

pages.cs.wisc.edu/.../6.RECURSION.ht... University of Wisconsin-Madison The original call causes 2 to be output, and then a recursive call is made, creating a clone with k == 1. That clone executes line 1: the if condition is false; line 4: ...

RECURSION

cs2420 | Introduction to Algorithms and Data Structures | Spring 2016

administrivia...

-assignment 4 due on Thursday at midnight

-partners?

-midterm next Tuesday

-exam review questions out later this week

-no office hours today

last time...

selection vs insertion

WORST: $O(N^2)$

AVERAGE: $O(N^2)$

> $O(N^2)$ BEST:

 $O(N^2)$

 $O(N^2)$

O(N)

selection vs insertion

WORST: $O(N^2)$ $O(N^2)$

AVERAGE: $O(N^2)$ $O(N^2)$

BEST: $O(N^2)$ O(N)

WHICH ONE PERFORMS BETTER IN PRACTICE?

- A) selection
- B) insertion

what we want...

- -a sorting algorithm that has subquadratic complexity
- -swapping adjacent items removes exactly 1 inversion

-what if we consider swapping nonadjacent pairs?



-removes inversions not involved with the swap

shellsort the simplest subquadratic sorting algorithm



- 1) set the gap size to N/2
- 2) consider the subarrays with elements at **gap size** from each other
- 3) do insertion sort on each of the subarrays
- 4) divide the gap size by 2
- 5) repeat steps 2 4 until the is gap size is <1



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WHAT DOES THIS LOOK LIKE?

HOV	V DO WE	DESCRIBE	INSERTION	SORT	WITH	RESPECT	TO SHE	ELLSOR	T í
				10					

```
void shellSort(int[] arr)
  for (gap = arr.length/2; gap > 0; gap /= 2)
    for(i = gap; i < arr.length; i++)</pre>
      val = arr[i];
      for(j = i-gap; j >= 0 && arr[j] > val; j -= gap)
        arr[j+qap] = arr[j];
      arr[j+gap] = val;
```

DIMINISHING GAP SEQUENCE -

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```

today...

-what is recursion? and some examples...

-driver methods

-the overhead of recursion

re · cur · sion

[ri-kur-zhuh n] **noun**

see recursion.

- -recursion is a problem solving technique in which the solution is defined in terms of a simpler (or smaller) version of the problem
 - -break the problem into smaller parts
 - -solve the smaller problems
 - -combine the results
- -a recursive method calls itself
- -some functions are easiest to define recursively

$$sum(N) = sum(N-1) + N$$

- -there must be at least one *base case* that can be computed without recursion
 - -any recursive call must make progress towards the base case!

```
sum(N) = sum(N-1) + N
```

```
public static int sum(int n) {
  if(n == 1)
    return 1;
  return sum(n-1) + n;
}
```

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HOW CAN WE SOLVE THE SAME PROBLEM WITHOUT RECURSION? WHICH IS BETTER, THE RECURSIVE SOLUTION OR THE ALTERNATIVE?

```
-how to compute N!

N! = N * N-1 * N-2 * ... * 2 * 1
```

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-how would you compute this using a for-loop?

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 - -what is recursive?

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- B) log N
- C) **N**
- D) N log N
- E) **N**²
- F) **N**³

WHAT IS THE COMPLEXITY OF THE FOR-LOOP METHOD?

- -how to compute N!N! = N * N-1 * N-2 * ... * 2 * 1
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WHAT IS THE COMPLEXITY OF THE RECURSIVE METHOD?

```
public static int divide(int a, int b)
{
    ...
}
HINT: 9/2 = 1 + (7/2)
```

-write a recursive method that computes A/B

- -do integer division
- -/ operator not allowed, can only use -
- -don't worry about negative input or divide-by-zero

```
public static int divide(int a, int b)
{
    ...
}
```

HINT: 9/2 = 1 + (7/2)

-recursion often seems like MAGIC -use this to your advantage

-when writing a recursive method, just assume that the function you're writing already works, so you can use it to help solve the problem

-once you've worked out the recursion, think about the base case, and you're done

driver methods

divide and conquer

- -divide and conquer is an important problem solving technique that makes use of recursion
 - -divide: smaller problems are solved recursively (except for base cases!)
 - -conquer: solutions to the subproblems form the solution to the original problem
- -typically, an algorithm containing more than one recursive call is referred to as divide and conquer
- -subproblems are usually disjoint (non-overlapping)



-write a recursive method to perform a binary search -assume an (ascending) sorted list



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-HINT

- -check if middle item is what we're looking for -if so, return true
- -else, figure out if item is the left or right half -repeat on that half

-base case(s)???

-recursive methods often have unusual parameters

-at the top level, we just want:

```
binarySearch(arr, item);
```

-but in reality, we have to call:

```
binarySearch(arr, item, 0, arr.length-1);
```

-driver methods are wrappers for calling recursive methods

- -driver makes the initial call to the recursive method, knowing what parameters to use
- -is *not* recursive itself

```
public static boolean binarySearch(arr, item) {
   return binarySearchRecursive(
        arr, item, 0, arr.length-1);
}
```

-another useful feature of driver methods is error checking (or, validity checks)

-do the error checking *only* in the driver method, instead of redundantly doing it every time in the recursion

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WHAT IS SOMETHING TO CHECK FOR IN OUR BINARY SEARCH METHOD?

```
public static boolean binarySearch(arr, item) {
   if (arr == null) // only check this once
     return false;

return binarySearchRecursive(
   arr, item, 0, arr.length-1);
}
```

overhead of recursion

- -every time a method is invoked, a unique "frame" is created
 - -contains local variables and state
 - -put on the call stack
- -when that method returns, execution resumes in the calling method
- -this is how methods know where to return to!

main

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-but each frame has different arguments

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factorial(4)
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recursion, beware

- -do not use recursion when a simple loop will do
 - -growth rates may be the same, but...
 - -...there is a lot of overhead involved in setting up the method frame
 - -way more overhead than one iteration of a for-loop
- -do not do redundant work in a recursive method
 -move validity checks to a driver method
- -too many recursive calls will overflow the call stack -stack stores state from all preceding calls

recap

4 recursion rules

- always have at least one case that can be solved without using recursion
- 2. any recursive call must progress toward a base case
- 3. always assume that the recursive call works, and use this assumption to design your algorithms
- 4. never duplicate work by solving the same instance of a problem in separate recursive calls

next time...

-reading

-chapters 7 & 8.5 - 8.8 (recursion, mergesort, & quicksort)

-homework

-assignment 4 due Thursday

-(short) midterm review on Thursday