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① Proofs by truth table:

a. $((p \to q) \to p) \to p$, show tautology

$\neg((P \to Q) \to P) \vee P$ b/c $A \to B \equiv \neg A \vee B$

$\neg(P \to Q) \to \neg P \vee P \equiv \underbrace{\neg P \to \neg Q}_{\text{True}} \to \neg P \vee P$

$\to \neg P \vee P$

b. $P \wedge q \to \neg p \vee \neg q$, show contradiction
- skip according to lecture

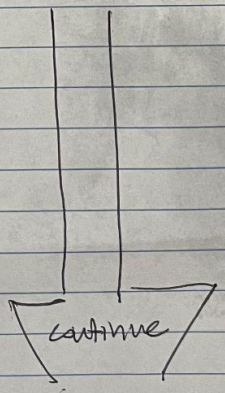
c. $(P \vee Q \vee R) \wedge (\neg P \vee \neg Q \vee R)$, show formula is satisfiable but not tautology

$\neg(P \vee Q \vee R) \wedge (\neg P \vee \neg Q \vee R)$

$(\neg P \vee \neg Q) \vee \neg R \quad \wedge (\neg P \vee \neg Q) \vee R$

$\neg P \vee \neg Q \equiv \neg P \vee \neg Q$

True



① continued

$(P \vee Q \vee R) \wedge (\neg P \vee \neg Q \vee R)$

$\neg (P \vee Q \vee R)$

Proofs by truth table ∠

P	Q	R	$P \vee Q \vee R$ (A)	$\neg P$	$\neg Q$	$\neg R$	$\neg P \vee \neg Q \vee R$ (B)	$A \wedge B$
F	F	F	F	T	T	T	T	F
F	T	F	T	T	F	T	T	T
F	T	T	T	T	F	F	T	T
F	F	T	T	T	T	F	T	T
T	F	F	T	F	T	T	T	T
T	F	T	T	F	T	F	T	T
T	T	F	T	F	F	T	T	T
T	T	T	T	F	F	F	F	F

2. DNF & CNF

a. P, Q, R

P	Q	R	Exactly one is true	or	All three are true	$P \vee Q \vee R$
F	F	F		F		
F	T	F		T		$\neg P \wedge Q \wedge \neg R$
F	F	T		T		$P \vee Q \vee \neg R$
F	T	T		F		$P \vee \neg Q \vee \neg R$
T	F	F		T		$P \wedge \neg Q \wedge \neg R$
T	F	T		F		$\neg P \vee Q \vee \neg R$
T	T	F		F		$\neg P \vee \neg Q \vee R$
T	T	T		T		$P \wedge Q \wedge R$

2. b. DNF

$(\neg P \wedge Q \wedge \neg R) \vee$
$(P \wedge \neg Q \wedge \neg R) \vee$
$(P \wedge Q \wedge R)$

c. CNF

$(P \vee Q \vee R) \wedge$
$(P \vee Q \vee \neg R) \wedge$
$(P \vee \neg Q \vee \neg R) \wedge$
$(\neg P \vee Q \vee \neg R) \wedge$
$(\neg P \vee \neg Q \vee R)$

3. Translation conditions into a logized formula

rules:

5 | integer P
or 25 ∤ integer q
25 | integer ∧ 125 | integer r

DNF Expression?

~~5 ∤ int~~ ∧ 25 | int

$(5 \mid int \wedge \sout{25 \nmid int} \wedge \neg(25 \mid int \wedge 125 \mid int)) \vee$
$(5 \mid int \wedge 25 \nmid int \wedge \neg(25 \mid int \wedge 125 \mid int)) \vee$

5 | int	25 | int	25 | int ∧ 125 | int	¬	Problem
F	F	F	T	X
F	T	F	F	X ~~P ∧ Q ∧ R~~
F	F	T	T	X
F	T	T	F	F
T	F	F	T	T $P \wedge \neg Q \wedge \neg R$
T	F	T	T	F
T	T	F	F	T $P \wedge Q \wedge \neg R$ ← only 2 clauses
T	T	T	T	X ~~P ∧ Q ∧ R~~

rewrite for clarity:

DNF: $(5 \mid int \wedge 25 \mid int \wedge \neg(25 \mid int \wedge 125 \mid int)) \vee$
$(5 \mid int \wedge 25 \nmid int \wedge \neg(25 \mid int \wedge 125 \mid int))$