Programming Tutorial

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 - Readability
 - Performance

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 - Problem: Array Division





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- 2 Problem Solving
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Good code depends on





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Good code depends on

- Correctness
- Readability



Good code depends on

- Correctness
- Readability
- Performance







Correctness

Code should be correct





- Code should be correct
- If code is incorrect then doesn't matter





- Code should be correct
- If code is incorrect then doesn't matter
 - how performant it is





- Code should be correct
- If code is incorrect then doesn't matter
 - how performant it is
 - how readable it is



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Boolean Satisfiability

```
1 def bool_sat(formula):
2   return True
```



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Boolean Satisfiability
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1 def bool_sat(formula):
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 \circ Above code is $\mathcal{O}(1)$ solution to an NP-Complete Problem



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 - how performant it is
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Boolean Satisfiability

```
1 def bool_sat(formula):
2  return True
```

- Above code is $\mathcal{O}(1)$ solution to an NP-Complete Problem
- But obviously not a correct solution







Testing

You must test your code





- You must test your code
- Easiest way to catch bugs





- You must test your code
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- Example





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```
Primality Checker
```

```
1 def is_prime(n):
2   if n in {0, 1}:
3    return False
4   for i in range(2, int(n ** 0.5)):
5    if n % i == 0:
6     return False
7   return True
```





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• What is wrong with above code?





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- Example

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Primality Checker
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```

- What is wrong with above code?
- Might not be obvious, but easy to see with testing



Test is_prime

```
1 for i in range(2, 11):
2  print(i, is_prime(i))
```



Test is_prime

```
for i in range(2, 11):
print(i, is_prime(i))
```

i	<pre>is_prime(i)</pre>	Correct?
2	True	√
3	True	\checkmark
4	True	×
5	True	\checkmark
6	True	×
7	True	\checkmark
8	True	×
9	True	X
10	False	\checkmark



4,

```
\textbf{Test is\_prime}
```

```
for i in range(2, 11):
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i	<pre>is_prime(i)</pre>	Correct?	
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4	True	X	6, 8, and 9
5	True	\checkmark	
6	True	X	
7	True	\checkmark	
8	True	X	
9	True	Х	
10	False	√	



```
Test is_prime
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i	is_prime(i)	Correct?	_
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3	True	✓ •	Incorrect for composite values 4,
4	True	X	6, 8, and 9
5	True	✓ ○	For these values
6	True	X	$int(n ** 0.5) = \lfloor \sqrt{n} \rfloor = 2$
7	True	\checkmark	
8	True	X	
9	True	X	
10	False	✓	



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Test is_prime
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3	True	•	Incorrect for composite values 4,
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5	True	✓ •	For these values
6	True	X	$int(n ** 0.5) = \lfloor \sqrt{n} \rfloor = 2$
7	True	✓ 。	Off-by-one error – add 1 to
8	True	X	range: int(n ** 0.5) + 1
9	True	X	
10	False	√	



Readability



Readability

Usually the most important thing after correctness





Readability

- Usually the most important thing after correctness
- Code will be read far more often than it is written



Readability

- Usually the most important thing after correctness
- Code will be read far more often than it is written
- Maintenance of code is often the highest expense



Syntax

• Follow a set of best-practices for your language. For example



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 - $\, \cdot \,$ In Python, there is PEP 8





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Ugly
```

```
c=(a+b)**0.5
```

$$L=[1,2,7]$$

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L = [1, 2, 7]
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Readable

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- Automatically formats your code according to some style guide



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Readable

- Easiest way to be consistent is to use a formatter
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 - Black, yapf, autopep8, etc., for Python
 - clang-format for C/C++



Structure

Syntactic readability (style) is not enough





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- Code should be structured for readability





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- Consider following problem





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Problem

Find the largest product of two 3-digit numbers that is a palindrome.



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- Syntactic readability (style) is not enough
- Code should be structured for readability
- Consider following problem

Problem

Find the largest product of two 3-digit numbers that is a palindrome.

Brute-Force Solution

Iterate through all 10^6 pairs of products and check if they are palindromic while keeping track of max.



Example of Bad Structure



Example of Bad Structure

```
Poorly Structured
   biggest = 0
   for a in range(100, 1000):
     for b in range(100, 1000):
       prod = a * b
       is_palindrome = True
       s = str(prod)
       n = len(s)
        # check if number is palindrome
       for i in range(n):
          if s[i] != s[n - i - 1]:
            is_palindrome = False
12
13
            break
       if not is_palindrome:
14
15
16
17
          continue
       else:
          if prod > biggest:
            biggest = prod
18 print(biggest)
```





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 code is hard to follow





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- code is hard to follow
- unnecessarily complex is_palindrome is essentially a sentinel value





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- code is hard to follow
- unnecessarily
 complex —
 is_palindrome
 is essentially a
 sentinel value
- code is imperative, not declarative







```
Better Structured
def is_palindrome(num):
  s = str(num)
  n = len(s)
  for i in range(n):
    if s[i] != s[n - i - 1]:
      return False
  return True
biggest = 0
for a in range(100, 1000):
  for b in range(100, 1000):
    if is_palindrome(a * b):
      biggest = max(biggest, a * b)
print(biggest)
```





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 code is much easier to follow





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- functionis_palindromeclearly conveysintent





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- code is much easier to follow
- function
 is_palindrome
 clearly conveys
 intent
- code is declarative





Example of Good Structure



Example of Good Structure

```
Well Structured
   def is_palindrome(num):
     s = str(num)
     n = len(s)
     for i in range(n):
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     return True
   def main():
     biggest = 0
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        for b in range(100, 1000):
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     print(biggest)
   if __name__ == "__main__":
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basically same code



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     print(biggest)
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```

- basically same code
- clearly describes what the program is doing overall







Comments

Code should be commented





- Code should be commented
- but not over commented





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Bad Comment
```

```
1 def i_sqrt(n):
2    i = 0
3    while i ** 2 < n:
4     i += 1 # increment i
5    return i</pre>
```





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Bad Comment

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Bad Comment

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3  while i ** 2 < n:
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5  return i</pre>
```

- it is clear that i += 1 increments i
- comment is superfluous and distracting





Good Comments



Good Comments

As a rule of thumb





- As a rule of thumb
 - good code describes what and how





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Good Comment
   def is_prime(n):
     if n == 1:
       return False
     elif n in \{2, 3\}:
       return True
     elif n % 2 == 0:
       return False
     else:
       # we need only check for odd factors
       # up to sqrt(n)
       for i in range(3, int(n ** 0.5) + 1, 2):
         if n % i == 0:
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           return False
       return True
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- As a rule of thumb
 - good code describes what and how
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Good Comment
   def is_prime(n):
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this comment
explains why the
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12        if n % i == 0:
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14        return True
```

- this comment
 explains why the
 code is iterating
 from 1 to
 int(n ** 0.5) + 1
- without this
 comment, reader
 would have to
 determine for
 themselves



Good Code

If your code is well-written, it will improve readability





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- Not just in terms of structure, but things like variable and function names





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Good Code is its Own Best Documentation





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- Well-written code often makes many comments unnecessary
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Good Code is its Own Best Documentation

Not an excuse to avoid comments



```
Good Names
Bad Names
def qs(a):
                                                       def quick_sort(array):
  if len(a) <= 1:
                                                         if len(array) <= 1:
    return a
                                                           return array
  else:
                                                         else:
    x, z = [], []
                                                           left, right = [], []
    y = a[0]
                                                           pivot = array[0]
    for p in a:
                                                           for ele in array:
                                                             if ele < pivot:
      if p < y:
                                                               left.append(ele)
        x.append(p)
      else:
                                                             else:
        z.append(p)
                                                               right.append(ele)
    return qs(x) + qs(z)
                                                           return quick_sort(left) + quick_sort(right)
```

these names convey





```
Bod Names

1 def qs(a):
2    if len(a) <= 1:
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8     if p < y:
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Good Names

def quick_sort(array):
    if len(array) <= 1:
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else:
    left, right = [], []
    pivot = array[0]

for ele in array:
    if ele < pivot:
    left.append(ele)
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    return quick_sort(left) + quick_sort(right)
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- these names convey
 - basically nothing





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- these names convey
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- these names convey
 - function is quicksort





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def quick_sort(array):
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else:
    left, right = [], []
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    if ele < pivot:
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    else:
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    return quick_sort(left) + quick_sort(right)
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- these names convey
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Good Names

def quick_sort(array):
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else:
    left, right = [], []
    pivot = array[0]
for ele in array:
    if ele < pivot:
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else:
    right.append(ele)
return quick_sort(right) + quick_sort(right)
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- these names convey
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- these names convey
 - function is quicksort
 - input is an array
 - left and right are
 partitions around
 pivot = array[0]





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- these names convey
 - function is quicksort
 - input is an array
 - left and right are
 partitions around
 pivot = array[0]
 - ele iterates through values of array







Miscellaneous 1

Impractical to create exhaustive list of best practices



- Impractical to create exhaustive list of best practices
 - and some practices are debateable





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 - Python flake8
 - \circ C++ clang-tidy





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- While it does depend on use case, it is still usually best to focus on readability over performance
 - it's easier to make slow code fast than to make confusing code understandable



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Problem





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Given an array of integers nums and a positive integer k, find whether it is possible to divide nums into sets of k consecutive numbers.





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- Naturally lends itself to a recursive solution:
 - remove x, x + 1, ..., x + k 1, then repeat
 - o if run out of elements before removing all k, then no solution







Writing a Solution

two observations





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 - ① we are removing the k smallest elements





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- suggests the use of a heap
- 2 suggests the use of a dictionary
- both solutions are valid



troduction 00000000000

Problem: Array Division





Heap Solution

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Problem: Array Division Heap Solution

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 - effectively removing c copies of x
 - then remove c copies of $x + 1, \ldots, x + k 1$ if possible
 - if not possible, return False
- After popping, need to return remaining elements to heap
- Keep track of remaining elements in array



Python Implementation





Python Implementation

```
popped = [] # store popped heap values
   prev, min count = heappop(heap)
 3
   for i in range(k - 1):
     if not heap: # ran out of elements
 5
       return False
 6
     else:
         value, count = heappop(heap)
8
         if value != prev + 1: # not consecutive
9
           return False
10
         else:
11
           count -= min_count
12
           prev = value
13
           if count > 0:
14
              popped.append((value, count))
15
   for val in popped:
16
       heappush(heap, val)
```





Dictionary Solution

Dictionary holds the count for each element





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- Decrease each count for $i \in \{x, x+1, \dots, x+k-1\}$ by count
- Remove entry if it becomes 0





Python Implementation





Python Implementation

```
while counts: # dictionary to store counts
     x = min(counts)
3
     min count = counts[x]
     del counts[x] # remove smallest
5
     for i in range(1, k):
6
          if counts[x + i] < min_count:</pre>
            return False
8
         else:
9
            counts[x + i] -= min count
10
            if counts[x + i] == 0:
11
              del counts[x + i]
```



