HW2

① Relations & their properties ($\mathbb{Z}$ integers)

a. $x \mid y$: "x divides y" where $y = x \cdot Z$
prove / show counter example for: reflexive, symmetric, transitive

Reflexive : xRx, $\forall x \in A$ (x relates to x for all values inside of A)

$xRx \Rightarrow x \mid x$ holds for all x's, $\therefore$ symmetric

$x \mid y$: Symmetric
if $(x,y) \in \mathbb{Z}$, then $(y,x) \in \mathbb{Z}$

$(2,4) \in \mathbb{Z}$, 4 is divisible by 2
$(4,2) \notin \mathbb{Z}$, 2 is not divisible by 4
$\therefore$ not symmetric

$x \mid y$: transitive
if y/x & z/y, then z/x

if $(x,y) \in R$ & $(y,z) \in R$, then $(x,z) \in R$
$\therefore$ R is transitive

b. $x \nmid y$ "x does not divide y" ; x not a factor of y

Reflexive: $xRx \Rightarrow x \mid x$ holds for all n
$\therefore$ not reflexive

Symmetric:
$(2,4) \notin R$, 4 is divisible by 2
$\therefore$ not symmetric

Transitive:
$(x,y) \notin R$ & $(y,z) \notin R$, then $(x,z) \notin R$

$\therefore$ Transitive

c. $x \equiv_n y$ "x is n-equivalent to y", x-y is a multiple of n

√ Reflexive if xRx for all n

$((a,b),(a,b)) \in R$, $ab = ab$ $\therefore$ Reflexive

√ Symmetric if $(a,b) \in R$, then $(b,a) \in R$

$((a,b)(c,d)) \in R$, then $((c,d)(a,b)) \in R$

for given condition: $ad = bc$ & $cb = da$

multiplication is commutative, $\therefore$ symmetric

√ Transitive

if $(a,b) \in R$ and $(b,c) \in R$, the (a,c) also in R

$((a,b),(c,d)) \in R$ and $((c,d),(e,f)) \in R$

assume:

$((a,b)(c,d)) \in R$ and $((c,d)(e,f)) \in R$

we get: $ad = cb$ & $cf = de$

implies: $a/b = c/d$ & $c/d = e/f$

so: $a/b = e/f$ or $af = be$

$\therefore ((a,b)(e,f)) \in R$

$\therefore$ transitive

$xy = ar^n + er^{\frac{n}{2}} + d$

② Karatsuba mult.

$a, d, e, \quad x \cdot y$?

a. $x = 53 \quad y = 47$

$a_1 = 53 \times 47$
$a_2 = 5 \times 4 = 20$
$d_2 = 3 \times 7 = 21$
$e_2 = (5+3)(4+7) - 20 - 21$
$= 47$ $\quad 88 - 41$

$53 \times 47 = 20 \times 10^2 + 47 \times 10 + 21 =$
$2000 + 470 + 21 = \underline{2491}$ a_1
491

b. $x = 4737 \quad y = 5345$

$d_1 = 37 \times 45$
$a_2 = 3 \times 4 = 12$
$d_2 = 7 \times 5 = 35$
$e_2 = (3+7)(4+5) - 12 - 35$
$= 43$ $\quad 90 \quad - 47$

$37 \times 45 = 12 \times 10^2 + 45 \times 10 + 35 = \underline{1685}$ d_1
$1200 + 450 + 35$

$e_1 = (53 + 37)(47 + 45) - 2491 - 1685$
$(90 \times 92) - 2491 - 1685 =$
$8280 - 2491 - 1685 = \underline{4104}$ e_1

$xy = (2491)10^4 + (1685)10^2 + 4104$
$24{,}910{,}000 \quad 168{,}500 + 4{,}104 = 25{,}082{,}604$

vs.

correct answer $= 25{,}319{,}265$

③ Horner's method

a. rewrite $3x^4 + 12x^3 + 15x^2 + 21x + 42$

Horner's Rule

3	12	15	21	42	$x = 11$
3×11	$\rightarrow 33$	$\overset{45 \times 11}{\rightarrow} 495$	$\overset{\times 11}{\rightarrow} 5610$	$\overset{\times 11}{}$ 61941	
3	45	510	5631	$\boxed{61{,}983}$	

$3 \cdot (11) \cdot (11) \cdot (11) \cdot (11) + 12 \cdot (11) \cdot (11) \cdot (11) + 15 \cdot (11) \cdot (11) + 21(11) + 42$

$43{,}923 + 15{,}927 + 1815 + 231 + 42$

$= 61{,}938$

b. 61,983

c 10 multiplications

d. 4 multiplications