1. $f = N(t+1) = a N(t)(1-N(t))$

$N(t+1) = N(t) = N^*$

$N^* = a N^* (1-N^*)$

$\frac{1}{a} = 1 - N^*$

$$\boxed{N^* = 1 - \frac{1}{a}}$$

$$\text{cause population is zero}$$
$$\text{so not considered}$$

for stability calculate $f' = \frac{dN(t+1)}{dN(t)}$ at $N^* = 1 - \frac{1}{a}$

$f' = a(1 - 2N(t))$

$= a\left[1 - 2\left(1 - \frac{1}{a}\right)\right]$

$= a\left[1 - 2 + \frac{2}{a}\right] = \left[\frac{2}{a} - 1\right]a$

$= 2 - a$

(a) for stable asymptote $\left|0 < 2-a < 1\right| \Rightarrow 1 < a < 2$
$0 < \lambda < 1$

$-2 < -a < -1$
$1 < a < 2$

(b) for stable oscillation $\left|-1 < 2-a < 0\right| \Rightarrow 2 < a < 3$
$-1 < \lambda < 0$

$-3 < -a < -2$
$3 > a > 2$

(c) for unstable oscillation $\left|2-a < -1\right| \Rightarrow a > 3$
$\lambda < -1$

$-a < -3$
$a > 3$

$f' \text{ at } N^* = 0$

$f' = @$

So, for $0 < a < 1$ it would be
stable asymptote

for $a < 0$, the $N(t+1)$ will become $-ve$
Hence not considering these value.

3. $N(t+1) = N(t) e^{r\left(\frac{k-N(t)}{k}\right)}$

$$N(t+1) = N(t) = N^*$$

$$1 = e^{r\left(1 - \frac{N^*}{k}\right)} = e^0$$

Comparing powers.

$$N(t) = \boxed{N^* = 0} \,k, \quad N^* = 0, k$$

~~$N^* + \cancel{A} \quad (N^* + n) e^{r}$~~

$$f = \frac{dN(t+1)}{dN(t)} = e^{r\left(\frac{k-N(t)}{k}\right)} + N(t)\, e^{r\left(\frac{k-N(t)}{k}\right)} r\left(\frac{-1}{k}\right)$$

$$= e^{r\left(\frac{k-N(t)}{k}\right)} \left[1 - \frac{r N(t)}{k}\right]$$

at $N(t) = k$ only to consider
                    k as $N(t) = 0$
                    is no population

$$= e^0 \left[1 - \frac{r \times k}{k}\right]$$

$$= 1 - r$$

$\left(N^*, N_{t+1}\right)$ So from condition, at $N^* = k$

when $-1 < 1 - r < 0 \Rightarrow 1 < r < 2$
exibibit oscillation.

also when $1 - r < -1$ $\Rightarrow$ $r > 2$
          • it also exibibt oscillation

at NPEO

4.

$$N(t) = \frac{k}{1 + Ce^{-rt}} \qquad \text{when} \quad C = \frac{k - N(0)}{N(0)}$$

$$1 + Ce^{-rt} = \frac{k}{N(t)}$$

$$e^{-rt} = \frac{1}{C}\left(\frac{k}{N(t)} - 1\right)$$

$$-rt = \log\left(\frac{k}{CN(t)} - \frac{1}{C}\right)$$

$$t = \frac{-1}{r}\log\left(\frac{CN(t)}{k - N(t)}\right)$$

$$*** \quad t = \frac{-1}{r}\log\left(\frac{k - N(t)}{CN(t)}\right)$$

$$t + 1 = \frac{-1}{r}\log\left(\frac{k - N(t+1)}{CN(t+1)}\right)$$

$$1 = \frac{-1}{r}\log\left(\frac{k - N(t+1)}{CN(t+1)}\right) + \frac{1}{r}\log\left(\frac{k - N(t+1)}{CN(t)}\right)$$

$$r = \log\left(\frac{\dfrac{k - N(t)}{CN(t)}}{\dfrac{k - N(t+1)}{CN(t+1)}}\right)$$

$$e^{r}\left(\frac{k - 1}{N(t+1)}\right) = \frac{k}{N(t)} - 1$$

$$\frac{ke^{r}}{N(t+1)} - e^{r} = \frac{k}{N(t)} - 1$$

$$\frac{1 e^r}{N(t+1)} - \frac{1}{N(t)} = \frac{(e^r - 1)}{k}$$

$$\frac{e^r}{N(t+1)} = \frac{1}{N(t)} + \frac{(e^r - 1)}{k}$$

$$\frac{1}{N(t+1)} = \frac{k + N(t)(e^r - 1)}{k e^r N(t)}$$

$$\therefore N(t+1) = \frac{k e^r N(t)}{k + N(t)(e^r - 1)}$$