Life Table

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Life Table

Life tables are a fundamental tool in survival analysis used to summarize the survival patterns of a population. They provide a method for estimating survival probabilities over discrete time intervals.

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Table 46 LIFE TABLE FOR NON-WHITE MALES IN THE UNITED STATES, 1959-61

Year of age	Mortality rate: No. dying per 1000 alive at beginning of year of age	Of 100,000 Males born alive: No. living at beginning of year of age	Of 100,000 Males born alive: No. dying during year of age	Total no. of years lived by each generation of 100,000 males: In year of age	Total no. of years lived by each generation of 100,000 males: In year of age and all later years	Average future lifetime: Average no. of years of life remaining at beginning of year of age
(1)	(2)	(3)	(4)	(5)	(6)	(7)
x to x+1	$1000q_x$	l_x	d_x	L_x	T_x	$\overset{o}{e}_x$
0-1	46.99	100,000	4,699	96,254	6,148,198	61.48
1-2	3.77	95,301	321	95,141	6,051,944	63.50
2-3	1.97	94,980	187	94,887	5,956,803	62.72
3-4	1.34	94,793	127	94,729	5,861,916	61.84
4-5	1.02	94,666	96	94,618	5,767,187	60.92
5-6	.87	94,570	83	94,528	5,672,569	59.98
6-7	.76	94,487	72	94,451	5,578,041	59.03
7-8	.68	94,415	64	94,383	5,483,590	58.08
8-9	.63	94,351	60	94,321	5,389,207	57.12
9-10	.60	94,291	57	94,263	5,294,886	56.15
10-11	.60	94,234	57	94,206	5,200,623	55.19
20-21	2.36	93,108	219	92,998	4,262,592	45.78
30-31	3.89	90,270	351	90,094	3,344,751	37.05
40-41	7.49	85,744	643	85,423	2,462,516	28.72
50-51	15.65	77,239	1,208	76,636	1,643,592	21.28
60-61	31.37	61,669	1,934	60,702	943,071	15.29
70-71	56.90	39,914	2,271	38,778	431,391	10.81
75-76	66.73	29,064	1,939	28,095	259,534	8.93
80-81	[illegible]	[illegible]	1,767	19,110	137,352	6.87
[illegible]	[illegible]	[illegible]	1,427	10,906	58,993	5.08
90-91	203.04	5,174	1,050	4,650	17,697	3.42

Life table

- The Life table method is very useful for a large sample, but the estimated results will depend on the chosen interval length.
- The larger the interval, the poorer the estimations.
- You should apply the Kaplan-Meier method if the sample is small.

Life Table

To construct a life table, two things are required

- Population living at all individuals ages in a selected year

- Number of deaths that occurred in these ages during the selected year

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Life Table

Person Years

- It is the sum of the number of years that each member in the study population is under observation

- The individuals are observed for different periods of time, the unit used for counting the observation time is person-year

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Key Components of Life Table

l_x: Number of individuals surviving to the start of the age interval x.

d_x: Number of individuals dying during the age interval x.

q_x: Probability of dying during the age interval x.

p_x: Probability of surviving through the age interval x.

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Life table

- observed data: D_x and P_x
- mortality rates: $M_x = D_x/P_x$
- mortality probabilities: $q_x = M_x/(1 + 0.5\, M_x)$
- survival probabilities: $p_x = 1 - q_x$
- number surviving: $l_x = l_{x-1}\, p_{x-1}$
- number dying: $d_x = l_x - l_{x+1}$

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Given data

Age	Deaths	Population
x	D_x	P_x
0	D_0	P_0
1	D_1	P_1
2	D_2	P_2
:	:	:
99	D_{99}	P_{99}
100 (=100+)	D_{100}	P_{100}

Life table, step 1: UK males 1998, numbers

Age	Deaths	Population
x	D_x	P_x
0	2327	364800
1	190	375100
2	117	369000
:	:	:
99	215	561
100	319	678

Life table, step 2: compute observed mortality rates by age, M_x

Mortality rate at age x

= Deaths at age x/Population at risk at age x

Deaths are recorded in period-age age-time spaces

Population at risk

either mid-year population aged x

or average of start and end of year population aged x

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Life table, step 2: compute observed mortality rates by age, M_x, variables

Age	Deaths	Population	Mortality rates
x	D_x	P_x	$M_x=D_x/P_x$
0	D_0	P_0	$M_0=D_0/P_0$
1	D_1	P_1	$M_1=D_1/P_1$
2	D_2	P_2	$M_2=D_2/P_2$
:	:	:	:
99	D_{99}	P_{99}	$M_{99}=D_{99}/P_{99}$
100 (=100+)	D_{100}	P_{100}	$M_{100}=D_{100}/P_{100}$

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Life table, step 2: compute observed mortality rates by age, M_x, UK males 1998, numbers

Age	Deaths	Population	Mortality rate
x	D_x	P_x	M_x
0	2327	364800	0.006379
1	190	375100	0.000507
2	117	369000	0.000317
:	:	:	:
99	215	561	0.382851
100	319	678	0.469674

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Life table, step 3: compute the probabilities of dying between age x and x+1, q_x

The equation

$$q_x = M_x/(1 + 0.5M_x)$$

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Life table, step 3: compute the probabilities of dying between age x and x+1, q_x, variables

Age	Mortality rate	Mortality probability
x	M_x	q_x
0	M_0	$q_0=M_0/(1+0.5M_0)$
1	M_1	$q_1=M_1/(1+0.5M_1)$
2	M_2	$q_2=M_2/(1+0.5M_2)$
:	:	
99	M_{99}	$q_{99}=M_{99}/(1+0.5M_{99})$
100 (=100+)	M_{100}	q_{100}= 1.0000

Life table, step 3: compute the probabilities of dying between age x and x+1, q_x, UK males 1998

Age	Mortality rate	Mortality probability
x	M_x	q_x
0	0.006379	0.006342
1	0.000507	0.000506
2	0.000317	0.000317
:	:	:
99	0.382851	0.321338
100 (=100+)	0.469674	1.000000

Life table, step 4: compute the probabilities of surviving from age x to x+1, p_x

Probability of survival from age x to x+1, p_x

$$p_x = (1 - q_x)$$

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Life table, step 4: compute the probabilities of surviving from age x to x+1, p_x, UK males 1998

Age	Mortality rate	Mortality probability	Survival probability
x	M_x	q_x	p_x
0	0.006379	0.006342	0.993658
1	0.000507	0.000506	0.999494
2	0.000317	0.000317	0.999683
:	:	:	:
99	0.382851	0.321338	0.678662
100	0.469674	1.000000	0.000000

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Life table, step 5: compute the numbers surviving to age x, l_x - concept

- Life tables have a **radix** (number base) = **hypothetical constant number born each year** into a stationary population
- **Usually radix = 100000** but you can use 10000 or 1000000 or 1 (in this case the survivors variable has a probability interpretation)
- **l_x = number of survivors of birth cohort** who have attained age x (exact age or birthday)
- The number surviving to age x is the number surviving to age x-1 times the probability of surviving from age x-1 to age x

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Life table, step 5: compute the numbers surviving to age x, l_x - formulae

- $l_x = l_{x-1} \times p_{x-1}$
  - e.g. $l_2 = l_1 p_1$
- We can include prior equations to obtain an expression for l_x which is linked to the radix
- $l_x = l_0 p_0 \times p_1 \times \ldots \times p_{x-1} = l_0 \times \Pi_{y=0,x-1} p_y$
- A picture can clarify this

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Life table, step 5: compute the numbers surviving to age x, l_x, UK males 1998

Age	Survival probability	Cohort survivors
x	p_x	l_x
0	0.993658	100000
1	0.999494	99366
2	0.999683	99315
:	:	:
99	0.678662	619
100	0.000000	420

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Life table, step 6: compute the numbers dying between ages x and x+1, d_x

- Some of the stationary birth cohort will die between the exact ages
- The number, d_x, is computed from
  - Successive cohort survivors
    - $d_x = l_x - l_{x+1}$
  - Multiplication of cohort survivors by mortality probability
    - $d_x = l_x \times q_x$

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Life table, step 6: compute the numbers dying between ages x and x+1, d_x

Age	Mortality probability	Cohort survivors	Cohort non-survivors
x	q_x	l_x	d_x
0	0.006342	100000	634
1	0.000506	99366	50
2	0.000317	99315	31
:	:	:	:
99	0.321338	619	199
100	1.000000	420	420

Life table, summary of formulae

- Step 1, observed data: D_x and P_x
- Step 2, mortality rates: $M_x = D_x/P_x$
- Step 3, mortality probabilities: $q_x = M_x/(1 + 0.5\ M_x)$
- Step 4, survival probabilities: $p_x = 1 - q_x$
- Step 5, number surviving: $l_x = l_{x-1}\ p_{x-1}$
- Step 6, number dying: $d_x = l_x - l_{x+1}$

Example_ Life Table

Age Group (x to x+5)	Number of Deaths (Dx)	population (Px)	Mortality rates (Mx)	Mortality probability (qx)	Survival probability (px)	Cohort survivors (lx)	Cohort non-survivors (dx)
0-4	2,500	36480	0.068531	0.06626	0.93374	100000	6626.027
5-9	1700	37510	0.045321	0.044317	0.955683	93373.97	4138.054
10-14	1200	36900	0.03252	0.032	0.968	89235.92	2855.549
15-19	700	35000	0.02	0.019802	0.980198	86380.37	1710.502
20-24	600	34600	0.017341	0.017192	0.982808	84669.87	1455.642
25-29	250	28750	0.008696	0.008658	0.991342	98280.8	850.916

Thank you