Introduction to Numerical Optimization - Project

Solar microgrid

A solar microgrid is a small-scale electrical grid composed of photovoltaic (PV) panels converting solar energy into electricity, a battery storing electricity, and a connection to the distribution network. A sketch of the system configuration considered is presented in Figure 1. We are interested in optimally sizing the microgrid, that is, identifying the capacity of components that minimize the total cost of the system over its lifetime, while satisfying the demand. The total cost of the microgrid is the sum of the overnight investment costs incurred when installing the various components and the operating costs incurred over the lifetime of the system. An agent designing the system faces a trade-off between building a large installation, which typically leads to high investment costs but low (variable) operating costs, and deploying a small installation, which often results in low investment costs but high (variable) operating costs.

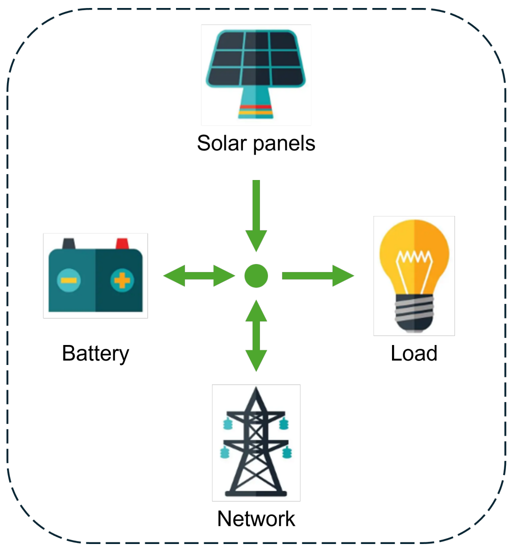

Figure 1: Microgrid configuration

Before formally introducing this benchmark problem, let us mention that we will use the notation $[\cdot]$ to indicate the physical unit of the symbol preceding it. In this project, $[W]$ denotes instantaneous power production in watts, $[W_p]$ denotes nameplate (manufacturer) power capacity, $[Wh]$ denotes energy in watt-hours and $[Wh_p]$ denotes nameplate (manufacturer) energy capacity. In addition, $[h]$ denotes the unit of time (hours) and [€] denotes the currency considered in this problem. Finally, in the following, the time period $[t, t+1[$ between any two successive time steps t and $t+1$ corresponds to one hour (i.e., $\Delta t = 1$ hour) during which the consumption and the production levels are assumed constant.

In the system, the solar panels and the grid connection provide electricity to supply the consumption of a load $P_t^C[W]$. On one hand, the solar panels harness solar radiation for electricity production. Formally, the quantity of power the panels produce $P_t^{PV}[W]$ at time t is proportional to the irradiance $i_t[Wh/Wh_p]$ at time t and to the installed (power) capacity of PV panels $C^{PV}[W_p]$ such that:

$$P_t^{PV} = \eta^{PV}\, i_t\, C^{PV} \tag{1}$$

where $\eta^{PV}[\%]$ is the efficiency of the panels. In addition, the power production can be curtailed by covering the panel. On the other hand, the microgrid is connected to the distribution network, from which it can buy electricity at the power $P_t^{G+}[W]$ or to which it can sell electricity at the power $P_t^{G-}[W]$ at any time t.

In order to make the most out of the solar panels installed, it is often necessary to store the energy produced when the sun shines in order to use it later. To do so, we can use batteries. Formally, a battery is an electrical device that can store energy up to a quantity $E^B[Wh_p]$, called (energy) capacity of the battery. Power can be charged or discharged in the battery during the time interval Δt with an efficiency $\eta^B[\%]$. In the case where energy is charged, the battery consumes energy that is produced. When the battery is discharged, it provides energy to the load or to the grid.

The PV panels and the battery are associated with an investment cost that is proportional to the installed capacity. In addition, the microgrid has marginal cost associated with the electricity that is bought from the distribution network but can also make revenues by selling electricity to the distribution network.

CO_2 emissions are also associated with the microgrid components. This aspect will be considered in a later stage of the project.

The objective is to find the capacity of the PV panels and the battery to install in order to minimize the sum of the investment and operation costs while ensuring that the power produced is effectively consumed in the microgrid. To this end, you will formulate the problem of controlling the microgrid during several years as a function of the two capacities you want to optimize. The different numerical values on the efficiencies are provided in Table 1 as well as the cost per unit of installed capacities and the buying and selling prices of electricity from the grid. In your model, you will consider that the hourly consumption and hourly irradiance of one typical day is repeated. The numerical values (in megawatt-hour) are provided in `data.jl`.

Symbol	Description	Value	
η^{PV}	PV panels efficiency	86	%
η^{B}	Battery efficiency	95	%
π^{PV}	Cost per installed capacity PV	800	€$/kW_p$
π^{W}	Cost per installed capacity of wind	1500	€$/kW_p$
π^{B}	Cost per installed capacity of battery	500	€$/kWh_p$
π^{G+}	Cost per unit of electricity bought	0.1	€$/kWh$
π^{G-}	Revenue per unit of electricity sold	0.02	€$/kWh$
θ^{PV}	CO_2 emissions of PV panels production	1000	$kgCO_2/kW_p$
θ^{B}	CO_2 emissions of battery production	150	$kgCO_2/kWh_p$
θ^{G}	CO_2 emissions of grid electricity production	0.1	$kgCO_2/kWh$

Table 1: Parameters of the problem

Model

1. Formulate the problem mathematically as a linear optimization model.

   Hint: add a constraint on the state of charge to ensure that the initial state has a feasible physical value.

2. Solve the problem simulating one year and five years of system operation. Plot the production and consumption of the different elements of the microgrid as well as the state of charge of the battery during a typical week. Discuss the results.

3. Solve the previous problem using the simplex algorithm and using a barrier method. Report the time to solve the problem as a function of the number of years over which the microgrid is operated. Discuss the results.

4. Reduce the number of variables in the problem by writing a model which avoids the use of the state of charge of the battery as an explicit variable. Is the solving performance any better? Why?

   Hint: the state of charge at each time period may be replaced by a function of the power charged and discharged at previous time steps.

Sensitivity analysis

5. Provide an interpretation of the dual variables associated with equality constraints in the linear optimization model.
6. Assume the π^B parameter may be changed by a value $\Delta\pi^B$. Give the interval outside of which the optimal basis would change. In this interval, give the new optimal solution for any value of $\Delta\pi^B$. Interpret the results.

Model reformulation

7. Reformulate the problem to determine the most sustainable microgrid, i.e. that minimizes the total CO_2 emissions. Answer Question 6 again for the parameter θ^G.
8. A wind turbine is added to the microgrid. The power produced by the wind turbine $P_t^W[W]$ at time t is proportional to the turbine nominal capacity $C_W[kW_p]$ to be determined and the wind power given in the data file; it cannot be curtailed. We consider three wind-availability scenarios, each with probability $\frac{1}{3}$. Formulate the objective function of the optimization problem in order to minimize the sum of total costs over the three scenarios together with the worst-case scenario cost. Solve the problem and discuss the results.

Submissions

Work by groups of two students. Send an email to gbailly@uliege.be with the names of the group members by October 15.

Submit on Gradescope your report and code by November 16 at 11:59 PM.
The report must be in PDF format and cannot exceed 6 pages in total. Do nice, readable, and interpretable plots (see this tutorial video).
Your code must be written in Julia. Make it easy to read.

Be ready to present your results on November 19. The presentation is a discussion around your code and report. You will be asked to modify parts of your code on the spot and to explain your results.