Programming Tutorial

Khalid Hourani

University of Houston

October 9, 2020



Table of Contents

- 1 Introduction
 - Correctness
 - Readability
 - Performance

- 2 Problem Solving
 - Problem: Array Division





Table of Contents

- 1 Introduction
 - Correctness
 - Readability
 - Performance

- 2 Problem Solving
 - Problem: Array Division



Good code depends on





Good code depends on



Good code depends on

- Correctness
- Readability



Good code depends on

- Correctness
- Readability
- Performance







Correctness

Code should be correct





- Code should be correct
- If code is incorrect then doesn't matter





- Code should be correct
- If code is incorrect then doesn't matter
 - how performant it is





- Code should be correct
- If code is incorrect then doesn't matter
 - how performant it is
 - how readable it is



- Code should be correct
- If code is incorrect then doesn't matter
 - how performant it is
 - how readable it is





Correctness

- Code should be correct
- If code is incorrect then doesn't matter
 - how performant it is
 - how readable it is

Boolean Satisfiability

```
1 def bool_sat(formula):
2   return True
```



Correctness

- Code should be correct
- If code is incorrect then doesn't matter
 - how performant it is
 - how readable it is

```
Boolean Satisfiability
```

```
1 def bool_sat(formula):
2  return True
```

 \circ Above code is $\mathcal{O}(1)$ solution to an NP-Complete Problem



Correctness

- Code should be correct
- If code is incorrect then doesn't matter
 - how performant it is
 - how readable it is

Boolean Satisfiability

```
1 def bool_sat(formula):
2  return True
```

- Above code is $\mathcal{O}(1)$ solution to an NP-Complete Problem
- But obviously not a correct solution







Testing

You must test your code





- You must test your code
- Easiest way to catch bugs





- You must test your code
- Easiest way to catch bugs
- Example





- You must test your code
- Easiest way to catch bugs
- Example





- You must test your code
- Easiest way to catch bugs
- Example

```
Primality Checker
```

```
1 def is_prime(n):
2   if n in {0, 1}:
3    return False
4   for i in range(2, int(n ** 0.5)):
5    if n % i == 0:
6     return False
7   return True
```





Testing

- You must test your code
- Easiest way to catch bugs
- Example

```
Primality Checker
```

```
1 def is_prime(n):
2    if n in {0, 1}:
3        return False
4    for i in range(2, int(n ** 0.5)):
5        if n % i == 0:
6        return False
7    return True
```

• What is wrong with above code?





- You must test your code
- Easiest way to catch bugs
- Example

```
Primality Checker
```

```
1 def is_prime(n):
2   if n in {0, 1}:
3    return False
4   for i in range(2, int(n ** 0.5)):
5    if n % i == 0:
6    return False
7   return True
```

- What is wrong with above code?
- Might not be obvious, but easy to see with testing



Test is_prime

```
1 for i in range(2, 11):
2  print(i, is_prime(i))
```



Test is_prime

```
for i in range(2, 11):
print(i, is_prime(i))
```

i	<pre>is_prime(i)</pre>	Correct?
2	True	√
3	True	\checkmark
4	True	×
5	True	\checkmark
6	True	×
7	True	\checkmark
8	True	×
9	True	X
10	False	\checkmark



4,

```
\textbf{Test is\_prime}
```

```
for i in range(2, 11):
print(i, is_prime(i))
```

i	<pre>is_prime(i)</pre>	Correct?	
2	True	√	
3	True	•	Incorrect for composite values
4	True	X	6, 8, and 9
5	True	\checkmark	
6	True	X	
7	True	\checkmark	
8	True	X	
9	True	Х	
10	False	√	



```
Test is_prime
```

```
1 for i in range(2, 11):
2 print(i, is_prime(i))
```

i	is_prime(i)	Correct?	_
2	True	✓	
3	True	✓ •	Incorrect for composite values 4,
4	True	X	6, 8, and 9
5	True	✓ ○	For these values
6	True	X	$int(n ** 0.5) = \lfloor \sqrt{n} \rfloor = 2$
7	True	\checkmark	
8	True	X	
9	True	X	
10	False	✓	



```
Test is_prime
for i in range(2, 11):
  print(i, is_prime(i))
```

i	<pre>is_prime(i)</pre>	Correct?	
2	True	√	
3	True	•	Incorrect for composite values 4,
4	True	X	6, 8, and 9
5	True	✓ •	For these values
6	True	X	$int(n ** 0.5) = \lfloor \sqrt{n} \rfloor = 2$
7	True	✓ 。	Off-by-one error – add 1 to
8	True	X	range: int(n ** 0.5) + 1
9	True	X	
10	False	√	



Readability



Readability

Usually the most important thing after correctness





Readability

- Usually the most important thing after correctness
- Code will be read far more often than it is written



Readability

- Usually the most important thing after correctness
- Code will be read far more often than it is written
- Maintenance of code is often the highest expense



Syntax

• Follow a set of best-practices for your language. For example



Syntax

- Follow a set of best-practices for your language. For example
 - $\, \cdot \,$ In Python, there is PEP 8





Syntax

- Follow a set of best-practices for your language. For example
 - In Python, there is PEP 8
 - In C, there is the Linux Kernel Style



Syntax

- Follow a set of best-practices for your language. For example
 - In Python, there is PEP 8
 - In C, there is the Linux Kernel Style
- The style chosen is less important than that you follow a consistent style



Syntax

- Follow a set of best-practices for your language. For example
 - In Python, there is PEP 8
 - In C, there is the Linux Kernel Style
- The style chosen is less important than that you follow a consistent style



- Follow a set of best-practices for your language. For example
 - In Python, there is PEP 8
 - In C, there is the Linux Kernel Style
- The style chosen is less important than that you follow a consistent style

```
Ugly
```

```
c=(a+b)**0.5
```

$$L=[1,2,7]$$

- Follow a set of best-practices for your language. For example
 - In Python, there is PEP 8
 - In C, there is the Linux Kernel Style
- The style chosen is less important than that you follow a consistent style

Ugly c=(a+b)**0.5 L=[1,2,7]

Easiest way to be consistent is to use a formatter



- Follow a set of best-practices for your language. For example
 - In Python, there is PEP 8
 - In C, there is the Linux Kernel Style
- The style chosen is less important than that you follow a consistent style

Ugly c=(a+b)**0.5

```
L = [1, 2, 7]
```

Readable

- Easiest way to be consistent is to use a formatter
- Automatically formats your code according to some style guide



- Follow a set of best-practices for your language. For example
 - In Python, there is PEP 8
 - In C, there is the Linux Kernel Style
- The style chosen is less important than that you follow a consistent style

Ugly c=(a+b)**0.5

```
C=(a+b)**0.5
L=[1,2,7]
```

Readable

- Easiest way to be consistent is to use a formatter
- Automatically formats your code according to some style guide
 - Black, yapf, autopep8, etc., for Python

- Follow a set of best-practices for your language. For example
 - In Python, there is PEP 8
 - In C, there is the Linux Kernel Style
- The style chosen is less important than that you follow a consistent style

Ugly

c=(a+b)**0.5 L=[1,2,7]

Readable

- Easiest way to be consistent is to use a formatter
- Automatically formats your code according to some style guide
 - Black, yapf, autopep8, etc., for Python
 - clang-format for C/C++



Structure

Syntactic readability (style) is not enough





Structure

- Syntactic readability (style) is not enough
- Code should be structured for readability





Structure

- Syntactic readability (style) is not enough
- Code should be structured for readability
- Consider following problem





Structure

- Syntactic readability (style) is not enough
- Code should be structured for readability
- Consider following problem





Structure

- Syntactic readability (style) is not enough
- Code should be structured for readability
- Consider following problem

Problem

Find the largest product of two 3-digit numbers that is a palindrome.



Structure

- Syntactic readability (style) is not enough
- Code should be structured for readability
- Consider following problem

Problem

Find the largest product of two 3-digit numbers that is a palindrome.

Brute-Force Solution

Iterate through all 10^6 pairs of products and check if they are palindromic while keeping track of max.



Example of Bad Structure



Example of Bad Structure

```
Poorly Structured
   biggest = 0
   for a in range(100, 1000):
     for b in range(100, 1000):
       prod = a * b
       is_palindrome = True
       s = str(prod)
       n = len(s)
        # check if number is palindrome
       for i in range(n):
          if s[i] != s[n - i - 1]:
            is_palindrome = False
12
13
            break
       if not is_palindrome:
14
15
16
17
          continue
       else:
          if prod > biggest:
            biggest = prod
18 print(biggest)
```





Example of Bad Structure

```
Poorly Structured
   biggest = 0
   for a in range(100, 1000):
     for b in range(100, 1000):
       prod = a * b
       is_palindrome = True
        s = str(prod)
       n = len(s)
        # check if number is palindrome
       for i in range(n):
          if s[i] != s[n - i - 1]:
            is_palindrome = False
12
13
            break
       if not is_palindrome:
14
15
          continue
       else:
          if prod > biggest:
            biggest = prod
18 print(biggest)
```

 code is hard to follow





Example of Bad Structure

```
Poorly Structured
   biggest = 0
   for a in range(100, 1000):
     for b in range(100, 1000):
       prod = a * b
       is palindrome = True
       s = str(prod)
       n = len(s)
       # check if number is palindrome
       for i in range(n):
         if s[i] != s[n - i - 1]:
           is_palindrome = False
           break
       if not is_palindrome:
14
15
         continue
       else:
         if prod > biggest:
           biggest = prod
  print(biggest)
```

- code is hard to follow
- unnecessarily complex is_palindrome is essentially a sentinel value





Example of Bad Structure

```
Poorly Structured
biggest = 0
for a in range(100, 1000):
  for b in range(100, 1000):
    prod = a * b
    is palindrome = True
    s = str(prod)
    n = len(s)
    # check if number is palindrome
    for i in range(n):
      if s[i] != s[n - i - 1]:
        is_palindrome = False
         break
    if not is_palindrome:
       continue
    else:
       if prod > biggest:
         biggest = prod
print(biggest)
```

- code is hard to follow
- unnecessarily
 complex —
 is_palindrome
 is essentially a
 sentinel value
- code is imperative, not declarative







```
Better Structured
def is_palindrome(num):
  s = str(num)
  n = len(s)
  for i in range(n):
    if s[i] != s[n - i - 1]:
      return False
  return True
biggest = 0
for a in range(100, 1000):
  for b in range(100, 1000):
    if is_palindrome(a * b):
      biggest = max(biggest, a * b)
print(biggest)
```





```
Better Structured
def is_palindrome(num):
  s = str(num)
  n = len(s)
  for i in range(n):
    if s[i] != s[n - i - 1]:
      return False
  return True
biggest = 0
for a in range(100, 1000):
  for b in range(100, 1000):
    if is_palindrome(a * b):
      biggest = max(biggest, a * b)
print(biggest)
```

 code is much easier to follow





```
Better Structured
   def is_palindrome(num):
     s = str(num)
     n = len(s)
     for i in range(n):
       if s[i] != s[n - i - 1]:
         return False
     return True
   biggest = 0
   for a in range(100, 1000):
     for b in range(100, 1000):
       if is_palindrome(a * b):
         biggest = max(biggest, a * b)
15 print(biggest)
```

- code is much easier to follow
- functionis_palindromeclearly conveysintent





```
Better Structured
   def is_palindrome(num):
     s = str(num)
     n = len(s)
     for i in range(n):
       if s[i] != s[n - i - 1]:
         return False
     return True
   biggest = 0
   for a in range(100, 1000):
     for b in range(100, 1000):
       if is_palindrome(a * b):
         biggest = max(biggest, a * b)
15 print(biggest)
```

- code is much easier to follow
- function
 is_palindrome
 clearly conveys
 intent
- code is declarative





Example of Good Structure



Example of Good Structure

```
Well Structured
   def is_palindrome(num):
     s = str(num)
     n = len(s)
     for i in range(n):
       if s[i] != s[n - i - 1]:
         return False
     return True
   def main():
     biggest = 0
     for a in range(100, 1000):
        for b in range(100, 1000):
          if is_palindrome(a * b):
            biggest = max(biggest, a * b)
15
16
17
     print(biggest)
   if __name__ == "__main__":
        main()
```





Example of Good Structure

```
Well Structured
   def is_palindrome(num):
     s = str(num)
     n = len(s)
     for i in range(n):
       if s[i] != s[n - i - 1]:
         return False
     return True
   def main():
     biggest = 0
     for a in range(100, 1000):
        for b in range(100, 1000):
          if is_palindrome(a * b):
            biggest = max(biggest, a * b)
15
16
17
     print(biggest)
   if __name__ == "__main__":
        main()
```

basically same code



Example of Good Structure

```
Well Structured
   def is_palindrome(num):
     s = str(num)
     n = len(s)
     for i in range(n):
       if s[i] != s[n - i - 1]:
         return False
     return True
   def main():
     biggest = 0
     for a in range(100, 1000):
       for b in range(100, 1000):
          if is_palindrome(a * b):
            biggest = max(biggest, a * b)
15
16
     print(biggest)
   if __name__ == "__main__":
       main()
```

- basically same code
- clearly describes what the program is doing overall







Comments

Code should be commented





- Code should be commented
- but not over commented





- Code should be commented
- but not over commented





- Code should be commented
- but not over commented

```
Bad Comment
```

```
1 def i_sqrt(n):
2    i = 0
3    while i ** 2 < n:
4     i += 1 # increment i
5    return i</pre>
```





Comments

- Code should be commented
- but not over commented

```
Bad Comment

1  def i_sqrt(n):
2   i = 0
3  while i ** 2 < n:
4   i += 1  # increment i
5  return i</pre>
```

o it is clear that i += 1 increments i





- Code should be commented
- but not over commented

```
Bad Comment

1  def i_sqrt(n):
2   i = 0
3  while i ** 2 < n:
4   i += 1  # increment i
5  return i</pre>
```

- it is clear that i += 1 increments i
- comment is superfluous and distracting





Good Comments



Good Comments

As a rule of thumb





- As a rule of thumb
 - good code describes what and how





- As a rule of thumb
 - good code describes what and how
 - good comments describe why





- As a rule of thumb
 - good code describes what and how
 - good comments describe why





- As a rule of thumb
 - good code describes what and how
 - good comments describe why

```
Good Comment
   def is_prime(n):
     if n == 1:
       return False
     elif n in \{2, 3\}:
       return True
     elif n % 2 == 0:
       return False
     else:
       # we need only check for odd factors
       # up to sqrt(n)
       for i in range(3, int(n ** 0.5) + 1, 2):
         if n % i == 0:
13
           return False
       return True
```





- As a rule of thumb
 - good code describes what and how
 - good comments describe why

```
Good Comment
   def is_prime(n):
     if n == 1:
       return False
     elif n in {2, 3}:
       return True
     elif n % 2 == 0:
       return False
     else:
       # we need only check for odd factors
       # up to sqrt(n)
       for i in range(3, int(n ** 0.5) + 1, 2):
         if n % i == 0:
13
           return False
       return True
```

this comment
explains why the
code is iterating
from 1 to
int(n ** 0.5) + 1



- As a rule of thumb
 - good code describes what and how
 - good comments describe why

```
Good Comment

1 def is_prime(n):
2    if n == 1:
3        return False
4    elif n in {2, 3}:
5        return True
6    elif n % 2 == 0:
7        return False
8    else:
9        # we need only check for odd factors
10        # up to sqrt(n)
11        for i in range(3, int(n ** 0.5) + 1, 2):
12        if n % i == 0:
13            return False
14        return True
```

- this comment
 explains why the
 code is iterating
 from 1 to
 int(n ** 0.5) + 1
- without this
 comment, reader
 would have to
 determine for
 themselves



Good Code

If your code is well-written, it will improve readability





- If your code is well-written, it will improve readability
- Not just in terms of structure, but things like variable and function names





- If your code is well-written, it will improve readability
- Not just in terms of structure, but things like variable and function names
- Well-written code often makes many comments unnecessary





- If your code is well-written, it will improve readability
- Not just in terms of structure, but things like variable and function names
- Well-written code often makes many comments unnecessary
- Often expressed as





- If your code is well-written, it will improve readability
- Not just in terms of structure, but things like variable and function names
- Well-written code often makes many comments unnecessary
- Often expressed as

Good Code is its Own Best Documentation





- If your code is well-written, it will improve readability
- Not just in terms of structure, but things like variable and function names
- Well-written code often makes many comments unnecessary
- Often expressed as

Good Code is its Own Best Documentation

Not an excuse to avoid comments



```
Good Names
Bad Names
def qs(a):
                                                       def quick_sort(array):
  if len(a) <= 1:
                                                         if len(array) <= 1:
    return a
                                                           return array
  else:
                                                         else:
    x, z = [], []
                                                           left, right = [], []
    y = a[0]
                                                           pivot = array[0]
    for p in a:
                                                           for ele in array:
                                                             if ele < pivot:
      if p < y:
                                                               left.append(ele)
        x.append(p)
      else:
                                                             else:
        z.append(p)
                                                               right.append(ele)
    return qs(x) + qs(z)
                                                           return quick_sort(left) + quick_sort(right)
```

these names convey





```
Bod Names

1 def qs(a):
2    if len(a) <= 1:
3        return a
4    else:
5        x, z = [], []
6    y = a[0]
7    for p in a:
8     if p < y:
9        x.append(p)
10    else:
11        z.append(p)
12    return qs(x) + qs(z)
```

```
Good Names

def quick_sort(array):
    if len(array) <= 1:
        return array

else:
    left, right = [], []
    pivot = array[0]

for ele in array:
    if ele < pivot:
    left.append(ele)
    else:
        right.append(ele)
    return quick_sort(left) + quick_sort(right)
```

- these names convey
 - basically nothing





```
Bad Names

1 def qs(a):
2    if len(a) <= 1:
3        return a
4    else:
5        x, z = [], []
6    y = a[0]
7    for p in a:
8    if p < y:
9        x.append(p)
10    else:
11    z.append(p)
12    return qs(x) + qs(z)
```

```
Good Names

def quick_sort(array):
    if len(array) <= 1:
        return array

else:
    left, right = [], []
    pivot = array[0]

for ele in array:
    if ele < pivot:
    left.append(ele)
    else:
        right.append(ele)
    return quick_sort(left) + quick_sort(right)
```

- these names convey
 - basically nothing

these names convey





```
Bod Names

1 def qs(a):
    if len(a) <= 1:
        return a
4 else:
        x, z = [], []
6     y = a[0]
7     for p in a:
        if p < y:
        x. append(p)
10     else:
11        z.append(p)
12     return qs(x) + qs(z)
```

```
Good Names

def quick_sort(array):
    if len(array) <= 1:
        return array
else:
    left, right = [], []
    pivot = array[0]
for ele in array:
    if ele < pivot:
    left.append(ele)
else:
    right.append(ele)
return quick_sort(left) + quick_sort(right)
```

- these names convey
 - basically nothing

- these names convey
 - function is quicksort





```
Bad Names

1 def qs(a):
2    if len(a) <= 1:
3        return a
4    else:
5        x, z = [], []
6    y = a[0]
7    for p in a:
8    if p < y:
9        x.append(p)
10    else:
11    z.append(p)
12    return qs(x) + qs(z)
```

```
Good Names

def quick_sort(array):
    if len(array) <= 1:
        return array
else:
    left, right = [], []
    pivot = array[0]
for ele in array:
    if ele < pivot:
    left. append(ele)
    else:
        right.append(ele)
    return quick_sort(left) + quick_sort(right)
```

- these names convey
 - basically nothing

- these names convey
 - function is quicksort
 - input is an array





```
Good Names

def quick_sort(array):
    if len(array) <= 1:
        return array
else:
    left, right = [], []
    pivot = array[0]
for ele in array:
    if ele < pivot:
    left.append(ele)
else:
    right.append(ele)
return quick_sort(right) + quick_sort(right)
```

- these names convey
 - basically nothing

- these names convey
 - function is quicksort
 - input is an array
 - left and right are
 partitions around
 pivot = array[0]





```
Bod Names

1 def qs(a):
1 if len(a) <= 1:
2 return a
4 else:
5 x, z = [], []
6 y = a[0]
7 for p in a:
8 if p < y:
9 x.append(p)
10 else:
11 z.append(p)
12 return qs(x) + qs(z)
```

- these names convey
 - basically nothing

- these names convey
 - function is quicksort
 - input is an array
 - left and right are
 partitions around
 pivot = array[0]
 - ele iterates through values of array







Miscellaneous 1

Impractical to create exhaustive list of best practices



- Impractical to create exhaustive list of best practices
 - and some practices are debateable





- Impractical to create exhaustive list of best practices
 - and some practices are debateable
- Good idea to look at resources like





- Impractical to create exhaustive list of best practices
 - and some practices are debateable
- Good idea to look at resources like
 - The Little Book of Python Anti-Patterns





- Impractical to create exhaustive list of best practices
 - and some practices are debateable
- Good idea to look at resources like
 - The Little Book of Python Anti-Patterns
 - The C++ Core Guidelines





- Impractical to create exhaustive list of best practices
 - and some practices are debateable
- Good idea to look at resources like
 - The Little Book of Python Anti-Patterns
 - The C++ Core Guidelines
- And to use a linter a static code analysis tool to flag bugs, style errors, etc, such as



- Impractical to create exhaustive list of best practices
 - and some practices are debateable
- Good idea to look at resources like
 - The Little Book of Python Anti-Patterns
 - The C++ Core Guidelines
- And to use a linter a static code analysis tool to flag bugs, style errors, etc, such as
 - Python flake8



- Impractical to create exhaustive list of best practices
 - and some practices are debateable
- Good idea to look at resources like
 - The Little Book of Python Anti-Patterns
 - The C++ Core Guidelines
- And to use a linter a static code analysis tool to flag bugs, style errors, etc, such as
 - Python flake8
 - \circ C++ clang-tidy





Highly case-by-case





- Highly case-by-case
 - specific use case may allow for less performant code





- Highly case-by-case
 - specific use case may allow for less performant code
 - e.g. some one-time scientific calculation





- Highly case-by-case
 - specific use case may allow for less performant code
 - e.g. some one-time scientific calculation
 - or may require more performant code





- Highly case-by-case
 - specific use case may allow for less performant code
 - e.g. some one-time scientific calculation
 - or may require more performant code
 - e.g. real-time financial analysis



- Highly case-by-case
 - specific use case may allow for less performant code
 - e.g. some one-time scientific calculation
 - or may require more performant code
 - e.g. real-time financial analysis
- Usually comes down to correct choice of algorithm and data structure



- Highly case-by-case
 - specific use case may allow for less performant code
 - e.g. some one-time scientific calculation
 - or may require more performant code
 - e.g. real-time financial analysis
- Usually comes down to correct choice of algorithm and data structure
 - e.g., if checking existence of an element, an array gives $\mathcal{O}(n)$ but a hash table gives $\mathcal{O}(1)$



Performance

Performance

- Highly case-by-case
 - specific use case may allow for less performant code
 - e.g. some one-time scientific calculation
 - or may require more performant code
 - e.g. real-time financial analysis
- Usually comes down to correct choice of algorithm and data structure
 - e.g., if checking existence of an element, an array gives $\mathcal{O}(n)$ but a hash table gives $\mathcal{O}(1)$
 - which might make the difference between $\mathcal{O}\!\left(n^2\right)$ and $\mathcal{O}\!\left(n\right)$ overall



Performance

- Highly case-by-case
 - specific use case may allow for less performant code
 - e.g. some one-time scientific calculation
 - or may require more performant code
 - e.g. real-time financial analysis
- Usually comes down to correct choice of algorithm and data structure
 - e.g., if checking existence of an element, an array gives $\mathcal{O}(n)$ but a hash table gives $\mathcal{O}(1)$
 - which might make the difference between $\mathcal{O}\!\left(n^2\right)$ and $\mathcal{O}\!\left(n\right)$ overall
- While it does depend on use case, it is still usually best to focus on readability over performance



Performance

Performance

- Highly case-by-case
 - specific use case may allow for less performant code
 - e.g. some one-time scientific calculation
 - or may require more performant code
 - e.g. real-time financial analysis
- Usually comes down to correct choice of algorithm and data structure
 - e.g., if checking existence of an element, an array gives $\mathcal{O}(n)$ but a hash table gives $\mathcal{O}(1)$
 - which might make the difference between $\mathcal{O}\!\left(n^2\right)$ and $\mathcal{O}\!\left(n\right)$ overall
- While it does depend on use case, it is still usually best to focus on readability over performance
 - it's easier to make slow code fast than to make confusing code understandable



Table of Contents

- 1 Introduction
 - Correctness
 - Readability
 - Performance

- 2 Problem Solving
 - Problem: Array Division



Problem





Problem

Problem

Given an array of integers nums and a positive integer k, find whether it is possible to divide nums into sets of k consecutive numbers.





Where to Start





Where to Start





1 nums =
$$[1, 2, 3, 3, 4, 4, 5, 6]$$
, $k = 4$





1 nums =
$$[1, 2, 3, 3, 4, 4, 5, 6]$$
, $k = 4$
 $[1, 2, 3, 4]$, $[3, 4, 5, 6]$



Where to Start

1 nums =
$$[1, 2, 3, 3, 4, 4, 5, 6]$$
, $k = 4$
• $[1, 2, 3, 4]$, $[3, 4, 5, 6]$

2 nums =
$$[3, 2, 1, 2, 3, 4, 3, 4, 5, 9, 10, 11]$$
, $k = 3$





1 nums =
$$[1, 2, 3, 3, 4, 4, 5, 6]$$
, $k = 4$
• $[1, 2, 3, 4]$, $[3, 4, 5, 6]$

2 nums =
$$[3, 2, 1, 2, 3, 4, 3, 4, 5, 9, 10, 11]$$
, $k = 3$
 $[1, 2, 3]$, $[2, 3, 4]$, $[3, 4, 5]$, $[9, 10, 11]$



Look at examples

1 nums =
$$[1, 2, 3, 3, 4, 4, 5, 6]$$
, $k = 4$
• $[1, 2, 3, 4]$, $[3, 4, 5, 6]$

2 nums =
$$[3, 2, 1, 2, 3, 4, 3, 4, 5, 9, 10, 11]$$
, $k = 3$
 $[1, 2, 3]$, $[2, 3, 4]$, $[3, 4, 5]$, $[9, 10, 11]$

• Hopefully notice a pattern – if $x = \min(A)$



- Look at examples
 - 1 nums = [1, 2, 3, 3, 4, 4, 5, 6], k = 4
 - [1, 2, 3, 4], [3, 4, 5, 6]
 - 2 nums = [3, 2, 1, 2, 3, 4, 3, 4, 5, 9, 10, 11], k = 3
 - [1,2,3], [2,3,4], [3,4,5], [9,10,11]
- Hopefully notice a pattern if $x = \min(A)$
 - then x, x + 1, ..., x + k 1 form one subarray





1 nums =
$$[1, 2, 3, 3, 4, 4, 5, 6]$$
, $k = 4$
 $\cdot [1, 2, 3, 4]$, $[3, 4, 5, 6]$
2 nums = $[3, 2, 1, 2, 3, 4, 3, 4, 5, 9, 10, 11]$, $k = 3$

- [1, 2, 3], [2, 3, 4], [3, 4, 5], [9, 10, 11]
- Hopefully notice a pattern if $x = \min(A)$ • then $x, x + 1, \ldots, x + k - 1$ form one subarray
- Naturally lends itself to a recursive solution:



1 nums =
$$[1, 2, 3, 3, 4, 4, 5, 6]$$
, $k = 4$
[1, 2, 3, 4], [3, 4, 5, 6]

2 nums =
$$[3, 2, 1, 2, 3, 4, 3, 4, 5, 9, 10, 11]$$
, $k = 3$

- Hopefully notice a pattern if $x = \min(A)$
 - then x, x + 1, ..., x + k 1 form one subarray
- Naturally lends itself to a recursive solution:
 - remove x, x + 1, ..., x + k 1, then repeat



- Look at examples
 - 1) nums = [1, 2, 3, 3, 4, 4, 5, 6], k = 4[1, 2, 3, 4], [3, 4, 5, 6]
 - 2 nums = [3, 2, 1, 2, 3, 4, 3, 4, 5, 9, 10, 11], k = 3
 - [1, 2, 3], [2, 3, 4], [3, 4, 5], [9, 10, 11]
- Hopefully notice a pattern if $x = \min(A)$
 - then x, x + 1, ..., x + k 1 form one subarray
- Naturally lends itself to a recursive solution:
 - remove x, x + 1, ..., x + k 1, then repeat
 - o if run out of elements before removing all k, then no solution







Writing a Solution

two observations





- two observations
 - ① we are removing the k smallest elements





- two observations
 - 1 we are removing the k smallest elements
 - 2 if we know some element x, we know the next element is x + 1





- two observations
 - 1 we are removing the k smallest elements
 - 2 if we know some element x, we know the next element is x + 1





- two observations
 - \bigcirc we are removing the k smallest elements
 - 2) if we know some element x, we know the next element is x + 1
- suggests the use of a heap





- two observations
 - \bigcirc we are removing the k smallest elements
 - 2) if we know some element x, we know the next element is x + 1
- suggests the use of a heap





- two observations
 - \bigcirc we are removing the k smallest elements
 - 2) if we know some element x, we know the next element is x + 1
- suggests the use of a heap
- 2 suggests the use of a dictionary





- two observations
 - 1 we are removing the k smallest elements
 - 2 if we know some element x, we know the next element is x + 1
- suggests the use of a heap
- 2 suggests the use of a dictionary
- both solutions are valid



troduction 00000000000

Problem: Array Division





Heap Solution

Heap Pop removes the smallest element





- Heap Pop removes the smallest element
- Need to account for removing the k smallest distinct elements





- Heap Pop removes the smallest element
- Need to account for removing the *k* smallest distinct elements
- Thus, set up (value, count) pairs





- Heap Pop removes the smallest element
- Need to account for removing the k smallest distinct elements
- Thus, set up (value, count) pairs
- Then, pop heap say smallest value is (x, c)





- Heap Pop removes the smallest element
- Need to account for removing the k smallest distinct elements
- Thus, set up (value, count) pairs
- Then, pop heap say smallest value is (x,c)
 - effectively removing c copies of x





- Heap Pop removes the smallest element
- Need to account for removing the k smallest distinct elements
- Thus, set up (value, count) pairs
- Then, pop heap say smallest value is (x, c)
 - effectively removing c copies of x
 - then remove c copies of $x+1, \ldots, x+k-1$ if possible



- Heap Pop removes the smallest element
- Need to account for removing the k smallest distinct elements
- Thus, set up (value, count) pairs
- Then, pop heap say smallest value is (x, c)
 - effectively removing c copies of x
 - then remove c copies of $x + 1, \ldots, x + k 1$ if possible
 - if not possible, return False





- Heap Pop removes the smallest element
- Need to account for removing the k smallest distinct elements
- Thus, set up (value, count) pairs
- Then, pop heap say smallest value is (x, c)
 - effectively removing c copies of x
 - then remove c copies of $x + 1, \ldots, x + k 1$ if possible
 - if not possible, return False
- After popping, need to return remaining elements to heap





- Heap Pop removes the smallest element
- Need to account for removing the k smallest distinct elements
- Thus, set up (value, count) pairs
- Then, pop heap say smallest value is (x, c)
 - effectively removing c copies of x
 - then remove c copies of $x + 1, \ldots, x + k 1$ if possible
 - if not possible, return False
- After popping, need to return remaining elements to heap
- Keep track of remaining elements in array



Heap Solution

- Heap Pop removes the smallest element
- Need to account for removing the k smallest distinct elements
- Thus, set up (value, count) pairs
- Then, pop heap say smallest value is (x, c)
 - effectively removing c copies of x
 - then remove c copies of $x + 1, \ldots, x + k 1$ if possible
 - if not possible, return False
- After popping, need to return remaining elements to heap
- Keep track of remaining elements in array
- Then push back onto heap







```
popped = [] # store popped heap values
   prev, min count = heappop(heap)
 3
   for i in range(k - 1):
     if not heap: # ran out of elements
 5
       return False
 6
     else:
         value, count = heappop(heap)
8
         if value != prev + 1: # not consecutive
9
           return False
10
         else:
11
           count -= min_count
12
           prev = value
13
           if count > 0:
14
              popped.append((value, count))
15
   for val in popped:
16
       heappush(heap, val)
```



oduction 000000000000000

Problem: Array Division





Dictionary Solution

Dictionary holds the count for each element





- Dictionary holds the count for each element
- o if x is min and D[x] = count





- Dictionary holds the count for each element
- o if x is min and D[x] = count
 - must have D[x + i] >= count for $i \in \{x, x + 1, ..., x + k - 1\}$





- Dictionary holds the count for each element
- o if x is min and D[x] = count
 - must have D[x + i] >= count for $i \in \{x, x + 1, ..., x + k - 1\}$
 - else return False





- Dictionary holds the count for each element
- o if x is min and D[x] = count
 - must have $D[x + i] \ge count$ for $i \in \{x, x + 1, \dots, x + k 1\}$
 - else return False
- Decrease each count for $i \in \{x, x+1, \dots, x+k-1\}$ by count



- Dictionary holds the count for each element
- o if x is min and D[x] = count
 - must have $D[x + i] \ge count$ for $i \in \{x, x + 1, \dots, x + k 1\}$
 - else return False
- Decrease each count for $i \in \{x, x+1, \dots, x+k-1\}$ by count
- Remove entry if it becomes 0





roduction

Problem: Array Division





```
while counts: # dictionary to store counts
     x = min(counts)
3
     min count = counts[x]
     del counts[x] # remove smallest
5
     for i in range(1, k):
6
          if counts[x + i] < min_count:</pre>
            return False
8
         else:
9
            counts[x + i] -= min count
10
            if counts[x + i] == 0:
11
              del counts[x + i]
```



