

# Lecture 1 : linear regression

By : Khalil idrissi

lecture 0 : intro  
lecture 1 : linear regression  
lecture 2 : SVMs  
lecture 3 : dealing with images  
lecture 4 : Neural network and backprop  
lecture 5 : CNN, transfer learning and behavioral cloning  
lecture 6 : autoencoders and image segmentation  
lecture 7 : object detection  
lecture 8 : RNN ,LSTM, GRU  
lecture 9 : decision trees, random forests, bagging, boosting, stacking  
lecture 10 : Variational AE and GANs  
lecture 11 : representation learning  
lecture 12 : PCA and K-means clustering  
lecture 13 : intro to Reinforcement learning

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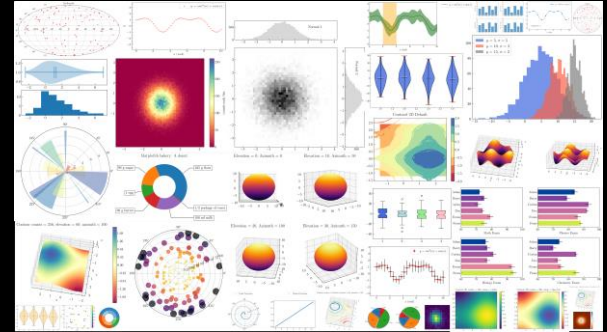
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# Today:

- 1- linear regression using OLS
- 2- linear regression using GD
- 3- working with sci-kit learn, numpy and matplotlib



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# Recap

# Recap

Machine learning



Approach for Ai

# Recap

Machine learning



Approach for Ai

Dataset

# Recap

Machine learning



Approach for Ai

Dataset





# Recap

Machine learning → Approach for Ai

Dataset { Training set  
Testing set

# Recap

Machine learning  Approach for Ai

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$$D = \{(x_i, y_i)\}_{i=1}^N$$

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Machine learning  $\longrightarrow$  Approach for Ai

Dataset  $\left\{ \begin{array}{l} \text{Training set} \\ \text{Testing set} \end{array} \right.$

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if  $y_i$  is real valued variable , then we are talking about regression



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if  $y_i$  is real valued variable , then we are talking about regression

if  $y_i$  is belongs to a set of classes , then we are talking about classification

Remember ?

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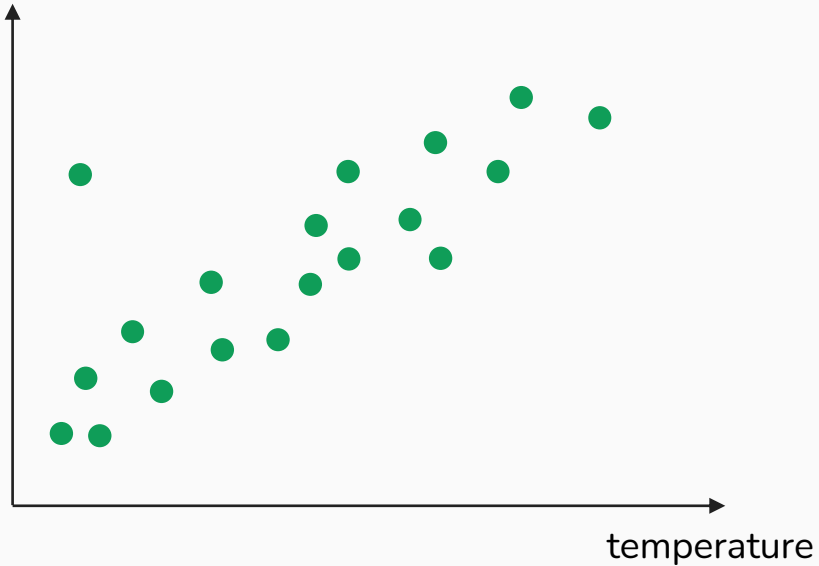


# Remember ?

Linear regression

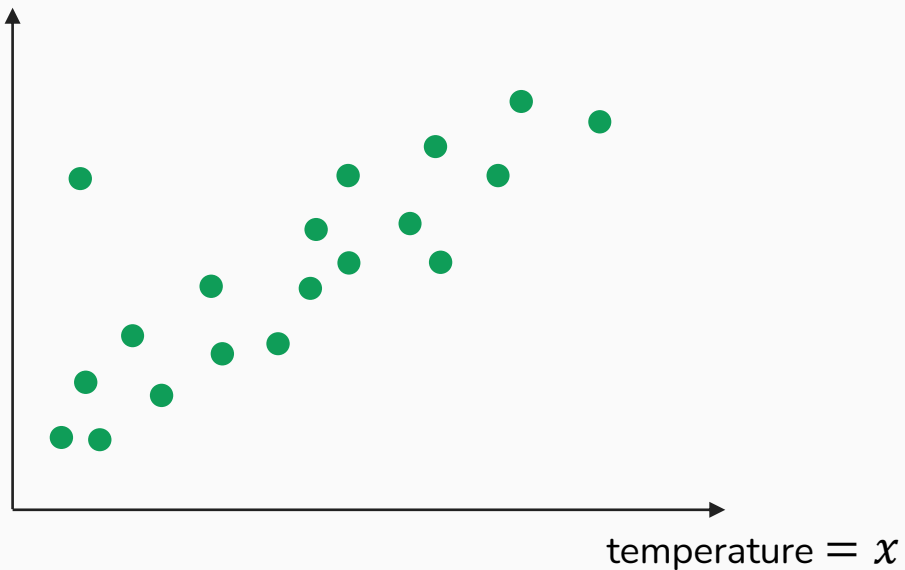
# 1- linear regression

ice cream sales  $= y$



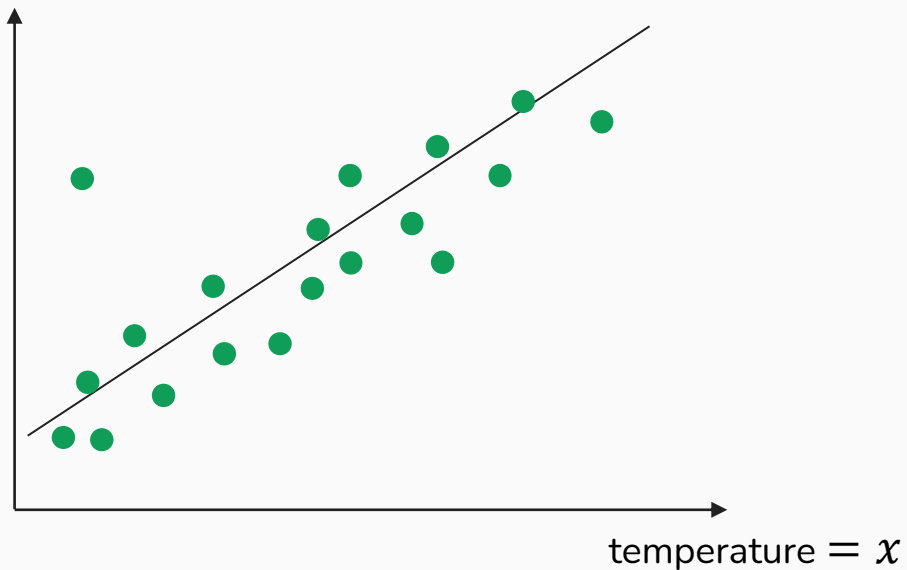
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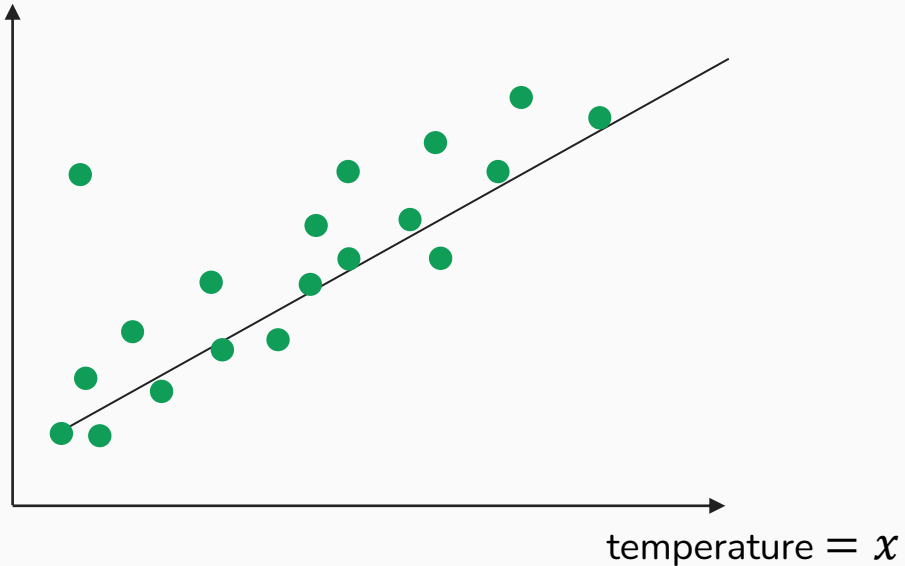
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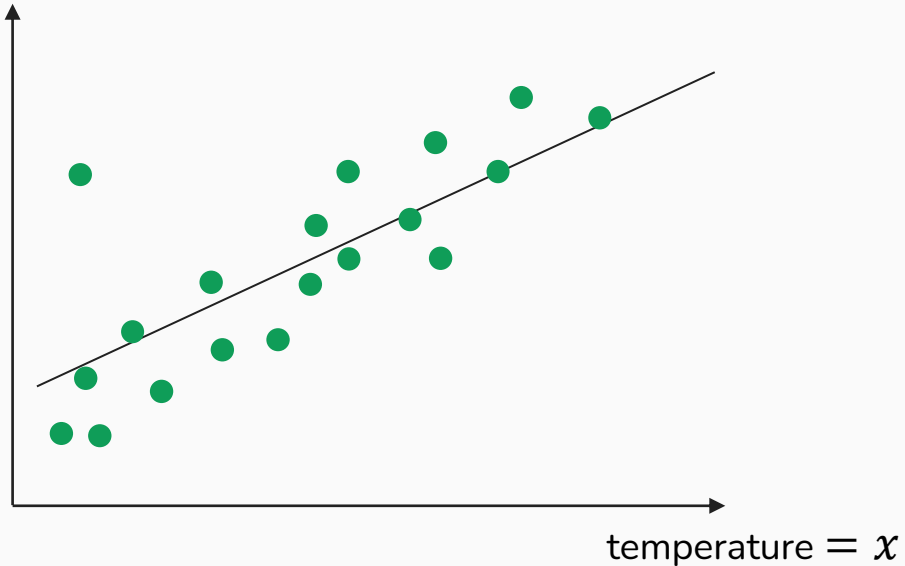
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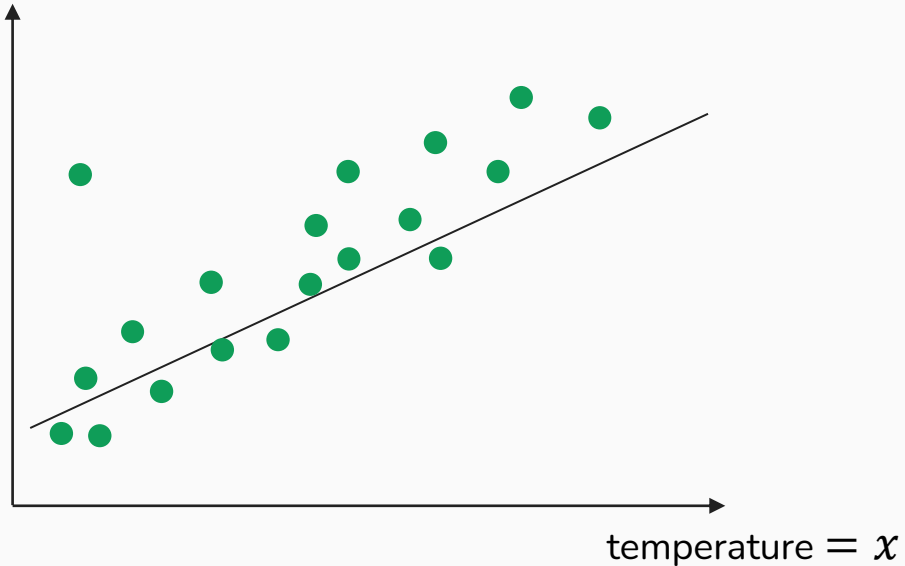
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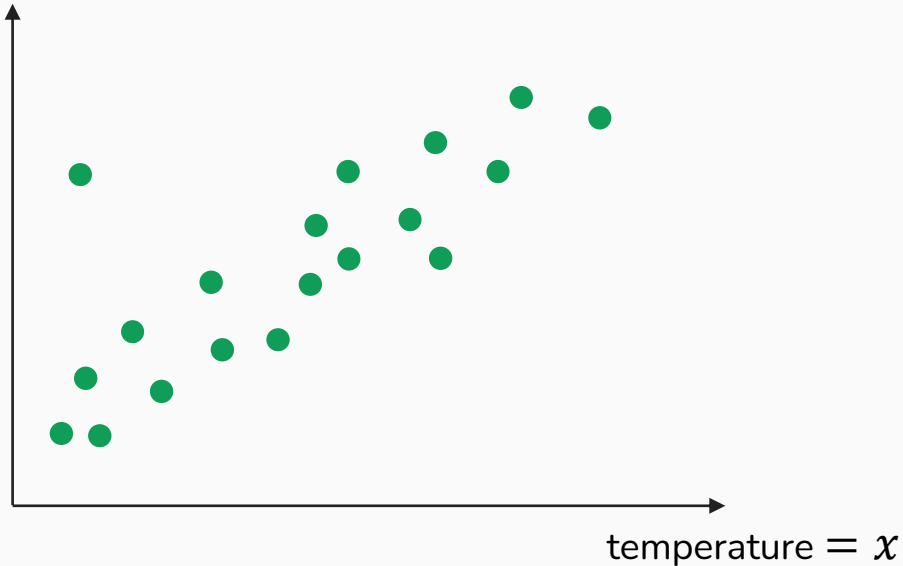
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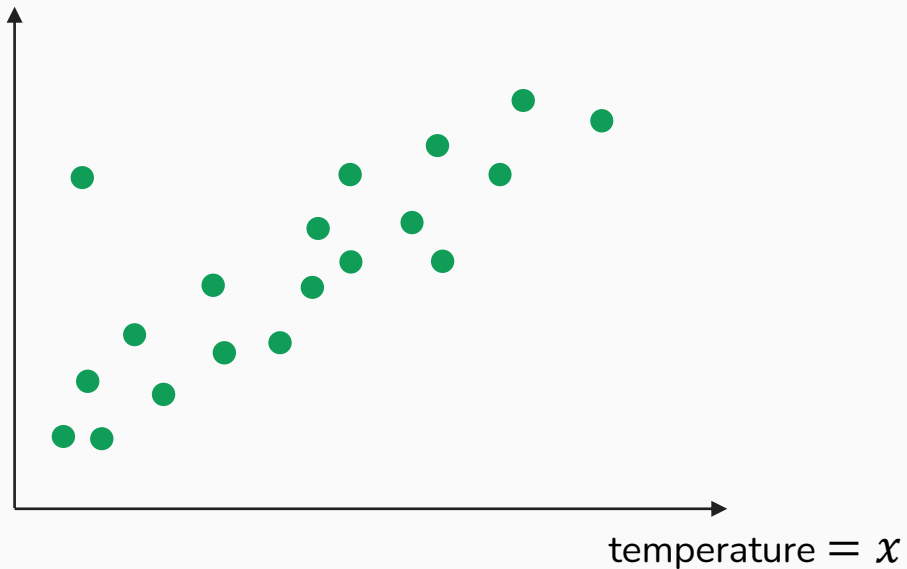
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We need to find the best fit line

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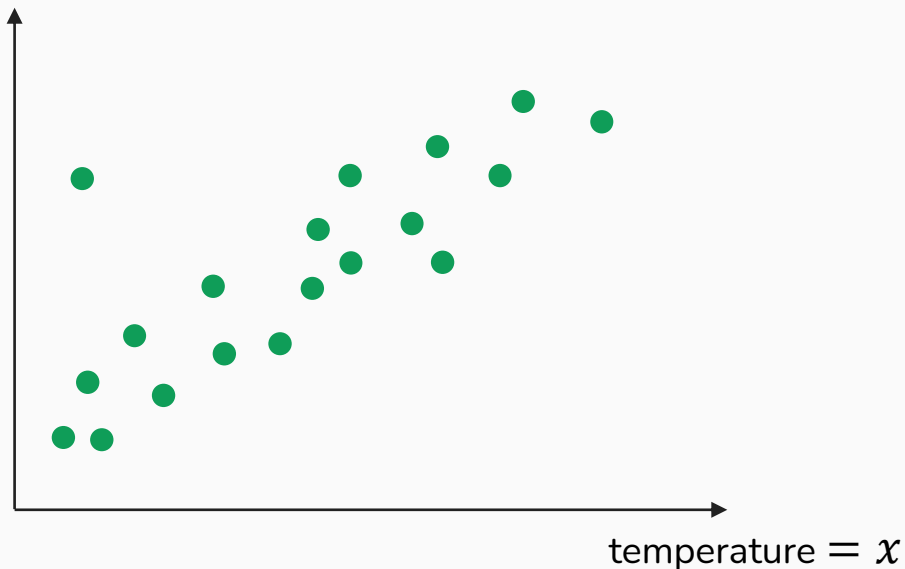
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1- using OLS (Ordinary Least Squares):

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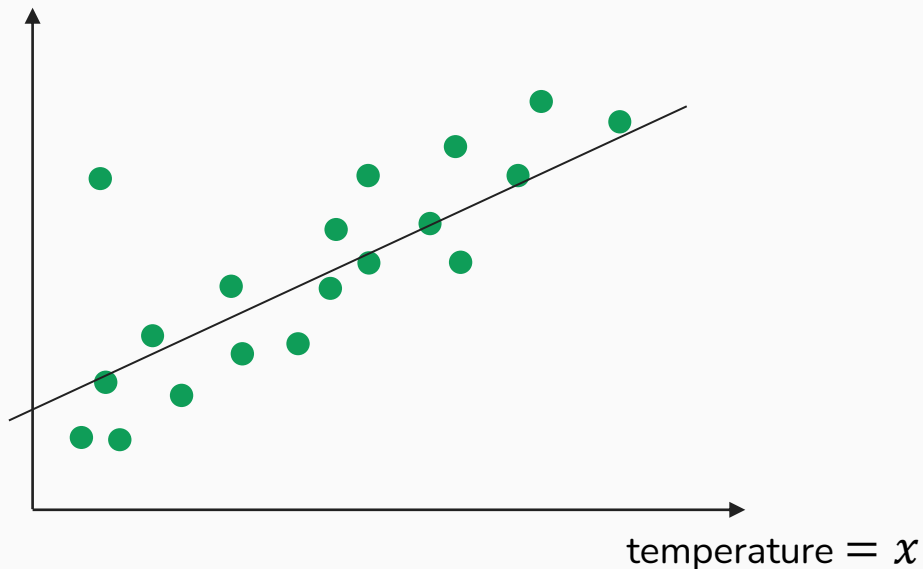


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Find  $a$  and  $b$  that minimizes the sum of squared residuals (SSR):

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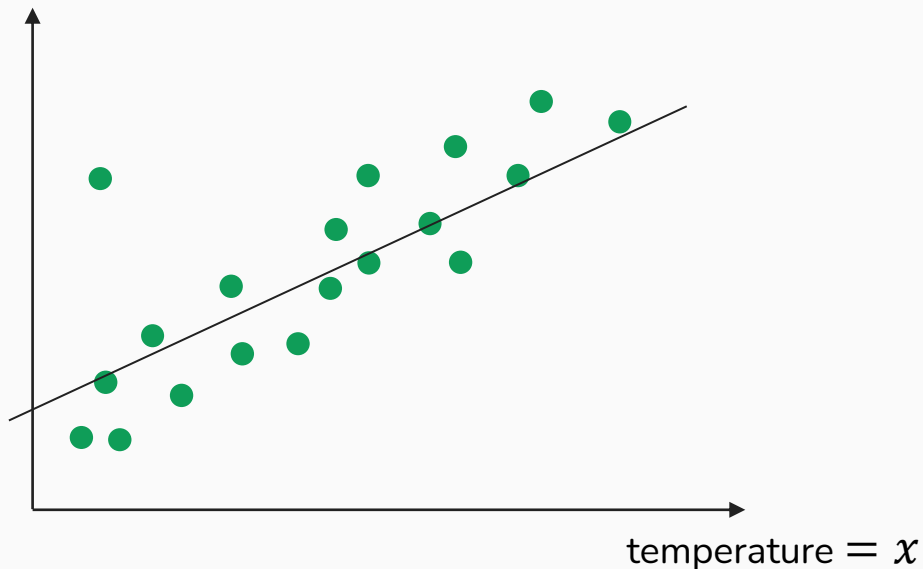


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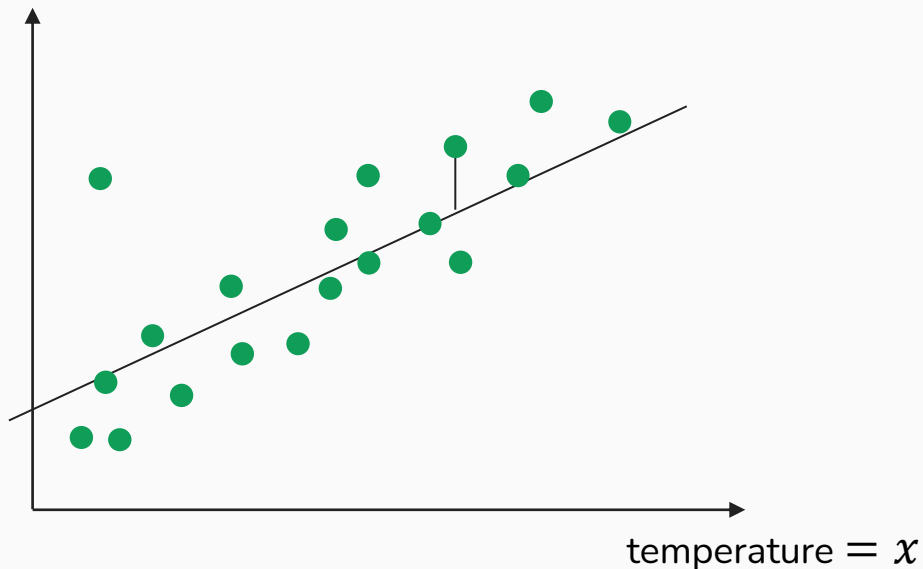
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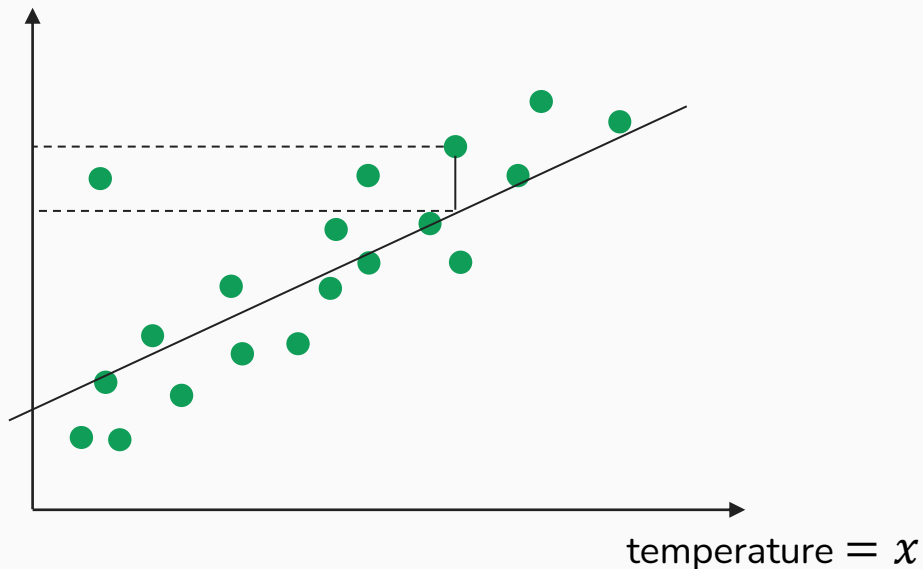
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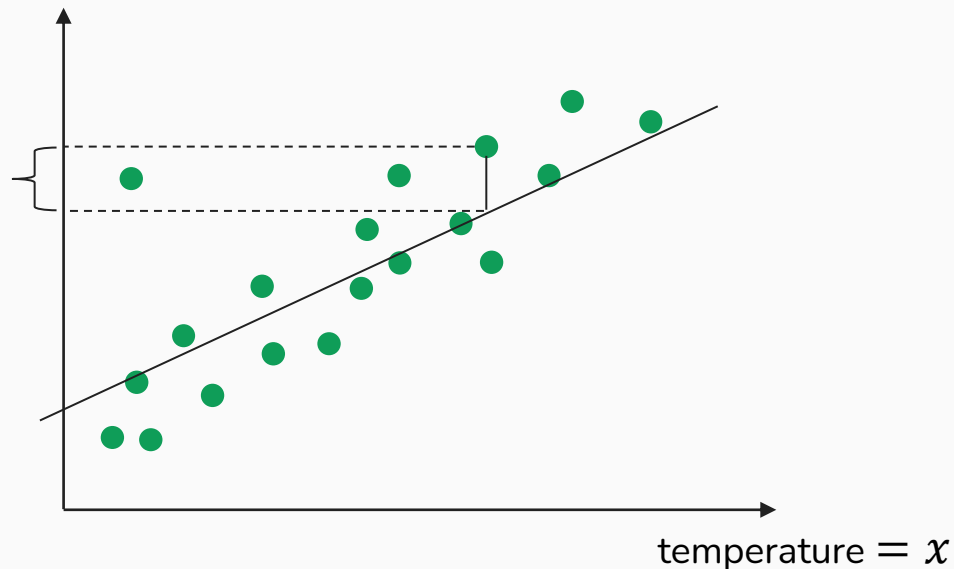
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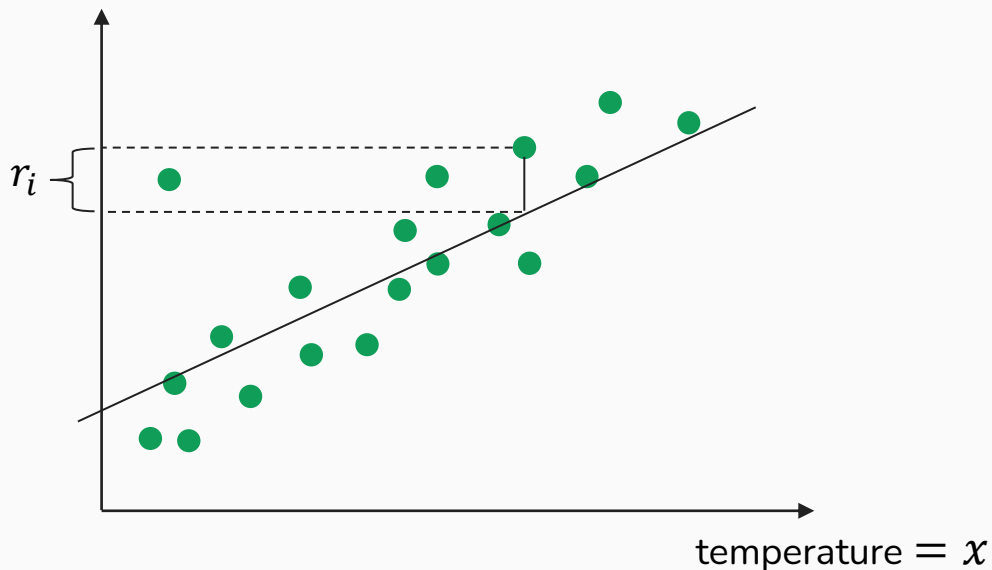
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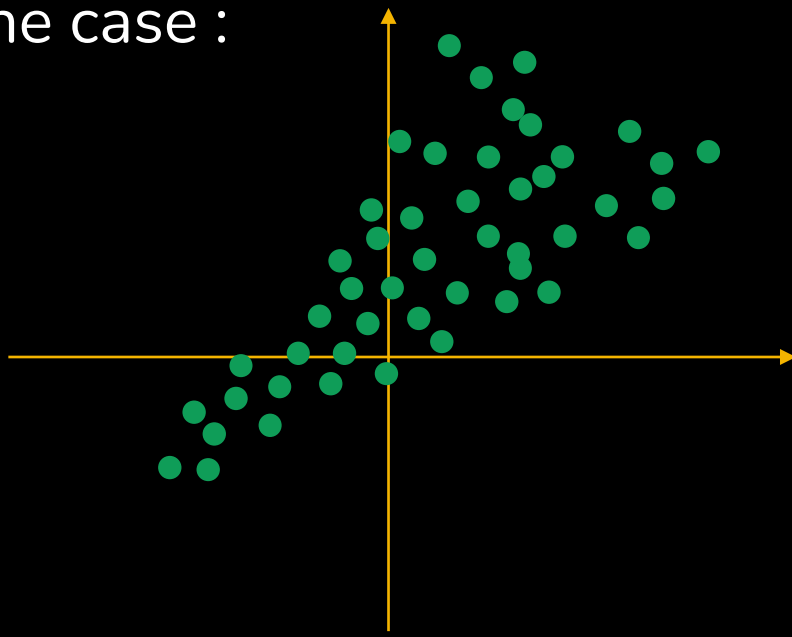
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The predicted line :  $\hat{y}(x) = ax + b + \varepsilon$

We often assume that  $\varepsilon$  has a gaussian distribution :  $N(0, \sigma^2)$

—————→ A constant variance aka homoscedasticity

If it is not the case :



—————→ the linear regression model is likely to give incorrect estimates



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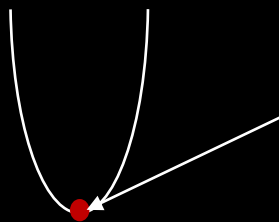
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To minimize  $y = ax^2 + bx + c$  :



$\left( \frac{-b}{2a}, \text{doesn't matter} \right)$

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$$\sum_{i=1}^n (y_i - ax_i)^2 - 2b \sum_{i=1}^n (y_i - ax_i) + b^2 \cdot n \text{ is minimized when } b = \frac{-b'}{2a}$$

À la fin on trouve  $w$  :

$$a = \frac{\sum_{i=0}^n (x_i - \bar{x})(y_i - \bar{y})}{\sum_{i=0}^n (x_i - \bar{x})^2}$$

De même on trouve  $b$  :

$$b = \frac{1}{n} \sum_{i=0}^n y_i - \frac{1}{n} \sum_{i=0}^n a x_i$$

$$\left\{ \begin{array}{l} \bar{x} = \frac{1}{n} \sum_{i=0}^n x_i \\ \bar{y} = \frac{1}{n} \sum_{i=0}^n y_i \end{array} \right.$$

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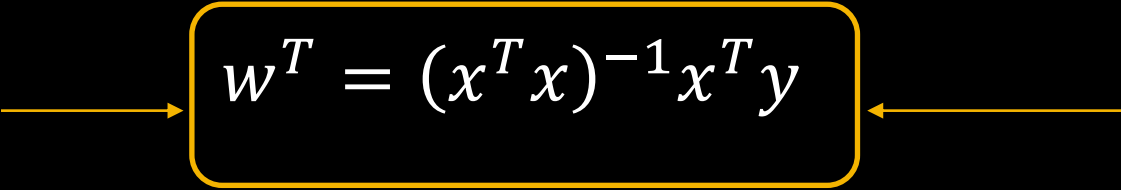
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Closed form equation  
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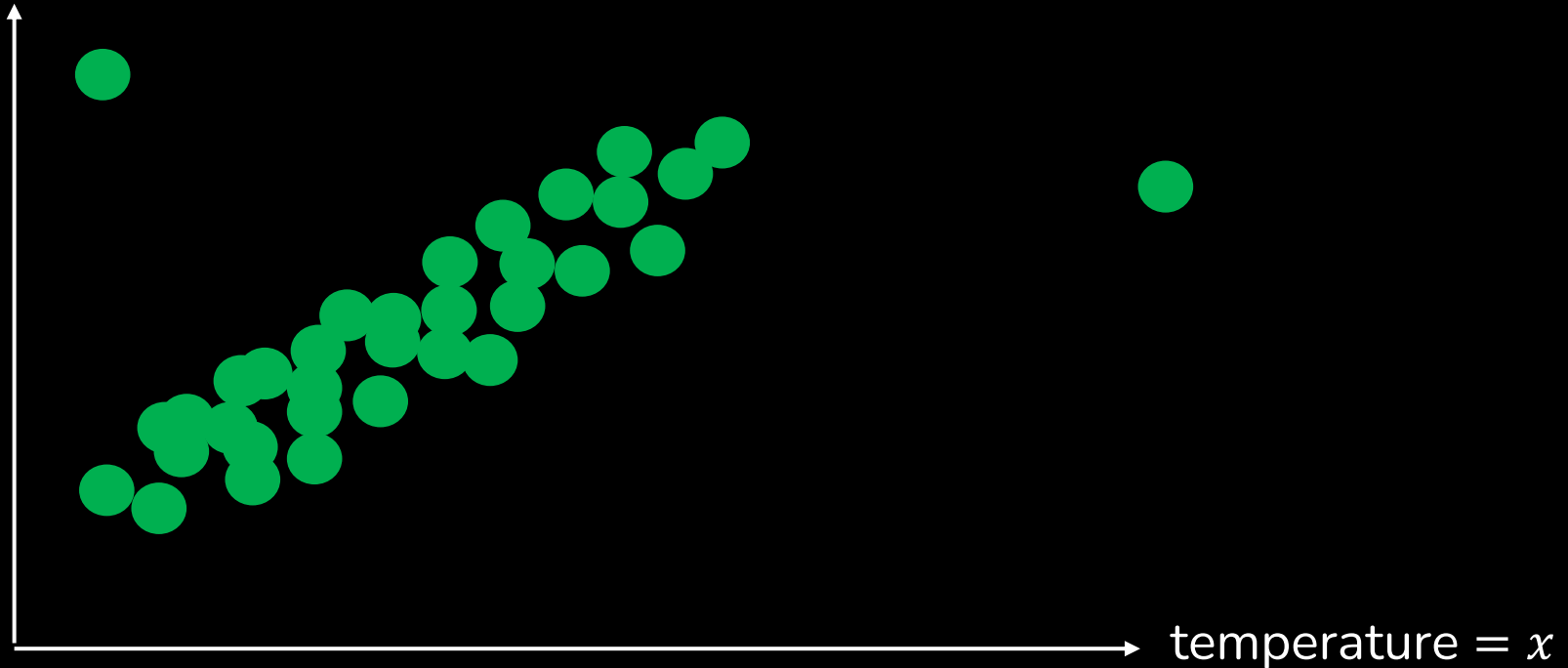
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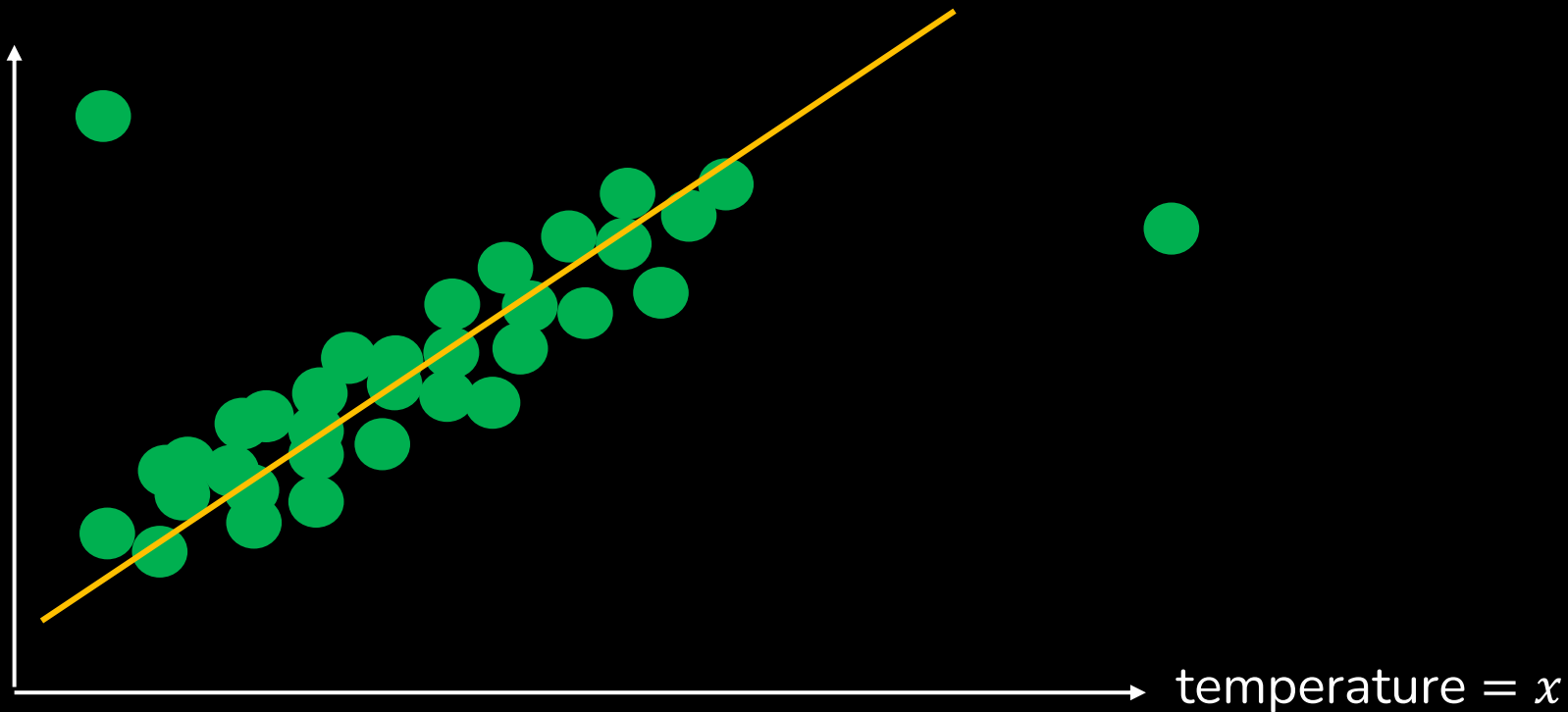
## 2- using gradient descent

ice cream sales =  $y$



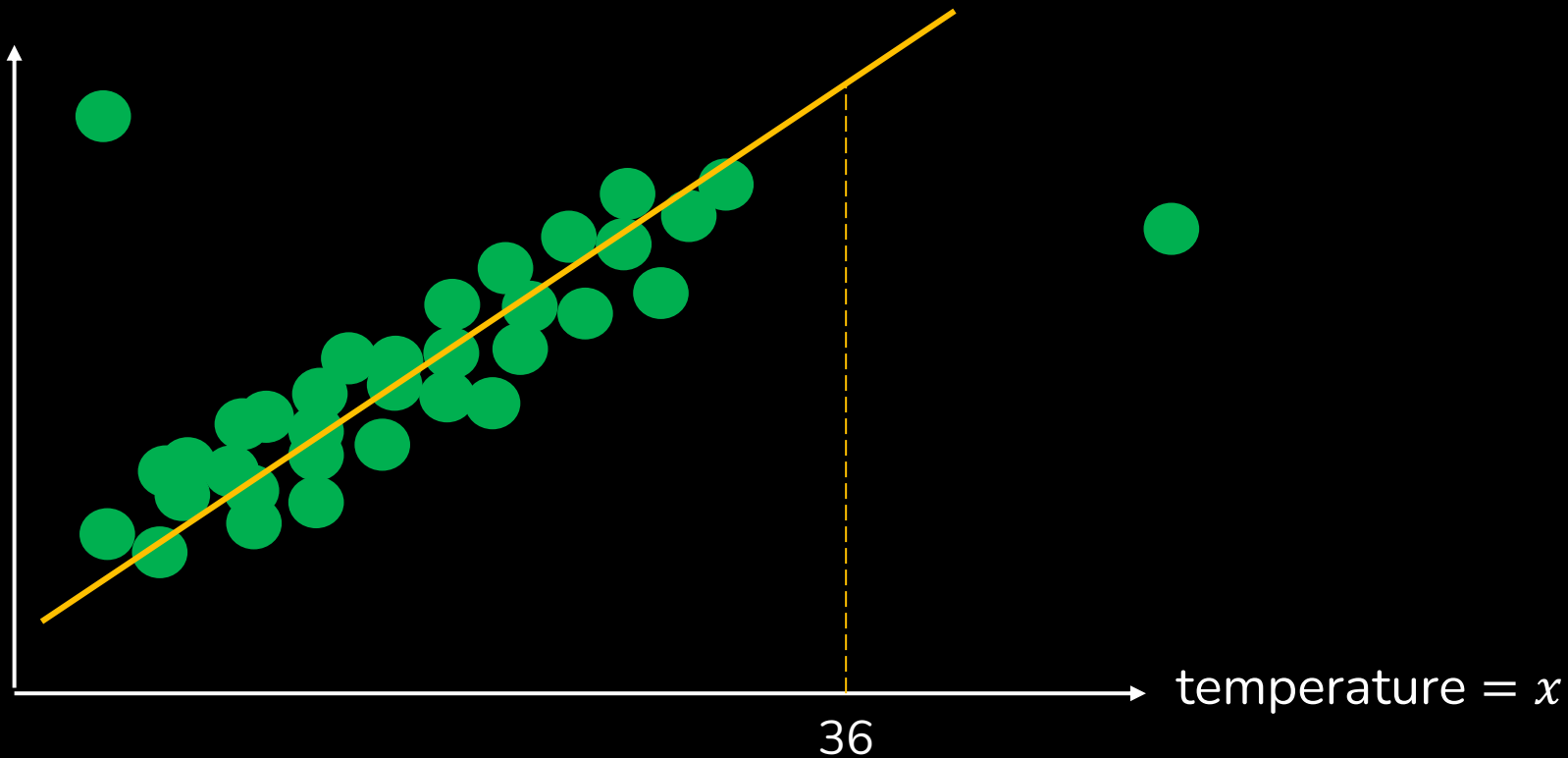
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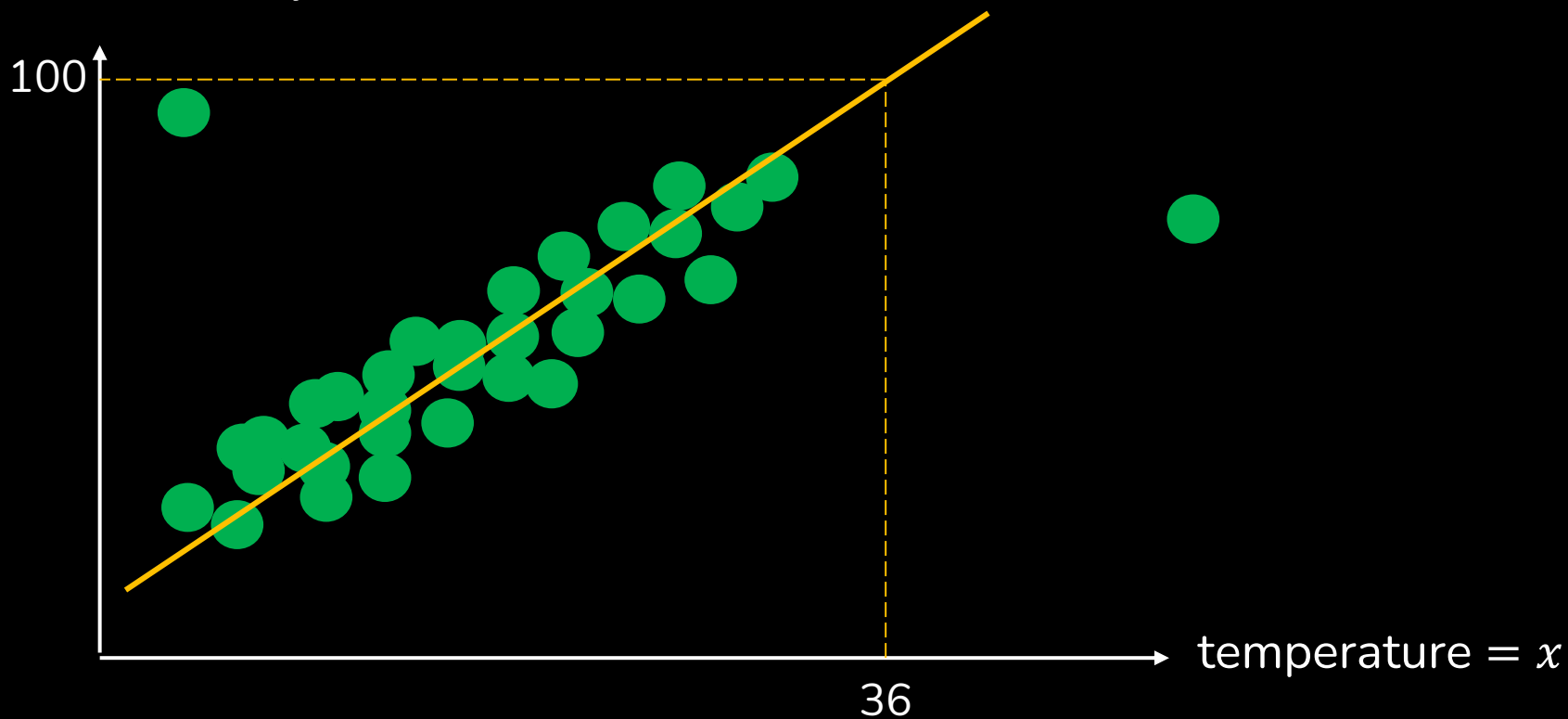
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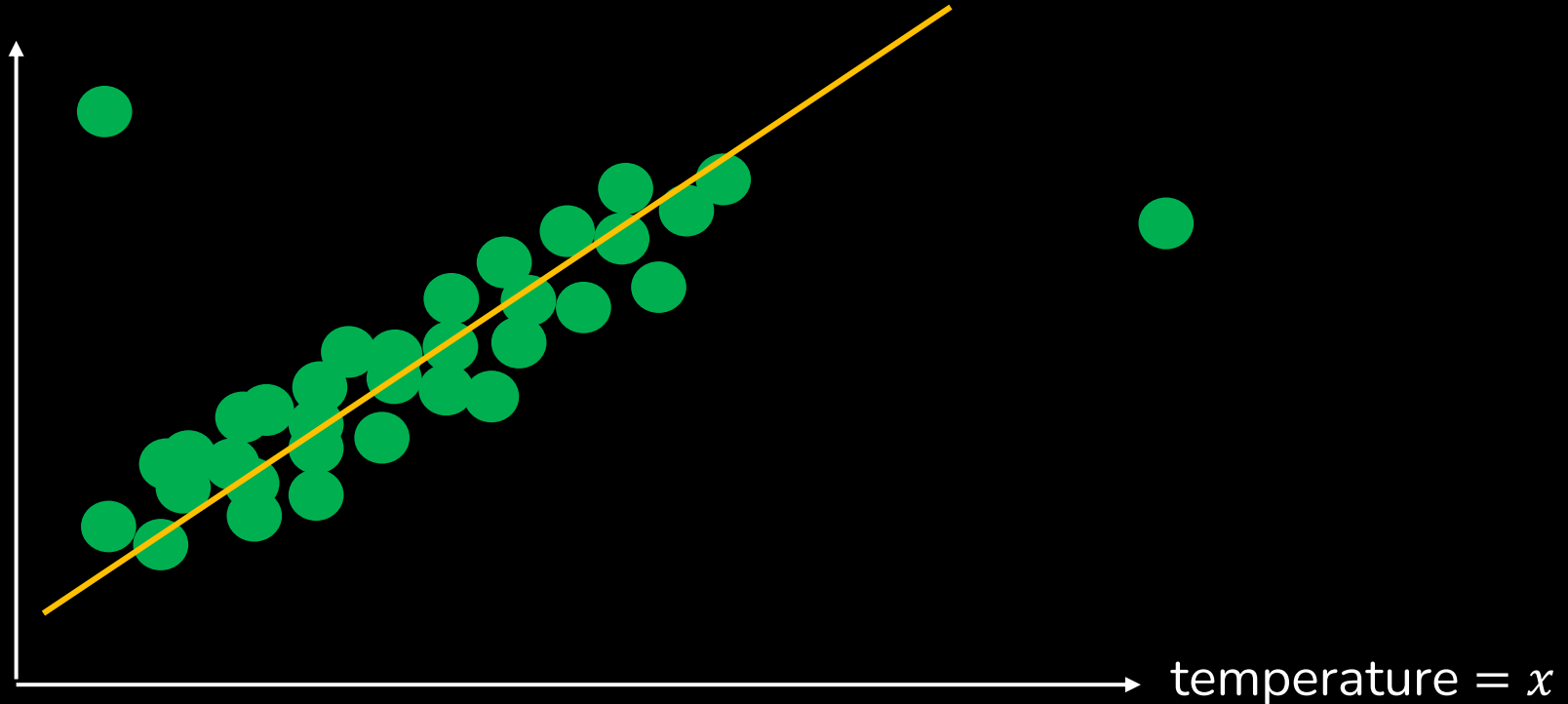
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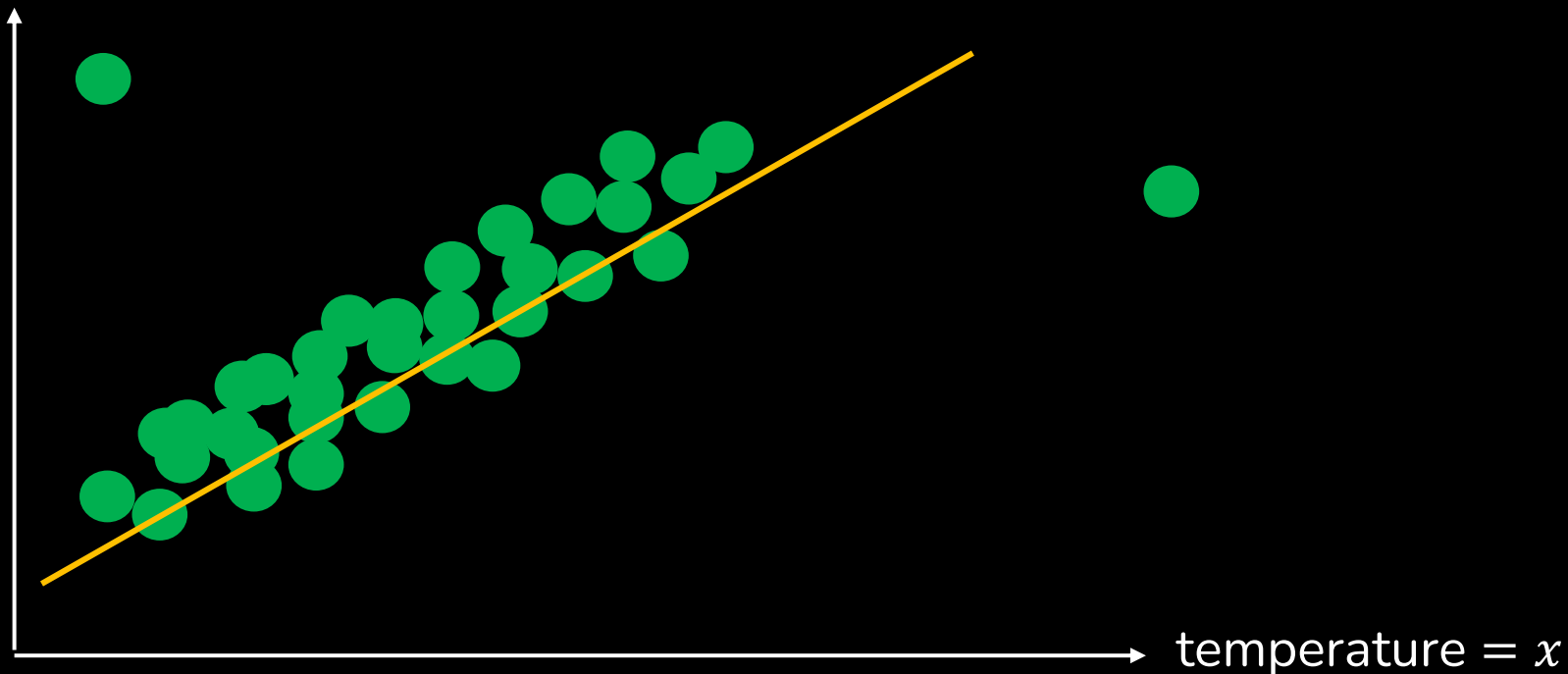
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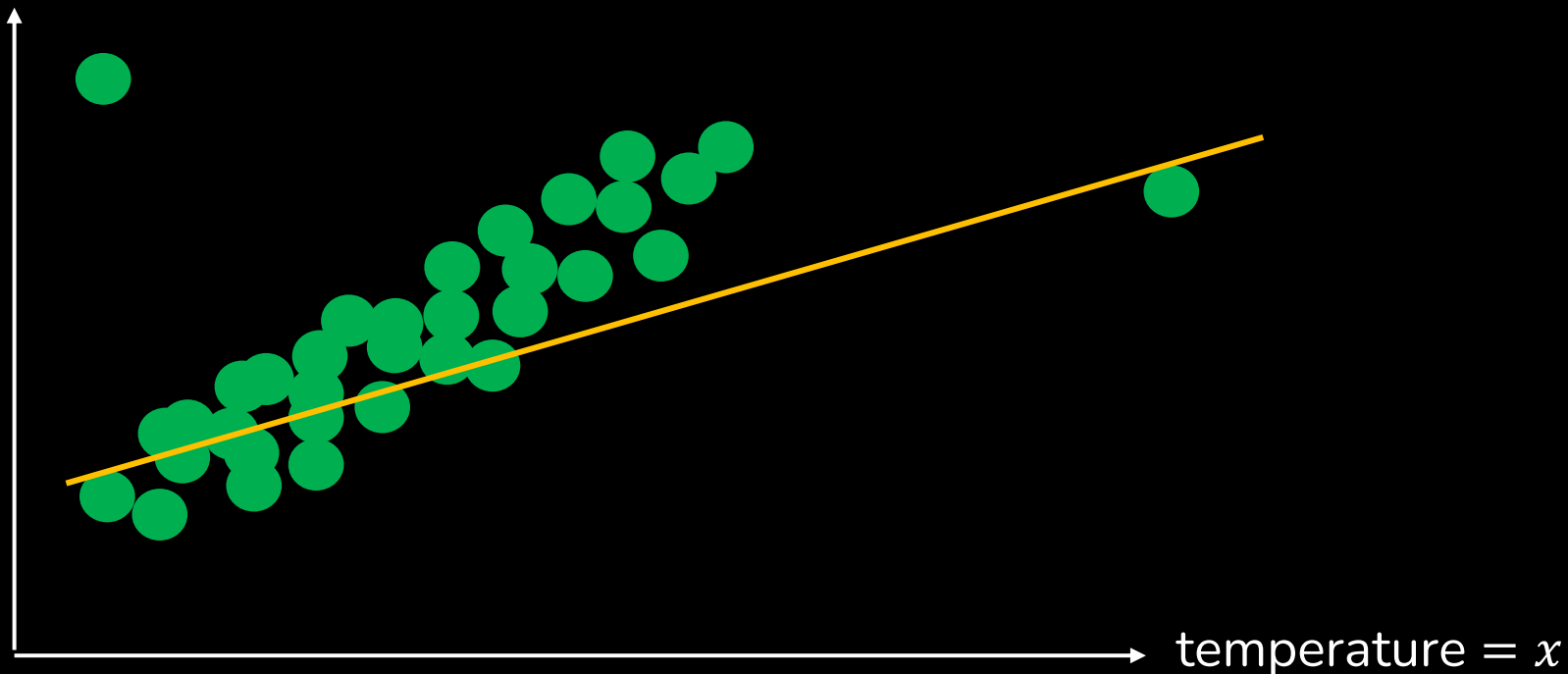
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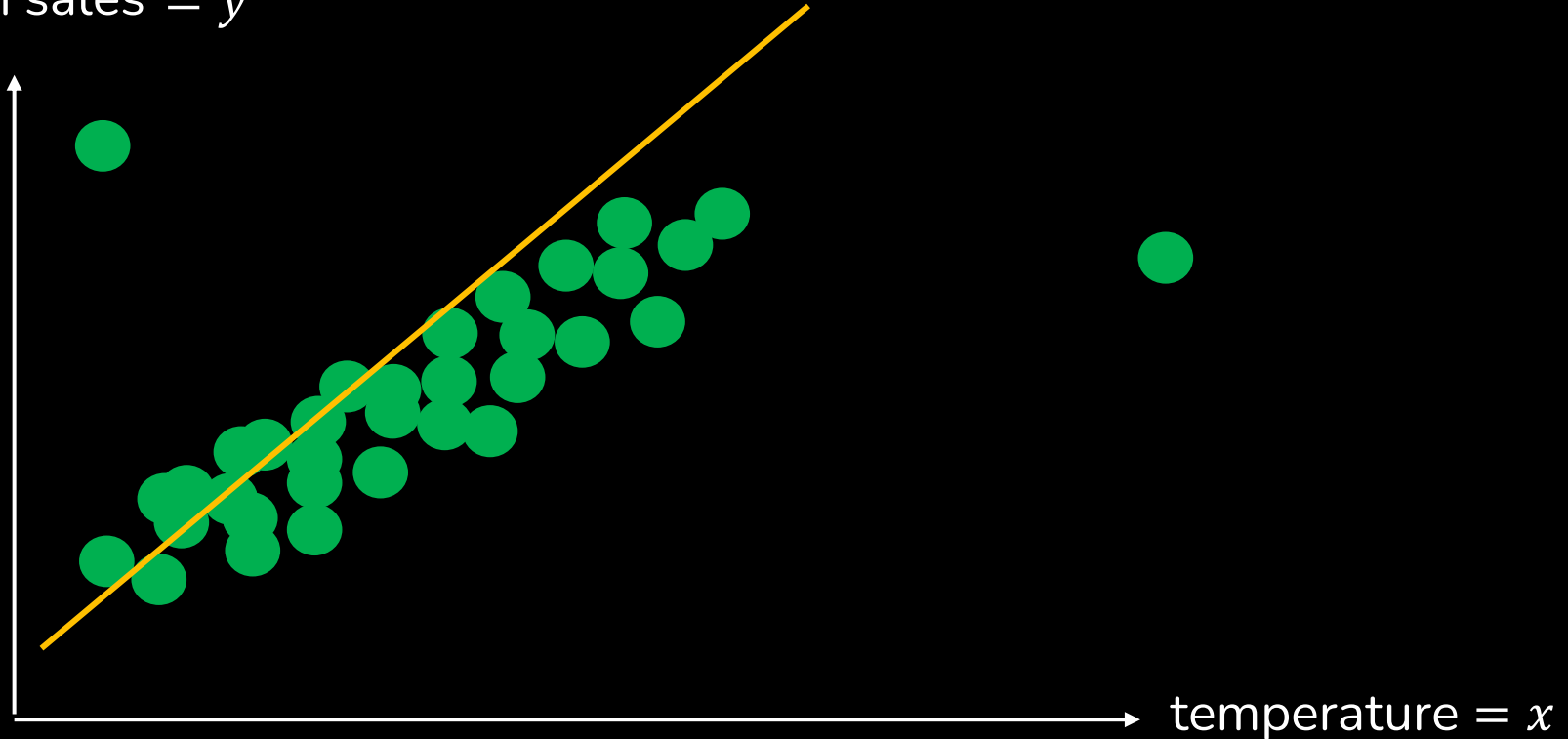
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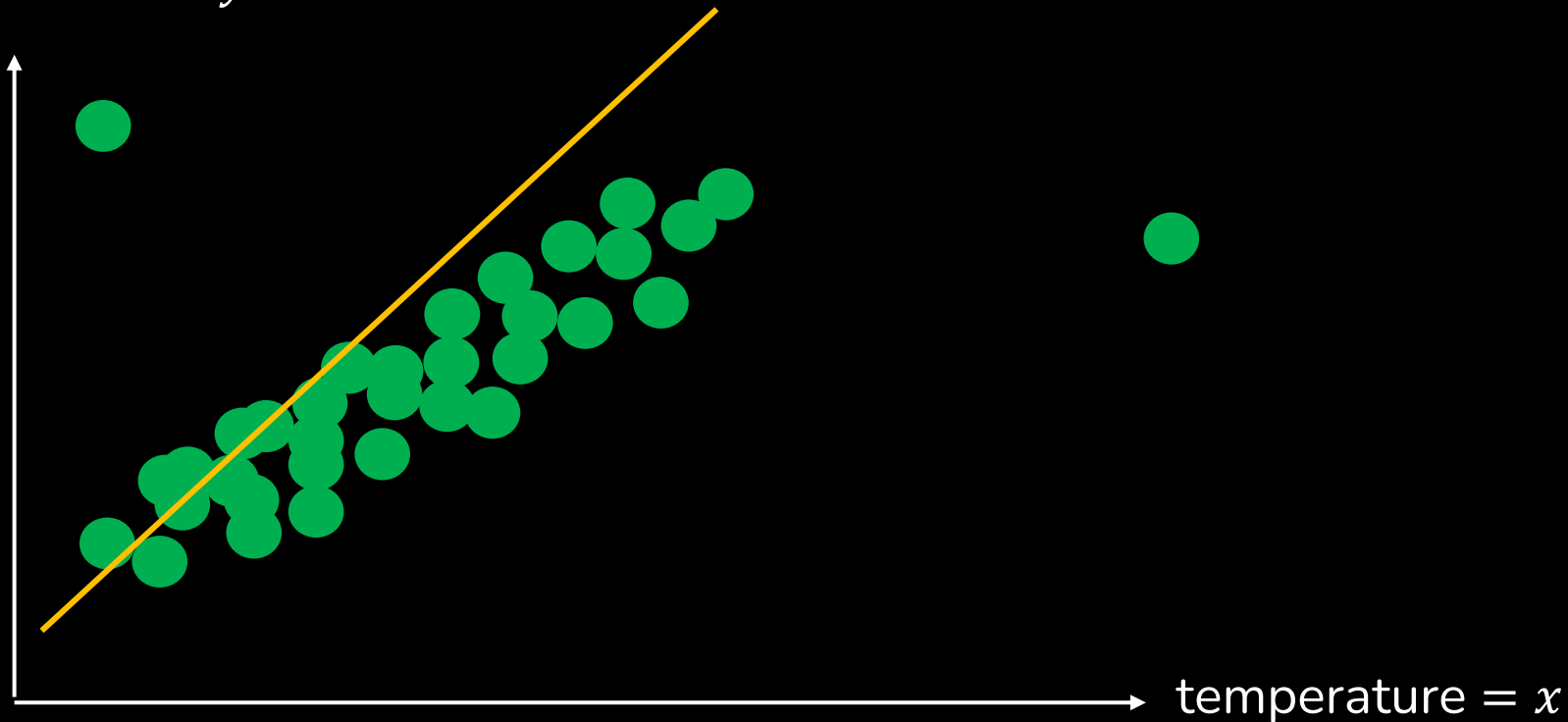
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Linear regression  
model

$$\hat{y}(x) = ax + b$$

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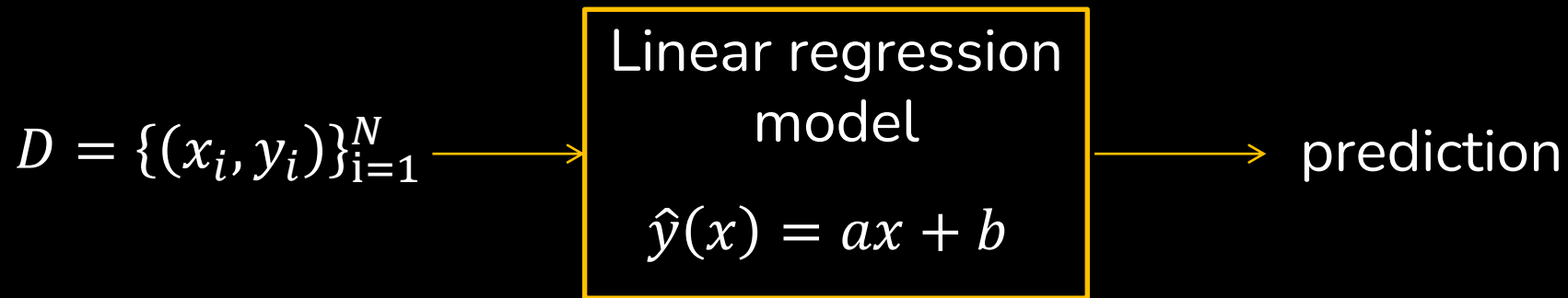
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temperature	Ice cream sales
30	56
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0	1
38	87
44	100
...	...
...	...

Linear regression  
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$$\hat{y}(x) = ax + b$$

prediction

$$a = 2, \quad b = 3$$

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$$\hat{y}(x) = 63$$



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Linear regression  
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$$\hat{y}(x) = 2.30 + 3$$

$$\hat{y}(x) = 63$$

$$MSE = \frac{1}{n} \sum_{i=1}^n (y_i - \hat{y}_i)^2$$

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$$\hat{y}(x) = 2.30 + 3$$

$$\hat{y}(x) = 63$$

$$MSE = \frac{1}{n} [(y_1 - \hat{y}_1)^2 + \dots]$$

$$a = 2, \quad b = 3$$

temperature	Ice cream sales
30	56
23	37
0	1
38	87
44	100
...	...
...	...

Linear regression  
model

$$\hat{y}(x) = 2.30 + 3$$

$$\hat{y}(x) = 63$$

$$MSE = 180$$

$$a = 2, \quad b = 3$$

temperature	Ice cream sales
30	56
23	37
0	1
38	87
44	100
...	...
...	...

Linear regression  
model

$$\hat{y}(x) = 2.30 + 3$$

$$\hat{y}(x) = 63$$

$$MSE = \frac{1}{n} \sum_{i=1}^n (y_i - \hat{y}_i)^2$$

$$MSE = 180$$

$$a = 2, \quad b = 3$$

temperature	Ice cream sales
30	56
23	37
0	1
38	87
44	100
...	...
...	...

Linear regression  
model

$$\hat{y}(x) = 2.30 + 3$$

$$\hat{y}(x) = 63$$

$$MSE = 180$$

$$a = 2, \quad b = 3$$

temperature	Ice cream sales
30	56
23	37
0	1
38	87
44	100
...	...
...	...

Linear regression  
model

$$\hat{y}(x) = 2.30 + 3$$

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$$MSE = 180$$

$$a = 2, \quad b = 3$$

temperature	Ice cream sales
30	56
23	37
0	1
38	87
44	100
...	...
...	...

Linear regression  
model

$$\hat{y}(x) = 2.30 + 3$$

$$\hat{y}(x) = 63$$

$$MSE = 180$$

$$\begin{cases} a = a - \eta \nabla MSE \\ b = b - \eta \nabla MSE \end{cases}$$



$$a = 2, \quad b = 3$$

temperature	Ice cream sales
30	56
23	37
0	1
38	87
44	100
...	...
...	...

Linear regression  
model

$$\hat{y}(x) = 2.30 + 3$$

$$\hat{y}(x) = 63$$

$$MSE = 180$$

$$\begin{cases} a = a - \eta \nabla MSE \\ b = b - \eta \nabla MSE \end{cases}$$

$$a = a - \eta \nabla MSE$$

$$MSE = 180$$

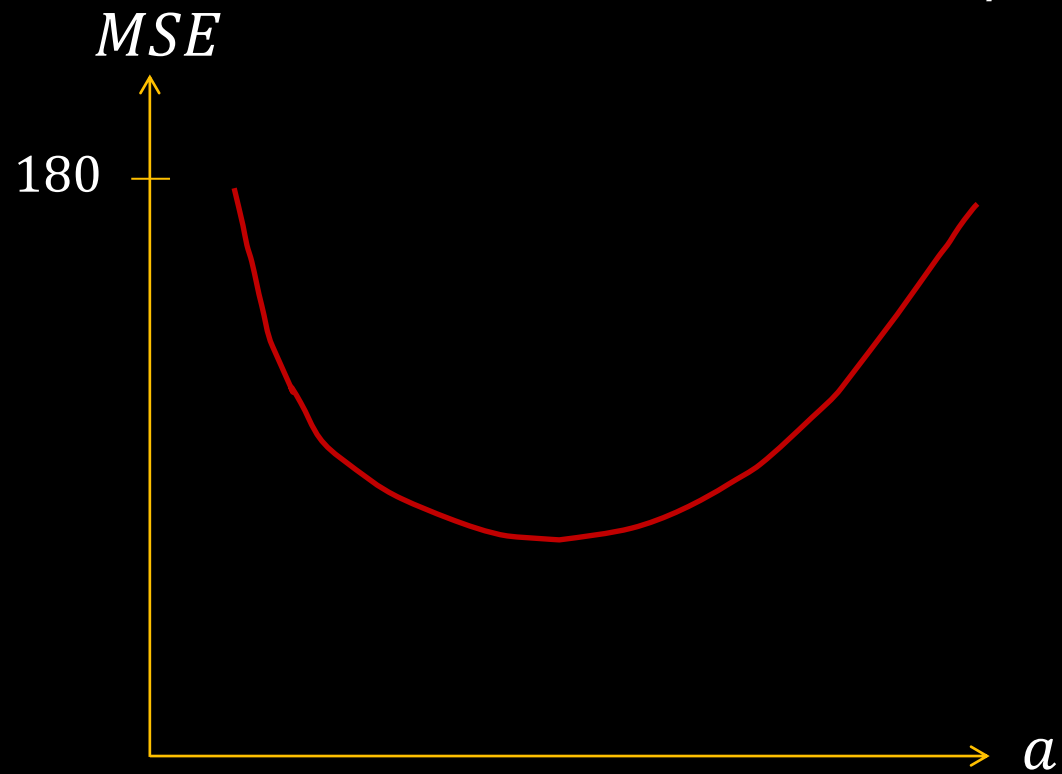
$MSE$



$a$

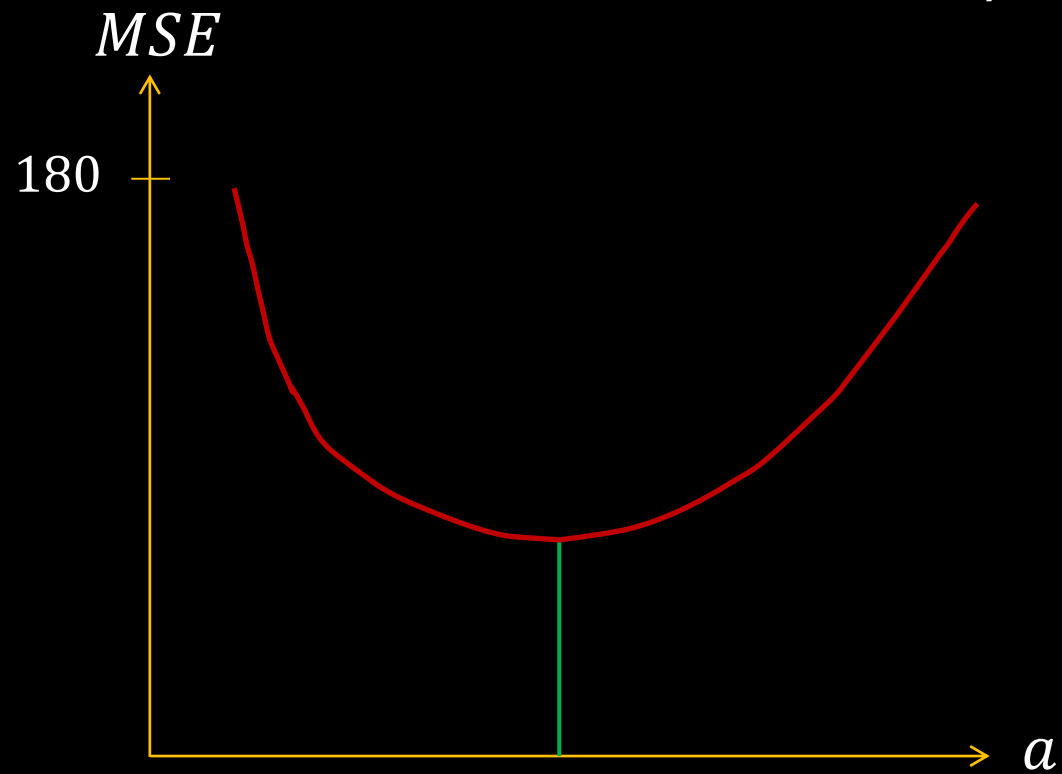
$$a = a - \eta \nabla MSE$$

$$MSE = 180$$



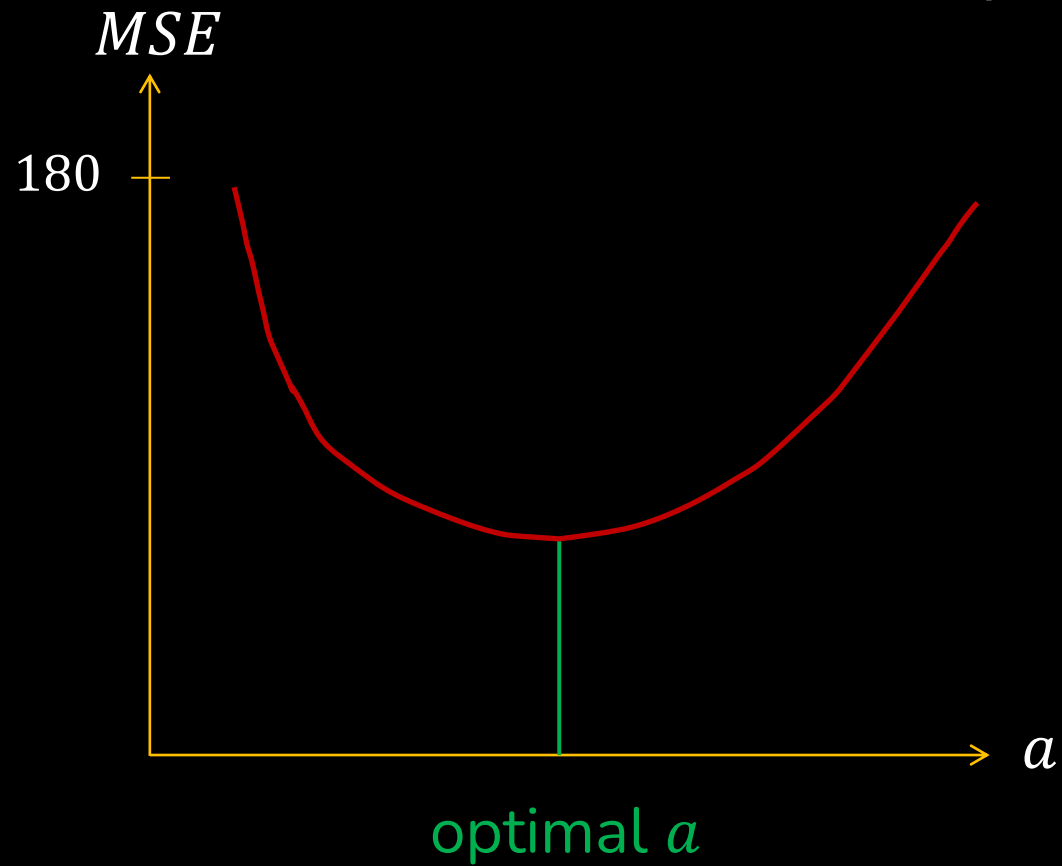
$$a = a - \eta \nabla MSE$$

$$MSE = 180$$



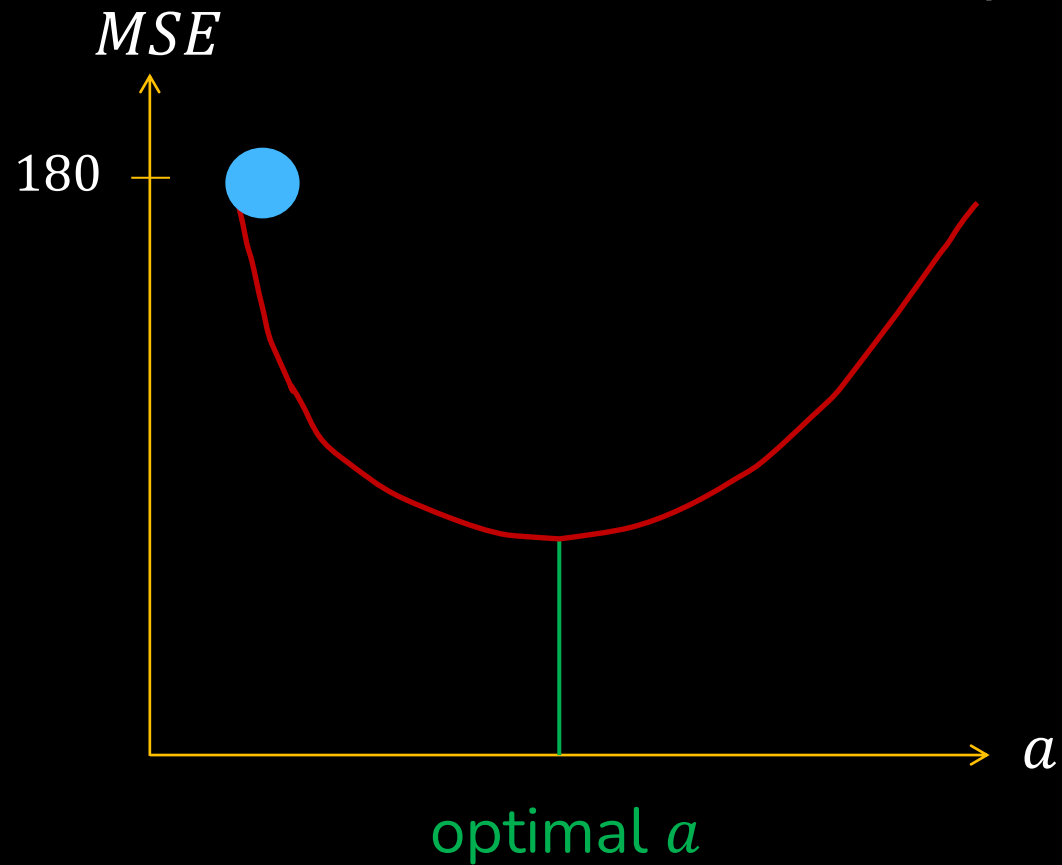
$$a = a - \eta \nabla MSE$$

$$MSE = 180$$



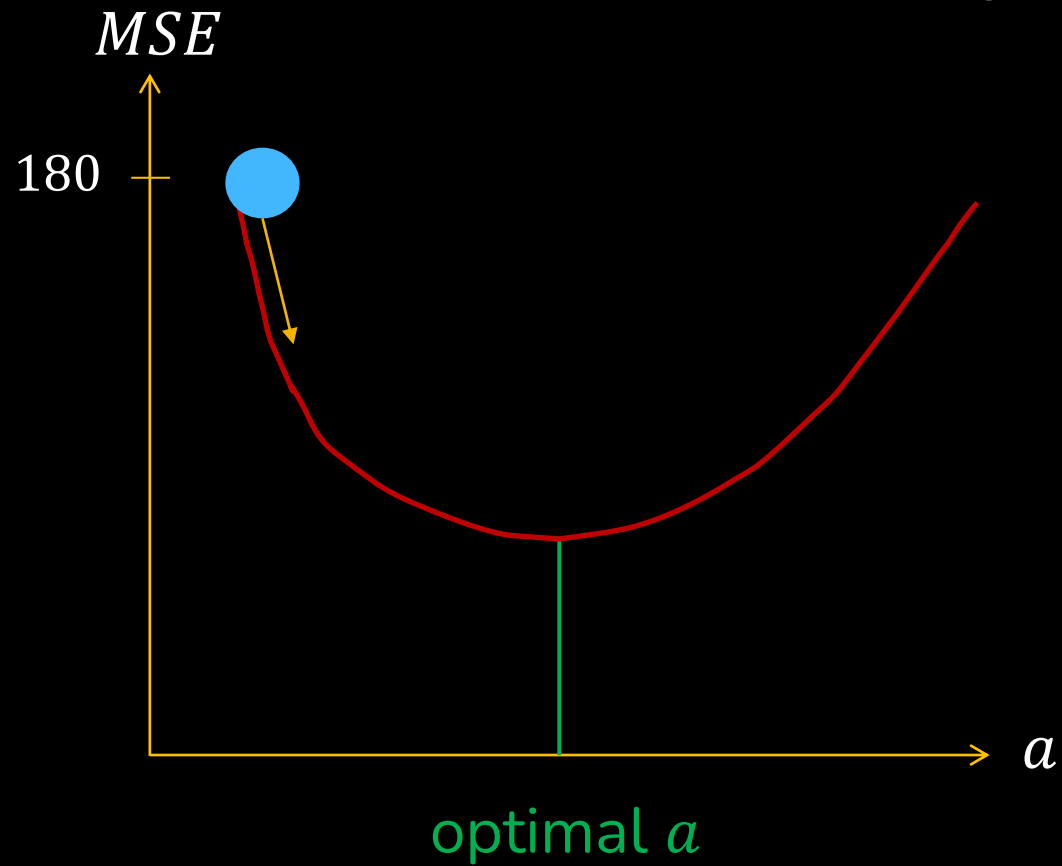
$$a = a - \eta \nabla MSE$$

$$MSE = 180$$



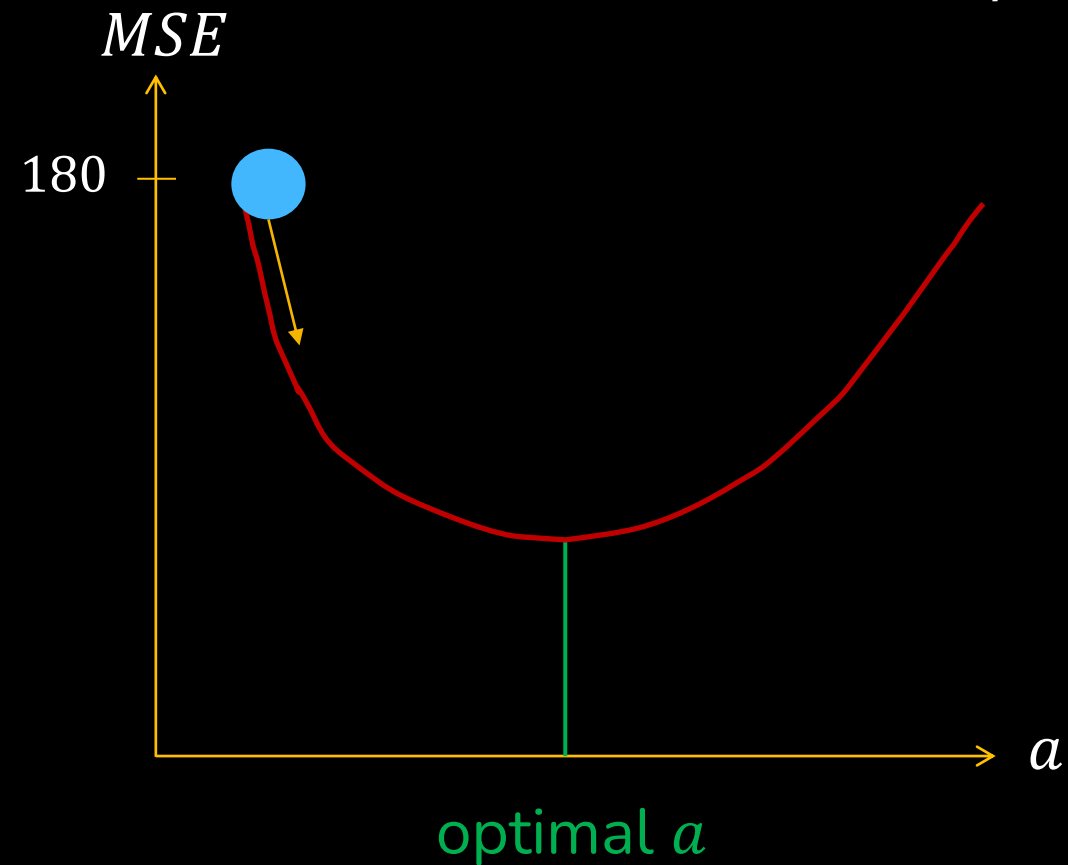
$$a = a - \eta \nabla \text{MSE}$$

$$\text{MSE} = 180$$



$$a = a - \eta \nabla MSE$$

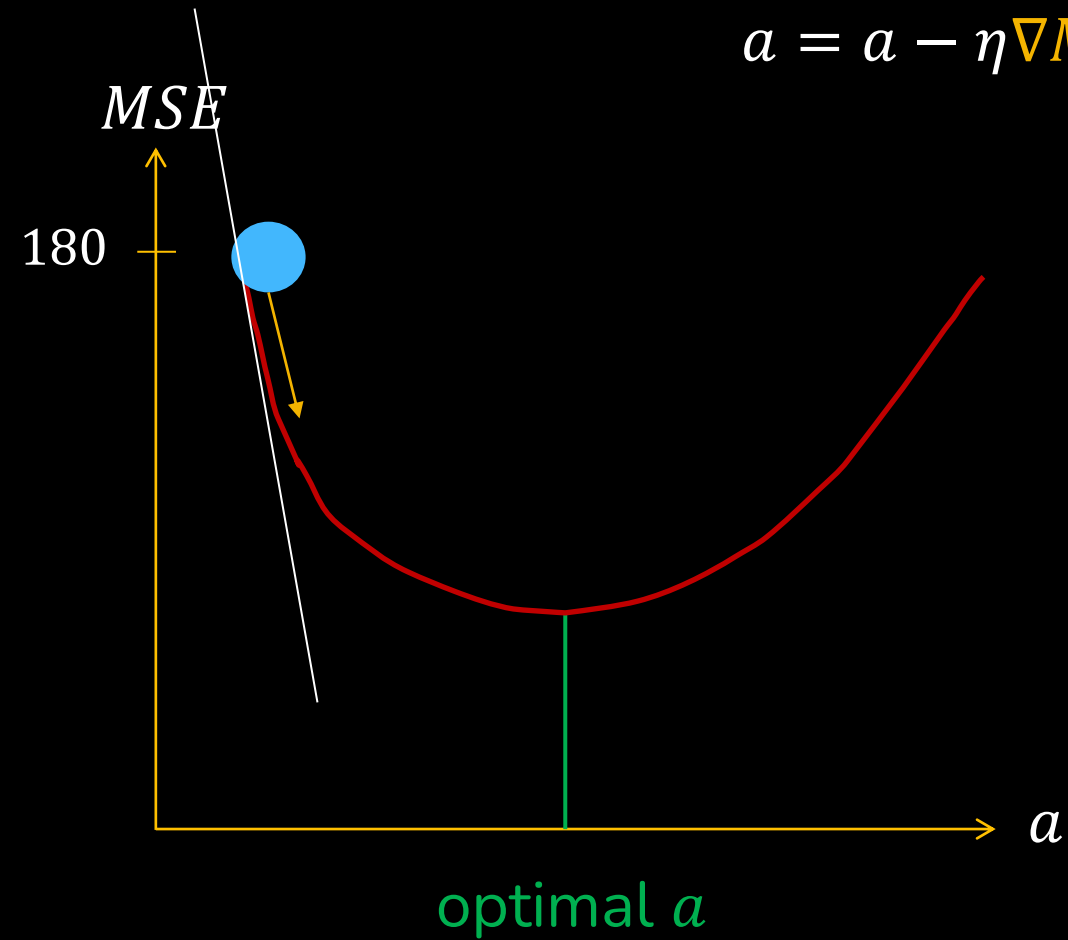
$$MSE = 180$$





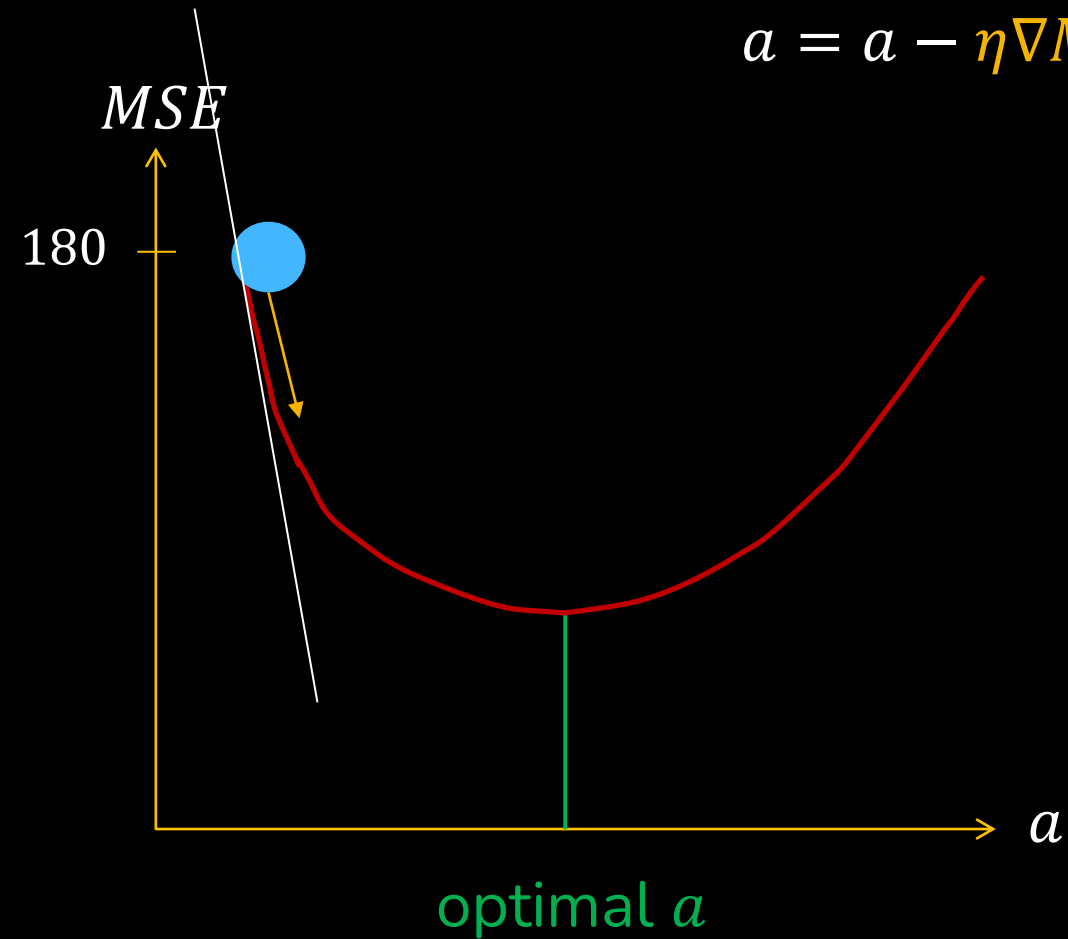
$$a = a - \eta \nabla MSE$$

$$MSE = 180$$



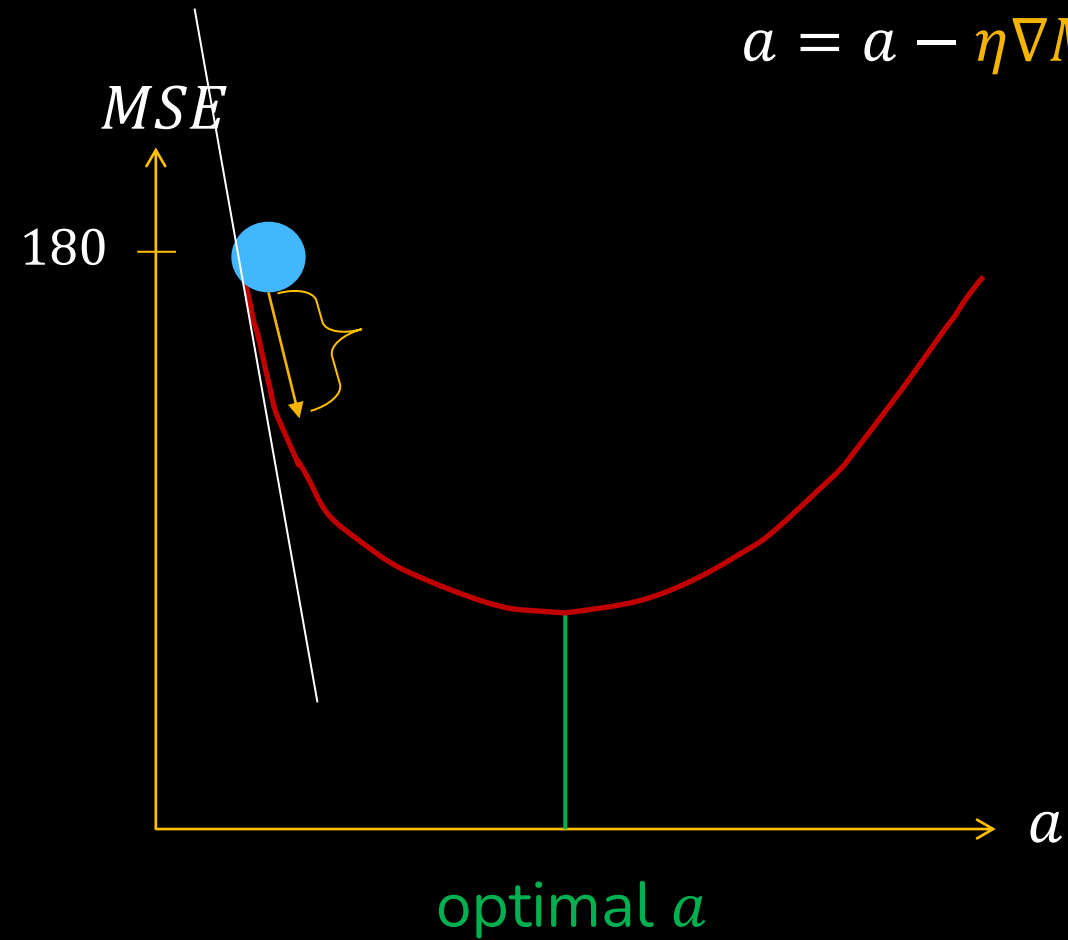
$$a = a - \eta \nabla MSE$$

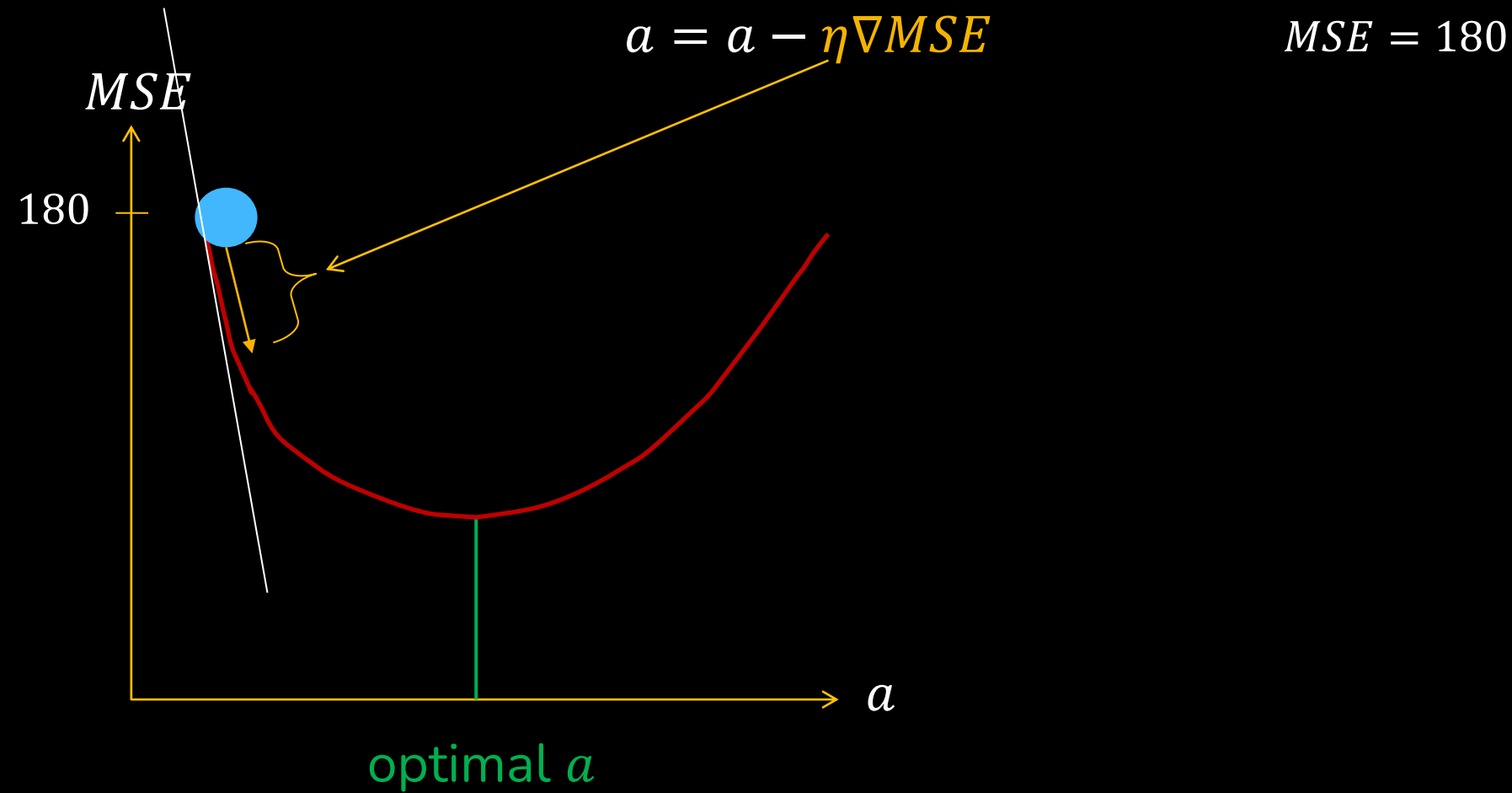
$$MSE = 180$$

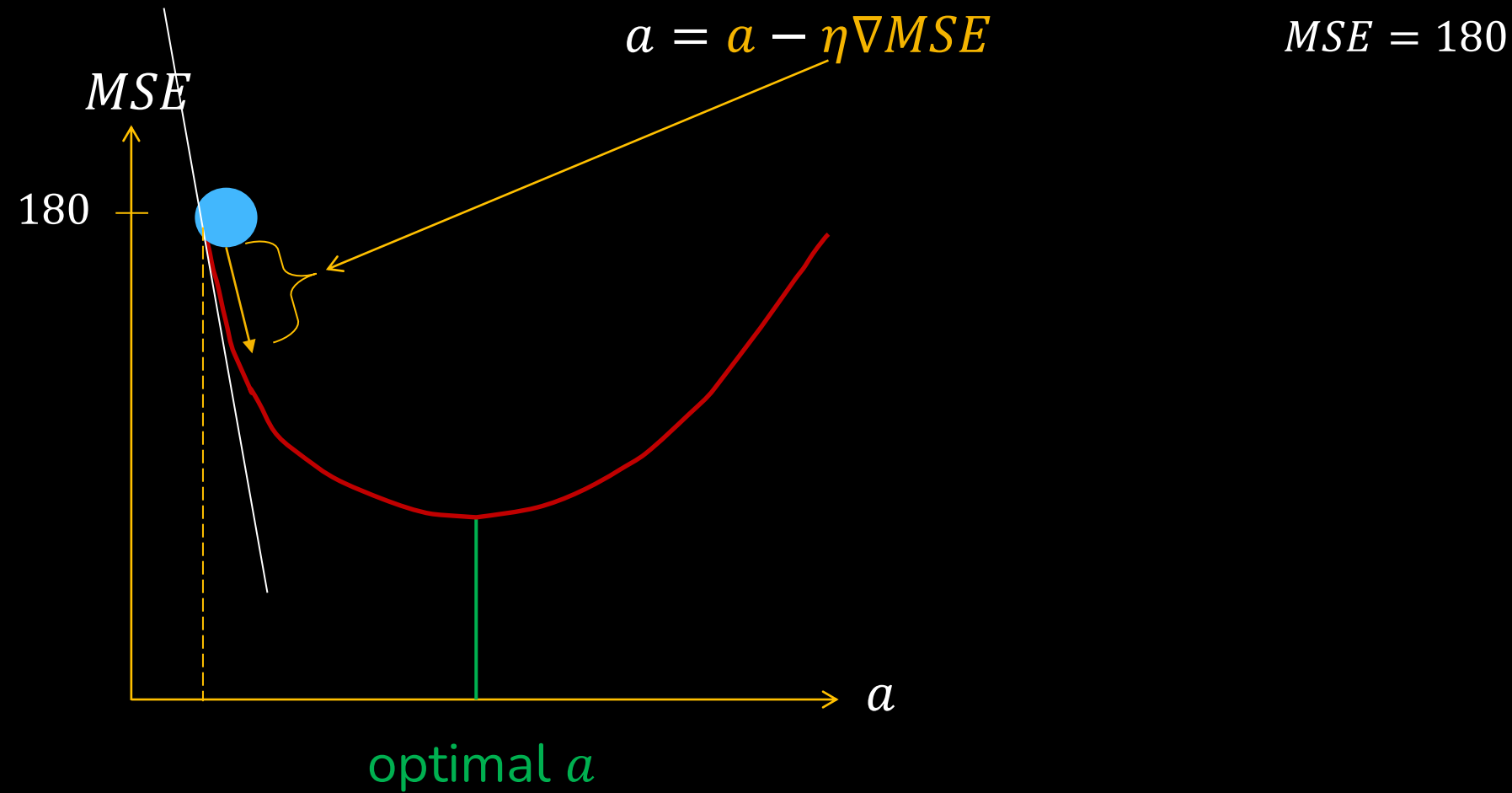


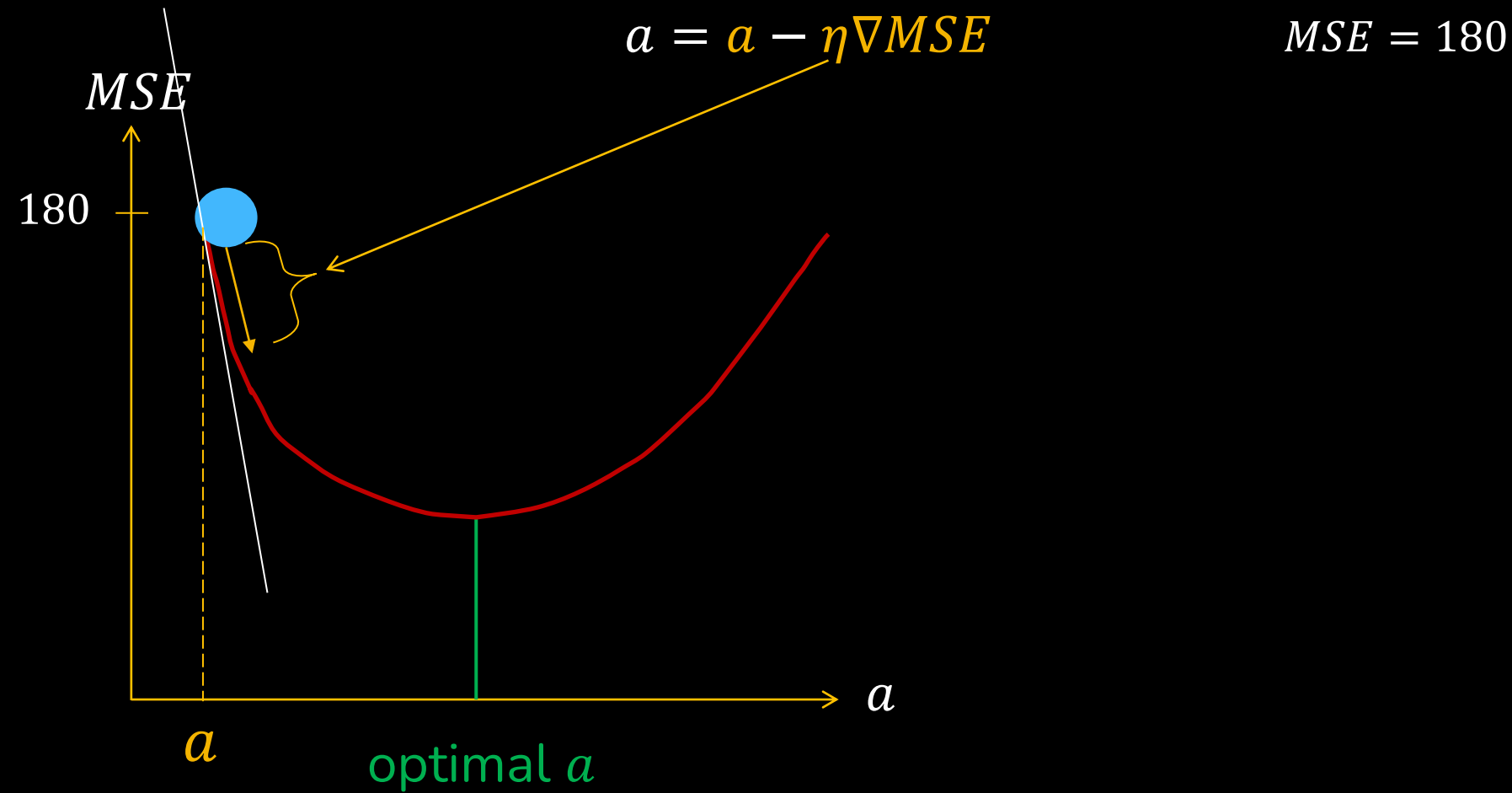
$$a = a - \eta \nabla MSE$$

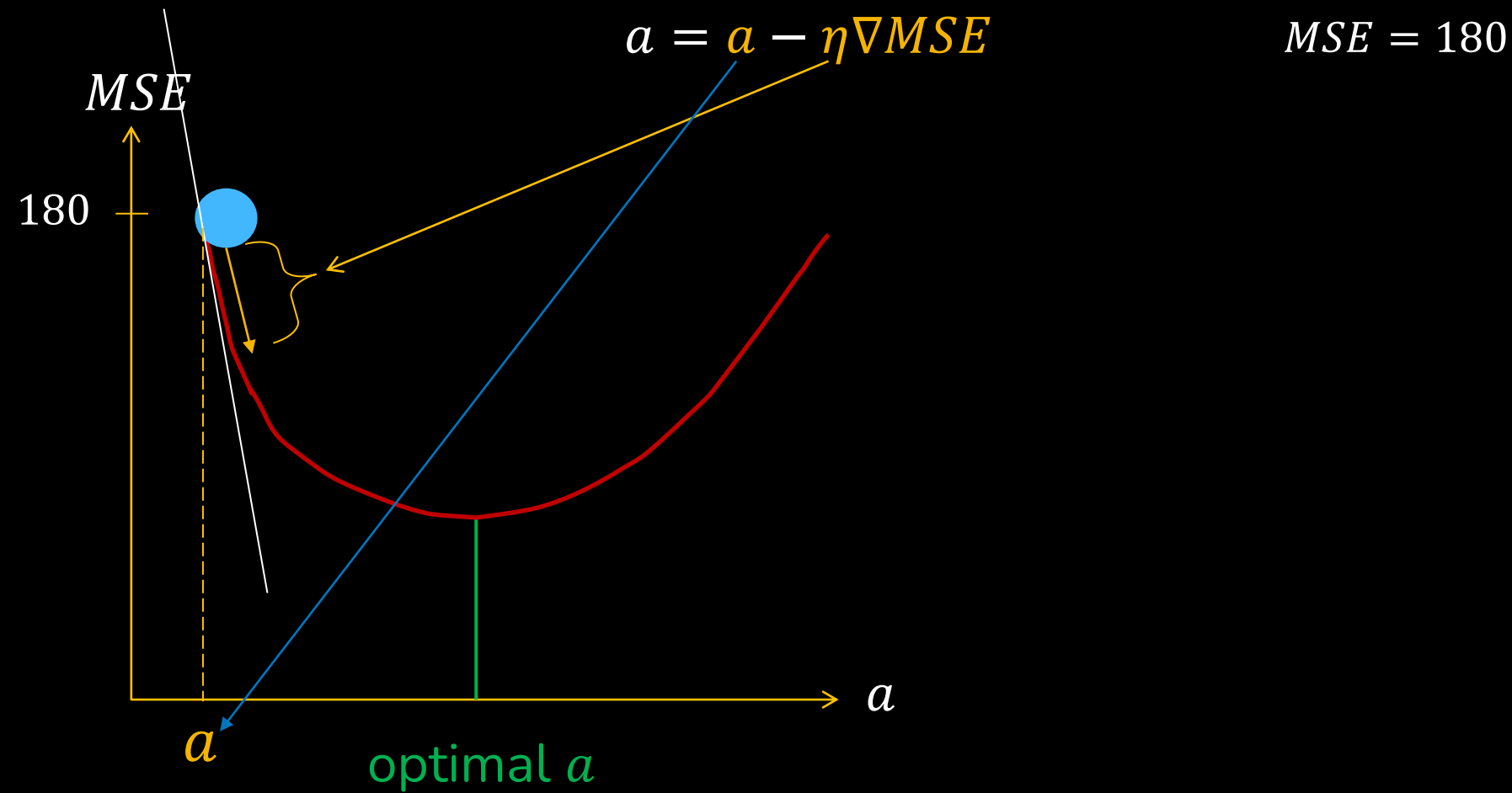
$$MSE = 180$$









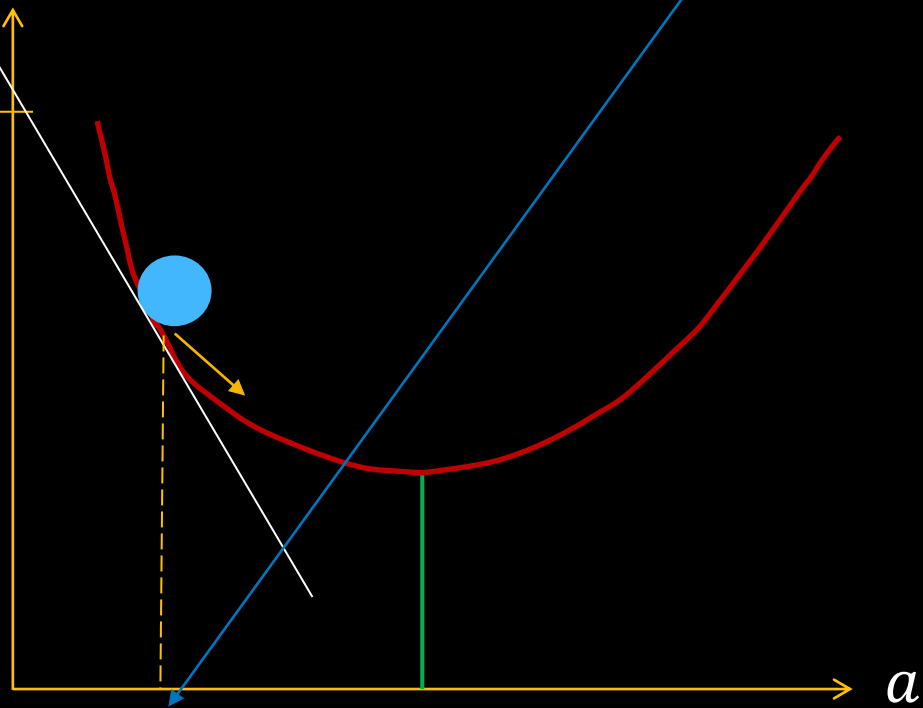


$MSE = 150$

$$a = a - \eta \nabla MSE$$

$MSE$

180



$a$

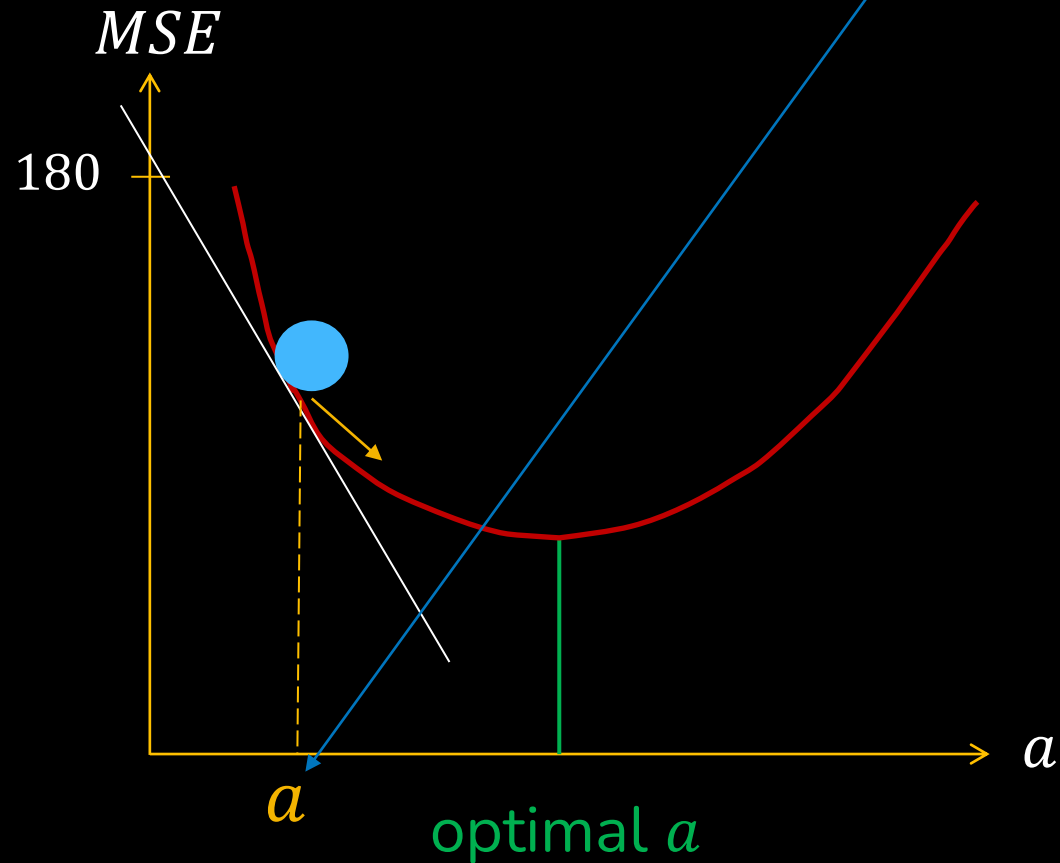
optimal  $a$

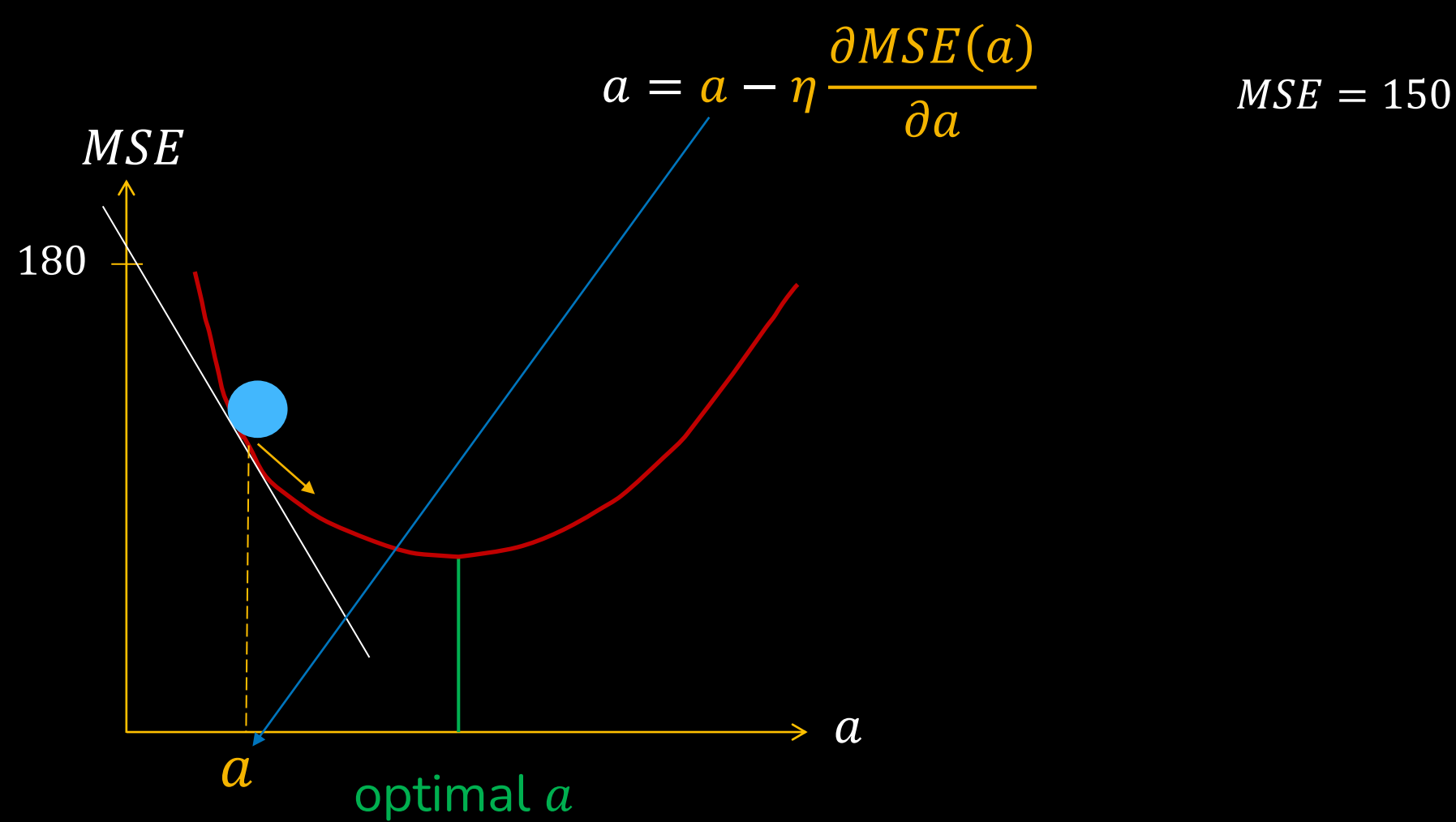


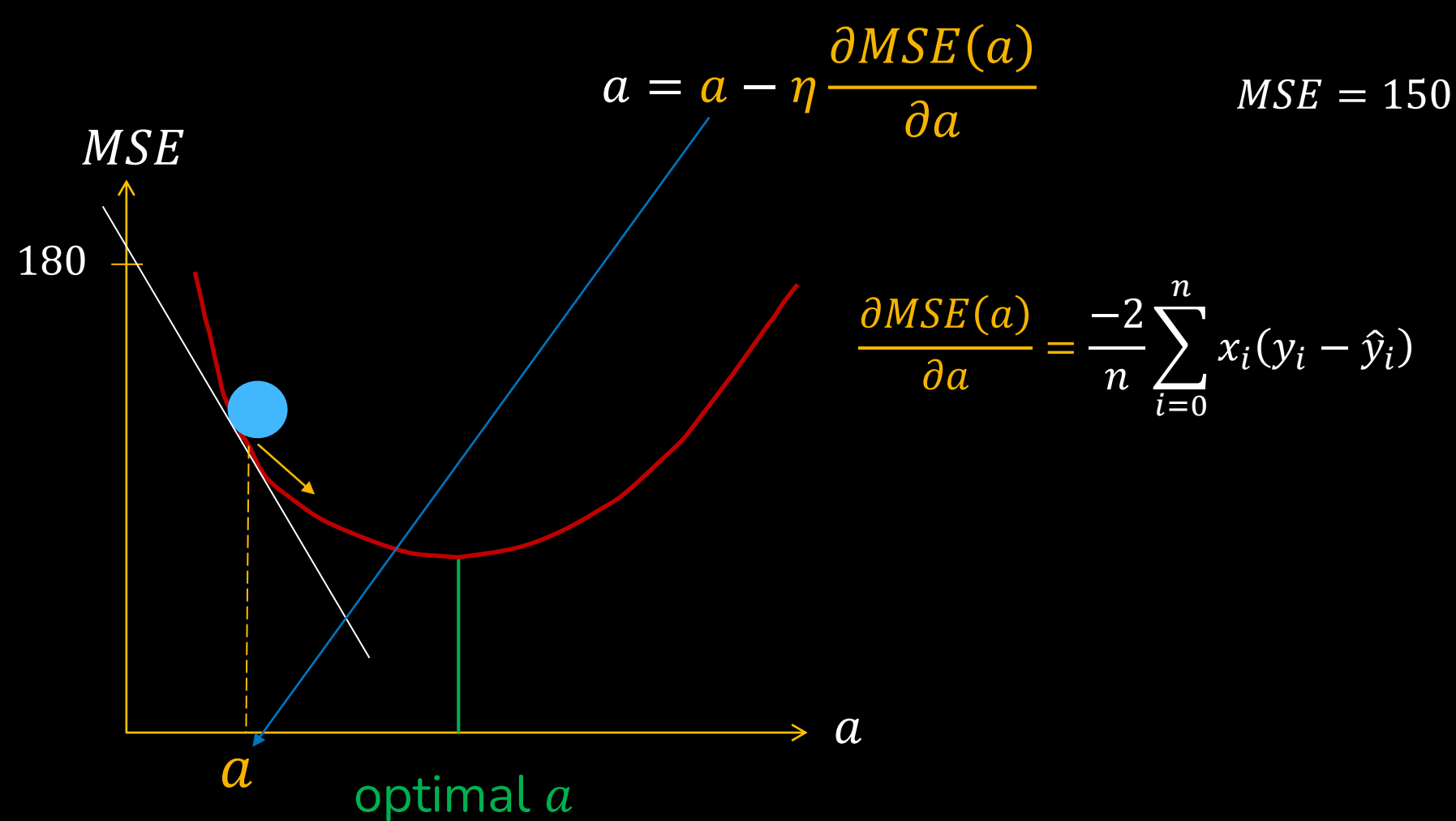
$$MSE = 150$$

$$a = a - \eta \nabla MSE$$

$$\nabla MSE = \begin{pmatrix} \frac{\partial MSE(a)}{\partial a} \\ \frac{\partial MSE(b)}{\partial b} \end{pmatrix}$$

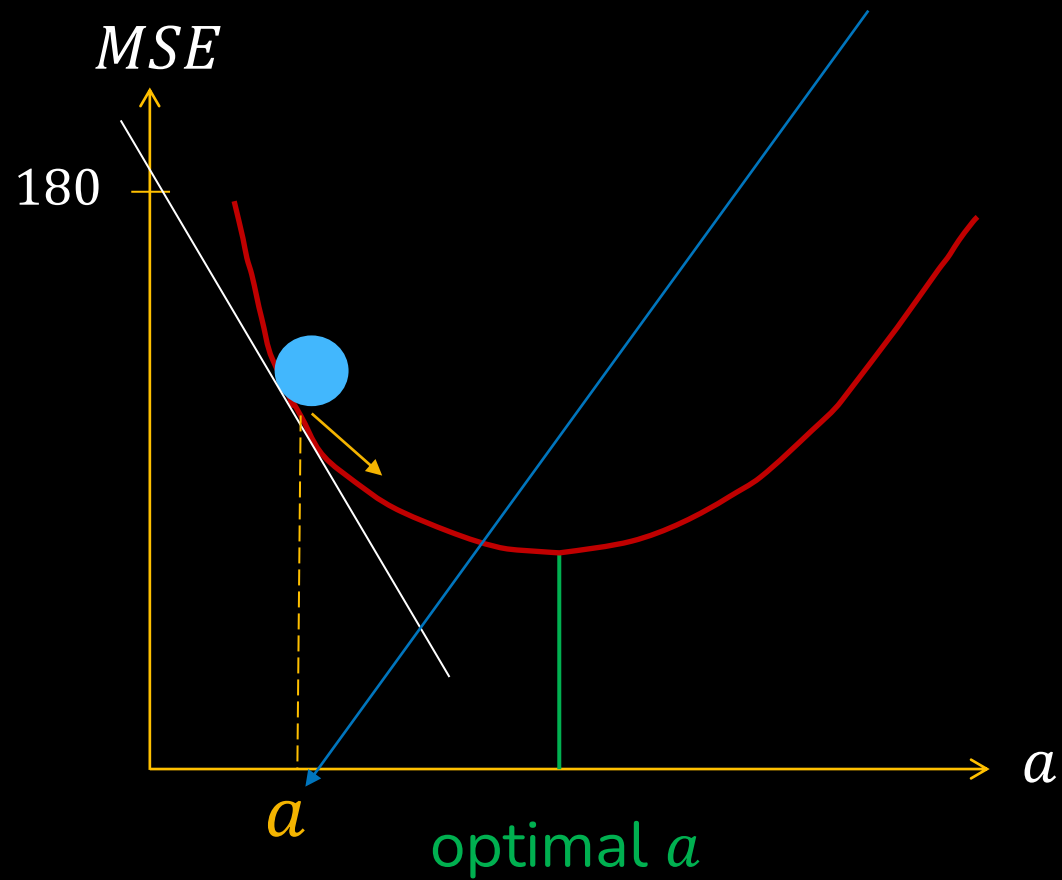






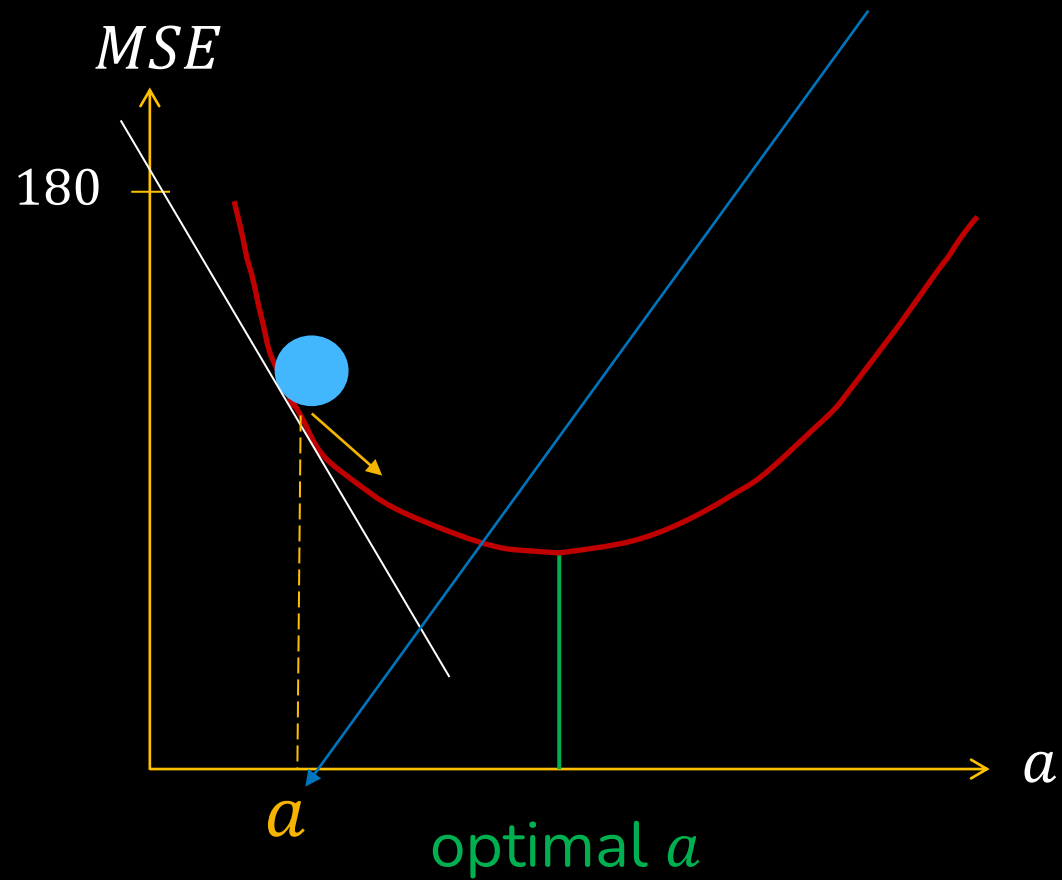
$$a = 2 - 0.2$$

$$MSE = 150$$



$$a = 1.8$$

$$MSE = 150$$



$$a = 2, \quad b = 3$$

temperature	Ice cream sales
30	56
23	37
0	1
38	87
44	100
...	...
...	...

Linear regression  
model

$$\hat{y}(x) = 2.30 + 3$$

$$\hat{y}(x) = 63$$

$$MSE = 180$$

$$\begin{cases} a = a - \eta \nabla MSE \\ b = b - \eta \nabla MSE \end{cases}$$

$$a = 1.8, \quad b = 3$$

temperature	Ice cream sales
30	56
23	37
0	1
38	87
44	100
...	...
...	...

Linear regression  
model

$$\hat{y}(x) = 1.8 * 30 + 3$$

$$\hat{y}(x) = 63$$

$$MSE = 180$$

$$\begin{cases} a = a - \eta \nabla MSE \\ b = b - \eta \nabla MSE \end{cases}$$

$$a = 1.8, \quad b = 3$$

temperature	Ice cream sales
30	56
23	37
0	1
38	87
44	100
...	...
...	...

Linear regression  
model

$$\hat{y}(x) = 1.8 * 30 + 3$$

$$\hat{y}(x) = 57$$

$$MSE = 180$$

$$\begin{cases} a = a - \eta \nabla MSE \\ b = b - \eta \nabla MSE \end{cases}$$



$$a = 1.8, \quad b = 3$$

temperature	Ice cream sales
30	56
23	37
0	1
38	87
44	100
...	...
...	...

Linear regression  
model

$$\hat{y}(x) = 1.8 * 30 + 3$$

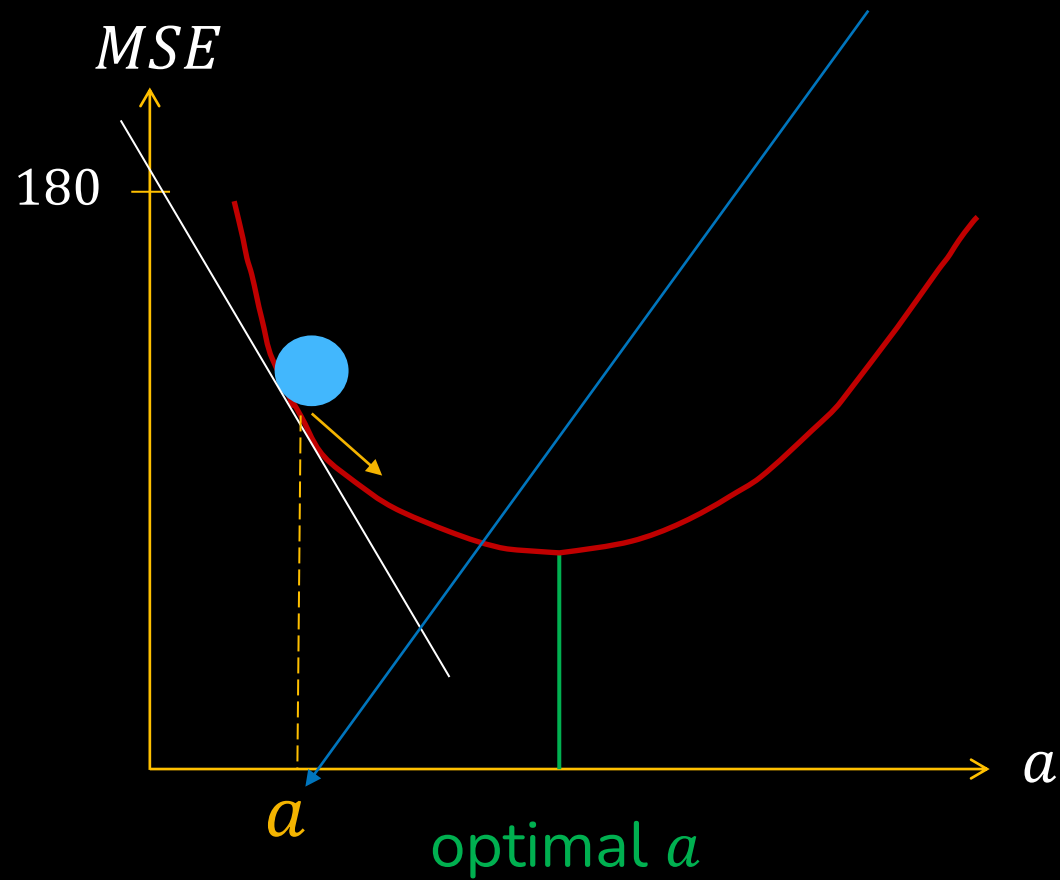
$$\hat{y}(x) = 57$$

$$MSE = 150$$

$$\begin{cases} a = a - \eta \nabla MSE \\ b = b - \eta \nabla MSE \end{cases}$$

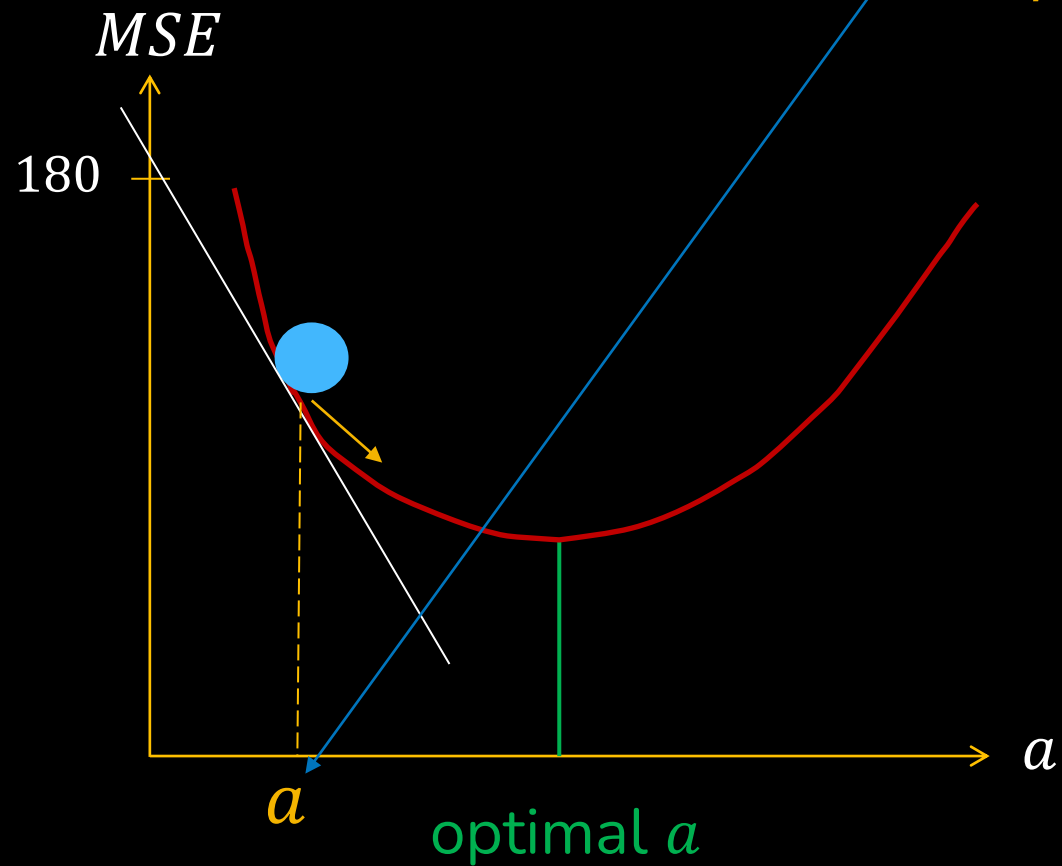
$$a = 1.8$$

$$MSE = 150$$



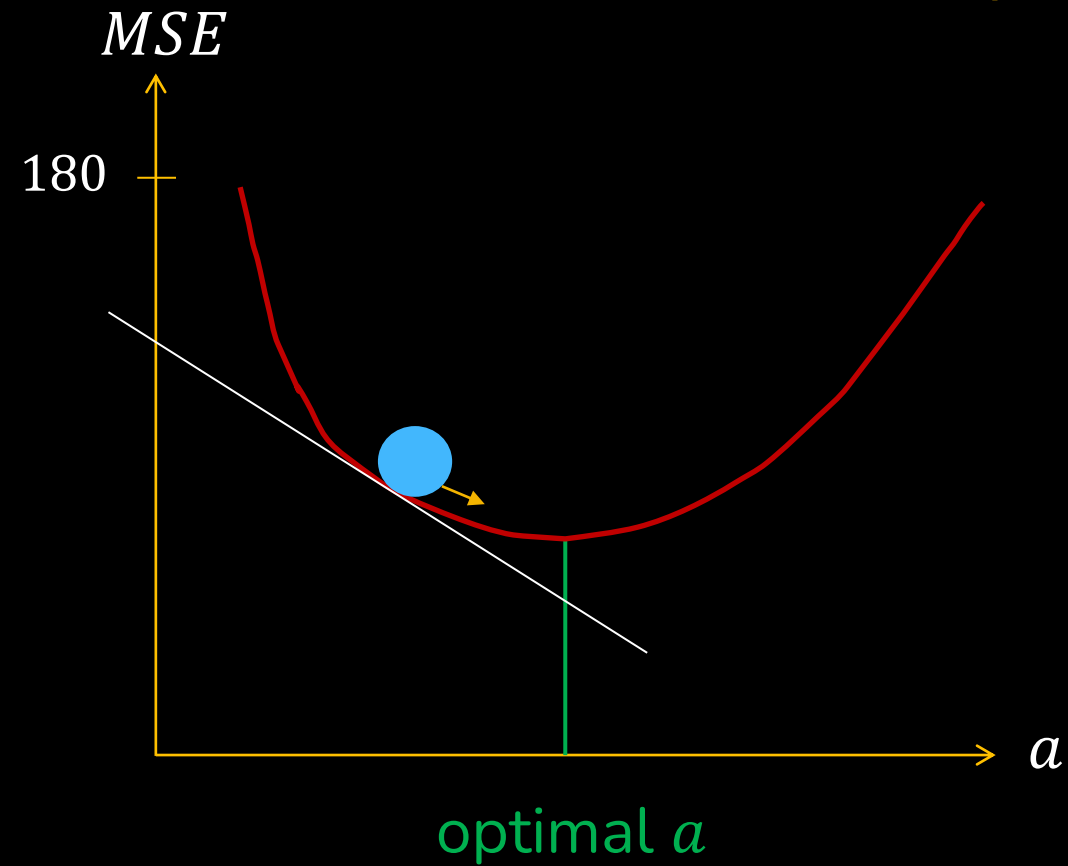
$$MSE = 150$$

$$a = 1.8 - \eta \nabla MSE$$



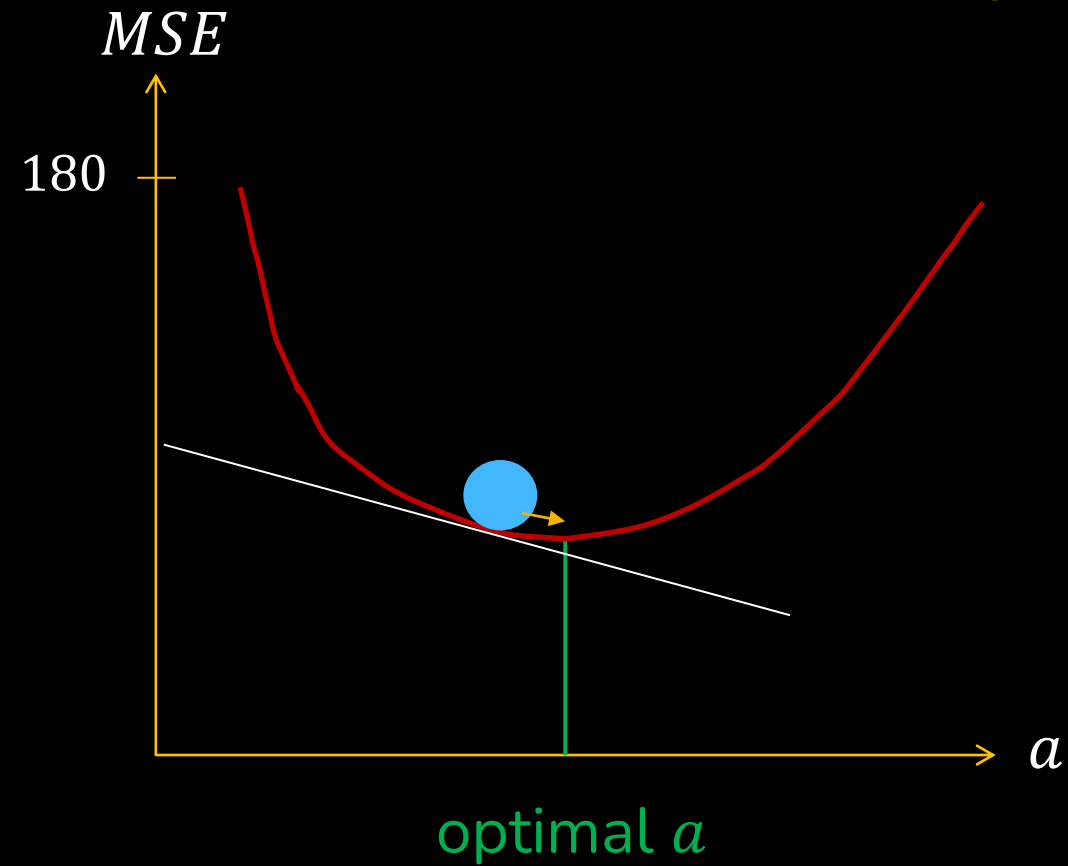
$$a = a - \eta \nabla MSE$$

$$MSE = 20$$



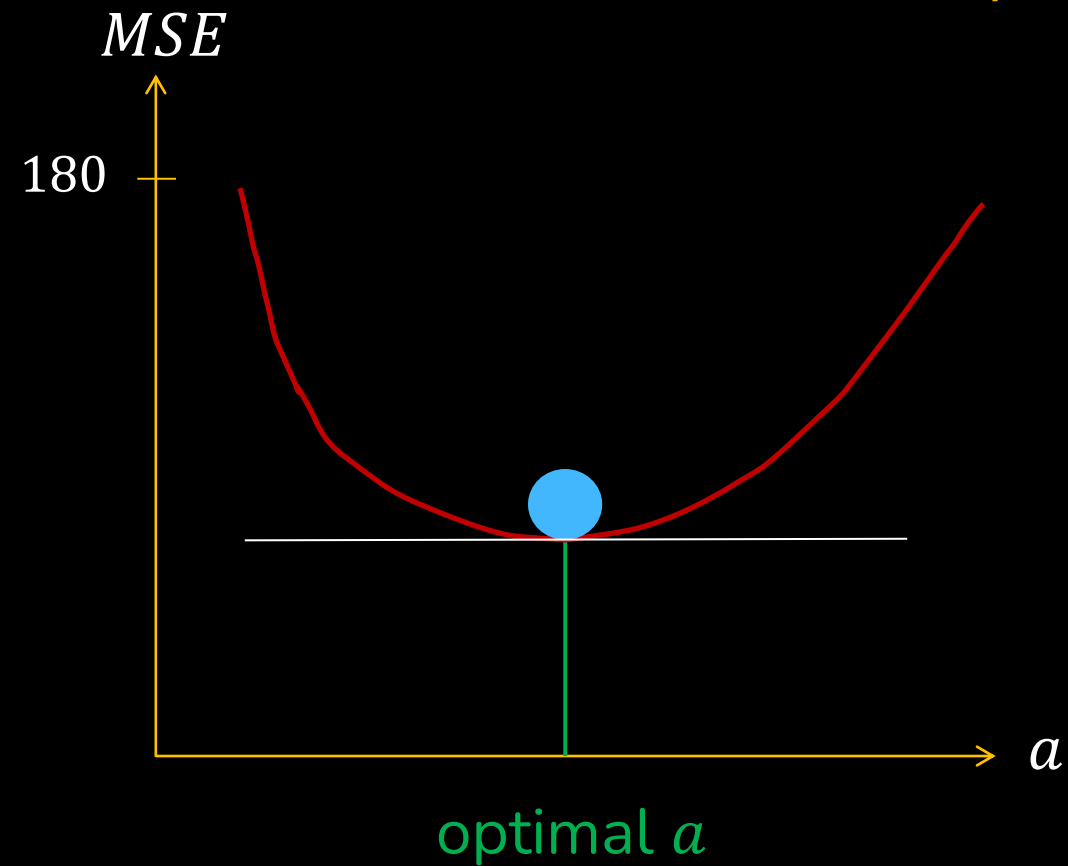
$$a = a - \eta \nabla MSE$$

$$MSE = 10$$



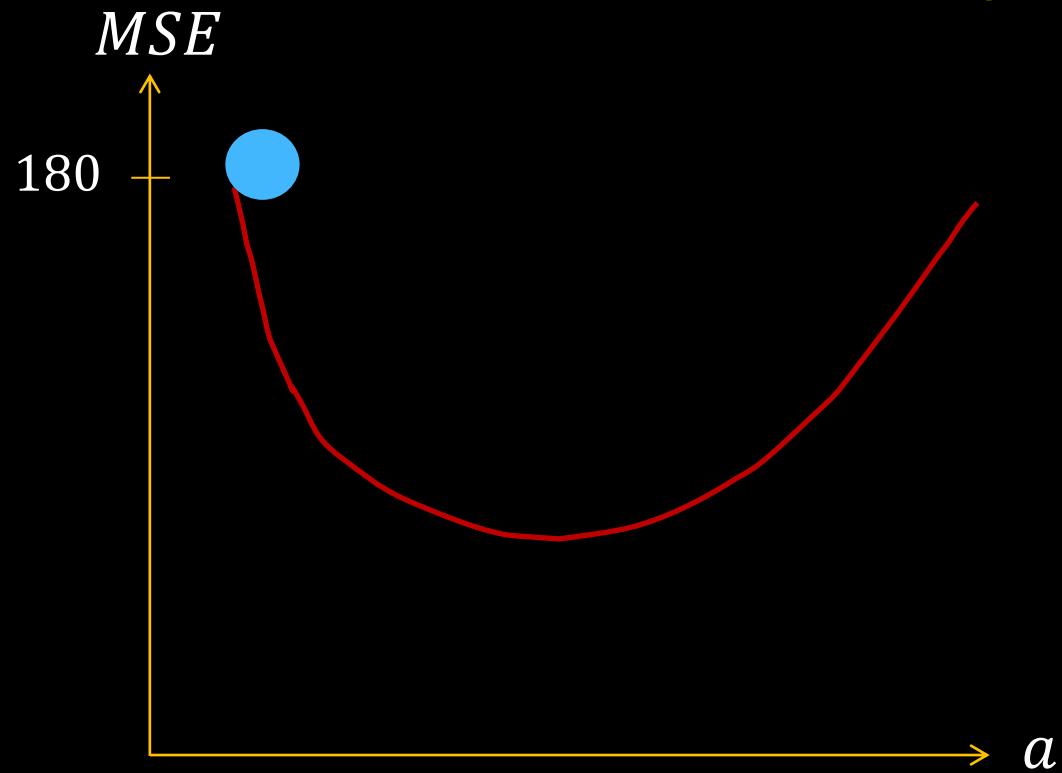
$$a = a - \eta \nabla MSE$$

$$MSE = 0.001$$



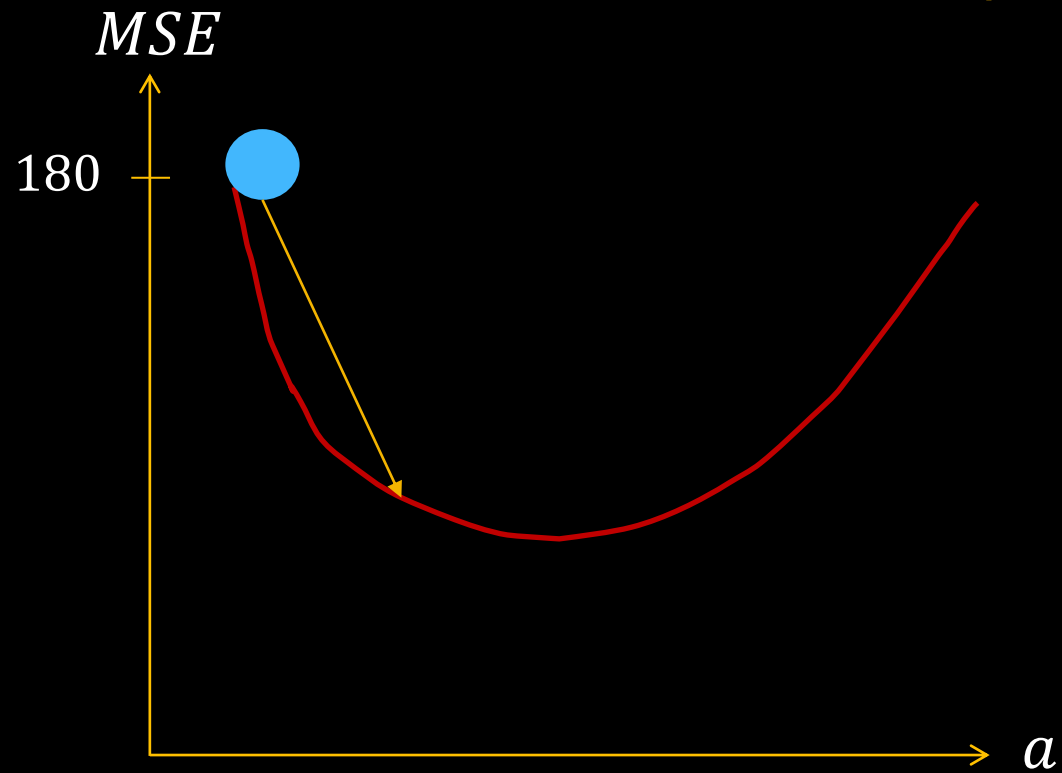
$$a = a - \eta \nabla MSE$$

$$MSE = 0.001$$



$$a = a - \eta \nabla MSE$$

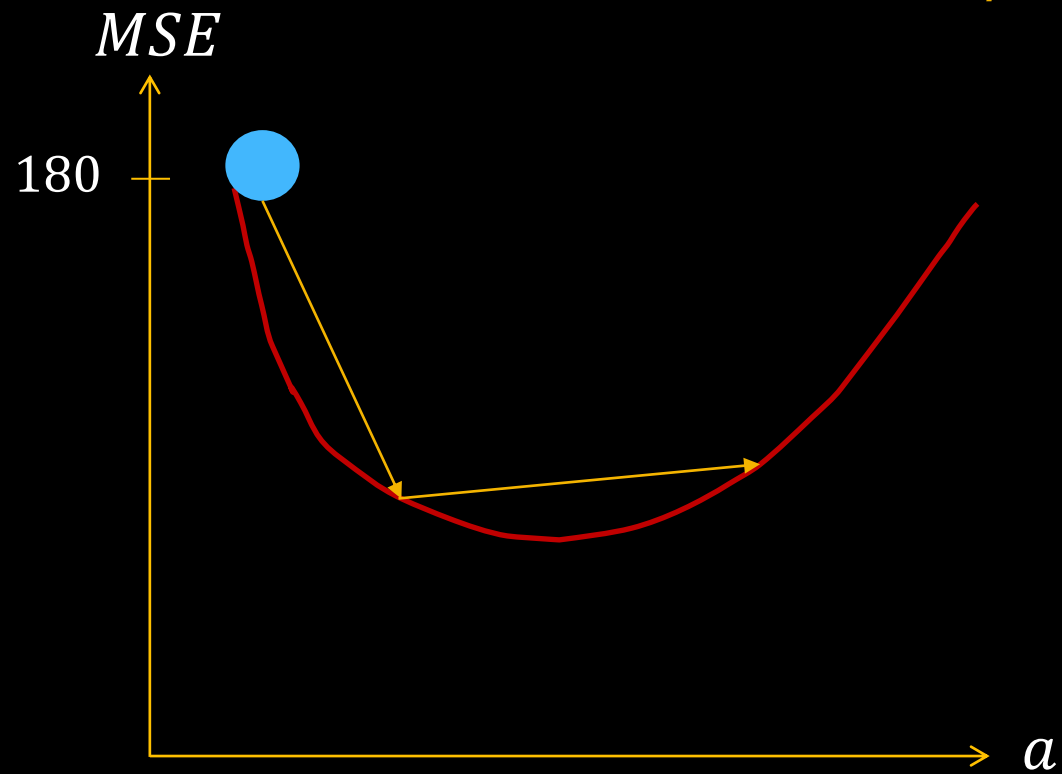
$$MSE = 0.001$$





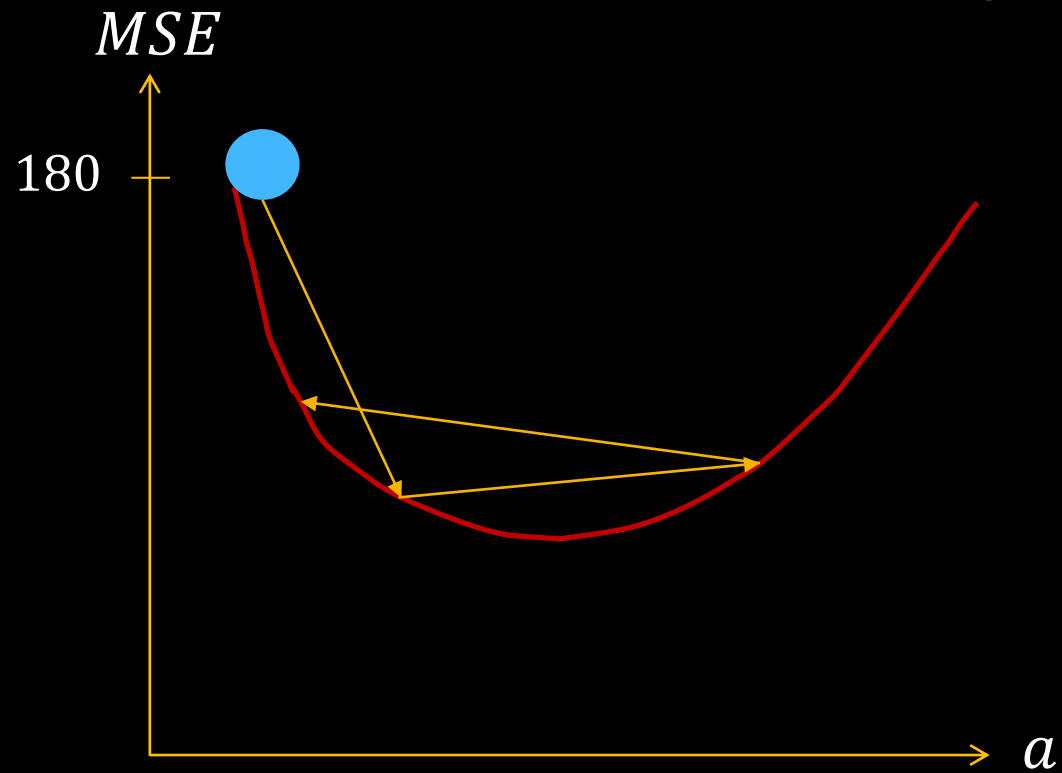
$$a = a - \eta \nabla MSE$$

$$MSE = 0.001$$



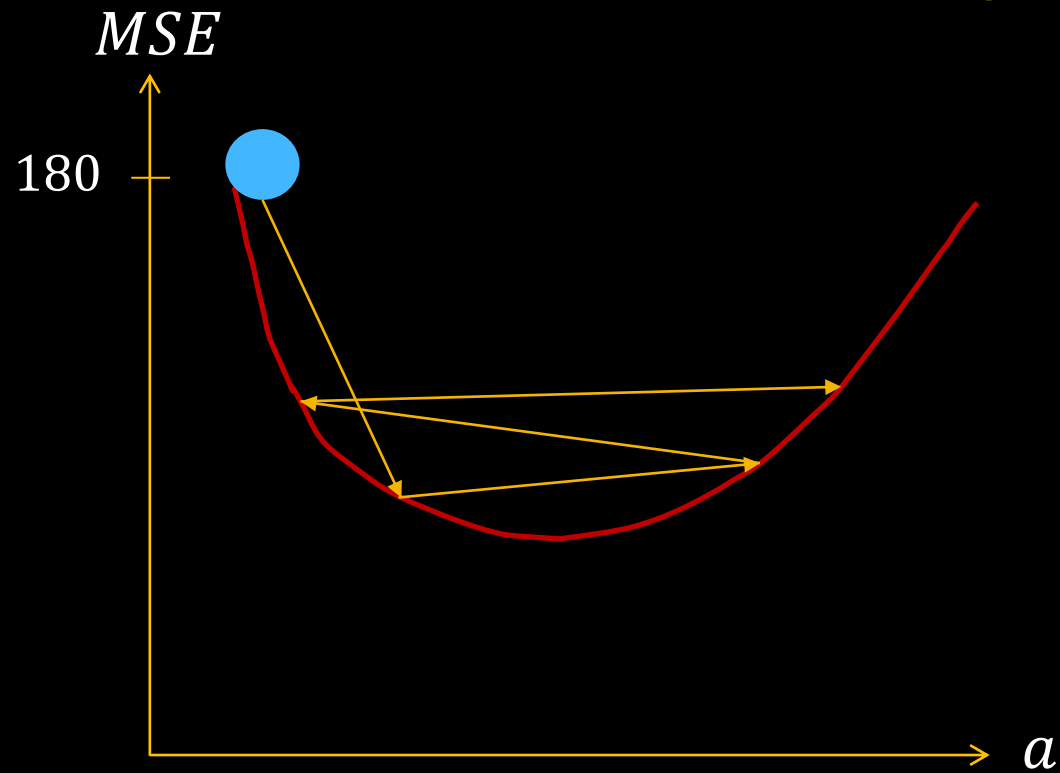
$$a = a - \eta \nabla MSE$$

$$MSE = 0.001$$



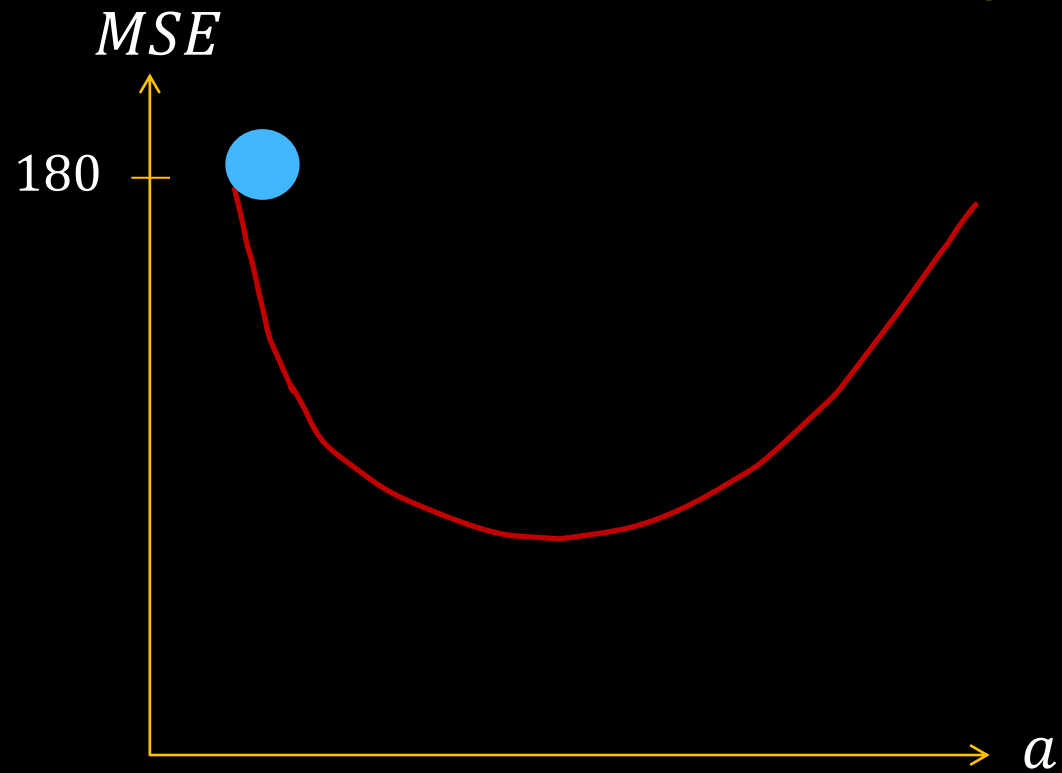
$$a = a - \eta \nabla MSE$$

$$MSE = 0.001$$



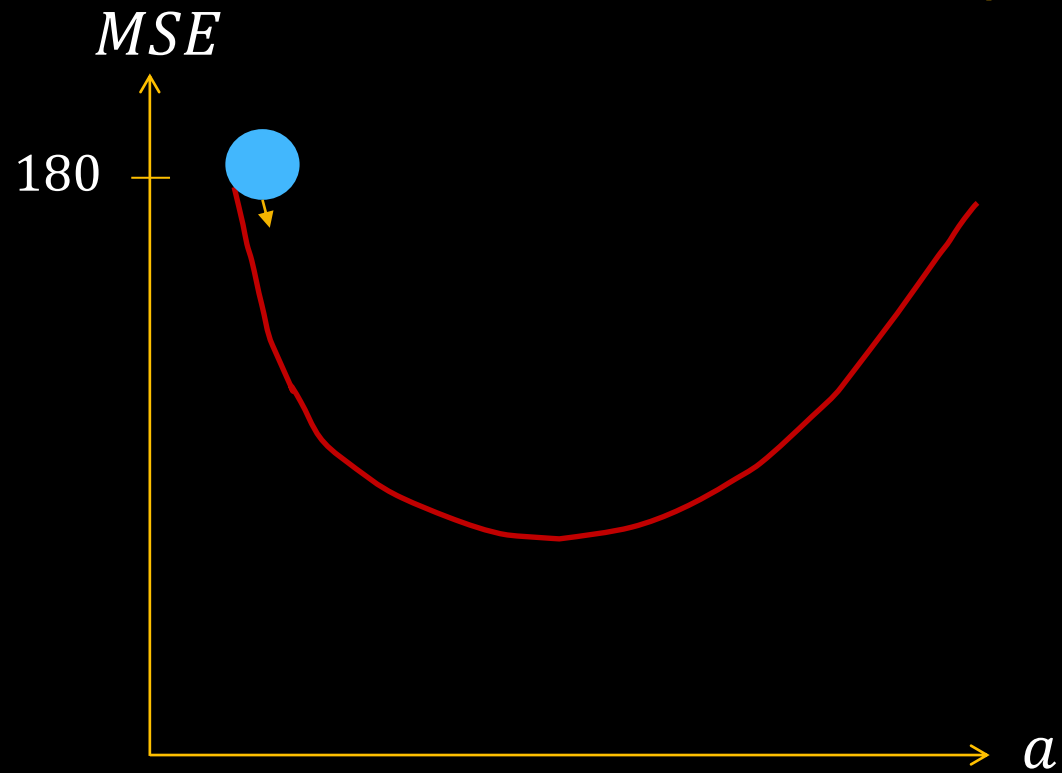
$$a = a - \eta \nabla MSE$$

$$MSE = 0.001$$



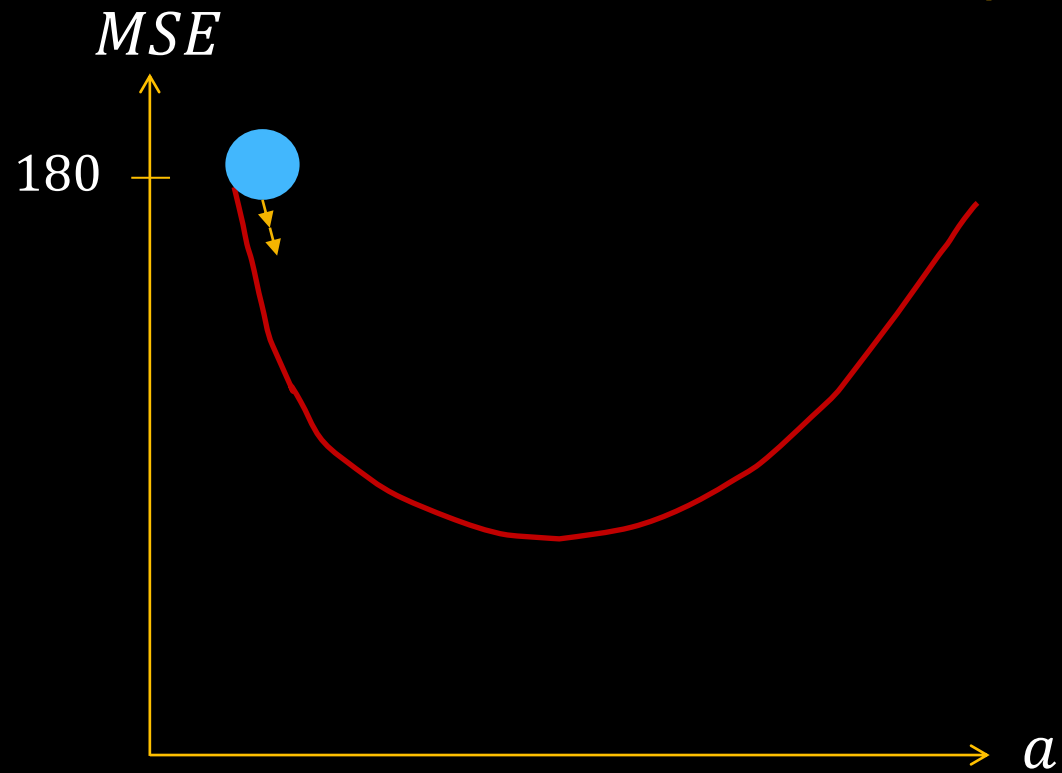
$$a = a - \eta \nabla MSE$$

$$MSE = 0.001$$



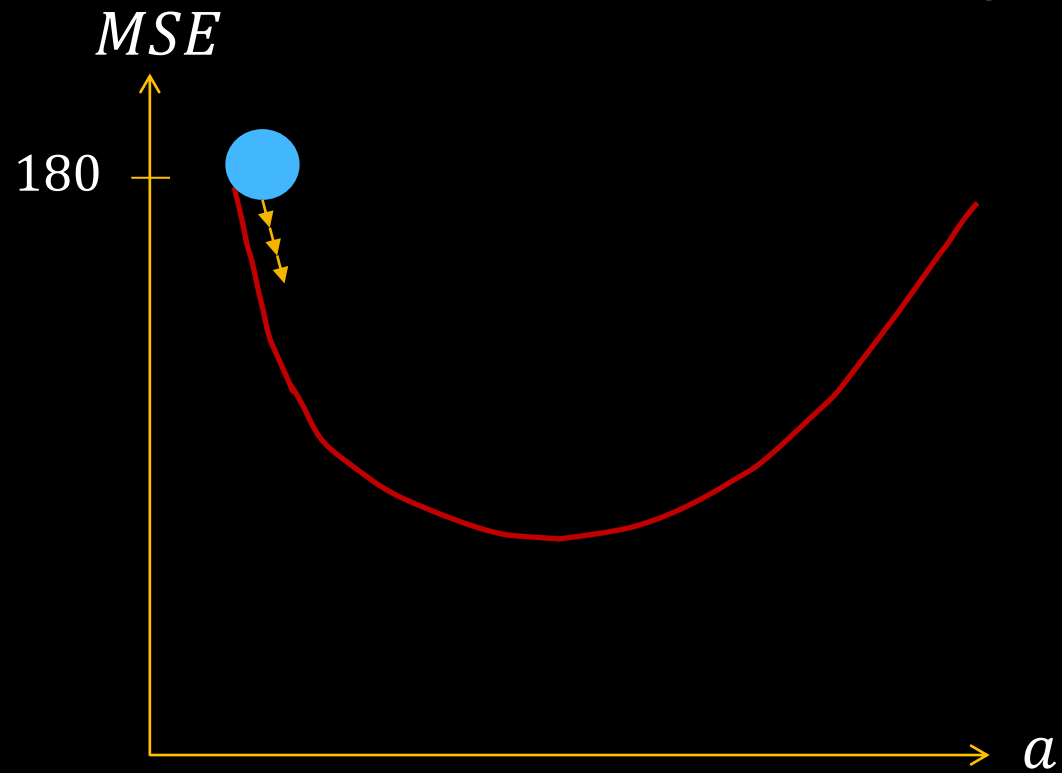
$$a = a - \eta \nabla MSE$$

$$MSE = 0.001$$



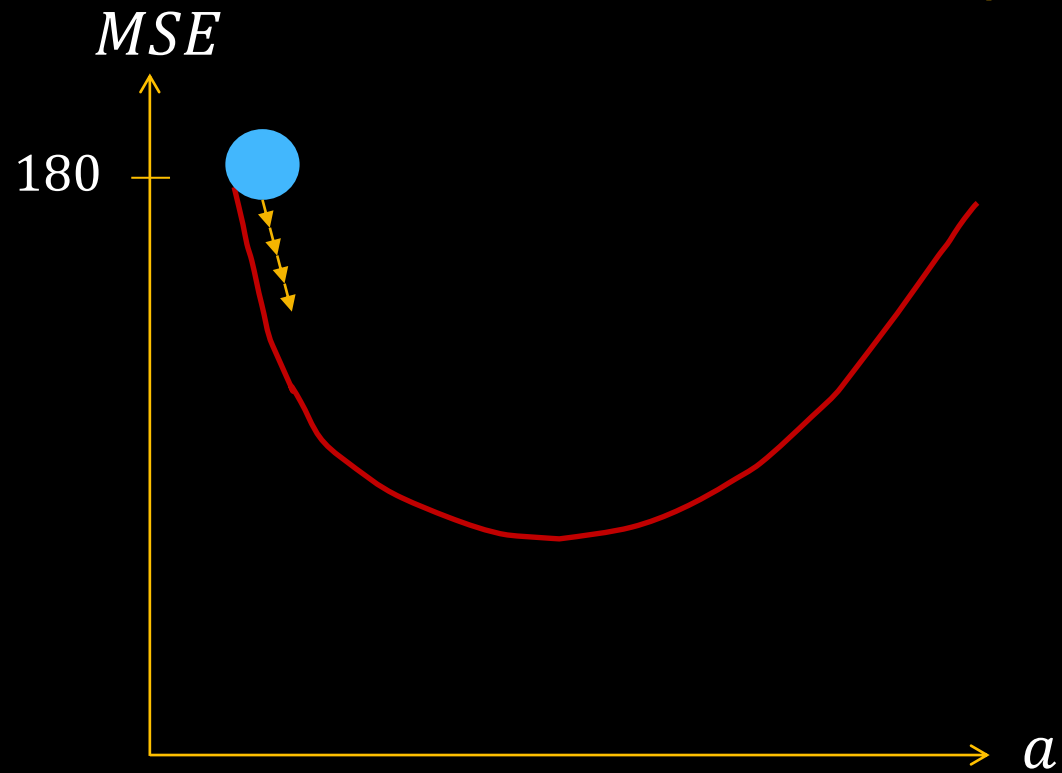
$$a = a - \eta \nabla MSE$$

$$MSE = 0.001$$



$$a = a - \eta \nabla MSE$$

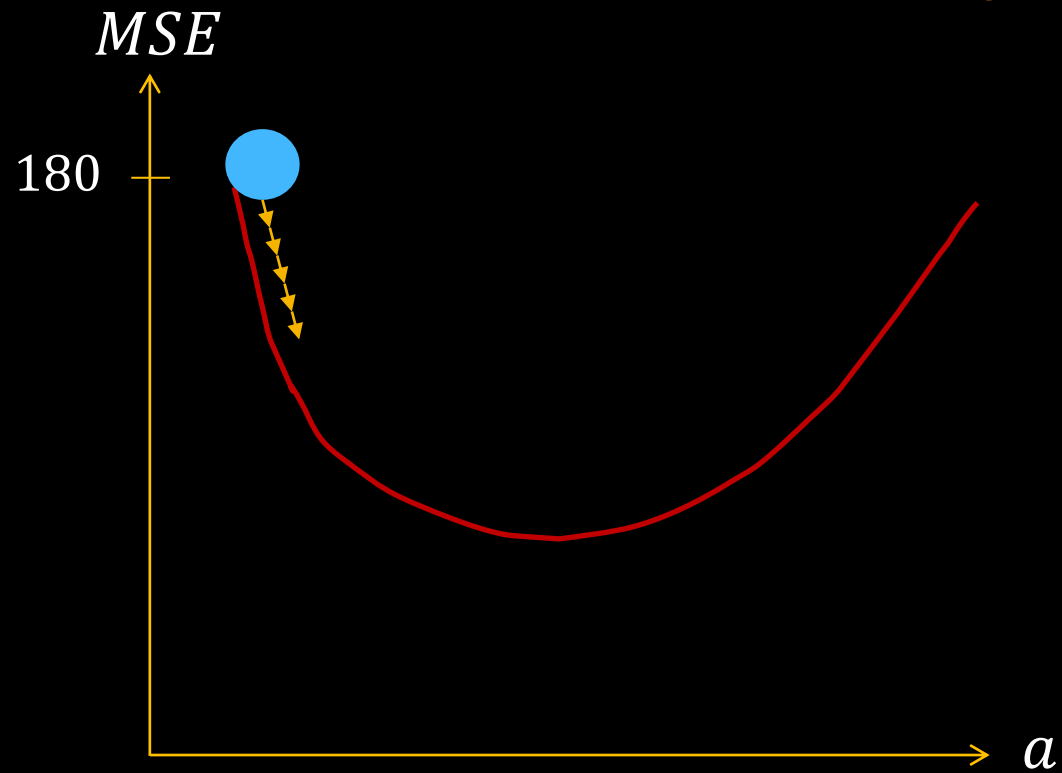
$$MSE = 0.001$$





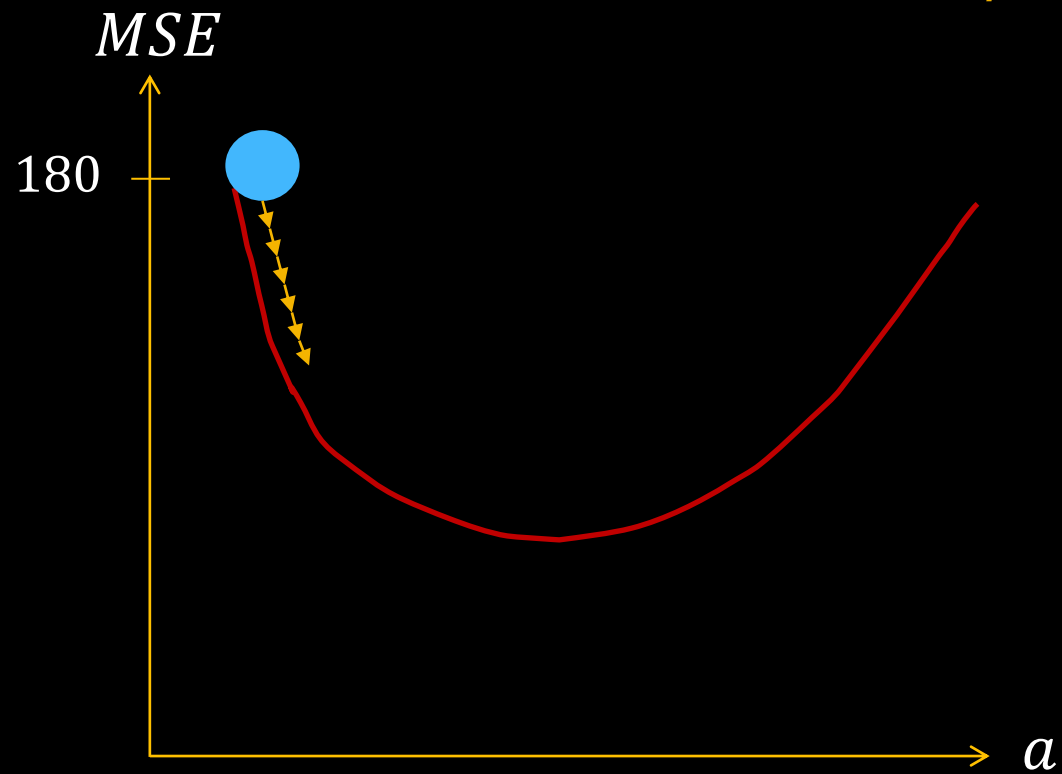
$$a = a - \eta \nabla MSE$$

$$MSE = 0.001$$



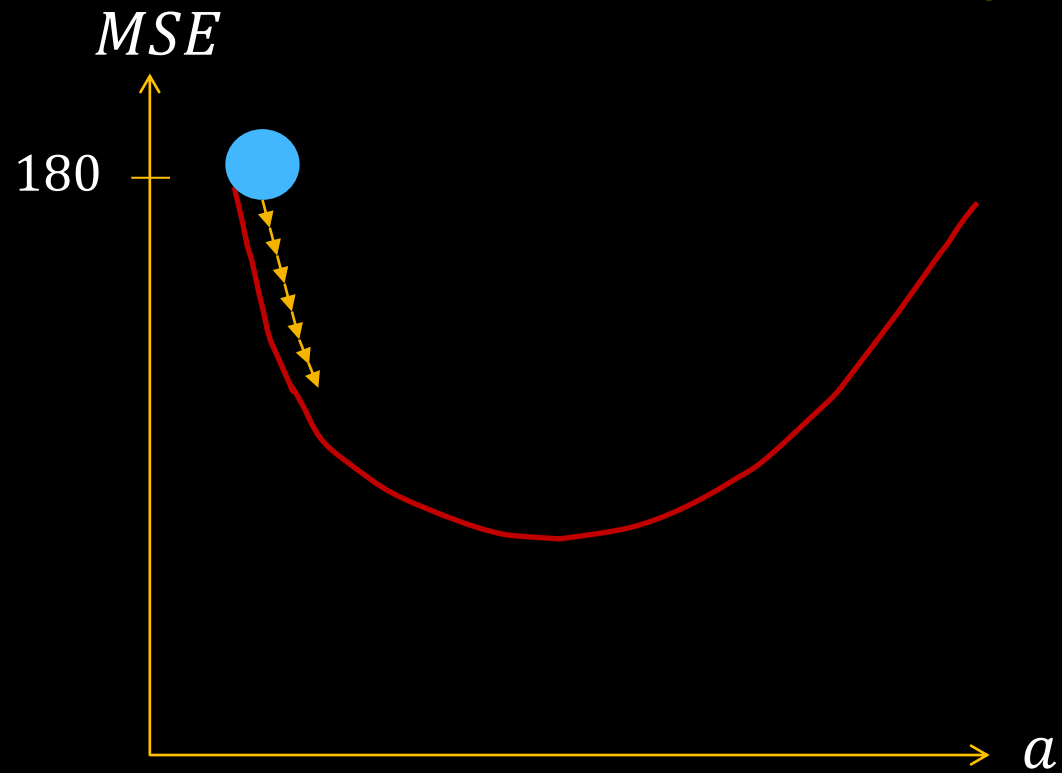
$$a = a - \eta \nabla MSE$$

$$MSE = 0.001$$



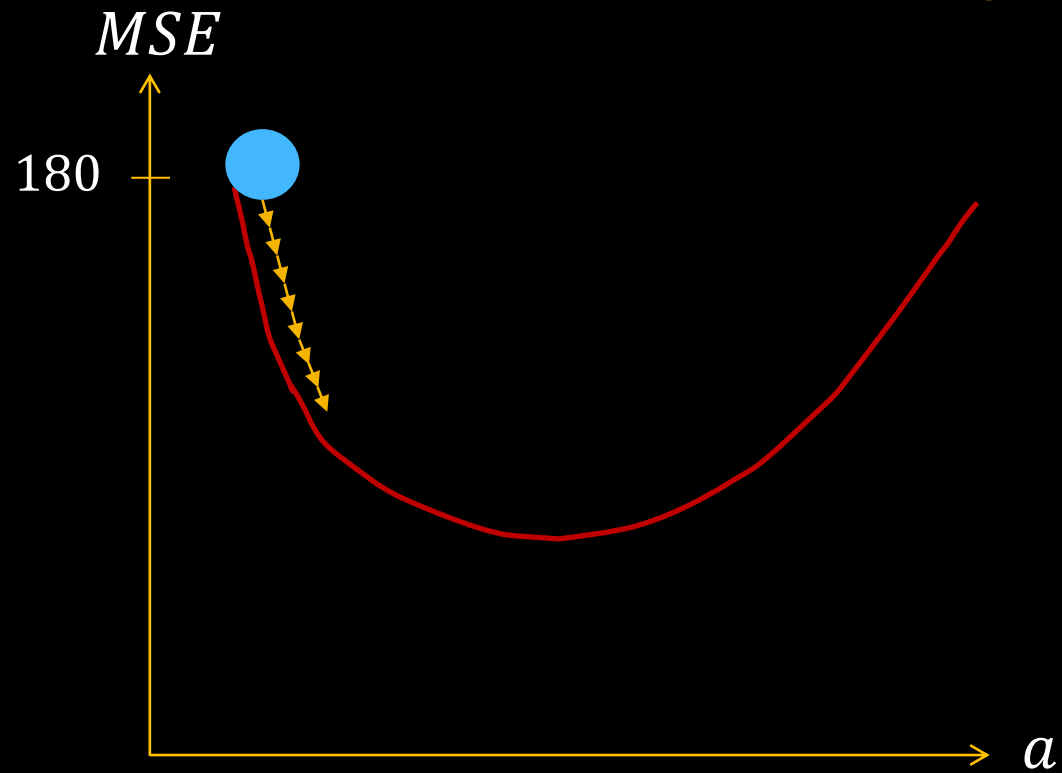
$$a = a - \eta \nabla MSE$$

$$MSE = 0.001$$



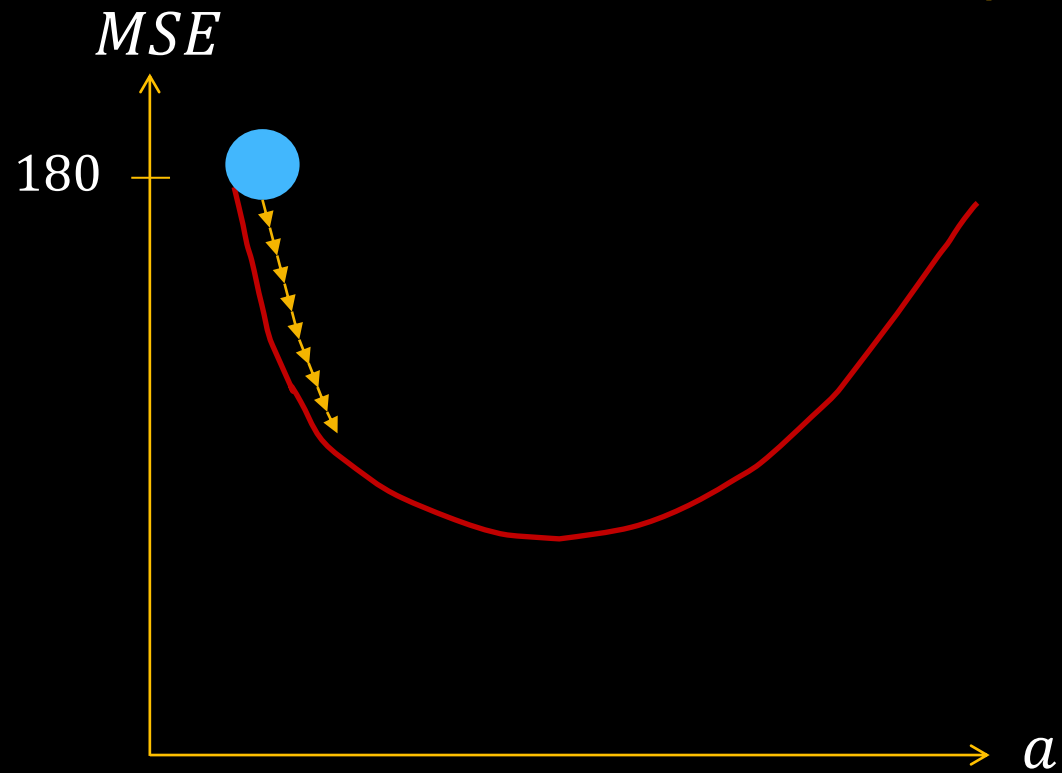
$$a = a - \eta \nabla MSE$$

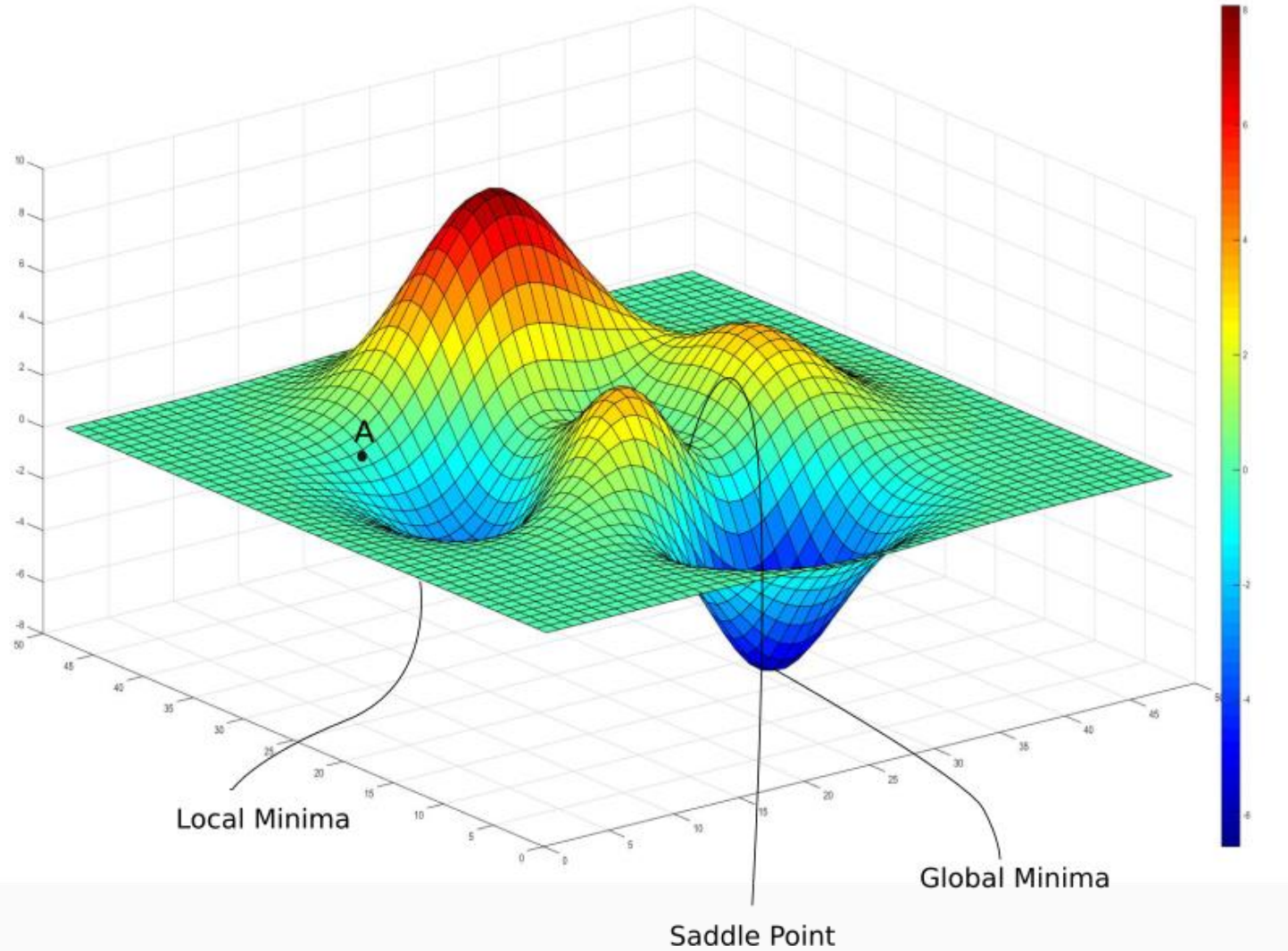
$$MSE = 0.001$$



$$a = a - \eta \nabla MSE$$

$$MSE = 0.001$$





temperature	Ice cream sales
30	56
23	37
0	1
38	87
44	100
...	...
...	...

$$\begin{cases} a = a - \eta \nabla MSE \\ b = b - \eta \nabla MSE \end{cases}$$

$$\frac{\partial MSE(a)}{\partial a} = \frac{-2}{n} \sum_{i=0}^n x_i (y_i - \hat{y}_i)$$

$$a = a - \eta \frac{\partial MSE(a)}{\partial a}$$

Vanilla gradient descent

temperature	Ice cream sales
30	56
23	37
0	1
38	87
44	100
...	...
...	...

$$\begin{cases} a = a - \eta \nabla MSE \\ b = b - \eta \nabla MSE \end{cases}$$

$$\frac{\partial MSE(a)}{\partial a} = \frac{-2}{n} x_i (y_i - \hat{y}_i)$$

$$a = a - \eta \frac{\partial MSE(a)}{\partial a}$$

stochastic gradient descent



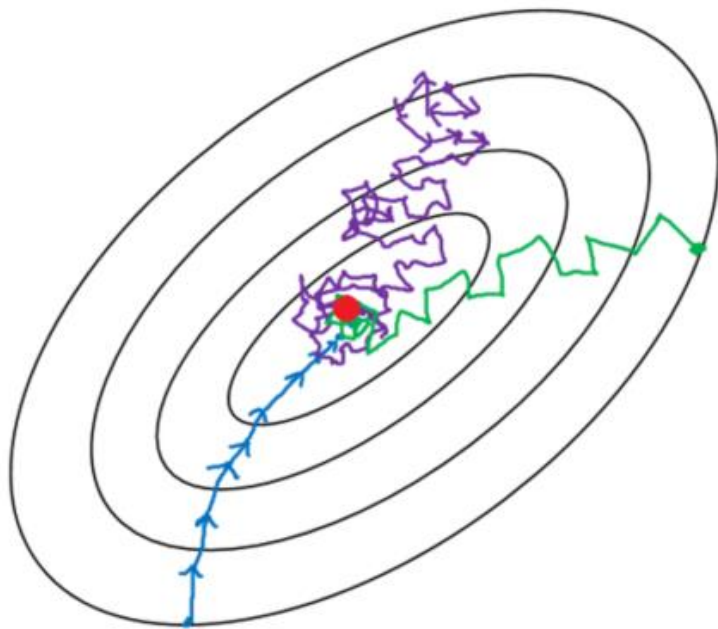
temperature	Ice cream sales
30	56
23	37
0	1
38	87
44	100
...	...
...	...

$$\begin{cases} a = a - \eta \nabla MSE \\ b = b - \eta \nabla MSE \end{cases}$$

$$\frac{\partial MSE(a)}{\partial a} = -\frac{2}{q} \sum_{i=0}^q x_i (y_i - \hat{y}_i)$$

$$a = a - \eta \frac{\partial MSE(a)}{\partial a}$$

Mini batch gradient descent



- Batch gradient descent
- Mini-batch gradient Descent
- Stochastic gradient descent

Q&A

END