GLASGOW COLLEGE UESTC

Final Exam Paper Solution

Physics II (Course Code)

Q1 Multiple choice

Choose the ONE alternative that best completes the statement or answer the question.

1. (C) An infinite plane with a circular hole of radius R in the middle. It is uniformly charged with surface density σ. The electric field at point P on the symmetric axis is [3]

(A) $E = \dfrac{\sigma}{2\varepsilon_0}$

(B) $E = \dfrac{\sigma}{2\varepsilon_0} - \dfrac{\sigma x}{2\varepsilon_0\sqrt{x^2+R^2}}$

(C) $E = \dfrac{\sigma x}{2\varepsilon_0\sqrt{x^2+R^2}}$

(D) $E = \dfrac{\sigma}{2\varepsilon_0} + \dfrac{\sigma x}{2\varepsilon_0\sqrt{x^2+R^2}}$

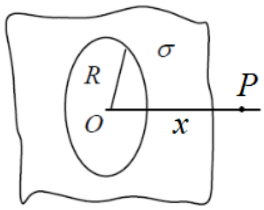

Figure Q1-1.

2. (A) Someone attempts to form an electrostatic field between two charged metal plates as shown in Figure Q1-2. The electric field lines are a series of concentric arcs on the plane perpendicular to the intersection line of two plates, and they are separated uniformly. which of the following statements is CORRECT? [3]

(A) It violates the circulation theorem of electrostatic field, so impossible.

(B) Electric field inside a capacitor is always uniform, so impossible.

(C) It violates Gauss's law of electrostatic field, so impossible.

(D) It is a possible situation for electrostatic field.

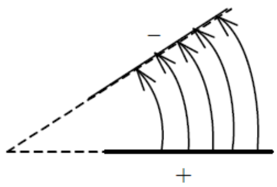

Figure Q1-2.

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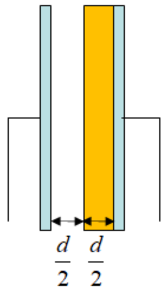
3. (B) An air capacitor has capacitance C_0, what is the capacitance when it is half filled with dielectric material (dielectric constant K=10)? [3]



(A) $C = 22C_0$ (B) $C = \frac{20}{11}C_0$

(C) $C = 11C_0$ (D) $C = \frac{10}{11}C_0$

Figure Q1-3.

4. (D) Which of the following motions is NOT possible for a point charge moving in a uniform magnetic field (only consider the action of magnetic force)? [3]

(A) free motion; (B) motion in helix;

(C) circular motion; (D) elliptical motion

5. (C) When using Ampere's law, how much current I_{in} passes through the area enclosed by the chosen path is important. How much is I_{in} in the figure? [3]

(A) $I_{in} = -2I$

(B) $I_{in} = 0$

(C) $I_{in} = 2I$

(D) $I_{in} = 4I$

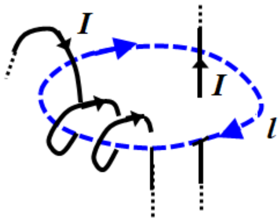


Figure Q1-5.

6. (B) Which of the following statements is CORRECT? [3]

(A) Moving conductor in magnetic field can produce displacement current.

(B) Displacement current is caused by change electric field.

(C) Electromagnetic wave travels at the same speed in any medium.

(D) The electric field and magnetic field caused by an electromagnetic wave is always parallel to each other.

7. (~~A~~) In reference frame S, a thin rod is at rest, the angle between the rod and x axis is θ. The angle is measured as $\theta' = 45°$ by another observer in frame S' which is moving with speed v relative to frame S along x axis. How much is θ? [3]

C

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(A) $\theta > 45°$

(B) $\theta = 45°$

(C) $\theta < 45°$

(D) All of the above would be possible.

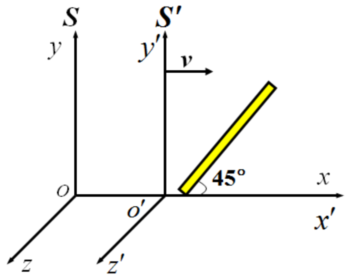

Figure Q1-7.

8. (D) In photoelectric effect experiment, firstly monochromatic light (wavelength λ) falls on the surface of metal, and the photocurrent I changes over voltage V as the solid lines in figure. Then let the intensity of light remain unchanged and decrease the wavelength, the relationship between V and I is shown by dashed lines. Which of the following figures shows the correct results? [3]

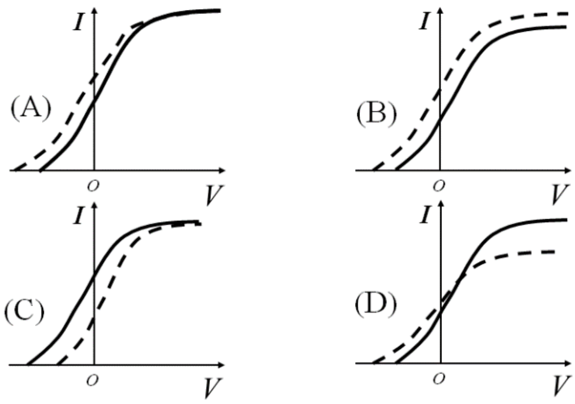

Figure Q1-8.

9. (C) If a microscopic particle is trapped in a range $\Delta x \approx 10^{-10}$ m, how much is the minimum uncertainty of its momentum, according to Heisenberg uncertainty principle? [3]

(A) $(\Delta p)_{\min} \approx 10^{-4}$ kg·m/s

(B) $(\Delta p)_{\min} \approx 10^{-14}$ kg·m/s

(C) $(\Delta p)_{\min} \approx 10^{-24}$ kg·m/s

(D) $(\Delta p)_{\min} \approx 10^{-34}$ kg·m/s

10. (B) According to the rules of quantum numbers, which of the following numbers (n, l, m_l, m_s) can represent the state of electron in atom? [3]

(A) (1, 1, 0, 0)

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(B) $(2,\ 1,\ 0,\ -\frac{1}{2})$

(C) $(2,\ 1,\ \frac{1}{2},\ \frac{1}{2})$

(D) $(2,\ 3,\ -1,\ \frac{1}{2})$

Q2 A spherical conductor A with radius R carries charge Q.

(a) Determine the electric field $\vec{E}$ inside and outside of the conductor. [5]

(b) How much is the total electric energy U due to the field? [5]

(c) If A is connected by wire to another spherical conductor B with radius $R'=2R$, which is very distanced and initially carrying no charge, how much charge do A and B carry at the equilibrium? [5]

(d) In process (c), does total energy increase or decrease? Explain it. [5]

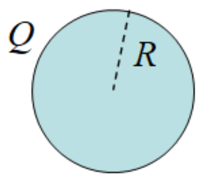

Figure Q2(a)(b).

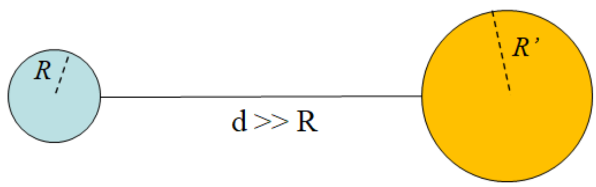

Figure Q2(c)(d).

Solution: (a) From the symmetry, we know the electric field directs radial outside. [1]

Choose a spherical Gaussian surface, and use Gauss's law:

$$\oint\!\!\!\!\oint \vec{E}\cdot d\vec{S} = E\cdot 4\pi r^2 = \frac{Q_{in}}{\varepsilon_0}$$ [2]

So electric field outside: $E = \dfrac{Q}{4\pi\varepsilon_0 r^2}$ [1]

electric field inside: $E = 0$ [1]

(b) The electric energy density is $u = \dfrac{1}{2}\varepsilon_0 E^2 = \dfrac{1}{2}\varepsilon_0 \left(\dfrac{Q}{4\pi\varepsilon_0 r^2}\right)^2$ [2]

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The total energy is $U = \int_R^\infty \frac{1}{2}\varepsilon_0 (\frac{Q}{4\pi\varepsilon_0 r^2})^2 \cdot 4\pi r^2 dr = \frac{Q^2}{8\pi\varepsilon_0 R}$ [3]

Or treat the isolated conductor as a capacitor: $C = \frac{Q}{V} = 4\pi\varepsilon_0 R$ [2]

So, the electric energy stored is $U = \frac{1}{2}CV^2 = \frac{1}{2}QV = \frac{Q^2}{8\pi\varepsilon_0 R}$ [3]

(c) When two conductors connected, they have the same electric potential:

$$\frac{Q_A}{4\pi\varepsilon_0 R} = \frac{Q_B}{4\pi\varepsilon_0 \cdot 2R}$$ [2]

and $Q_A + Q_B = Q$ [1]

So we have $Q_A = \frac{Q}{3}$, $Q_B = \frac{2Q}{3}$ [2]

(d) Two conductors are very far away from each other, we can calculate their electric energy separately. The total energy will be:

$$U' = \frac{Q_A^2}{8\pi\varepsilon_0 R} + \frac{Q_B^2}{8\pi\varepsilon_0 R'} = \frac{Q^2}{24\pi\varepsilon_0 R} < U$$

So, the electric energy decreases. [3]

Because the electric charge can move freely in the system, the final state must be a minimum energy state, i.e., comparing to the initial state (all charges on conductor A) the electric energy decreases. [2]

Q3 A square coil with l=10 cm for each side lies on a plane, and a long straight wire intersects the plane perpendicularly at point P (on the extension line of one side of coil).

(a) Determine the magnetic field $\vec{B}$ at point P if the coil carries current I=2.0 A. [7]

(b) Determine the mutual inductance between the coil and the straight wire. [4]

(c) Suppose the straight wire and the coil both carry a current I, where the directions are shown in Figure Q3(c). Draw on the figure to show the magnetic field lines by the straight current, and direction of the magnetic forces acting on each side of the coil. [5]

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(d) If the straight wire carries current I=2.0 A, and the coil is rotating about AP axis with constant angular velocity ω=10 rad/s, how much is the maximum EMF in the coil? [9]

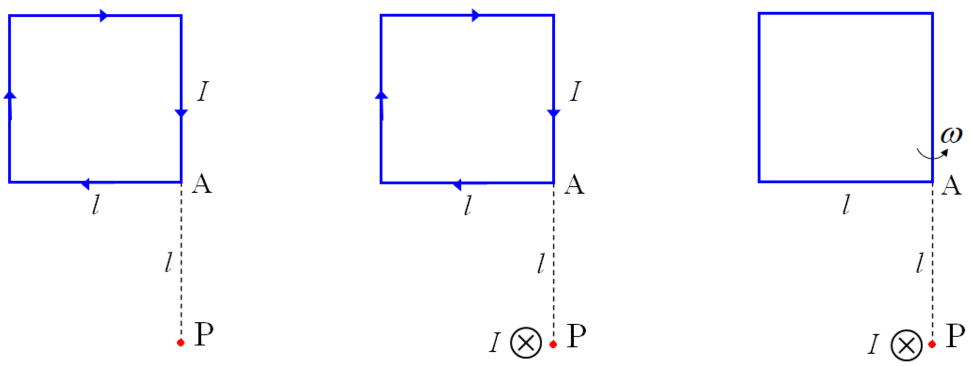

Figure Q3(a)(b). ***Figure Q3(c).*** ***Figure Q3(d).***

Solution: (a) For each side of the coil, a straight current causes magnetic field:

$$B = \frac{\mu_0 I}{4\pi a}\left|\cos\theta_1 - \cos\theta_2\right| \qquad [3]$$

So, the total field is

$$B = 0 + \frac{\mu_0 I}{4\pi l\sqrt{2}} + \frac{\mu_0 I}{4\pi l}\left(\frac{1}{\sqrt{2}} - \frac{2}{\sqrt{5}}\right) - \frac{\mu_0 I}{4\pi l}\frac{1}{2\sqrt{5}} = \frac{\mu_0 I}{4\pi l}\left(\sqrt{2} - \frac{\sqrt{5}}{2}\right) = 5.92\times10^{-5} T \qquad [3]$$

The direction points outward $\odot$. [1]

(b) Suppose there is a current on straight wire, the magnetic field lines are parallel to the plane of square coil, so no magnetic flux through the coil. [2]

The mutual inductance $M = \frac{\Phi_B}{I} = 0$. [2]

(c) As shown in figure, 1 point for each direction. [5]

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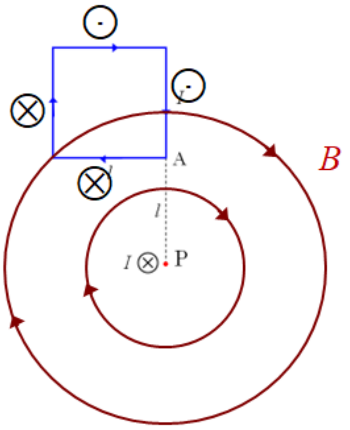

(d) Suppose the coil rotates angle $\theta = \omega t + \varphi$ from the position in figure, the magnetic flux is:

$$\Phi_B = \int_l^{2l} \frac{\mu_0 I}{2\pi r} \cdot l \sin\theta dr = \frac{\mu_0 Il \ln 2}{2\pi} \sin\theta = \frac{\mu_0 Il \ln 2}{2\pi} \sin(\omega t + \varphi) \qquad [4]$$

And the EMF is

$$\varepsilon = -\frac{d\Phi_B}{dt} = -\frac{\mu_0 \omega Il \ln 2}{2\pi} \cos(\omega t + \varphi) \qquad [3]$$

The maximum value of EMF is

$$\varepsilon_{\max} = \frac{\mu_0 \omega Il \ln 2}{2\pi} = 2.77 \times 10^{-7}\,\text{V} \qquad [2]$$

Q4 (a) An electron moves at v=0.99c. Determine its momentum, kinetic energy and de Broglie wavelength. [7]

(b) If a photon has the same wavelength as the electron in (a), how much is its energy? Is it possible for this photon to cause significant Compton effect? [5]

(c) A hydrogen atom in ground state can absorb a photon and jump to the second exited state, how much energy is required for the photon? Is it a visible light photon? [6]

(d) The wave function of a microscopic particle can be expressed as $\psi = \begin{cases} C\sin^2 \pi x, & 0 \le x \le 1 \\ 0, & x < 0 \text{ or } x > 1 \end{cases}$, where C is a constant to be determined. What is

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the probability to find the particle in region $0 < x < \frac{1}{3}$? (The probability distribution is given by $|\psi|^2$) [7]

Solution: (a) The relativity factor: $\gamma = \frac{1}{\sqrt{1-v^2/c^2}} = 7.09$

So, the momentum of electron:

$$p = \gamma m_0 v = 7.09 \times 9.11 \times 10^{-31} \times 0.99 \times 3 \times 10^8 = 1.92 \times 10^{-21} kg \cdot m/s \quad [3]$$

the kinetic energy of electron:

$$E_k = (\gamma - 1)m_0 c^2 = 6.09 \times 9.11 \times 10^{-31} \times (3 \times 10^8)^2 = 4.99 \times 10^{-13} J = 3.12 MeV \quad [2]$$

The de Broglie wavelength

$$\lambda = \frac{h}{p} = \frac{6.626 \times 10^{-34}}{1.92 \times 10^{-21}} = 3.45 \times 10^{-13} m \quad [2]$$

(b) For a photon, the energy satisfies: $E = \frac{hc}{\lambda} = 5.72 \times 10^{-13} J = 3.58 MeV$. [3]

This is a typical high energy photon (X ray or γ ray), so it's possible to cause significant Compton effect. [2]

(c) Energy levels of Hydrogen atom: $E_n = -\frac{13.6eV}{n^2}$ [2]

Transition from the n=1 state to the n=3 state:

$$\Delta E = E_3 - E_1 = (1 - \frac{1}{9}) \cdot 13.6eV = 12.1eV \quad [2]$$

It is not a visible light photon (1.66eV~3.11eV). [2]

(d) Normalization condition:

$$\int_{-\infty}^{\infty} |\psi|^2 dx = \int_0^1 C^2 \sin^4 \pi x dx = \frac{3}{8} C^2 = 1 \quad [2]$$

So, we know $C = \sqrt{\frac{8}{3}}$ [1]

The probability to find the particle in region $0 < x < \frac{1}{3}$:

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$$p = \int_0^{\frac{1}{3}} C^2 \sin^4 \pi x \mathrm{d}x = \frac{8}{3}\int_0^{\frac{1}{3}} \sin^4 \pi x \mathrm{d}x$$ [2]

$$= \frac{1}{3} - \frac{3\sqrt{3}}{8\pi} = 0.127$$ [2]

End of question paper