GLASGOW COLLEGE UESTC

Final Exam Solution

Q1 Multiple choice

Choose the ONE alternative that best completes the statement or answer the question.

1. (C) Consider a uniformly charged semi-circular object (half of a thin ring) with charge Q and radius R, what is the electric field at the center point o? [3]

(A) $\frac{Q}{4\pi\varepsilon_0 R^2}$ (B) $\frac{Q}{2\pi\varepsilon_0 R}$

(C) $\frac{Q}{2\pi^2\varepsilon_0 R^2}$ (D) $\frac{Q}{4\varepsilon_0 R^2}$

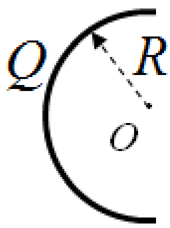

Figure Q1-1.

2. (B) Which of the following statement is NOT true about electrostatic field? [3]

(A) Electric field originates from positive charge and terminates on negative charge.

(B) Where the electric field line density is higher, the electric field is smaller there.

(C) Electric field lines never cross with one another.

(D) The direction of electric field is tangential with the field lines.

3. (D) Put a charge q nearby a conducting sphere initially carrying no charge. What is the electric potential of the sphere? (V=0 at infinity) [3]

(A) $\frac{q}{4\pi\varepsilon_0 R}$ (B) $\frac{Qb}{4\pi\varepsilon_0 R^2}$

(C) $\frac{q}{4\pi\varepsilon_0 (R-b)}$ (D) $\frac{q}{4\pi\varepsilon_0 b}$

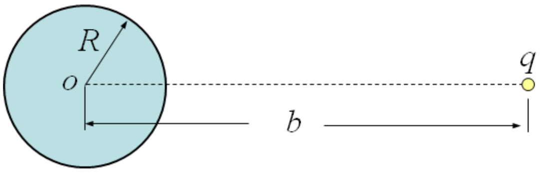

Figure Q1-3.

4. (B) There are some statements about Hall Effect. Which of them is NOT correct? [3]

(A) There will be a voltage across a current-carrying conductor if it is placed in a magnetic field.

(B) The charges can escape from the metal surface when they absorb enough energy.

(C) There are more than one force acting on the moving charges.

(D) Hall Effect can be used to distinguish p-type and n-type semiconductors.

5. (C) A circuit consists of two arc-shaped wires and two straight wires as shown in figure. If it carries current I, how much is the magnetic field produced at the center O? [3]

(A) $\dfrac{\mu_0 I}{4r}$ (B) $\dfrac{\mu_0 I}{4\pi r}$

(C) $\dfrac{\mu_0 I}{12r}$ (D) $\dfrac{\mu_0 I}{12\pi r}$

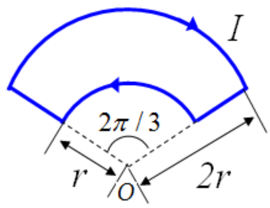

Figure Q1-5.

6. (A) About Maxwell's equations and their explanations, which of the following statement is CORRECT? [3]

(A) $\oint \vec{E}\cdot d\vec{S} = \dfrac{Q_{in}}{\varepsilon_0}$, which means electric field has sources.

(B) $\nabla\cdot\vec{B} = 0$, which means magnetic field is conservative.

(C) $\nabla\times\vec{E} = -\dfrac{\partial\vec{B}}{\partial t}$, which means changing electric field can produce magnetic field.

(D) $\oint \vec{B}\cdot d\vec{l} = \mu_0 I + \mu_0\varepsilon_0 \dfrac{d\Phi_E}{dt}$, which is a general statement of Faraday's law.

7. (C) Two spaceships move with same speed v=0.80c relative to the Earth, but in opposite directions. What is the speed of one ship relative to the other? [3]

(A) 0.640c; (B) 0.925c;

(C) 0.976c; (D) 0.983c.

8. (C) There are some statements about photoelectric effect and Compton effect, which of them is CORRECT? [3]

(A) Bohr won Nobel prize because of his work on photoelectric effect.

(B) In photoelectric effect, an electron absorbs the energy of many photons so that it can escape from the metal surface.

(C) The process of Compton effect can be described as: an electron is ejected out of atom by an X-ray photon in an elastic collision.

(D) In Compton effect we have energy conservation equation: $\frac{hc}{\lambda_0} = \frac{hc}{\lambda} + \frac{1}{2}mv^2$.

9. (D) A particle trapped in an infinitely deep square potential well of length L, has a wave function $\psi(x) = \sqrt{\frac{2}{L}}\sin(\frac{2\pi}{L}x)$. What is the probability to find the particle in region $0 < x < \frac{L}{3}$? [3]

(A) 0.264

(B) 0.303

(C) 0.333

(D) 0.402

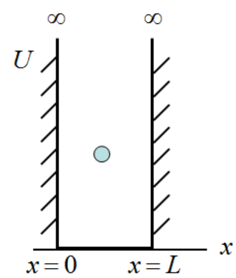

Figure Q1-9.

10. (A) According to the rules of quantum numbers of electrons inside atom, which of the following electron configuration is possible? [3]

(A) $1s^2 2s^2 2p^5 3s^2$;

(B) $1s^2 2s^3 2p^3$;

(C) $1s^2 2s^2 2p^6 2d^2$;

(D) $2s^2 2p^8 3s^2$.

Q2 A cylindrical capacitor consists of two coaxial conductor thin shells (R_1, R_2). The shells have radius R_1 and R_2 respectively, and equal length L ($L >> R_2$). Initially the conductor shells carry equal and opposite charge $+Q$ and $-Q$.

(a) Use Gauss's law to determine the electric field $\vec{\boldsymbol{E}}$ in the space. [7]

(b) What is the capacitance, and how much energy U is stored in the capacitor? [6]

(c) If the capacitor is filled with dielectric material (dielectric constant K), and the charge Q on capacitor doesn't change. How does the capacitance, voltage and energy storage change? [6]

(d) If Q is adjustable, the breakdown field for air capacitor is E_m. How much is the maximum energy storage in air capacitor? [6]

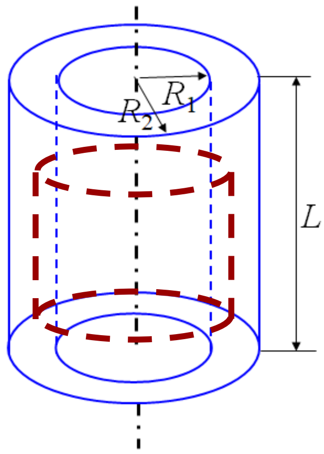

Figure Q2.

Solution: (a) Gauss's law $\oint \vec{E}\cdot d\vec{S} = \frac{Q_{in}}{\varepsilon_0}$, and choose proper Gaussian surface. [3]

The electric field between two shells: $E = \frac{Q}{2\pi\varepsilon_0 Lr}$ [2]

The electric field at other position: $E = 0$ [2]

(b) The electric potential difference V_{12} between two conductors

$$V = \int_{R_1}^{R_2} \vec{E}\cdot d\vec{r} = \frac{Q}{2\pi\varepsilon_0 L}\ln\frac{R_2}{R_1}$$ [2]

So, the capacitance is $C = \frac{Q}{V_{12}} = \frac{2\pi\varepsilon_0 L}{\ln(R_2/R_1)}$ [2]

The energy storage is $U = \frac{1}{2}CV^2 = \frac{Q^2}{4\pi\varepsilon_0 L}\ln\frac{R_2}{R_1}$ [2]

(c) When filled with dielectric material, the capacitance is $C' = KC = \frac{2\pi K\varepsilon_0 L}{\ln(R_2/R_1)}$ [2]

The voltage is $V' = \frac{Q}{C'} = \frac{Q}{2\pi K\varepsilon_0 L}\ln\frac{R_2}{R_1}$ [2]

And energy storage $U' = \frac{1}{2}QV' = \frac{Q^2}{4\pi K\varepsilon_0 L}\ln\frac{R_2}{R_1}$ [2]

(d) The field nearby the inner shell has the maximum value $E_m = \frac{Q}{2\pi\varepsilon_0 LR_1}$ [2]

So, the maximum charge storage is $Q = 2\pi\varepsilon_0 LR_1 E_m$ [2]

The maximum energy stored in capacitor:

$$U = \frac{1}{2}\frac{Q^2}{C} = \frac{1}{2}\frac{\ln(R_2/R_1)}{2\pi\varepsilon_0 L}\cdot(2\pi\varepsilon_0 LR_1 E_m)^2 = \pi\varepsilon_0 LR_1^2 E_m^2 \ln\frac{R_2}{R_1}$$ [2]

Q3 Consider a circular loop (radius R) with a current I.

(a) Calculate the magnetic field produced at point P (distance x from center o) on its axis. [6]

(b) How much is the magnetic moment μ? [4]

(c) If another identical loop is placed on point P, what is the mutual inductance M between these two loops? (Suppose $x >> R$, and two loops are coaxially placed) [5]

(d) Suppose current I changes as $I = I_0 e^{-\beta t}$, where I_0, β are constants. What is the EMF ε in the right loop? If it is a conductor loop, why is an induction current in it? [5]

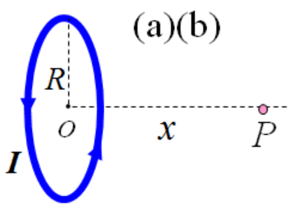

Figure Q3(a)(b).

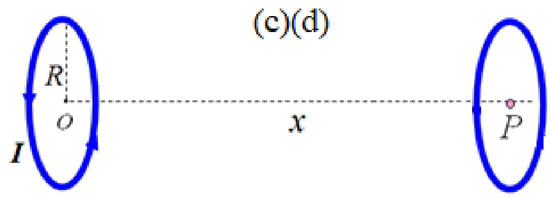

Figure Q3(c)(d).

Solution: (a) From the symmetry, magnetic field has only axial component. [2]

Choose an infinitesimal current, we can obtain an integral for the total field:

$$B = \int \frac{\mu_0 I dl}{4\pi r^2} \sin\varphi \quad [2]$$

$$= \frac{\mu_0 I \sin\varphi}{4\pi r^2} \cdot 2\pi R = \frac{\mu_0 I R^2}{2r^3} = \frac{\mu_0 I R^2}{2(R^2+x^2)^{3/2}} \quad [2]$$

(b) Magnetic moment is defined as $\mu = IS$ [2]

So $\mu = \pi R^2 I$ [2]

(c) If $x >> R$, $B = \frac{\mu_0 I R^2}{2(R^2+x^2)^{3/2}} \approx \frac{\mu_0 I R^2}{2x^3}$, it can be treated as a constant nearby point P. [1]

The mutual inductance $M = \frac{\Phi_B}{I}$ [2]

$$= \frac{B \cdot \pi R^2}{I} = \frac{\mu_0 \pi R^4}{2x^3} \quad [2]$$

(d) Faraday's law: $\varepsilon = -\frac{d\Phi_B}{dt} = \frac{-\beta \mu_0 I_0 \pi R^4}{2x^3} e^{-\beta t}$ [3]

Changing magnetic field will produce an induction electric field in the space, which will motivate the carriers in right loop to make an induction current. [2]

Q4 A laser pointer (P=5W) sends red laser with a wavelength $\lambda = 650\text{nm}$ to a white wall 5m away from it.

(a) The laser makes a uniform circular light spot on the wall with diameter D=3mm. What is the intensity there? What are the maximum electric and magnetic fields at the wall? [6]

(b) What is energy of each photon in the laser? Can this photon be absorbed by a ground state hydrogen atom, and why? [5]

(c) If an electron has de Broglie wavelength $\lambda_1 = 650\text{nm}$, what is its speed v_1? If it has de Broglie wavelength $\lambda_2 = 0.001\text{nm}$, what is its speed v_2 and kinetic energy E_k ?[10]

(d) If the uncertainty in position of an electron is $\Delta x = 650\text{nm}$, what is the minimum uncertainty in its momentum Δp_x, by Heisenberg uncertainty principle? [4]

Solution: (a) The intensity is $I = \dfrac{P}{\pi(D/2)^2} = 7.07\times10^5\,\text{W/m}^2$ [2]

We have $I = \bar{S} = \dfrac{E_{\max}B_{\max}}{2\mu_0}$ and $E_{\max} = B_{\max}\cdot c$ [2]

Then $E_{\max} = \sqrt{2\mu_0 c\bar{S}} = 2.31\times10^4\,\text{V/m}$ [1]

$$B_{\max} = \frac{E_{\max}}{c} = 7.70\times10^{-5}\,\text{T}$$ [1]

(b) Energy of photon:

$$E = \frac{hc}{\lambda} = 3.06\times10^{-19}\,J = 1.91eV$$ [2]

Energy gap for hydrogen atom:

$$\Delta E = (1-\frac{1}{n^2})E_1 = \left(1-\frac{1}{n^2}\right)\times13.6\text{eV}$$ [2]

$E = 1.91eV$ can't match the formula, so it can't be absorbed by ground state H atom. [1]

(c) de Broglie wavelength: $\lambda = \dfrac{h}{p} = \dfrac{h}{mv}$ [2]

For $\lambda_1 = 650\text{nm}$, we have $v_1 = \dfrac{h}{m_e\lambda_1} = \dfrac{6.63\times10^{-34}}{9.11\times10^{-31}\times650\times10^{-9}} = 1.12\times10^3\,m/s$ [2]

For $\lambda_2 = 0.001\text{nm}$, it's a relativistic particle, so $\lambda_2 = \dfrac{h\sqrt{1-v_2^2/c^2}}{m_e v_2}$ [2]

Solve it we have $v_2 = c \cdot \sqrt{\dfrac{h^2}{h^2 + m_e^2 c^2 \lambda_2^2}} = 0.925c = 2.77 \times 10^8 \text{m/s}$ [2]

Relativistic factor $\gamma = \dfrac{1}{\sqrt{1 - v_2^2 / c^2}} = 2.63$

Kinetic energy $E_k = (\gamma - 1) m_e c^2 = 1.34 \times 10^{-13} \text{J} = 0.835 \text{MeV}$ [2]

(d) Heisenberg uncertainty principle $\Delta x \Delta p \geq \hbar$ [2]

So, the minimum uncertainty in momentum is $(\Delta p)_{\min} = \dfrac{\hbar}{\Delta x} = 1.63 \times 10^{-28} \text{kg} \cdot \text{m/s}$ [2]

【Using $\Delta x \Delta p \geq h$ is also acceptable】