PHYS 310 Work and Results

Khalil El Achi

Table of Content

1 Interpolation

1.1 Lagrange Interpolation

1.1.1 Manual Function

The first attempt to code a second-degree Lagrange Interpolation of the data sets modeling

$$y = x^2$$

is by utilizing a self-coded function:

```
import numpy as np

import matplotlib.pyplot as plt
```

```python
from scipy.interpolate import lagrange

# Starting with the manual Interpolation


x2=np.array([1, 3, 5])

y2=np.array([0.8, 10, 23.5])


x_lagrange=np.arange(0,10,0.1)


def lagrange_interpol(x,x2, y2):
       P1 = (y2[0]*(x-x2[1])*(x-x2[2]))/((x2[0]-x2[1])*(x2[0]-x2[2]))
       P2 = (y2[1]*(x-x2[0])*(x-x2[2]))/((x2[1]-x2[0])*(x2[1]-x2[2]))
       P3 = (y2[2]*(x-x2[0])*(x-x2[1]))/((x2[2]-x2[0])*(x2[2]-x2[1]))
       y = P1 + P2 + P3
       return y


x1= np.arange(0,10, 0.1)
y1 = np.array([])
for i in x1:
       y1 = np.append(y1, lagrange_interpol(i,x2,y2))


plt.plot(x2[0],y2[0], 'bo',label='P1')
plt.plot(x2[1],y2[1], 'mo',label='P2')
plt.plot(x2[2],y2[2], 'ko',label='P3')
plt.plot(x1,y1, color='r')
plt.grid()
plt.legend()
```

```
plt.title('Lagrange Interpolation of y=x^2')
plt.xlabel('x')
plt.ylabel('y')
plt.show()
```

The outputted Graph recovers the data points and the result of the interpolation:

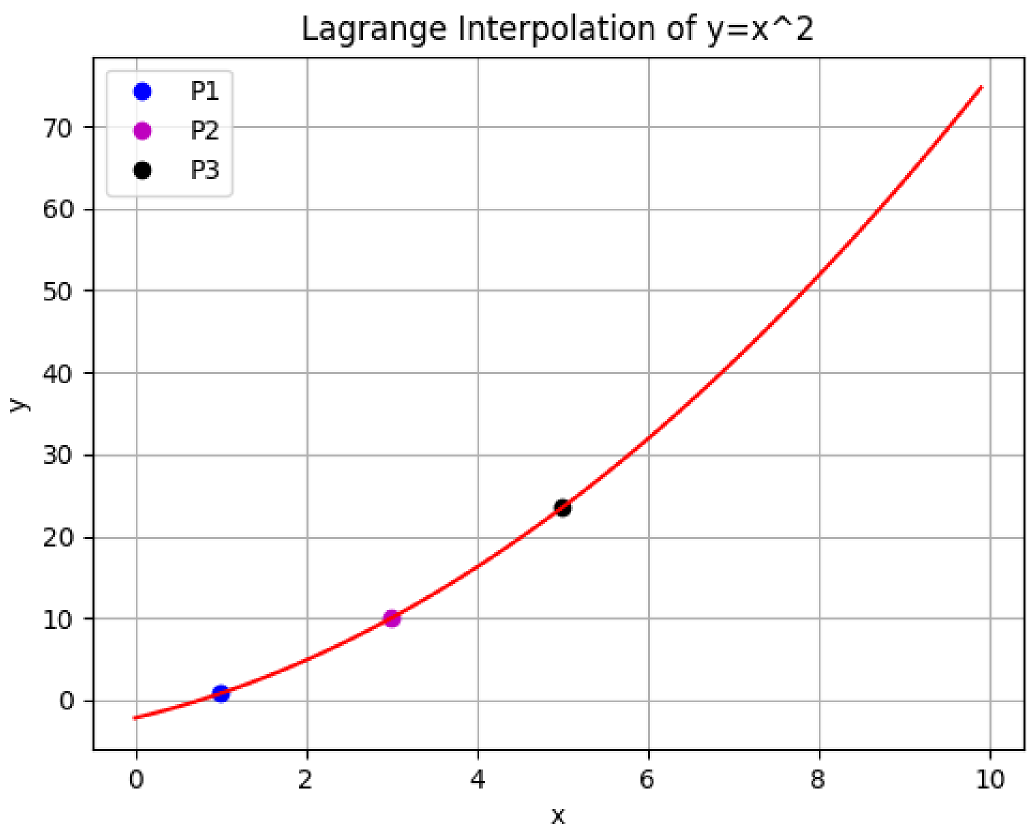

Figure 1: Lagrange Interpolation using a Manual Function

1.2 Atkins Method

Attempting to Model

$$y = \frac{1}{1+x}$$

using both the Atkins Method of Polynomials and the built in function in Scipy library.

```
        import numpy as np
        import matplotlib.pyplot as plt
        from scipy.interpolate import lagrange
# Newton/Atkins Method And Quadratic Splines
def Atkins_method(x,x1,y1):
        p12 = (x-x1[1])/(x1[0] - x1[1]) * y1[0] + (x-x1[0])/(x1[1]-x1[0]) * y1[1]
        p23 = (x-x1[2])/(x1[1] - x1[2]) * y1[1] + (x-x1[1])/(x1[2]-x1[1]) * y1[2]
        p34 = (x-x1[3])/(x1[2] - x1[3]) * y1[2] + (x-x1[2])/(x1[3]-x1[2]) * y1[3]
        p123 = (x-x1[2])/(x1[0] - x1[2]) * p12 + (x-x1[0])/(x1[2]-x1[0]) * p23
        p234 = (x-x1[3])/(x1[1] - x1[3]) * p23 + (x-x1[1])/(x1[3]-x1[1]) * p34
        p1234 = (x-x1[3])/(x1[0] - x1[3]) * p123 + (x-x1[0])/(x1[3]-x1[0]) * p234
        return p1234

# Modeling equation f(x) = 1/1+x


x_f=np.array([1, 2, 3, 4])
y_f=np.array([0.45, 0.35, 0.23, 0.18])


x_r=np.linspace(np.min(x_f),np.max(x_f),50)
y_quadratic = interp1d(x_f, y_f, kind='quadratic')
y_r=np.array([])
for i in x_r:
        y_r = np.append(y_r,Atkins_method(i,x_f,y_f))
y_real=1/(1+x_r)
plt.figure()
plt.plot(x_f[0],y_f[0], 'bo',label='P1')
plt.plot(x_f[1],y_f[1], 'mo',label='P2')
```

```
plt.plot(x_f[2],y_f[2], 'ko',label='P3')
plt.plot(x_f[3],y_f[3], 'ro',label='P4')
plt.plot(x_r,y_r, color='b',label='Atkins Interpolation')
plt.plot(x_r,y_real,color='k',label='Real Function')
plt.plot(x_r,y_quadratic(x_r),color='m',label='Quadratic Splines')
plt.grid()
plt.legend()
plt.title('Atkins Method for Interpolating y=1/1+x')
plt.xlabel('x')
plt.ylabel('y')
plt.show()


# Scipy Python Library
f_lag= lagrange(x2,y2)
plt.figure
plt.plot(x1,f_lag(x1), 'g' , x2, y2 , 'mo')
plt.grid()
plt.legend()
plt.title('Lagrange Interpolation of y=x^2')
plt.xlabel('x')
plt.ylabel('y')
plt.show()
```

Here one can see when the function doesn't follow an apparent trend or when the data points are not too far off and don't cover a wide range the estimated interpolation is very far from the actual function being modeled. This is seen below.

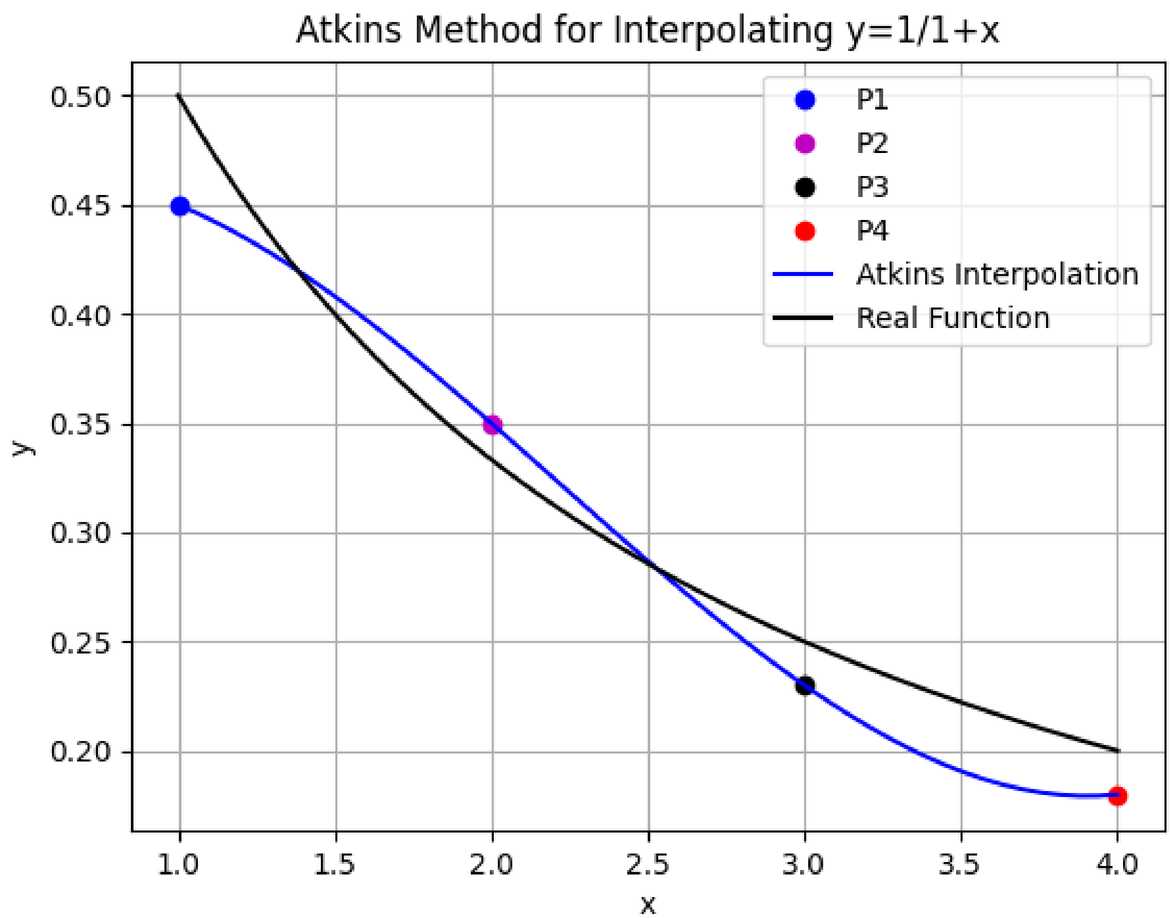

Figure 2: Bad Interpolation for $y = \frac{1}{1+x}$

1.3 Quadratic Splines

Using the built in function scipy.interpole from SciPy, one can produce better results using Quadratic Splines.

```
from scipy.interpolate import interp1d

y_quadratic = interp1d(x_f, y_f, kind='quadratic')
```

This will output a quadratic splines estimate between the data points and is a more accurate estimate than Lagrange interpolation as the figure below shows.

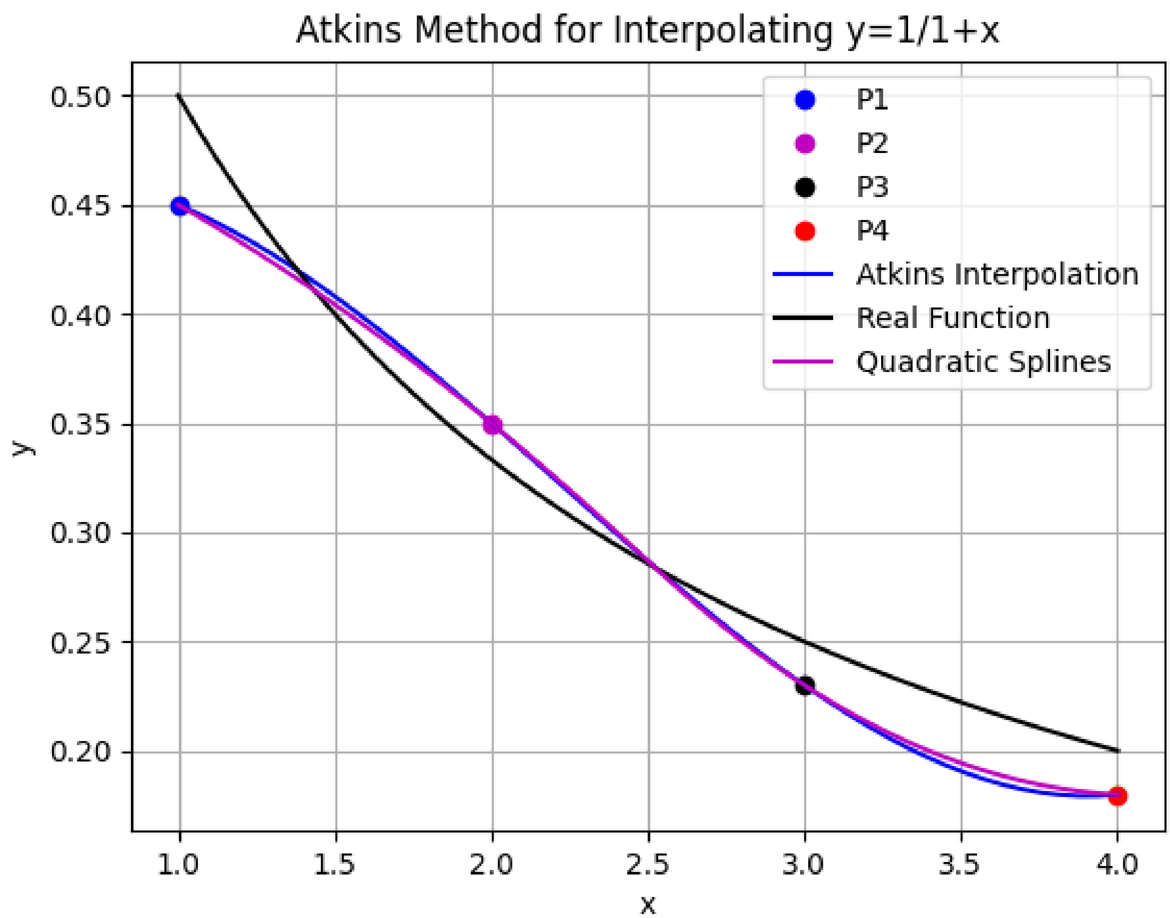

Figure 3: Quadratic Splines and Lagrange Interpolation for $y = \frac{1}{1+x}$

Additionally, below is the code used for the function lagrange in scipy.interpole that automatically outputs the Lagrange Interpolation of the the Data point. Below is the code and the output for Data Sets modeling $y = \frac{1}{1+x}$.

```
# Scipy Python Library
f_lag= lagrange(x_f,y_f)
plt.figure
plt.plot(x_r,f_lag(x1), 'g' , x_f, y_f , 'mo')
plt.plot(x_r,y_real,color='k')
plt.grid()
plt.legend()
```

```
plt.title('Lagrange Interpolation of y=x^2')
plt.xlabel('x')
plt.ylabel('y')
plt.show()
```

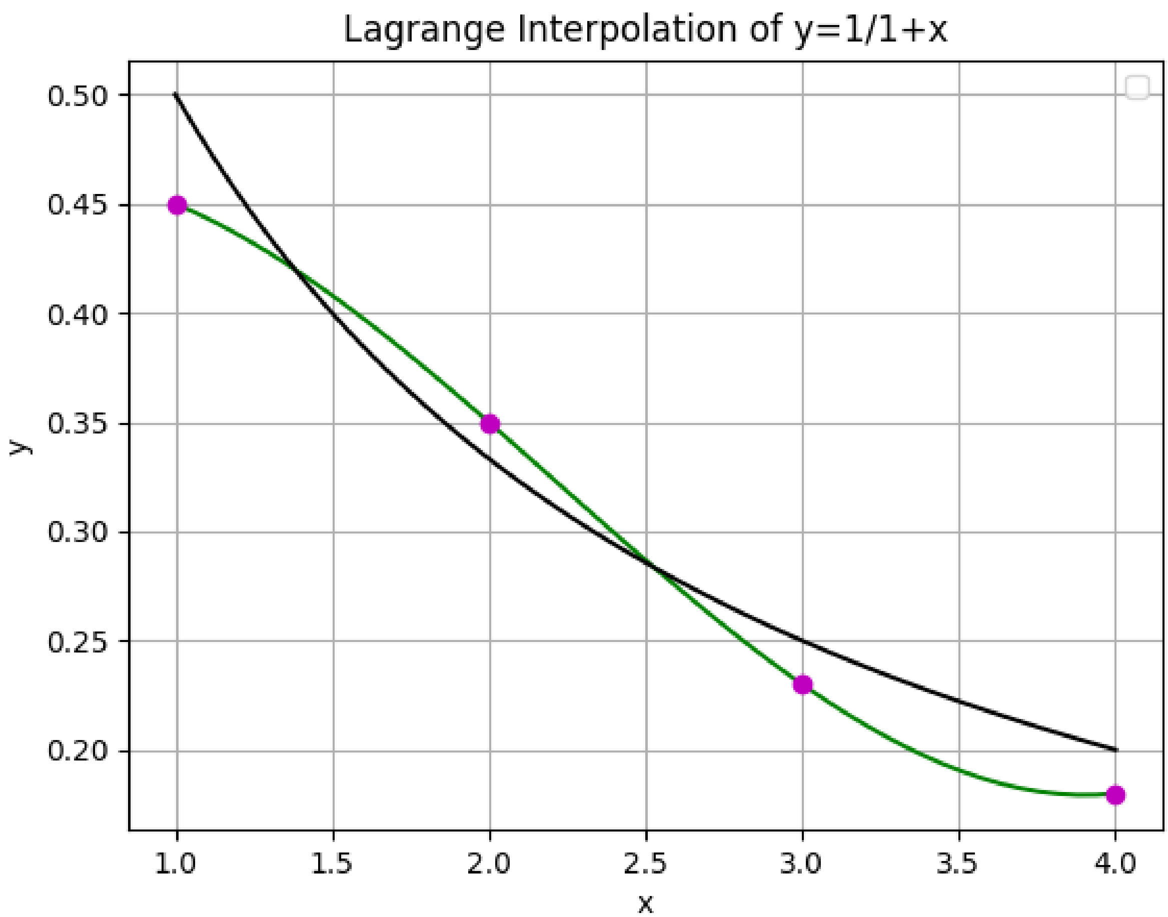

Figure 4: Lagrange Interpolation Using the Built-in function in SciPy Library for $y = \frac{1}{1+x}$

The output is identically the same as the ones above. The only difference that will help the interpolation is introducing more points in different ranges to have a better estimate towards the trend of the points.