

### Heapify Operations (heapifyMax and heapifyMin):

Time Complexity:  $O(\log n)$

These functions are called recursively to maintain the heap property of a subtree rooted with node  $i$  assuming subtrees are already heapified. Each call affects the children of a node and their children, leading to a logarithmic time complexity.

### Building Heaps (buildMaxHeap and buildMinHeap):

Time Complexity:  $O(n)$

Building a heap from an unsorted array is done in linear time. This is achieved by calling heapify for all non-leaf nodes in reverse level order.

### Extracting N Largest/Smallest Elements (getNMaxElements and getNMinElements):

Time Complexity:  $O(N \log n)$

For each of the  $N$  elements extracted, a heapify operation ( $O(\log n)$ ) is performed. Therefore, the total complexity is  $O(N \log n)$ .

### Calculate Average Rate (calculateAvgRate):

Time Complexity:  $O(n)$

This function iterates over all elements to sum their values and then calculates the average, which requires linear time.

### Calculate Differences (calculateDifferenceM):

Time Complexity:  $O(n)$

Similar to calculateAvgRate, this function iterates over all elements to adjust their values based on the average rate.

### Maximum Subsequence Sum (maxSubsequenceSum):

Time Complexity:  $O(n)$

This algorithm is a variation of Kadane's algorithm, which finds the maximum sum contiguous subarray within a one-dimensional array of numbers. It runs in linear time.

### Reading CSV Data (readCSV):

Time Complexity:  $O(n)$

Assuming the file read operation itself is not the bottleneck, the time complexity is linear with respect to the number of lines in the CSV file.

### Main Function Complexity:

The main function calls buildMaxHeap, buildMinHeap, getNMaxElements, getNMinElements, calculateAvgRate, calculateDifferenceM, and maxSubsequenceSum. The dominant terms in the overall complexity would be the heap building and the extraction of  $N$  elements, both  $O(n)$  and  $O(N \log n)$  respectively.