

Statistical Inference Course Project Part1

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Part ONE

The project consists of two parts:

- **A simulation exercise.**
- Basic inferential data analysis.

investigate the exponential distribution in R and compare it with the Central Limit Theorem

The exponential distribution can be simulated in R with `rexp(n, lambda)` where `lambda` is the rate parameter.

The mean of exponential distribution is $1/\lambda$ and the standard deviation is also $1/\lambda$.

Set $\lambda = 0.2$ for all of the simulations. You will investigate the distribution of averages of 40 exponentials.

Note that you will need to do a thousand simulations.

Illustrate via simulation and associated explanatory text the properties of the distribution of the mean of 40 exponentials. You should

- Show the sample mean and compare it to the theoretical mean of the distribution.
- Show how variable the sample is (via variance) and compare it to the theoretical variance of the distribution.
- Show that the distribution is approximately normal.

install needed packages

```
library(cowplot)
```

```
##
## *****
```

```
## Note: As of version 1.0.0, cowplot does not change the
```

```
## default ggplot2 theme anymore. To recover the previous
```

```
## behavior, execute:  
## theme_set(theme_cowplot())
```

```
## *****
```

```
library(ggplot2)
```

assigning values

```
lambda <- 0.2  
sample_num <- 40  
sim_num <- 1000
```

calculate the 40 samples 1000 times

```
set.seed(2) #for reproducibility purposes  
mean_sim_num <- replicate(1000, mean(rexp(sample_num, rate = lambda)))
```

Show the sample mean and compare it to the theoretical mean of the distribution.

The mean of exponential distribution is $1/\lambda$

```
1/lambda
```

```
## [1] 5
```

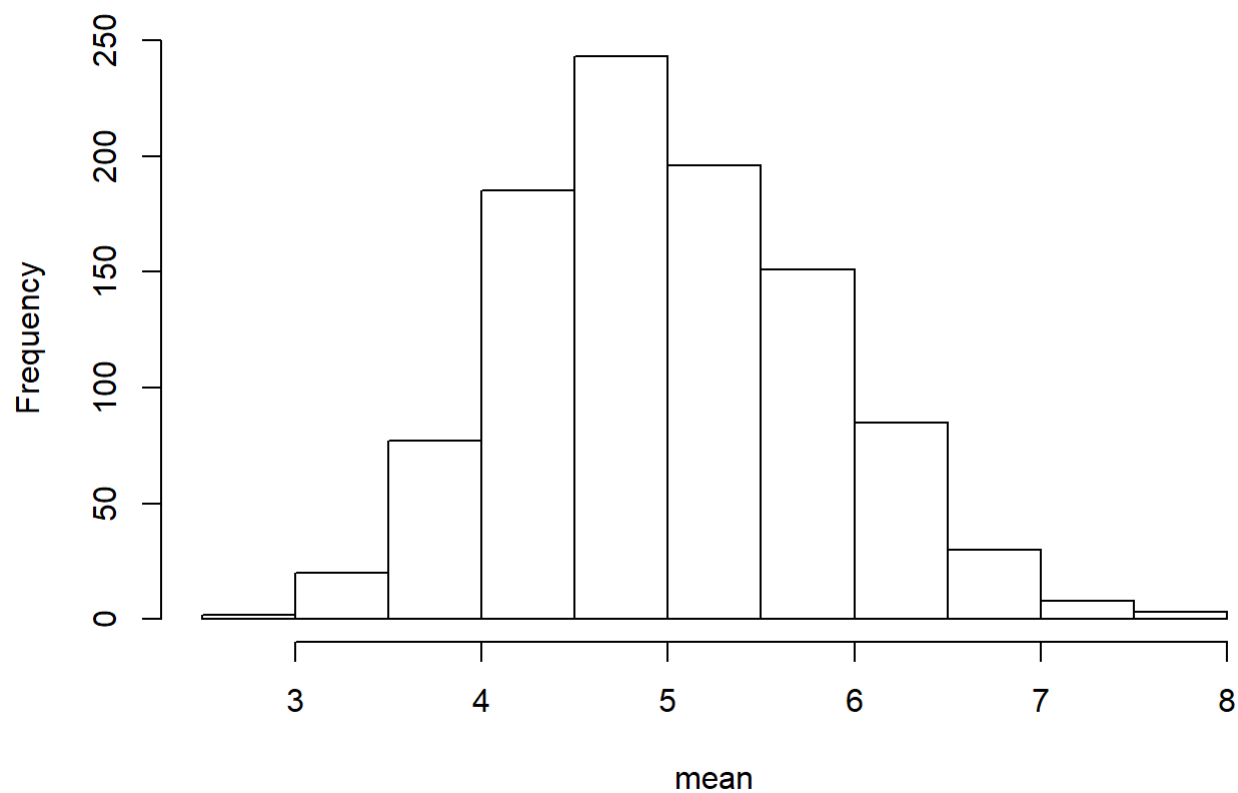
```
# The mean othe 40 samples 1000 times  
mean(mean_sim_num)
```

```
## [1] 5.016356
```

visualization

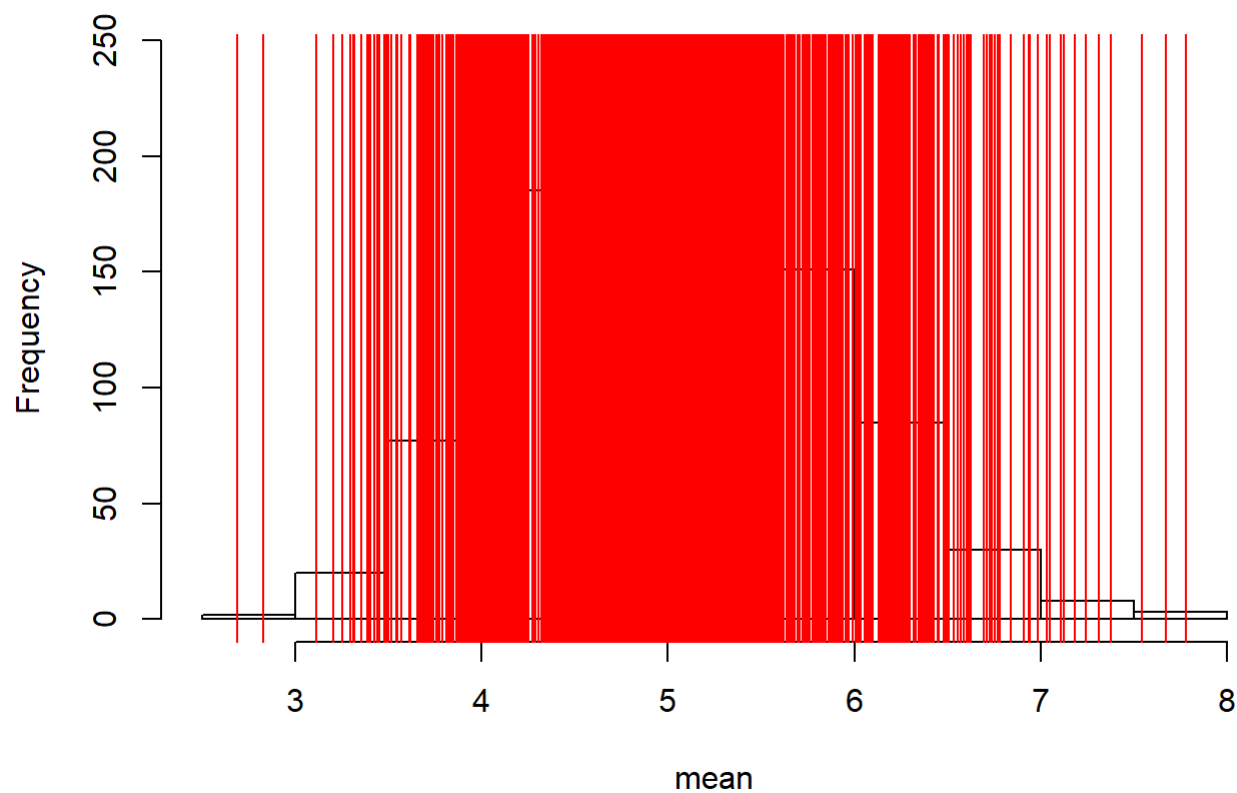
```
hist(mean_sim_num, xlab = "mean", main = "Exponential Function Simulations")
```

Exponential Function Simulations



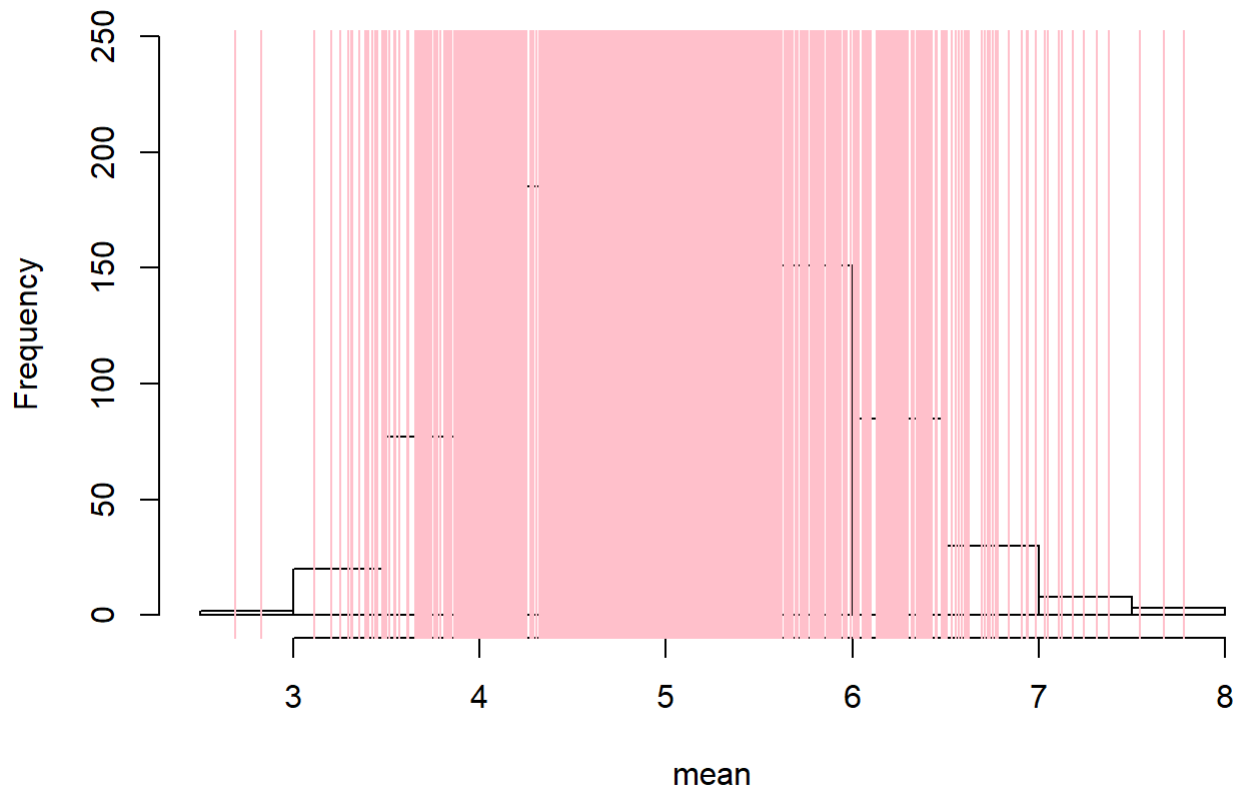
```
hist(mean_sim_num, xlab = "mean", main = "Exponential Function Simulations")  
abline(v = mean_sim_num, col = "red")
```

Exponential Function Simulations



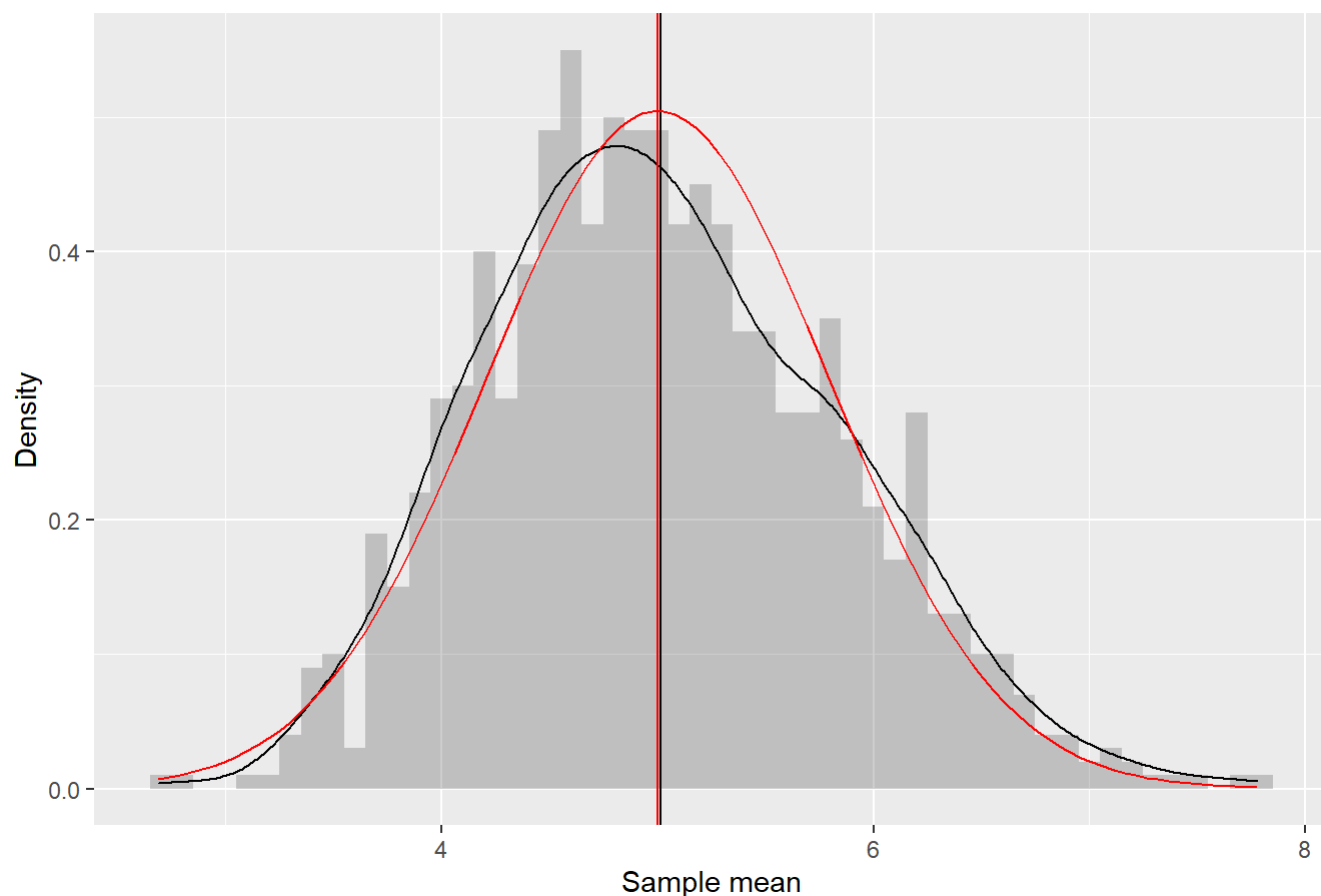
```
hist(mean_sim_num, xlab = "mean", main = "Exponential Function Simulations")
abline(v = mean_sim_num, col = "pink")
```

Exponential Function Simulations



```
ggplot(data = as.data.frame(mean_sim_num), aes(x = mean_sim_num)) +
  geom_histogram(binwidth = 0.1, aes(y = ..density..), alpha = 0.3) +
  geom_density(color = 'black') + #density curve of the sample distribution
  geom_vline(xintercept = mean(mean_sim_num), color = 'black') +
  stat_function(fun = dnorm, args = list(mean = 1/lambda, sd = 1/lambda/sqrt(sample_num)),
  color = 'red') +
  geom_vline(xintercept = 1/lambda, color = 'red') +
  xlab('Sample mean') +
  ylab('Density') +
  ggtitle('Histogram of 1000 simulations for mean of 40 samples')
```

Histogram of 1000 simulations for mean of 40 samples



Show how variable the sample is (via variance) and compare it to the theoretical variance of the distribution.

standard deviation of distribution

```
sd_dv_dist <- sd(mean_sim_num)
sd_dv_dist
```

```
## [1] 0.818004
```

standard deviation from analytical expression

```
sd_dv_theory <- (1 / lambda) / sqrt(sample_num)
sd_dv_theory
```

```
## [1] 0.7905694
```

variance of distribution

```
variance_dist <- sd_dv_dist^2
variance_dist
```

```
## [1] 0.6691305
```

variance from analytical expression

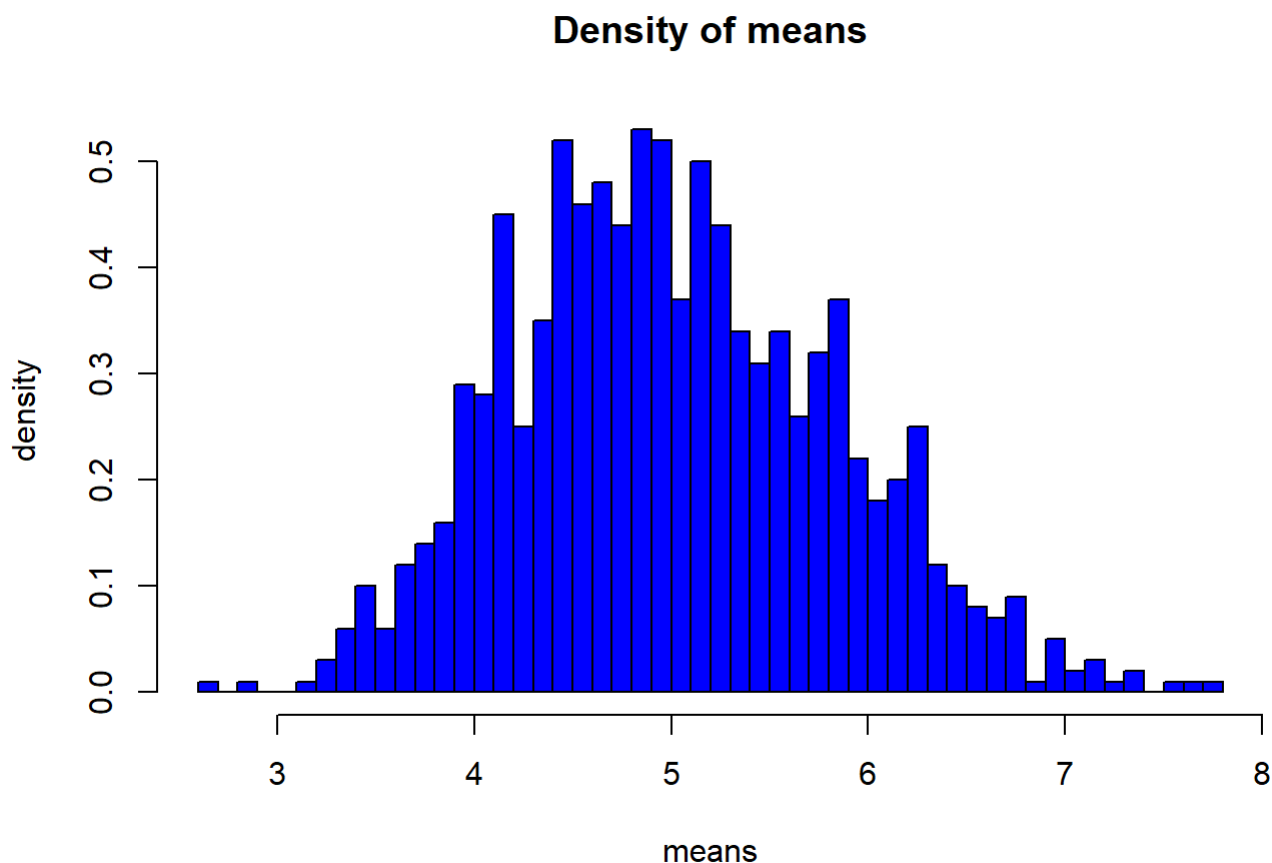
```
variance_theory <- ((1 / lambda)*(1 / sqrt(sample_num)))^2
variance_theory
```

```
## [1] 0.625
```

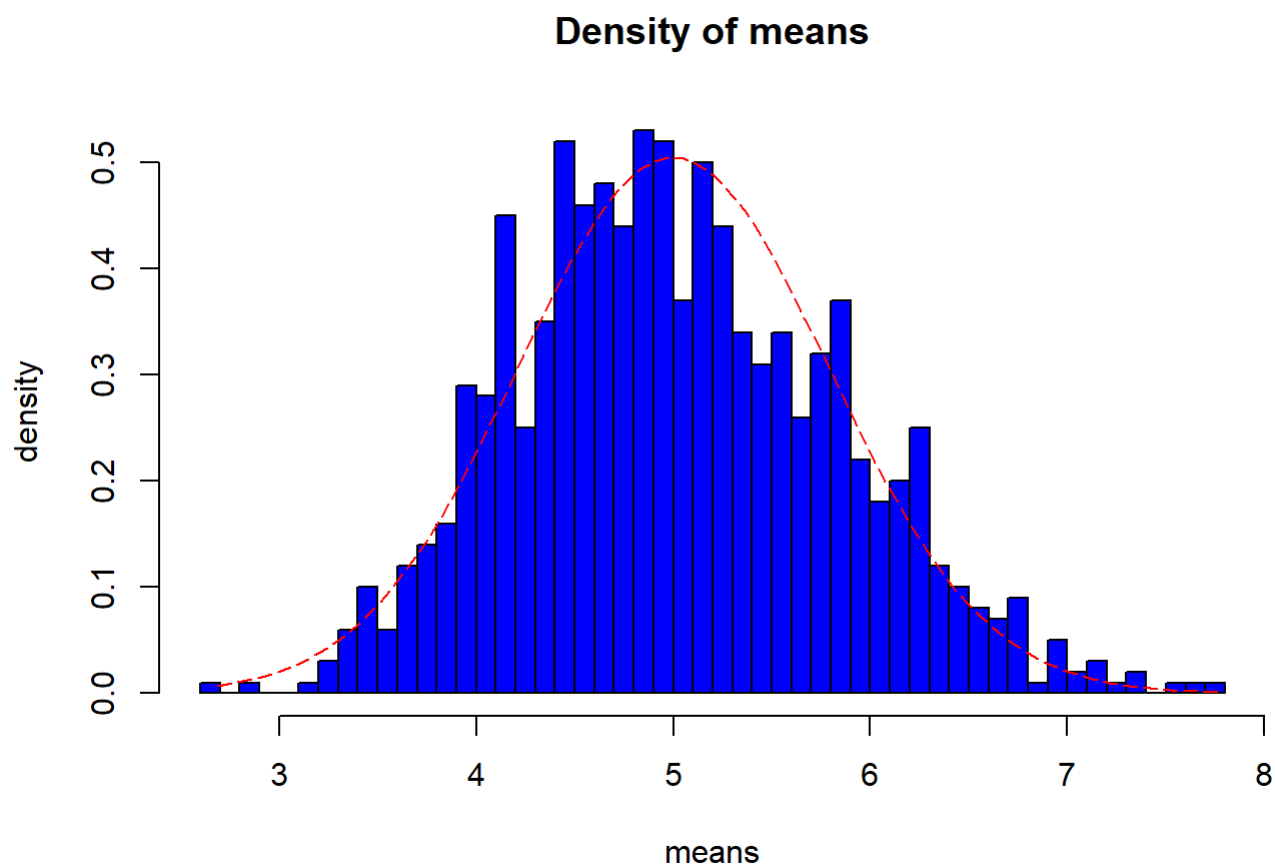
Show that the distribution is approximately normal.

```
xfit <- seq(min(mean_sim_num), max(mean_sim_num), length=100)
yfit <- dnorm(xfit, mean=1/lambda, sd=(1/lambda/sqrt(sample_num)))
```

```
hist(mean_sim_num,breaks=sample_num,prob=T,col="blue",xlab = "means",main="Density of means",ylab="density")
```



```
hist(mean_sim_num,breaks=sample_num,prob=T,col="blue",xlab = "means",main="Density of means",ylab="density")
lines(xfit, yfit, pch=22, col="red", lty=5)
```



compare the distribution of averages of 40 exponentials to a normal distribution

```
qqnorm(mean_sim_num)
qqline(mean_sim_num, col = 2)
```


Normal Q-Q Plot

