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MA3: Binary trees

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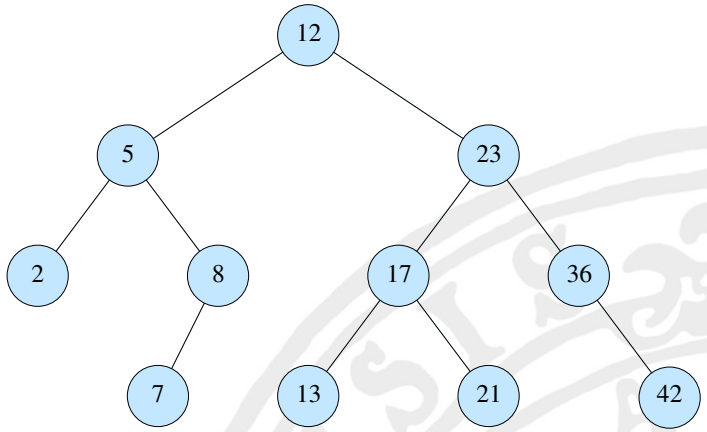
Binary trees

Definition:

A set of nodes that is either empty or consists of three three disjoint sets:

- One with one node called *the root*
- One called *the left subtree* which is a binary tree
- One called *the right subtree* which is a binary tree



Inorder traversal: 2, 5, 7, 8, 12, 13,17, 21, 23, 36, 42

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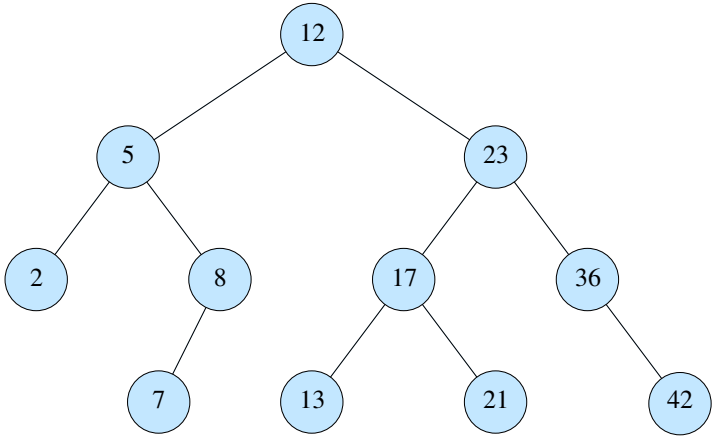
Binary *search* trees

A *binary search tree* is a binary tree where the nodes are ordered:



The data in every node is greater than all data in its left subtree and less than all data in its right subtree.

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Binary search trees

This is a powerful structure for storing data with an order relationship.

The operations

- searching data
- inserting
- removing data

can be done in $\Theta(\log n)$ time on the average.

We will here look at these algorithms.

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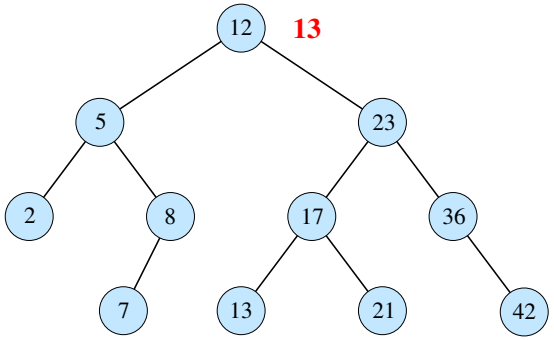
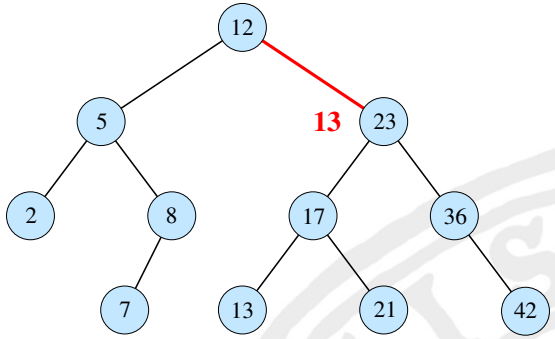
A class for binary search trees

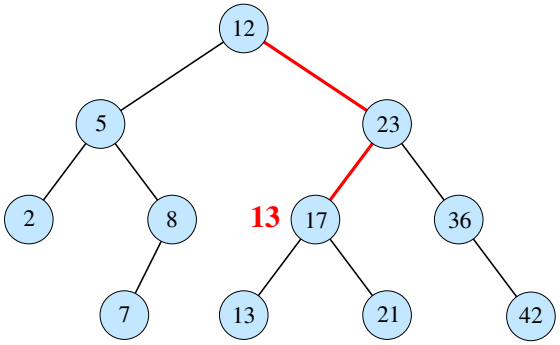
```
class BST:
    class Node:
        def __init__(self, key,
                     left = None,
                     right = None):
            self.key = key
            self.left = left
            self.right = right

    def __init__(self, root = None)
        self.root = root

     . . . .
```

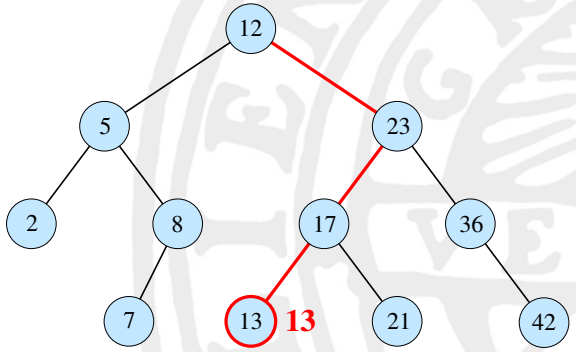
Searching

Searching starts at the root and is guided by the values in the nodes:









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A method for searching

```
def contains(self, k):
    n = self.root
    while n and n.key != k:
        if k < n.key:
            n = n.left
        else:
            n = n.right
    return n
```

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Traversal: Counting nodes

Use a recursive help method:

```
def size(self):
    return self._size(self.root)

def _size(self, r):
    if r:
        return 1 + self._size(r.left) + self._size(r.right)
    else:
        return 0
```

Note that `r == None` is a much better base case than `r.left == None` and `r.right == None`.

We will later see how this could be done using a *generator*.

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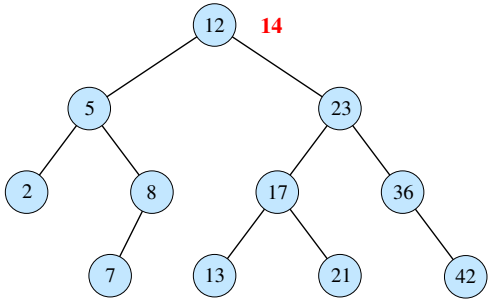
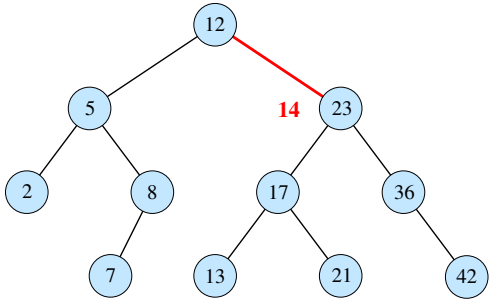
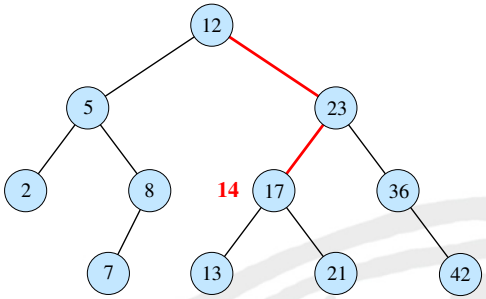
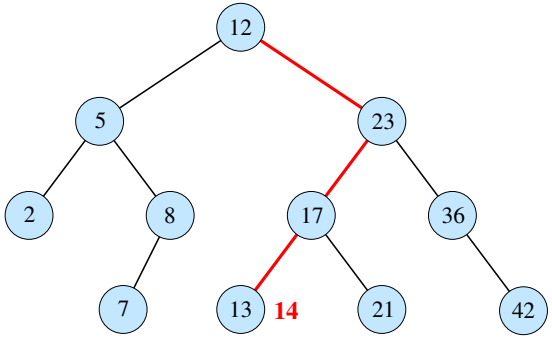
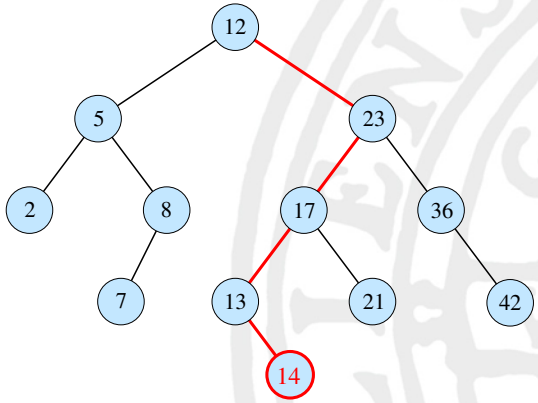
Inserting in binary search tree











Code

```
def insert(self, key):
    self.root = self._insert(self.root, key)

def _insert(self, r, key):
    if r is None:
        return self.Node(key)
    elif key < r.key:
        r. left = self._insert(r.left, key)    # Insert in the left subtree
    elif key > r.key:
        r.right = self._insert(r.right, key)   # Insert in the right subtree
    else:
            pass
    return r
```

Modify the (sub-)tree with root in r and return the root of the modified (sub-)tree.

Note that the help method always have to return the root of the modified subtree regardless if it is the new node or not.

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Modified insertion code

Suppose we want to know if a new node was inserted or not.

```
def insert(self, key):
    self.root = self._insert(self.root, key)

def _insert(self, r, key):

    if r is None:
           return self.Node(key)

    elfi key < r.key:
        r. left = self._insert(r.left, key)    # Insert in the left subtree
    elif key > r.key:
        r.right = self._insert(r.right, key)   # Insert in the right subtree
    else:
           pass

    return r
```

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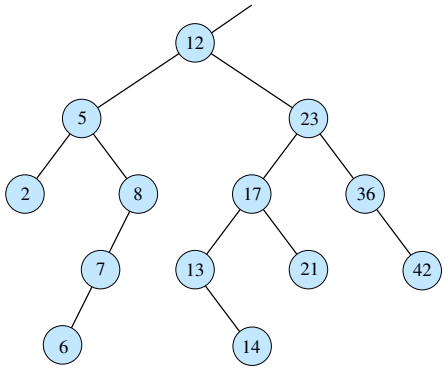
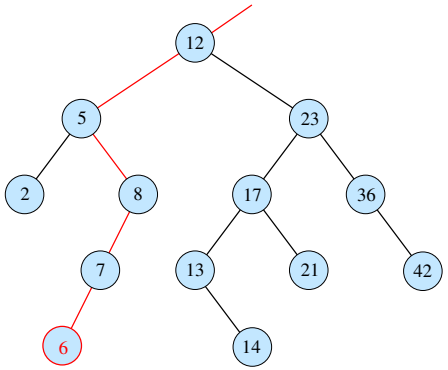
Modified insertion code

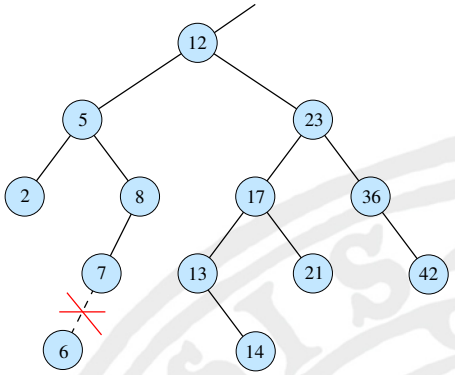
```
def insert(self, key):
        self.root, result = self._insert(self.root, key)
        return result

    def _insert(self, r, key):
        if r is None:
            return self.Node(key), True
        elif key < r.key:
            r.left, result = self._insert(r.left, key)
        elif key > r.key:
            r.right, result = self._insert(r.right, key)
        else:
            result = False # Already there
        return r, result
```

Remove

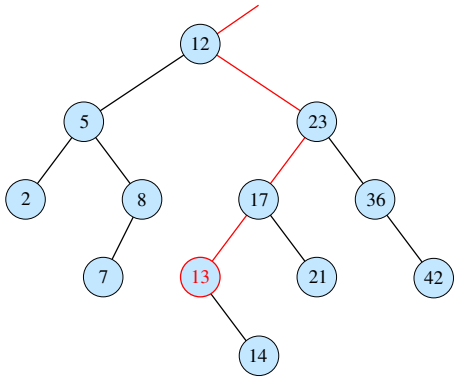
If the node to be removed has no children. Example: remove 6.

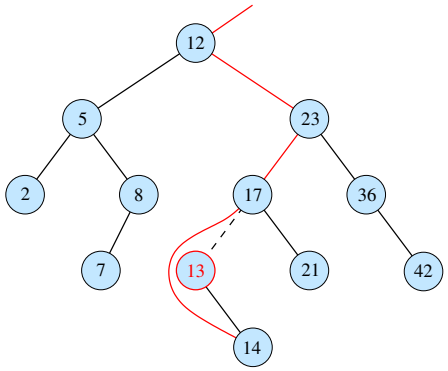


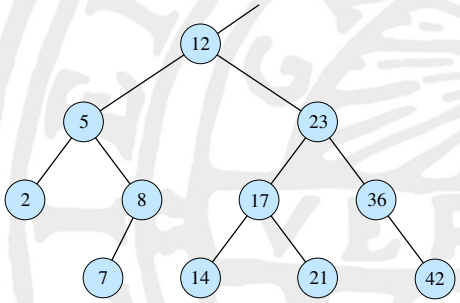




If the node to be removed has one child. Example: remove 13.







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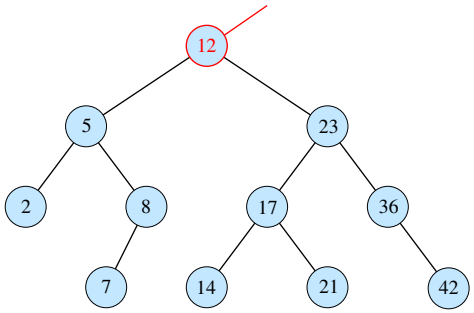
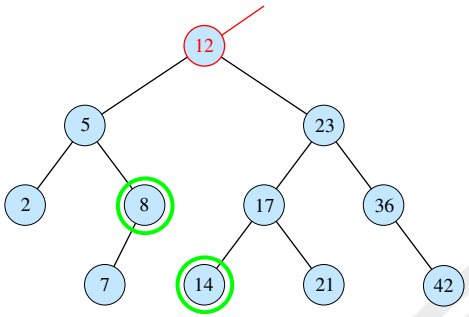
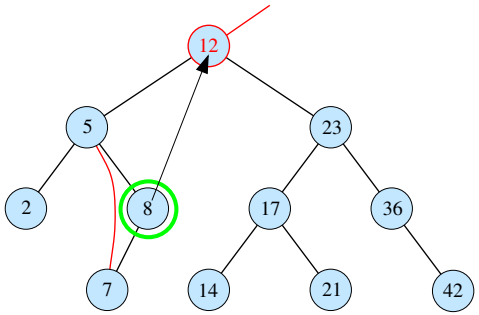
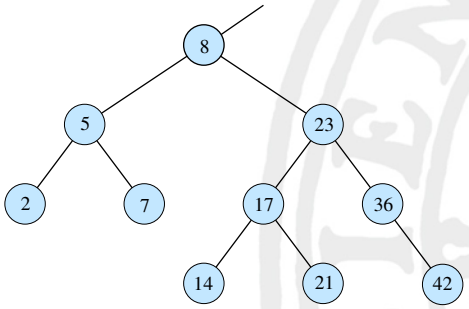
Remove – the harder case

Removing a key in a node with two children. Example: remove 12.

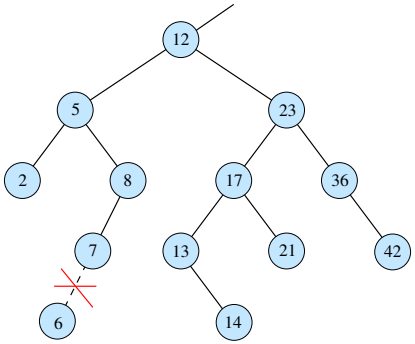


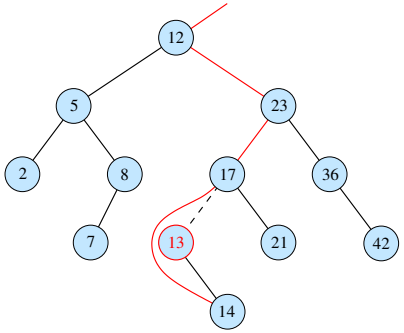


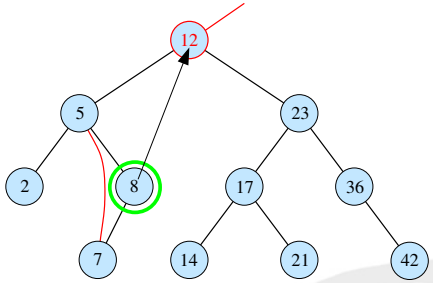




Coding hints for remove








```
def _remove(self, r, key):
    pass
```

Takes a reference to a node (`r`) that is the root in a subtree. Removes `key` from that subtree and returns a reference to the root in the resulting subtree.

```
def _get_largest(self, r):
    pass
```

Find the largest and then then call `_remove` recursively

```
def _remove_largest(self, r):
    pass
```

Removes the node with the largest key and returns a tuple with that key and a reference to the root in the resulting subtree.

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The end