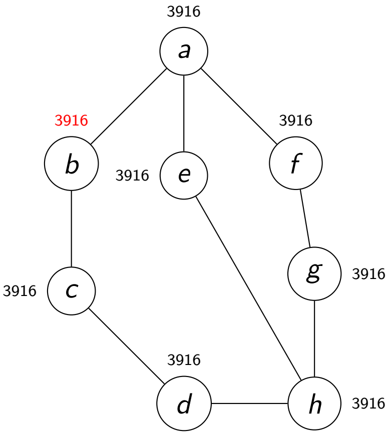

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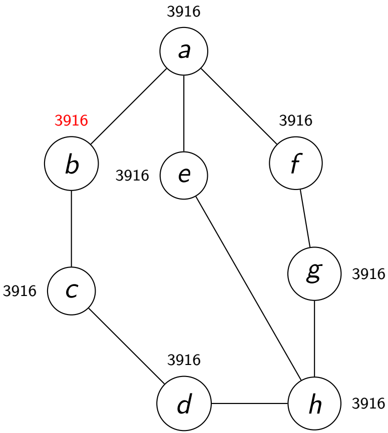

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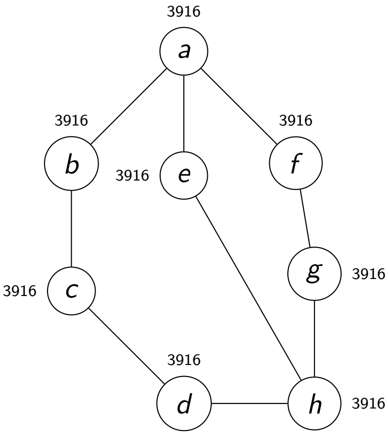

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  - after some $\mathcal{O}(n \log n/k + D)$ rounds, aggregate all IDs located on machine M and send them to machine M_1
- Finally, machine M_1 counts the number of distinct IDs (which is the number of connected components) and broadcasts it to all machines

Theorem

With high probability, the above algorithm correctly counts the number of connected components with

$$T_\ell = \mathcal{O}((m/k + \Delta \log n) \log n + k)$$
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- maximum runtime is therefore $\mathcal{O}(\Delta \log n)$.

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- apply node distribution lemma on the degree of nodes

$$T_\ell = \mathcal{O}\left(\frac{1}{k}\left(\sum_{i=1}^{n}(d(v_i)\log n)\right) + \log n \cdot \Delta \log n\right)$$
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- in the worst case, a machine sends/receives messages from all other machines, totaling

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- thus, $\mathcal{O}(n \log n)$ broadcasts initiated with high probability

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    - when exactly one fragment remains, it forms an MST

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- algorithm then continues on remaining subgraphs

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- the machine then merges fragments

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  - merge all matched edges and any edge where exactly one endpoint is matched

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  - optimal, up to polylog factors!

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Algorithm	Round complexity	Local runtime
Flooding	$\tilde{O}\left(\frac{n}{k} + D\right)$	$\tilde{O}\left(\frac{m}{k} + \Delta + k\right)$
Filtering	$\tilde{O}\left(\frac{n}{k}\right)$	$\tilde{O}\left(\frac{m}{k} + n\right)$
Borůvka-Style	$\tilde{O}\left(\frac{n}{k}\right)$	$\tilde{O}\left(\frac{m+n}{k} + \Delta + k\right)$
Randomized CC	$\tilde{O}\left(\frac{n}{k^2}\right)$	$\tilde{O}\left(\frac{m+n}{k} + \Delta + k\right)$

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- Implement algorithms and compare wall-clock time with T_ℓ and T_c
  - e.g. are there constants α and β (for a given set of hardware) such that wall-clock time is $\alpha T_\ell + \beta T_c$?