1. $f = N(t+1) = aN(t)(1-N(t))$

$$N(t+1) = N(t) = N^*$$

$$N^* = aN^*(1-N^*)$$

$$\frac{1}{a} = 1 - N^*$$

$$\boxed{N^* = 1 - \frac{1}{a}}, 0$$

for stability calculate $f' = \frac{dN(t+1)}{dN(t)}$ at $N^* = 1 - \frac{1}{a}$

$$f' = a(1 - 2N(t))$$

$$= a\left[1 - 2\left(1 - \frac{1}{a}\right)\right]$$

$$= a\left[1 - 2 + \frac{2}{a}\right] = \left[\frac{2}{a} - 1\right]a$$

$$= 2 - a$$

(a) for stable asymptote $0 < \lambda < 1$: $\boxed{0 < 2-a < 1} \Rightarrow 1 < a < 2$

$-2 < -a < -1$
$1 < a < 2$

(b) for Stable oscillation $-1 < \lambda < 0$: $\boxed{-1 < 2-a < 0} \Rightarrow 2 < a < 3$

$-3 < -a < -2$
$3 > a > 2$

(c) for unstable oscillation $\lambda < -1$: $\boxed{2-a < -1} \Rightarrow a > 3$

$-a < -3$
$a > 3$

at $N^* = 0$

$f' = a$

So, for $0 < a < 1$ it would be stable asymptote

for $a < 0$, the $N(t+1)$ will become -ve hence not considering these value.

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3. $N(t+1) = N(t)\, e^{r\left(\frac{k - N(t)}{k}\right)}$

$N(t+1) = N(t) = N^*$

$1 = e^{r\left(1 - \frac{N^*}{k}\right)} = e^0$

Comparing powers.

$N^* = 0, k$

$$f = \frac{dN(t+1)}{dN(t)} = e^{r\left(\frac{k-N(t)}{k}\right)} + N(t)\, e^{r\left(\frac{k-N(t)}{k}\right)} r\left(\frac{-1}{k}\right)$$

$$= e^{r\left(\frac{k-N(t)}{k}\right)}\left[1 - \frac{r N(t)}{k}\right]$$

at $N(t) = k$ only to consider k as $N(t) = 0$ is no population

$$= e^0\left[1 - \frac{r \times k}{k}\right]$$

$$= 1 - r$$

So from condition, at $N^* = k$

when $-1 < 1 - r < 0 \Rightarrow 1 < r < 2$

exhibit oscillation.

also when $1 - r < -1 \Rightarrow r > 2$

it also exhibit oscillation

at $N^* = 0$ No oscillation would take place for $N^* = 0$ as e^r will be always +ve

$2N^* n t)$

$+ \frac{2}{a}$

a^2

$+ \frac{2nt}{a}$

$nt)$

$2nt)$

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4. $$N(t) = \frac{k}{1+Ce^{-rt}}$$ when $$C = \frac{k - N(0)}{N(0)}$$

$$1 + Ce^{-rt} = \frac{k}{N(t)}$$

$$e^{-rt} = \frac{1}{C}\left(\frac{k}{N(t)} - 1\right)$$

$$-rt = \log\left(\frac{k}{CN(t)} - \frac{1}{C}\right)$$

$$\cancel{t = \frac{1}{r}\log\left(\frac{CN(t)}{k - N(t)}\right)}$$

$$t = \frac{-1}{r}\log\left(\frac{k - N(t)}{CN(t)}\right)$$

$$t+1 = \frac{-1}{r}\log\left(\frac{k - N(t+1)}{CN(t+1)}\right)$$

$$1 = \frac{-1}{r}\log\left(\frac{k - N(t+1)}{CN(t+1)}\right) + \frac{1}{r}\log\left(\frac{k - N(t)}{CN(t)}\right)$$

$$r = \log\left(\frac{\frac{k - N(t)}{\cancel{C}N(t)}}{\frac{k - N(t+1)}{\cancel{C}N(t+1)}}\right)$$

$$e^{r}\left(\frac{k}{N(t+1)} - 1\right) = \frac{k}{N(t)} - 1$$

$$\frac{ke^{r}}{N(t+1)} - e^{r} = \frac{k}{N(t)} - 1$$

$$\frac{\cancel{1}e^{r}}{N(t+1)} - \frac{\cancel{1}}{N(t)} = \frac{(e^{r}-1)}{K}$$

$$\frac{e^{r}}{N(t+1)} = \frac{1}{N(t)} + \frac{(e^{r}-1)}{K}$$

$$\frac{1}{N(t+1)} = \frac{K + N(t)(e^{r}-1)}{K e^{r} N(t)}$$

$$\boxed{\therefore N(t+1) = \frac{K e^{r} N(t)}{K + N(t)(e^{r}-1)}}$$