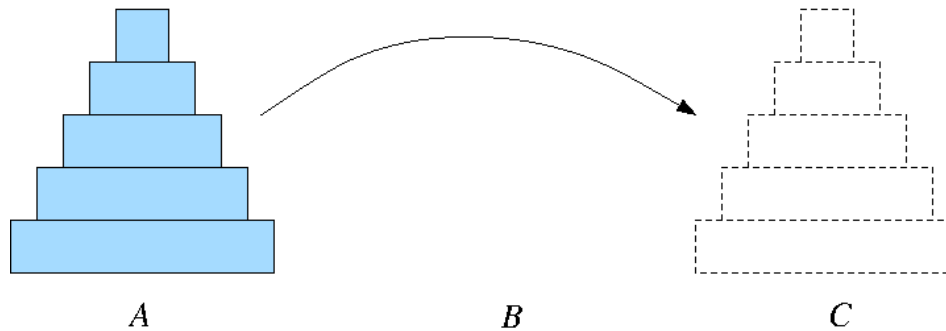


Tower of Hanoi

Tom Smedsaas

Complexity of the problem with the
tower of Hanoi

Tower of Hanoi

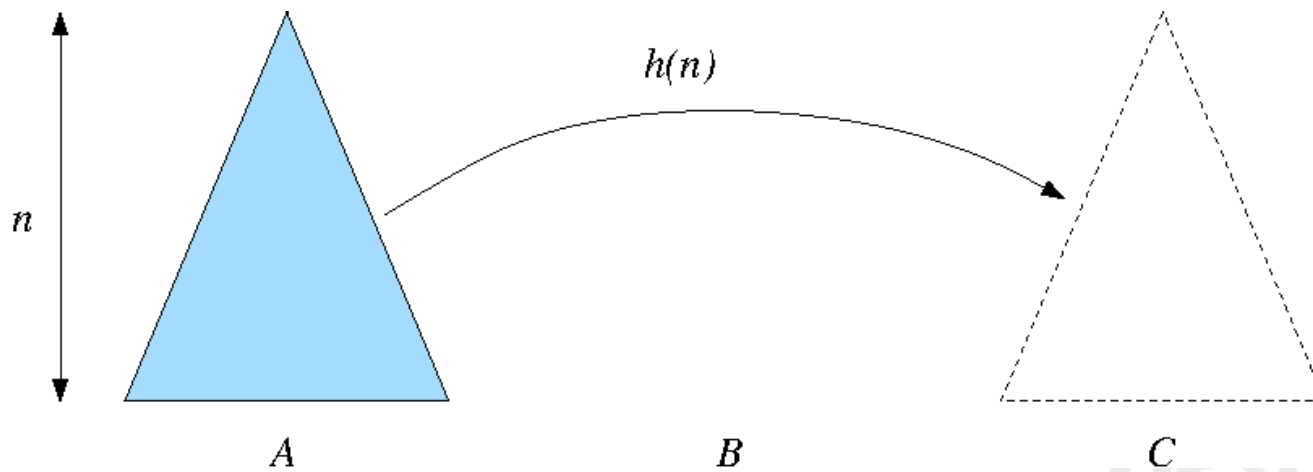


A stack of tiles must be moved from A to C using the following rules:

1. Only one tile may be moved at a time and
2. A larger tile must never be placed on top of a smaller one.

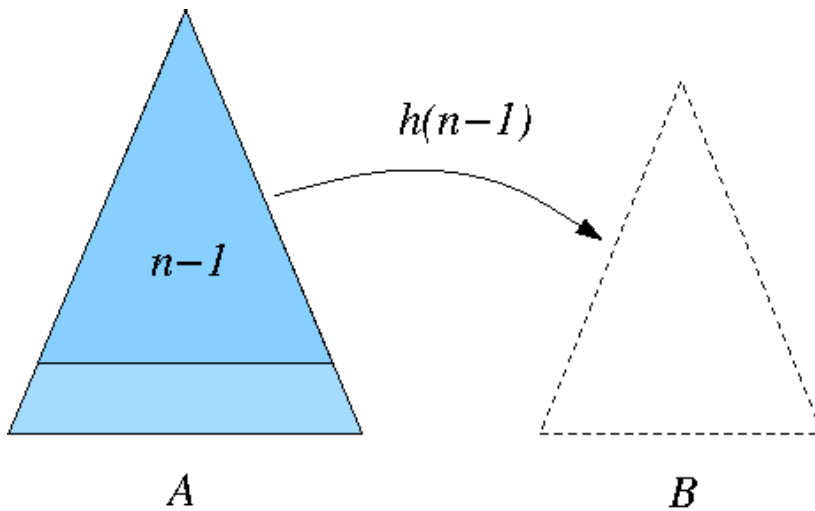
Tower of Hanoi

Let $h(n)$ stand for the problem of moving n tiles.



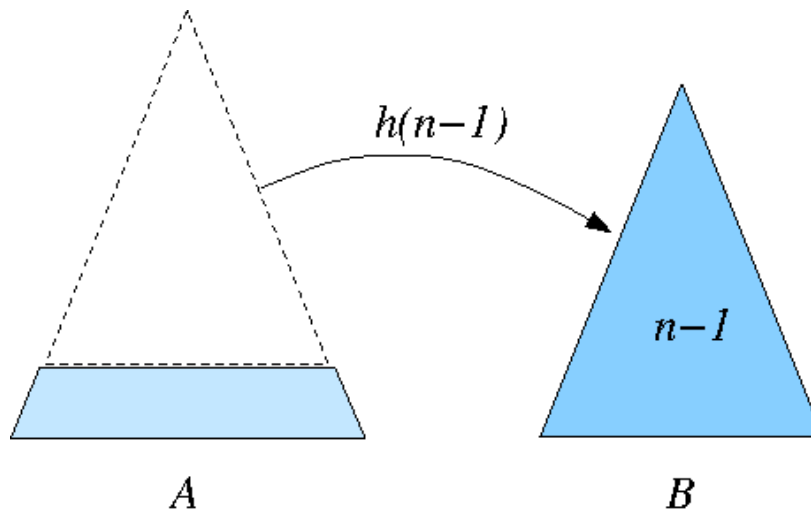


Tower of Hanoi



C

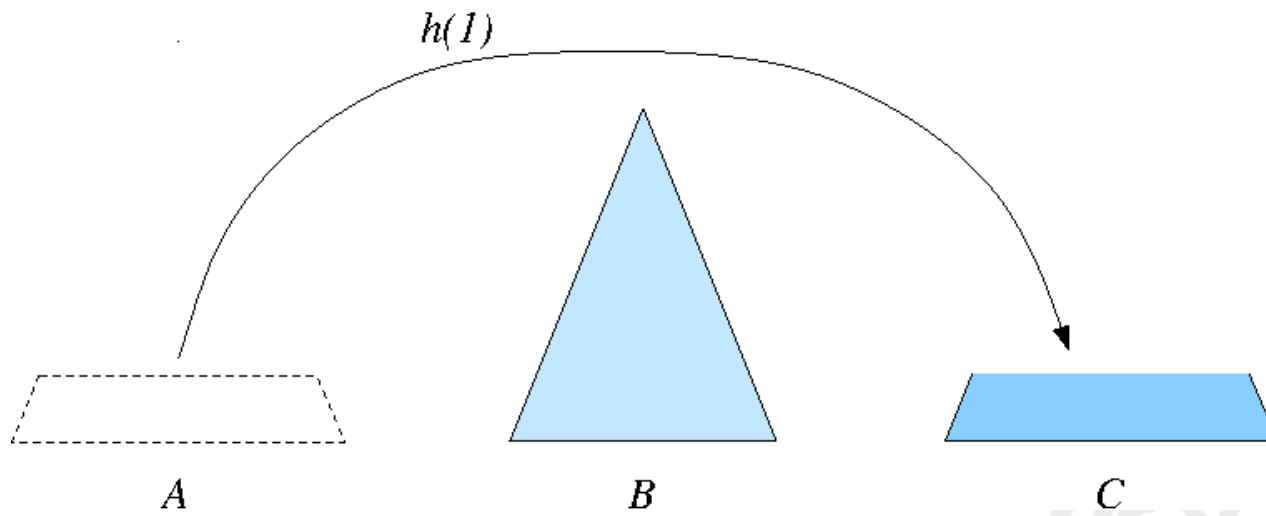
Tower of Hanoi



C

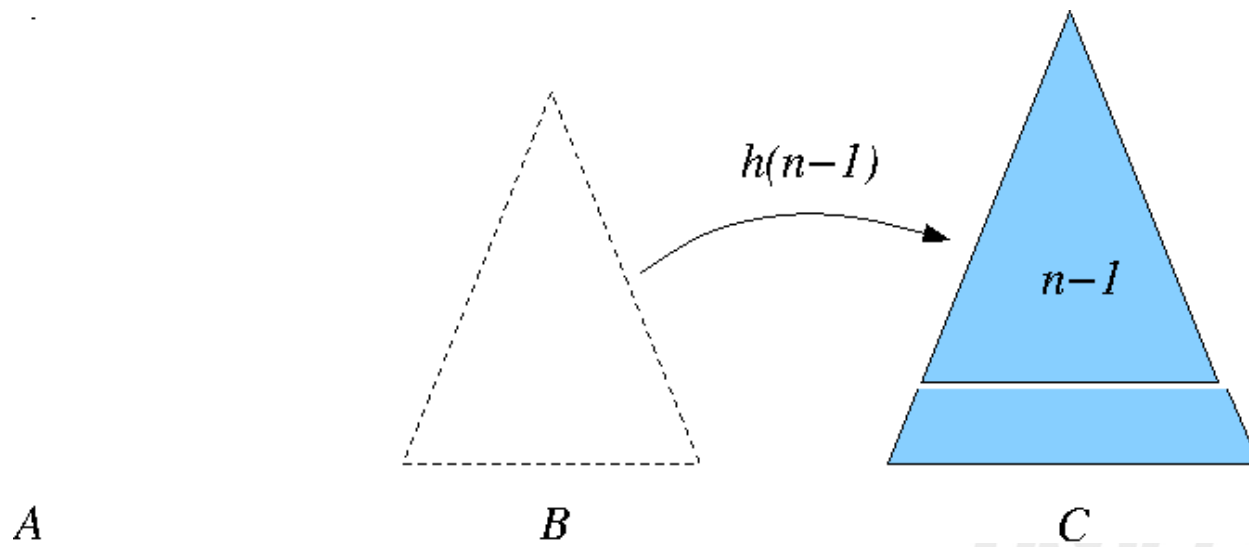


Tower of Hanoi





Tower of Hanoi



Complexity analysis

We solve a problem of size n by solving two problems of size $n-1$ and one problem of size 1.

If $h(n)$ stands for the number of tile moves then becomes

$$\begin{aligned}h(n) &= 2h(n-1) + 1 = \\&= 2(2h(n-2) + 1) + 1 = 4h(n-2) + 2 + 1 = \dots = \\&= 2^k h(n-k) + 2^{k-1} + 2^{k-2} + \dots + 2 + 1 = \\&= 2^{n-1} h(1) + 2^{n-2} + \dots + 2 + 1 = \\&= 2^{n-1} \cdot 1 + 2^{n-2} + \dots + 2 + 1 = 2^n - 1\end{aligned}$$

Hopples for large n .



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The end