

# **MERGESORT & QUICKSORT**

cs2420 | Introduction to Algorithms and Data Structures | Spring 2016

**administrivia...**

- assignment 4 due tonight at midnight
- assignment 5 is out
- no lab on Monday
- midterm next Tuesday

**last time...**

**re · cur · sion**

[ri-**kur**-zhuh n]

***noun***

see *recursion*.

**-recursion** is a problem solving technique in which the solution is defined in terms of a simpler (or smaller) version of the problem

- break the problem into smaller parts

  - solve the smaller problems*

  - combine the results*

- a recursive method calls itself

- some functions are easiest to define recursively

$$\text{sum}(N) = \text{sum}(N-1) + N$$

- there must be at least one *base case* that can be computed without recursion

  - any recursive call must make progress towards the base case!

# a simple example

$$\text{sum}(N) = \text{sum}(N-1) + N$$

```
public static int sum(int n) {  
    if (n == 1)  
        return 1;  
    return sum(n-1) + n;  
}
```

FIX TO HANDLE ZERO OR  
NEGATIVE VALUES. . .



HOW CAN WE SOLVE THE SAME PROBLEM WITHOUT RECURSION?  
WHICH IS BETTER, THE RECURSIVE SOLUTION OR THE ALTERNATIVE?

- recursive methods often have unusual parameters

- at the top level, we just want:

- ```
binarySearch(arr, item);
```

- but in reality, we have to call:

- ```
binarySearch(arr, item, 0, arr.length-1);
```

- driver methods** are wrappers for calling recursive methods

- driver makes the initial call to the recursive method, knowing what parameters to use

- is *not* recursive itself

- ```
public static boolean binarySearch(arr, item) {  
    return binarySearchRecursive(  
        arr, item, 0, arr.length-1);  
}
```



# recursion, beware

- do not use recursion when a simple loop will do**
  - growth rates may be the same, but...
  - ...there is a lot of overhead involved in setting up the method frame
    - way more overhead than one iteration of a for-loop*
- do not do redundant work in a recursive method
  - move validity checks to a driver method
- too many recursive calls will overflow the call stack
  - stack stores state from all preceding calls

# 4 recursion rules

1. always have at least one case that can be solved without using recursion
2. any recursive call must progress toward a base case
3. always assume that the recursive call works, and use this assumption to design your algorithms
4. never duplicate work by solving the same instance of a problem in separate recursive calls

**today...**

-mergesort

-quicksort

-midterm stuff

# mergesort

divide and conquer

# but first, merging...

- say we have two sorted lists, how can we efficiently merge them?

- idea:** compare the first element in each list to each other, take the smallest and add to the merged list

  - AND... repeat

# but first, merging...

-say we have two sorted lists, how can we efficiently merge them?

-**idea:** compare the first element in each list to each other, take the smallest and add to the merged list

-AND... repeat

## WHAT DOES THIS LOOK LIKE?

# mergesort

- 1) divide the array in half
- 2) sort the left half
- 3) sort the right half
- 4) merge the two halves together



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WHAT IS MISSING HERE?

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HOW DO WE SORT?



WHAT IS MISSING HERE?

# mergesort

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HOW DO WE SORT?  
CAN WE AVOID SORTING? HOW?

WHAT IS MISSING HERE?

# mergesort

1) divide the array in half

~~2) sort the left half~~

~~3) sort the right half~~

4) merge the two halves together

2) take the left half, and go back to step 1

3) take the right half, and go back to step 1

# mergesort

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2) take the left half, and go back to step 1 UNTIL???

3) take the right half, and go back to step 1 UNTIL???

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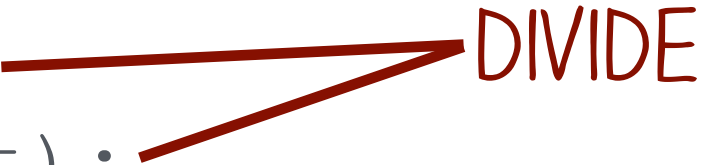
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WHAT DOES THIS LOOK LIKE?

```
void mergesort(int[] arr, int left, int right)
{
    int mid = (left + right) / 2;
    mergesort(arr, left, mid);
    mergesort(arr, mid+1, right);
    merge(arr, left, mid+1, right);
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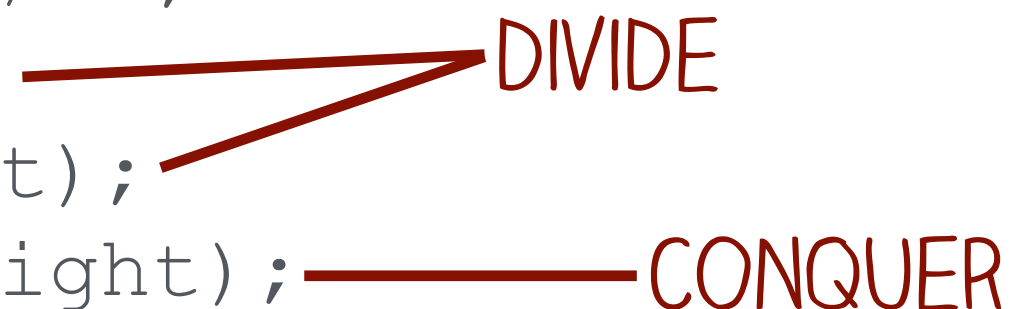




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}
```

The diagram illustrates the Divide and Conquer strategy for mergesort. The word "DIVIDE" is written in red, with two arrows pointing to the recursive calls `mergesort(arr, left, mid);` and `mergesort(arr, mid+1, right);`. The word "CONQUER" is also written in red, with an arrow pointing to the `merge(arr, left, mid+1, right);` call.

```
void mergesort(int[] arr, int left, int right)
{
    int mid = (left + right) / 2;
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```



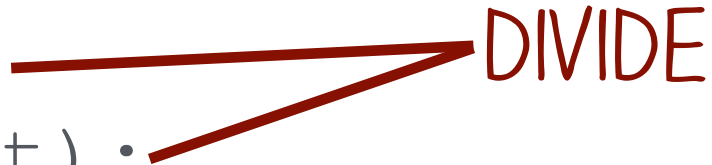
## WHAT ARE WE MISSING?

```
void mergesort(int[] arr, int left, int right)
{
    // arrays of size 1 are already sorted
    if(start >= end)
        return;

    int mid = (left + right) / 2;
    mergesort(arr, left, mid);
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    merge(arr, left, mid+1, right);
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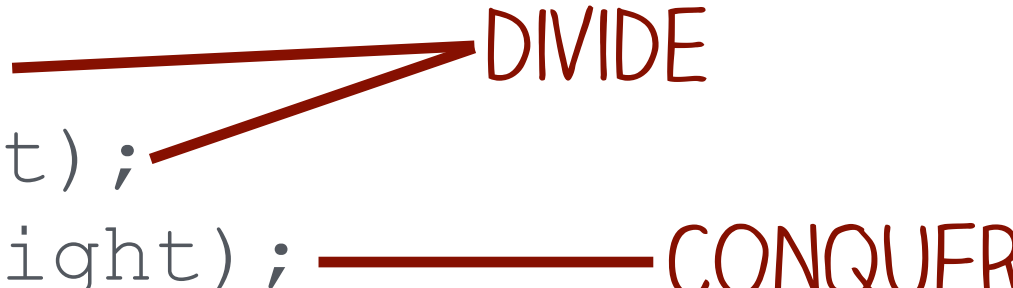
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DIVIDE

CONQUER

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WHAT IS THE COMPLEXITY OF THE DIVIDE STEP?

- A) **c**
- B) **log N**
- C) **N**
- D) **N log N**
- E) **N<sup>2</sup>**
- F) **N<sup>3</sup>**

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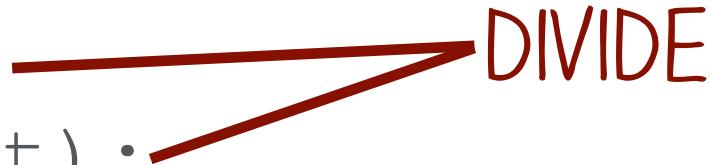
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}
```


DIVIDE

CONQUER

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}
```

 **DIVIDE**

 **CONQUER**

WHAT IS THE COMPLEXITY OF THE CONQUER STEP?

- A) **c**
- B) **log N**
- C) **N**
- D) **N log N**
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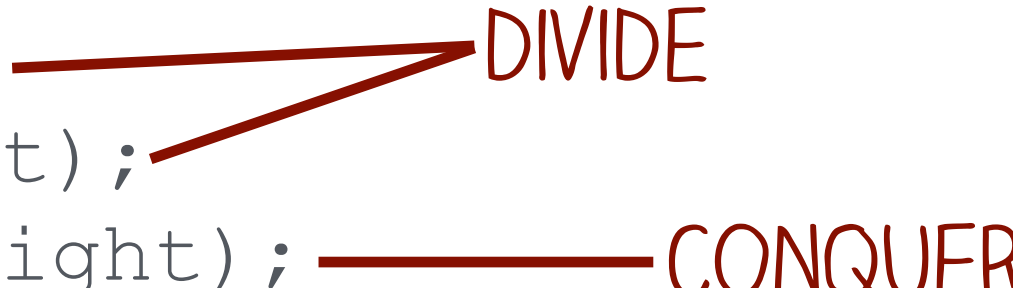
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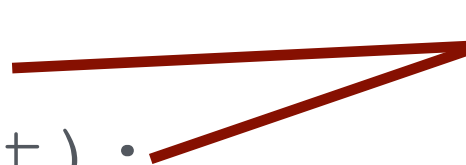



WHAT IS THE COMPLEXITY OF MERGESORT?

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 **DIVIDE**

 **CONQUER**

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IS THIS THE WORST || AVERAGE || BEST-CASE?

# merging sorted arrays

- easy concept, tricky code...
- lots of special cases:
  - keep track of two indices to step through both arrays (the “front” of each array)
    - indices do not necessarily move at the same speed*
  - have to stop the loop when either index reaches the end of their array
  - the two arrays are not necessarily the same size
  - what to do when you reach the end of one array but not the other?
  - copy from temp back into the array

```

void Merge(int[] arr, start, mid, end)
{
    // create temp array for holding merged arr
    int[] temp = new int[end - start + 1];

    int i1 = 0, i2 = mid;
    while(i1 < mid && i2 < end)
    {
        put smaller of arr[i1], arr[i2] into temp;
    }

    copy anything left over from larger half to temp;
    copy temp over to arr;
}

```

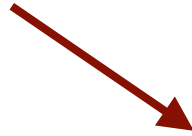
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    copy anything left over from larger half to temp;
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}
```

IS THERE ANYTHING WE SHOULD DO DIFFERENT?

## ALLOCATE TEMP IN DRIVER METHOD!



```
void Merge(int[] arr, int[] temp, start, mid, end)
{
    int i1 = 0, i2 = mid;
    while(i1 < mid && i2 < end)
    {
        put smaller of arr[i1], arr[i2] into temp;
    }

    copy anything left over from larger half to temp;
    copy temp over to arr;
}
```

## IS THERE ANYTHING WE SHOULD DO DIFFERENT?

# notes on merging

- the major disadvantage of mergesort is that the merging of two arrays requires an extra, temporary array
- this means that mergesort requires 2x as much space as the array itself
  - can be an issue if space is limited!
  - an *in-place* mergesort exists, but is complicated and has worse performance
- to achieve the overall running time of  **$O(N \log N)$**  it is critical that the running time of the merge phase be linear



# mergesort variation

- it is a good idea to invoke insertion sort when the subarray size reaches a small enough threshold

- why???

- HINT:** *what is the complexity of insertion sort? of mergesort? what are other runtime considerations?*

- the real threshold depends on several things:

- hardware / OS / compiler

- input characteristics

# quicksort

another divide and conquer

# quicksort

- 1) select an item in the array to be the ***pivot***
- 2) ***partition*** the array so that all items less than the pivot are to the left of the pivot, and all the items greater than the pivot are to the right
- 3) sort the left half
- 4) sort the right half

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**NOTE: after partitioning, the pivot is in its final position!**

# quicksort

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WHAT DO YOU NOTICE?

# quicksort

- 1) select an item in the array to be the ***pivot***
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- ~~3) sort the left half~~
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- ~~3) sort the left half~~
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- 3) take the left half, and go back to step 1 UNTIL???
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# quicksort

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- ~~4) sort the right half~~      4) take the right half, and go back to step 1 **UNTIL???**

WHAT DOES THIS LOOK LIKE?



```
void quicksort(int[] arr, int left, int right)
{
    int pivot_index = partition(arr, left, right);
    quicksort(arr, left, pivot_index-1);
    quicksort(arr, pivot_index+1, right);
}
```

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void quicksort(int[] arr, int left, int right)
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    int pivot_index = partition(arr, left, right);
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WHAT ARE WE MISSING?

```
void quicksort(int[] arr, int left, int right)
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## WHAT IS THE DIVIDE STEP?

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```

WHAT IS THE DIVIDE STEP?

WHAT IS THE CONQUER STEP?

# quick note...

- a huge benefit of quicksort is that it can be done **in-place**
  - ie. you can do the sort within the original array
- mergesort requires an extra, temporary array for merging
- however, in-place partitioning for quicksort requires some careful thought...

# in-place partitioning

- 1) select an item in the array to be the ***pivot***
- 2) swap the pivot with the last item in the array (*just get it out of the way*)
- 3) step from left to right until we find an item  $>$  pivot  
*-this item needs to be on the **right** of the partition*
- 4) step from right to left until we find an item  $<$  pivot  
*-this item needs to be on the **left** of the partition*
- 5) swap items
- 6) continue until left and right stepping cross
- 7) swap pivot with left stepping item

# in-place partitioning

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- 5) swap items
- 6) continue until left and right stepping cross
- 7) swap pivot with left stepping item

WHAT DOES THIS LOOK LIKE?



```

find pivot, swap with right_bound;

L = left_bound, R = right_bound - 1;
while (L <= R)
{
    if (arr[L] <= pivot)
    {
        L++; continue; // find next item > pivot
    }

    if (arr[R] >= pivot)
    {
        R--; continue; // find its "swapping partner"
    }

    swap(arr, L, R); // partners found, swap them
    L++; R--;
}

// point where L met R is the pivot location
swap(arr, L, right_bound); // put pivot back

```

# choosing a pivot

- the median of all array items is the best possible choice... why?
  - is time-consuming to compute
  - finding true median is  **$O(N)$**
- it is important that we avoid the worst case
  - what IS the worst case(s)?
- middle array item is a safe choice... why?
- median-of-three*: pick a few random items and take median
  - why not the first, middle, and last items?
- random pivot*: faster than median-of-three, but lower quality

# choosing a pivot

- any nonrandom pivot selection has some devious input that causes  $O(N^2)$
- trade-off between quality of pivot and time to select
- selection cost should always be  $O(c)$ 
  - ie. it should not depend on  $N$ !

# quicksort complexity

- performance of quick sort heavily depends on which array item is chosen as the pivot
- best case:** pivot partitions the array into two equally-sized subarrays *at each stage* —  $O(N \log N)$
- worst case:** partition generates an empty subarray *at each stage* —  $O(N^2)$
- average case:** bound is  $O(N \log N)$ 
  - proof is quite involved, see the textbook if you are curious

# quicksort vs mergesort

- both are  $O(N \log N)$  in the average case
- mergesort is also  $O(N \log N)$  in the *worst* case
  - so, why not always use mergesort?
- mergesort requires  $2N$  space
  - and, copying everything from the merged array back to the original takes time
- quicksort requires no extra space
  - thus, no copying overhead!
  - but, in  $O(N^2)$  worst case <wha wha>

- both are divide and conquer algorithms (recursive)
- mergesort sorts “on the way up”
  - after the base case is reached, sorting is done as the calls return and merge
- quicksort sorts “on the way down”
  - once the base case is reached, that part of the array is sorted
- though quicksort is more popular, it is not always the right choice!

# sorting summary

|                | Best          | Average       | Worst         | Notes                                       |
|----------------|---------------|---------------|---------------|---------------------------------------------|
| Selection Sort | $O(N^2)$      | $O(N^2)$      | $O(N^2)$      | Never used in practice                      |
| Insertion Sort | $O(N)$        | $O(N^2)$      | $O(N^2)$      | Takes advantage of “sortedness”             |
| Shellsort      | $O(N \log N)$ | $O(N^{1.25})$ | $O(N^{1.5})$  | Depends on gap sizes                        |
| Mergesort      | $O(N \log N)$ | $O(N \log N)$ | $O(N \log N)$ | 2x space overhead, guaranteed $O(N \log N)$ |
| Quicksort      | $O(N \log N)$ | $O(N \log N)$ | $O(N^2)$      | Depends on pivot                            |



**midterm**

# topics

## -Java basics

- variables, types
- control flow
- reference types
- classes, methods

## -OOP

- inheritance
- polymorphism
- interfaces
- super

# topics

- generics

  - wild cards

  - generic classes

- comparators

- algorithm analysis

  - growth rates

  - Big-O

  - determining complexity of loops and algorithms

# topics

-Collections

-Iterators

-recursion

-selection sort

-insertion sort

-shellsort

-mergesort

-quicksort

BE ABLE TO REASON ABOUT  
PERFORMANCE AND BEHAVIOR OF EACH!

# test format

- problems may be of the following types:
  - short answer
  - determining output of code
  - writing code
  - filling in missing code
  - multiple choice
  - true / false

**next time...**

- no lab on Monday

- midterm on Tuesday in class**

- reading

  - chapter 17

  - chapter 3

    - <http://opendatastructures.org/ods-java/>*

- homework

  - assignment 5 due Thursday