GLASGOW COLLEGE UESTC

Exam paper Solution

Physics II

Q1 Multiple choice

1. (D)

2. (C)

3. (D)

4. (B)

5. (A)

6. (B)

7. (C)

8. (C)

9. (A)

Q2 Consider a hemi-spherical shape object with radius R. Try to obtain the electrostatic quantities at the centre point o of its circular bottom surface.

(a) If electric charge distributes ONLY on the bottom surface with a uniform density σ.

(i) What is the electric field $\vec{E}_1$ at point o? [3]

(ii) How much is the electric potential V_1 at point o? [8]

(b) If electric charge distributes ONLY on the hemi-spherical surface with a uniform density σ.

(i) What is the electric field $\vec{E}_2$ at point o? [10]

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(ii) How much is the electric potential V_2 at point o? [4]

Hint: Consider the electric field and potential about a uniformly charged thin ring.

Figure Q2.

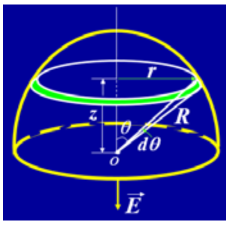
SOLUTION: (a) (i) From the symmetry, electric field at point o is $\vec{E}_1 = 0$ [3]

(ii) Choose a ring-shape infinitesimal, its charge is $dQ = \sigma \cdot 2\pi r dr$ [3]

Let V=0 at infinity, the electric potential due to dQ is $dV = \dfrac{dQ}{4\pi\varepsilon_0 r} = \dfrac{\sigma dr}{2\varepsilon_0}$ [2]

So the total potential is $V_1 = \int_0^R \dfrac{\sigma dr}{2\varepsilon_0} = \dfrac{\sigma R}{2\varepsilon_0}$ [3]

(b)



(i) Choose a ring-shape infinitesimal, now its charge is $dQ = \sigma \cdot 2\pi r \cdot R d\theta$ [3]

The electric field produced by dQ is pointing downward, it can be expressed as

$$dE = \frac{dQ}{4\pi\varepsilon_0 R^2}\cos\theta$$ [2]

Notice that $r = R\sin\theta$, we have $dE = \dfrac{\sigma \sin\theta\cos\theta d\theta}{2\varepsilon_0} = \dfrac{\sigma}{4\varepsilon_0}\sin 2\theta d\theta$ [2]

So the total electric field is $E = \dfrac{\sigma}{4\varepsilon_0}\int_0^{\frac{\pi}{2}} \sin 2\theta d\theta = \dfrac{\sigma}{4\varepsilon_0}$ [2]

The direction of electric field is perpendicular to the bottom, pointing downward. [1]

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(ii) All the charges have the same distance *R* to point *o*, the electric potential is

$$V = \frac{Q}{4\pi\varepsilon_0 R} = \frac{\sigma \cdot 2\pi R^2}{4\pi\varepsilon_0 R} = \frac{\sigma R}{2\varepsilon_0} \qquad [4]$$

Q3 (a) A sphere centres at point *O* with radius *R*, it is uniformly charged and the charge density per unit volume is ρ.

(i) What is the electric field outside the sphere? [6]

(ii) What is the electric field inside the sphere? [6]

(b) Suppose there is a spherical hole on the sphere of case (a), the hole centres at *O'* with radius *R'*, the distance between *O* and *O'* is *a*.

(i) What is the electric field inside the hole? [8]

(ii) How much is the total electric energy in the space of hole? [5]

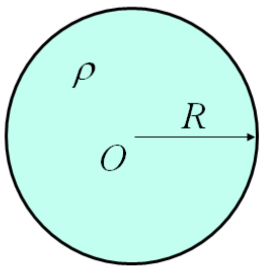

Figure Q3 (a)

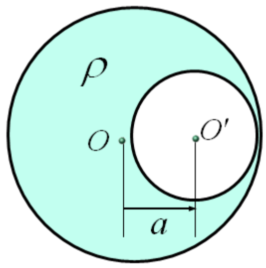

Figure Q3 (b)

SOLUTION: (a) (i) From the symmetry, electric field is along radial direction. [2]

Choose a spherical Gaussian surface with radius *r* (*r*>*R*) centered at O, Gauss's law:

$$\oint EdS = E \cdot 4\pi r^2 = \frac{Q_{in}}{\varepsilon_0} = \frac{4\pi\rho R^3}{3\varepsilon_0} \qquad [2]$$

So the electric field outside the sphere is $E = \dfrac{\rho R^3}{3\varepsilon_0 r^2}$ [2]

(ii) From the symmetry, electric field is also along radial direction. [2]

Choose a spherical Gaussian surface with radius *r* (*r*<*R*) centered at O, Gauss's law:

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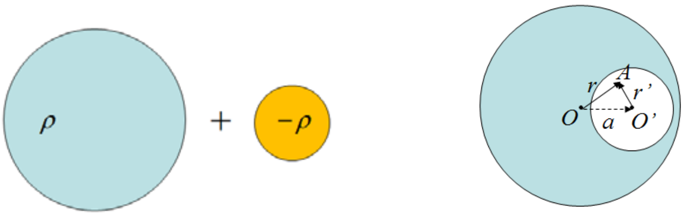
$$\oint EdS = E \cdot 4\pi r^2 = \frac{Q_{in}}{\varepsilon_0} = \frac{4\pi\rho r^3}{3\varepsilon_0}$$ [2]

So the electric field outside the sphere is $E = \dfrac{\rho r}{3\varepsilon_0}$ [2]

(b) (i) The system is equivalent to a combination of two spheres as figure: [3]



For a point A in the hole, the electric field produced by two spheres is:

$$\vec{E} = \frac{\rho\vec{r}}{3\varepsilon_0} - \frac{\rho\vec{r}'}{3\varepsilon_0} = \frac{\rho\vec{a}}{3\varepsilon_0}$$ [3]

It is a uniform field in the hole, points from O to O'. [2]

(ii) Electric energy density is $u = \dfrac{1}{2}\varepsilon_0 E^2 = \dfrac{\rho^2 a^2}{18\varepsilon_0}$ [3]

So the total electric energy in the space of hole is $U = u \times \dfrac{4\pi R'^3}{3} = \dfrac{2\pi\rho^2 a^2 R'^3}{27\varepsilon_0}$ [2]

Q4 Rotating charged objects can be related to circular currents. Consider a uniformly charged thin disk rotating about its central axis. The surface density of charge is σ, radius of disk is R, mass of disk is m, and angular velocity is ω.

(a) What is the magnetic moment $\vec{\mu}$ of rotating disk? [8]

(b) What is the magnetic field $\vec{B}$ produced at the centre of disk? [8]

(c) If the disk is released at rest in external magnetic field produced by a long straight current I, initially its magnetic moment $\vec{\mu}$ has a small angle θ with the external field, analyze its motion. Suppose the distance d between the disk and straight current satisfies $d >> R$, consider only magnetic forces. [9]

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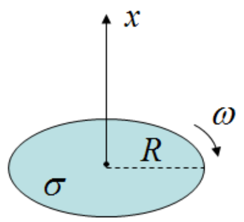

Figure Q4.

SOLUTION: (a) Rotating charges cause circular current: $I = \frac{\omega}{2\pi} Q$ [2]

Choose a ring-shape infinitesimal, its charge is $dQ = \sigma \cdot 2\pi r dr$,

so it causes a current $dI = \sigma\omega r dr$ [2]

The magnetic moment of this circular current is $d\mu = SdI = \pi\sigma\omega r^3 dr$ [2]

So the total magnetic moment of the rotating disk is

$\mu = \int_0^R \pi\sigma\omega r^3 dr = \frac{\pi\sigma\omega R^4}{4}$, its direction points to –x axis. [2]

(b) Use Biot-Savart law, the magnetic field produced by circular current *I* at its center:

$B = \int_0^{2\pi r} \frac{\mu_0}{4\pi} \frac{Idl}{r^2} = \frac{\mu_0 I}{2r}$ [3]

Similar to case (a), choose a ring-shape infinitesimal, its charge is $dQ = \sigma \cdot 2\pi r dr$,

and it causes a current $dI = \sigma\omega r dr$ [2]

So the total magnetic field at the center is

$B = \int_0^R \frac{\mu_0}{2r} \sigma\omega r dr = \frac{\mu_0 \sigma\omega R}{2}$, its direction points to –x axis. [3]

(c) Magnetic field produced by a long straight current *I* is $B = \frac{\mu_0 I}{2\pi d}$ [2]

If the distance *d* between the disk and straight current satisfies $d >> R$, it can be treated as uniform field in the range of disk.

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A magnetic torque is exerted on the disk: $\vec{\tau} = \vec{\mu} \times \vec{B}$, when the angle θ between $\vec{\mu}$ and $\vec{B}$ is small, we have $\tau = \mu B \sin\theta \approx \mu B\theta$ [3]

Under the action of magnetic torque, there will be a simple harmonic motion:

$$\tau = \mu B\theta = \frac{\pi\sigma\omega R^4}{4} \times \frac{\mu_0 I}{2\pi d} \times \theta = -\frac{1}{4} mR^2 \frac{d^2\theta}{dt^2}$$

$$\Rightarrow \frac{d^2\theta}{dt^2} + \frac{\sigma\omega\mu_0 IR^2}{2md}\theta = 0$$

The period of the simple harmonic motion is $T = 2\pi\sqrt{\frac{2md}{\sigma\omega\mu_0 IR^2}}$. [4]

End of question paper