Suicide Rate Prediction in Nordic Countries

Linh Nguyen, Duy Nguyen, Aysun Yozgyur



Table of content

- 1. Introduction
- 2. Suicide rate prediction for Nordic countries
 - a. Prior Justification
 - b. Results Convergence diagnostics
 - c. Results Models comparison
 - d. Posterior predictive assessment
- 3. Suicide rate prediction for Age group
 - a. Results Models comparison
 - o. Results Convergence diagnostics
 - c. Posterior predictive assessment
- 4. Suicide rate prediction for Finland
- 5. Conclusion

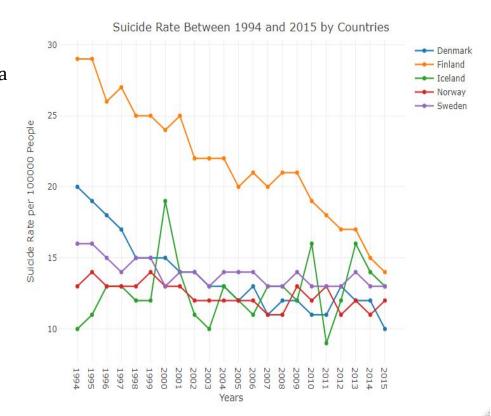
1. Introduction

Introduction

- Nordic countries including Finland, Norway, Sweden,
 Denmark and Iceland, have been always spotted to be
 in the top ten of the World Happiness Report recent
 years. However, According to the United Nations,
 happiness is about wellbeing: income (GDP per capita),
 social support, healthy life expectancy, freedom,
 generosity and the absence of corruption.
- Gallup conducted a survey of positive and negative emotions (e.g. experiencing respect, pain or worry or feeling well-rested or sad) in which the Nordic countries did not score anywhere near the top (the top three comprised Paraguay, Panama and Guatemala)

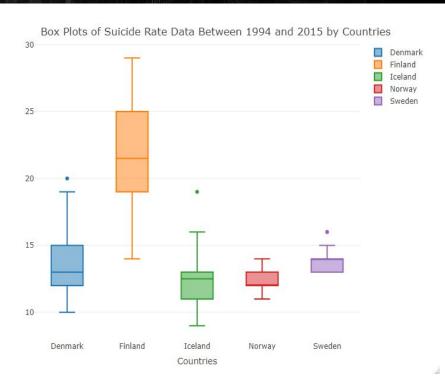


- The suicide dataset used in this analysis is obtained from Kaggle, an online community of data scientists and machine learners where data is provided for the users.
- The dataset reports suicide rates overview from 1985 to 2016 across countries in the world. The analysis will use a subset covering the suicide data of Nordic countries from 1994 to 2014.
- Can we analyze the number of the suicides in Nordic countries and give a prediction on the suicide rate in the future by building a Bayesian model?
 - by countries
 - by age-groups

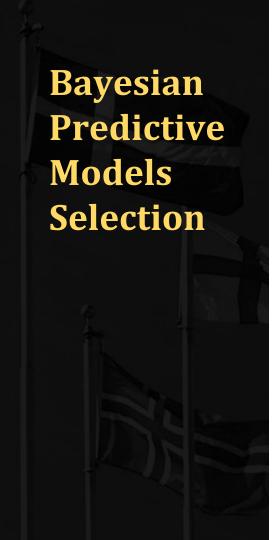


2. SUICIDE RATE PREDICTION FOR NORDIC COUNTRIES

Data exploration (Suicide cases per 100k Population)



The boxplots display the mean and variance of the datasets



Normal Model

- Normal Separate model
- Normal hierarchical model

Negative Binomial Model

- Separate Negative Binomial Model
- Hierarchical Separate Negative Binomial Model

Prior Justification

Normal Models:

• Normal Separate model used a prior distribution following the normal distribution with $mu(\mu)$ is the mean of the dataset and $sigma(\sigma)$ is the variance of the dataset

yij ~
$$N(\mu j, \sigma j)$$

• Normal hierarchical model with weakly informative priors - mean (μ) and variance (σ) values drawn from Cauchy and normal distribution

yij ~ N(
$$\mu$$
j, σ)
 σ ~ Cauchy(0,4)
 μ j ~ N(μ 0, σ 0)
 μ 0 ~ N(12, 10) σ 0 ~ Cauchy(0,4)

Negative Binomial Models:

For negative binomial distribution, instead of using the parameterization described in Gelman et al. (2013) that seek for probability parameter p, following a beta distribution with shape parameters alpha(α) and beta(β), we use the alternative parameterizations according to Stan documentation .

The alternative parameterization is a log alternative parameterization which use mean(μ) \in R+ and phi(ϕ) \in R+ $\mathbb{E}[Y] = \mu \quad \text{and} \quad \mathrm{Var}[Y] = \mu + \frac{\mu^2}{\phi}$

• Negative Separate model

yij ~ neg_binomial(
$$\mu$$
j, ϕ j)
 μ j ~ $N(14.89,10)$
 ϕ j ~ $N(63.35.10)$

Negative hierarchical model

yij ~ neg_binomial(
$$\mu$$
j, ϕ)
 μ j ~ $N(14.89,10)$
 ϕ ~ $N(63.35,10)$

ResultsConvergence diagnostics

To analyse the convergence of the different models, we have calculated the R $\hat{}$ values and the effective sample sizes (ESS). An R $\hat{} \le 1.05$ and ESS ≥ 100 for a chain is considered good, and we can say that the chain has converged.

Results: The R ^ values for all variables in 4 all equal to 1.0. The effective sample sizes are large enough when all are larger than 100. Based on this, we can conclude the convergence has been reached and therefore we can make conclusion about the target distributions as the estimates are reliable

Models	max_Rhat	min_ESS
Normal Separate	1.001931	1863
Normal hierarchical	1.003021	1542
Separate Negative	1.003806	1601
Hierarchical Separate Negative	1.001964	1833

Results Model Comparison

As a part of model selection, we computed the PSIS (Pareto-Smoothed Importance Sampling) or pareto k diagnostics and elpd (expected log predictive density) values

To compare how each of the models fit and describe the data, we will use Loo package. This tool lets us calculate the difference in the log predictive density expected between the models and the standard error of that difference. The model that best describes the data will have the highest ELPD which will

be used as a baseline	elpd_diff	se_diff		
Normal Separate	0.0	0.0		
Normal hierarchical	-29.2	6.8		
Separate Negative	-46.0	7.1		
Hierarchical Separate Negative	-75.8	9.8		

Model 1 is selected



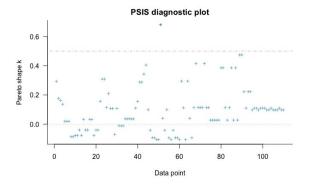
Model 1: Separate Normal



Model 2: Hierarchical Normal

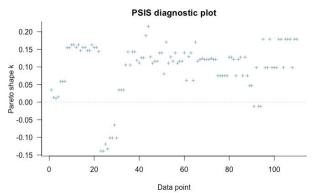
All the k-values are under 0.7 for Separate Normal Model and Hierarchical Normal Model which indicates that the elpd_loo

estimate of these 2 are reliable.

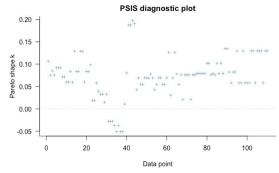


PSIS diagnostic plot 0.3 0.2 0.1 -0.1 -0.2 20 100 Data point

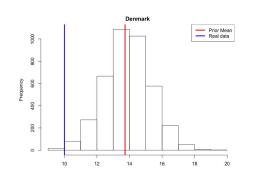
Model 3: Separate Negative Binomial

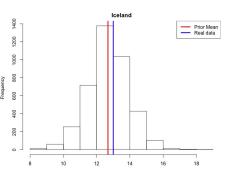


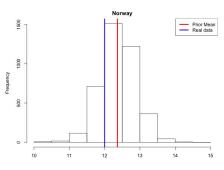
Model 4: Hierarchical Negative Binomial

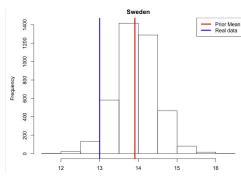


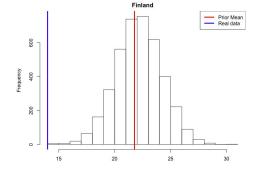
Posterior predictive distribution 2015 (Using 1994-2014 data)









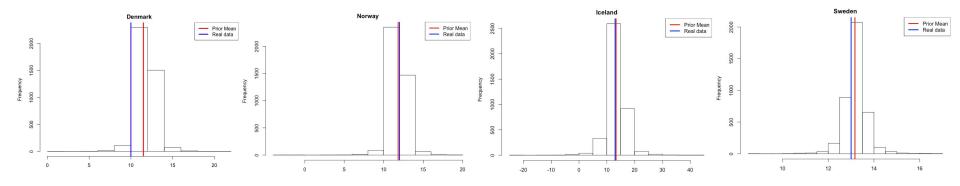


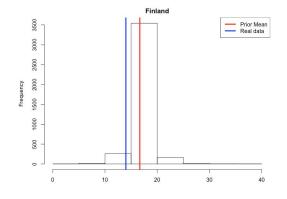
Actual data	2015
Denmark	10 (*)
Finland	14 (*)
Iceland	13
Norway	12
Sweden	14

The figures above present the posterior predictive distribution of the **number of suicides per 100k population in 2015** according to different countries in Nordic region compared with the real data in 2015.

(*) Prediction for Denmark and Finland are quite far from actual data. Iceland, Sweden and Norway have shown better result.

Posterior predictive distribution with different prior





Actual data	2015
Denmark	10
Finland	14
Iceland	13
Norway	12
Sweden	14

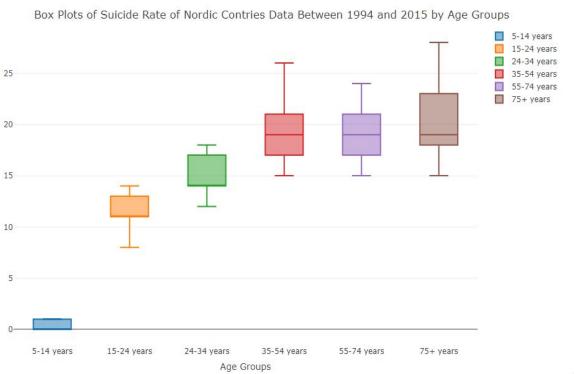
Considering the fact that there, suicide & mental health awareness has been promoted strongly and many action has been taken by the Nordic Governents. We changed the weakly informative prior for the intercept by taking the mean value of last 5 years (2010-2014) to predict for 2015 instead of the whole dataset 1994 to 2014.

The result look much better than the previous one.

3. SUICIDE RATE PREDICTION FOR DIFFERENT AGE GROUPS IN NORDIC REGION

Data exploration

The same approach will be used as Countries level rate prediction



ResultsConvergence diagnostics

To analyse the convergence of the different models, we have calculated the R $\hat{}$ values and the effective sample sizes (ESS). An R $\hat{} \leq 1.05$ and ESS ≥ 100 for a chain is considered good, and we can say that the chain has converged.

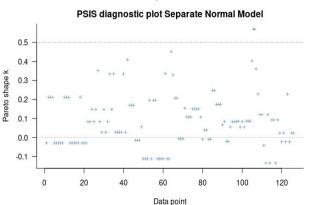
Models	max_Rhat	min_ESS
Normal Separate	1.003931	1478
Normal hierarchical	1.007018	1323
Separate Negative	1.003164	1586
Hierarchical Separate Negative	1.004141	1621

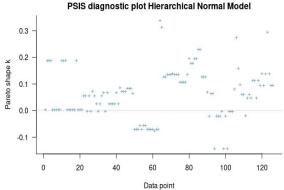
Results Model Comparison

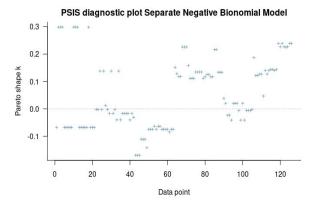
Using loo_compare function, the results of elpd values in the assessment of suicide rates by age groups also imply that **Separate Normal Model** is the most suitable model.

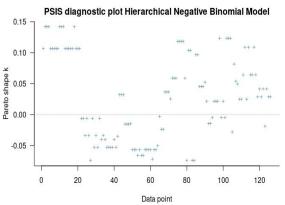
	elpd_diff	se_dif
Normal Separate	0.0	0.0
Normal hierarchical	-24.5	5.7
Separate Negative	-27.7	4.7
Hierarchical Separate Negative	-64,7	6.8

Model 1 is selected

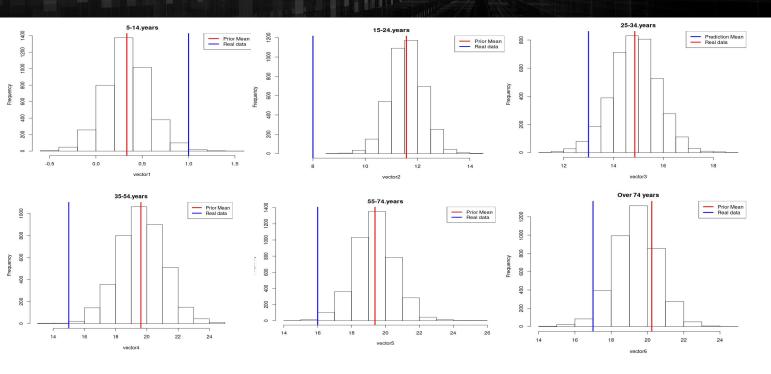








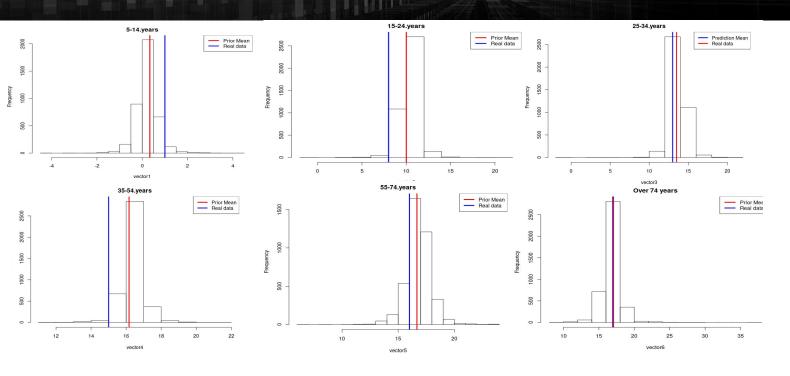
Posterior predictive distribution



2015	Actual data
5-14 years	1
15-24 years	8 (*)
25-34 years	13
35-54 years	15
55-74 years	16
Over 74y	17

The figures above present the posterior prediction of the number of suicides by age groups in 2015 for all Nordic countries.

Posterior predictive distribution with a different prior



Another prior from the last 5 years data (2010-2014) were used to generate the second posterior predictive distribution. The results show better performance in predicting the number of suicides.

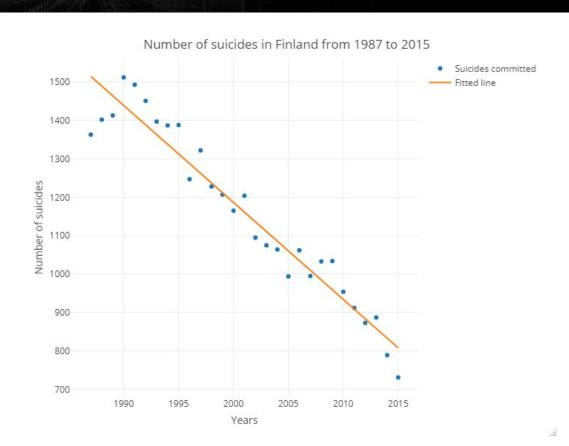
4. Suicide rate prediction for Finland

Data exploration

Why Finland?

- Higher suicide rates with a decreasing trend
- The highest among other Nordic countries each year
- Larger data available, from 1987 to 2015
- Where we live!

Linear Model with Gaussian residuals using time as the predictor



Prior Justification

The approximate historical mean yearly number of suicides committed is 1177. Hence, we set the slope so that the following holds for the prior probability for β :

$$P(-588.5 < \beta < 588.5) = 0.99$$

That is the same with

$$P(\beta) < 0.005$$

Indicating that we should refer to the standard normal table. The corresponding z value is 2.58, and the following calculation has been done:

$$z = \frac{\beta - \mu}{\sigma_{\beta}}$$

Where μ =0 and $\sigma_{\beta} = \frac{-588.5}{-2.58} = 228.10$

Therefore, weakly informative prior for the slope β can be written as $\beta \sim \text{normal}(0,228.10)$.

Similarly, we wrote the weakly informative prior for the intercept α using normal distribution in which the mean value μ equals to the mean value of the number of suicides from 1987 to 2014, and the standard deviation can be set to 100 to define a weakly-informative prior. Hence, the weakly informative prior for the intercept α can be written as $\alpha \sim \text{normal}(1177,100)$.

Results Convergence diagnostics

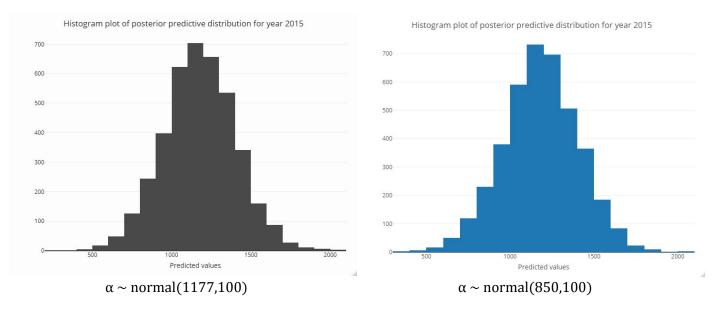
As can be seen, all the Rhat values are one, meaning that chains have converged. The mean value of the posterior predictive distribution for the year 2015 is 1178.

The results obtained using the linear model with Gaussian residual model differ from the actual number of suicides committed in Finland in 2015 (the actual number of suicides is 731). The output of the sampling can be seen in the figure below:

	mean	se_mean	sd	2.5%	25%	50%	75%	97.5%	n_eff	Rhat
alpha	1183.02	2.46	101.15	987.20	1116.23	1182.05	1251.27	1377.72	1686	1
beta	0.00	0.00	0.06	-0.11	-0.04	0.00	0.03	0.10	1672	1
sigma	222.38	0.74	31.63	170.55	199.65	219.41	240.84	292.68	1814	1
mu[1]	1175.40	0.63	42.02	1093.17	1147.62	1175.87	1202.89	1258.18	4457	1
mu[2]	1175.39	0.63	42.05	1093.05	1147.58	1175.88	1202.88	1258.23	4453	1
mu[3]	1175.39	0.63	42.07	1093.01	1147.63	1175.88	1202.85	1258.27	4449	1
mu [4]	1175.38	0.63	42.09	1093.01	1147.62	1175.89	1202.83	1258.35	4445	1
mu[5]	1175.38	0.63	42.11	1093.01	1147.63	1175.90	1202.84	1258.47	4441	1
mu[6]	1175.38	0.63	42.13	1093.02	1147.62	1175.89	1202.86	1258.60	4437	1
mu[7]	1175.37	0.63	42.15	1093.02	1147.60	1175.88	1202.85	1258.71	4433	1
mu[8]	1175.37	0.63	42.17	1093.03	1147.57	1175.86	1202.82	1258.71		1
mu[9]	1175.37			1092.93						1
mu[10]	1175.36	0.63	42.21	1092.83	1147.49	1175.84	1202.86	1258.60	4420	1
mu[11]	1175.36	0.64	42.24	1092.73	1147.45	1175.85	1202.86	1258.64	4416	1
	1175.35	0.64	42.26	1092.71	1147.40	1175.84	1202.83	1258.69	4412	1
	1175.35			1092.72					4408	1
mu[14]	1175.35			1092.72					4403	1
	1175.34			1092.72						1
	1175.34			1092.72						1
	1175.33	0.64		1092.72					4391	1
	1175.33	0.64		1092.67					4386	1
	1175.33	0.64		1092.61						1
	1175.32			1092.55						1
	1175.32			1092.49						1
	1175.32			1092.43						1
	1175.31			1092.47						1
	1175.31			1092.52					4361	1
	1175.30	0.64		1092.52					4357	1
-	1175.30	0.65		1092.35					4352	1
	1175.30	0.65		1092.19		Printed to the Control of the Control			0.0000000000000000000000000000000000000	1
	1175.29	0.65		1092.22					4344	1
ypred	1178.07		229.12				1331.46		3618	1
1p	-159.45	0.04	1.25	-162.78	-160.02	-159.13	-158.55	-158.01	1228	1

Posterior predictive assessment

The population of
Finland in 2015 was
5.481.122, meaning that
our prediction of 1178
suicides indicates that
suicide rate is 21 per
100.000 people, that is
pretty close to the
outcomes of separate
and hierarchical models.



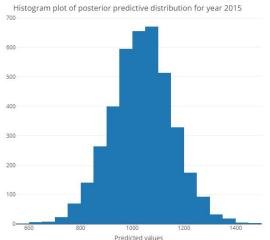
The mean value of the posterior predictive distribution for the year 2015 is 1178 and the histogram plot of the distribution can be seen above.

We believe there are many precautions taken by the authorities in order to decrease the suicide rates. We changed the weakly informative prior for the intercept from the mean value of all the available data to the last three years' mean value, that is 850. We fit another stan model using the weakly informative prior for α as $\alpha \sim \text{normal}(850,100)$.

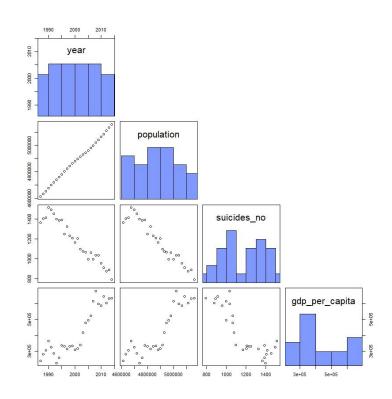
As can be seen from the histogram above, there is no significant change in posterior predictions if a different prior is used.

Multiple Linear Regression Model

- Correlation plot of the variables used
 - There is a strong linear relation between the year and population
- Model: suicide number ~ year + gdp/capita
- Prior Selection: Using the same fashion as before in the model with single predictor



Mean Value of Predictions: ∼1035



Multiple Linear Regression Model

New Model: $y \sim year + population + gdp/year$

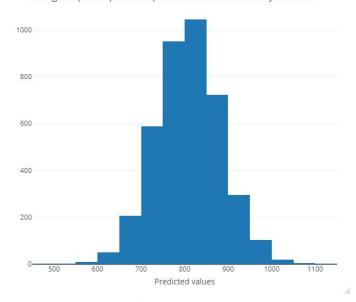
 $\label{lem:prior} \mbox{Prior Selection: Using the same fashion as before in the model with single predictor}$

Convergence problem of the chains solved by setting max_treedepth = 15

• All R^ are 1, and ESS ≥ 1

	mean	se_mean	sd	2.5%	25%	50%	75%	97.5%	n_eff	Rhat
alpha	1177.89	2.75	101.61	977.37	1107.90	1177.17	1245.61	1377.12	1361	1
beta[1]	3.35	0.01	0.45	2.49	3.05	3.36	3.65	4.21	1415	1
beta[2]	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1438	1
beta[3]	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1564	1
sigma	63.98	0.25	10.03	47.68	56.89	63.05	69.70	86.51	1578	1
mu[1]	1514.43	0.52	25.45	1465.50	1496.94	1514.35	1531.49	1563.91	2409	1
mu[2]	1487.55	0.57		1436.81	1469.64	1487.91	1504.82	1538.88	2090	1
mu[3]	1460.65	0.57	25.61	1410.59	1443.00	1460.86	1477.52	1511.21	2028	1
mu[4]	1431.02	0.69	29.06	1374.80	1411.15	1431.72	1449.69	1488.94	1775	1
mu[5]	1405.38	0.45			1390.64				2253	1
mu[6]	1381.41	0.29	17.44	1347.75	1369.58	1381.14	1392.98	1416.01	3497	1
mu[7]	1362.20	0.42		1318.51	1347.74	1361.99	1376.29	1406.13	2694	1
	1334.62	0.35		1296.59	1322.28	1334.28	1347.11	1374.16	2979	1
mu[9]	1304.85				1295.07				3862	1
mu[10]	1280.20	0.23			1270.94				3957	1
mu[11]	1256.05	0.32	16.70	1222.59	1245.15	1255.85	1267.04	1289.92	2778	1
mu[12]	1228.75			1195.37	1217.89	1228.60	1239.64	1261.41	2611	1
mu[13]	1205.21	0.39	18.24	1168.32	1193.32	1205.26	1217.22	1240.81	2214	1
mu[14]	1186.59	0.57	24.12	1137.86	1170.44	1186.74	1202.53	1233.48	1796	1
mu[15]	1165.85	0.60	25.12	1115.20	1149.07	1165.81	1182.37	1214.79	1731	1
mu[16]	1144.53	0.56	23.55	1096.93	1128.91	1144.43	1160.09	1190.19	1744	1
mu[17]	1122.28	0.30		1092.34	1112.47	1122.37	1132.53	1151.37	2509	1
mu[18]	1100.31	0.21		1074.91	1091.53	1100.28	1108.94	1126.31	3950	1
mu[19]	1079.04	0.22		1052.19	1069.97	1078.94	1088.12	1106.22	3901	1
mu[20]	1055.64	0.24	14.78		1046.01				3895	1
mu[21]	1025.48	0.43	23.23	979.63	1010.19	1025.13	1040.73	1071.50	2880	1
mu[22]	994.52	0.63	30.93	933.84	973.60	994.29	1014.41	1055.84	2385	1
mu[23]	973.99		19.38	936.10	961.05	973.77	986.46	1012.98	3636	1
mu[24]	949.33	0.33	19.06	910.87	936.92	949.46	961.69	987.47	3339	1
mu[25]	917.21	0.38	22.67	872.99	902.16	917.09	931.78	963.26	3537	1
mu[26]	890.63	0.42	21.98	845.92	876.67	890.69	905.17	934.75	2702	1
mu[27]	857.58	0.45	23.67	809.72	842.76	857.93	873.23	905.00	2738	1
mu[28]	827.19		25.66	775.97				878.48	2435	1
ypred[1]	809.11			663.54		808.75		957.89	2899	1
1p	-126.03	0.05	1.67	-130.06	-126.93	-125.70	-124.80	-123.77	1272	1

Histogram plot of posterior predictive distribution for year 2015



Mean Value of the predictions: ~809

CONCLUSION

- Separate Normal Model can estimate the number of suicide rate using informative prior with mean of last 5 years
- A linear model with Gaussian residuals for Finland has been created and results show similarity to the ones obtained with separate and hierarchical models. Also, the difference between the posterior draws has been analyzed with the usage of different priors.
- A linear model with multiple predictors has been constructed and the best results for Finland are achieved with this model using year, population and gdp/capita information.
- As the potential improvements for the results obtained, the number of predictors could be increased and informative priors can be used instead of weakly informative priors. Finding more observations would increase the accuracy of the results obtained.
- More detailed about the project at
 https://github.com/khalinguy/SuicideRatePrediction-BayesianModelling

Additional Information

```
data {
 int<lower=0> N;
                            // number of data points
  int<lower=0> K:
                   // number of groups
  int<lower=1,upper=K> x[N]; // group indicator
  vector[N] y;
parameters {
 vector[K] mu;
                            // group means
  vector<lower=0>[K] sigma; // group stds
model {
  y \sim normal(mu[x], sigma[x]);
generated quantities {
 vector[K] y_state;
  vector[N] log_lik;
  for (i in 1:N)
   log_lik[i] = normal_lpdf(y[i] | mu[x[i]], sigma[x[i]]);
  for (i in 1:K)
   y_state[i]=normal_rng(mu[i], sigma[i]);
```

Separate Normal Model

```
int<lower=0> N;
                           // number of data points
 int<lower=0> K:
                           // number of groups
 int<lower=1,upper=K> x[N]; // group indicator
 vector[N] y;
parameters {
 real mu0:
                           // prior mean
 real<lower=0> sigma0;
                           // prior std
 vector[K] mu;
                            // group means
 real<lower=0> sigma:
                           // common std
model {
 mu0 ~ normal(14.89,18.39); // weakly informative prior
                            // weakly informative prior
 sigma0 \sim cauchy(0,4);
 mu ~ normal(mu0, sigma0); // population prior with unknown parameters
 sigma \sim cauchy(0,4);
                            // weakly informative prior
 y \sim normal(mu[x], sigma);
generated quantities {
 real vpred;
 real mupred:
 vector[K] y_state;
 vector[N] log_lik;
 mupred = normal_rng(mu0, sigma0);
 ypred = normal_rng(mupred, sigma);
 for (i in 1:N)
   log_lik[i] = normal_lpdf(y[i] | mu[x[i]], sigma);
 for (i in 1:K)
   y_state[i]=normal_rng(mu[i], sigma);
```

Hierarchical Normal Model

Additional Information

```
data {
 int<lower=0> N;
                            // number of data points
 int<lower=0> K:
                           // number of groups
  int<lower=1,upper=K> x[N]; // group indicator
  int<lower=0> y[N];
parameters {
  real<lower=0> mu[K];
  real<lower=0> phi[K];
model {
 mu ~ normal(14.89,10); //weekly informative prior
 phi ~ normal(63.35,10); // weekly informative prior
 y ~ neg_binomial_2(mu[x], phi[x]); // likelihood
generated quantities {
  real<lower=0> y_rep[K];
  vector[N] log_lik;
  for (i in 1:N)
   log_lik[i] = neg_binomial_2_lpmf(y[i] | mu[x[i]], phi[x[i]]);
  for (i in 1:K)
   y_rep[i] = neg_binomial_2_rng(mu[i], phi[i]);
```

```
data {
 int<lower=0> N;
                           // number of data points
 int<lower=0> K;
                           // number of groups
 int<lower=1,upper=K> x[N]; // group indicator
 int<lower=0> y[N];
parameters {
 real mu;
 real<lower=0> phi[K];
model {
 mu ~ normal(14.89,10); //weekly informative prior
 phi ~ normal(63.35,10); // weekly informative prior
 y ~ neg_binomial_2(mu, phi[x]); // likelihood
generated quantities {
 int<lower=0> y_rep[K];
 vector[N] log_lik;
 for (i in 1:N)
   log_lik[i] = neg_binomial_2_lpmf(y[i] | mu, phi[x[i]]);
 for (i in 1:K)
   y_rep[i] = neg_binomial_2_rng(mu, phi[i]);
```

Separate Negative Binomial Model

Hierarchical Negative Binomial Model

Additional Information

```
data {
  int<lower=0> N; // number of data points
 vector[N] x; // observation year
 vector[N] y; // observation number of drowned
  real xpred; // prediction year
 real psbeta;
  real pmualpha:
  real psalpha;
parameters {
  real alpha;
 real beta;
  real<lower=0> sigma;
transformed parameters {
  vector[N] mu = alpha + beta*x:
model
  alpha ~ normal(pmualpha,psalpha);
  beta ~ normal(0,psbeta);
  v ~ normal(mu, sigma):
generated quantities {
 real ypred = normal_rng(alpha + beta * xpred, sigma);
```

Linear Regression Model with one predictor

```
data {
  int<lower=0> N; // number of data points
  int<lower=0> K; // number of predictors
 int<lower=0> P; //number of predictions
 matrix[N, K] x; // predictor matrix
 vector[N] y; // observation number of suicides
 matrix[1,K] xpred; // prediction year
 vector[K] psbeta;
 real pmualpha;
 real psalpha;
parameters {
 real alpha;
 vector[K] beta;
 real<lower=0> sigma;
transformed parameters {
 vector[N] mu = alpha + x*beta;
model |
  alpha ~ normal(pmualpha,psalpha);
 for(i in 1:K)
    beta[i] ~ normal(0,psbeta[i]);
 y ~ normal(x * beta + alpha, sigma); // likelihood
generated quantities {
 real ypred[P] = normal_rng(xpred*beta + alpha, sigma);
```

Linear Regression Model with multiple predictors