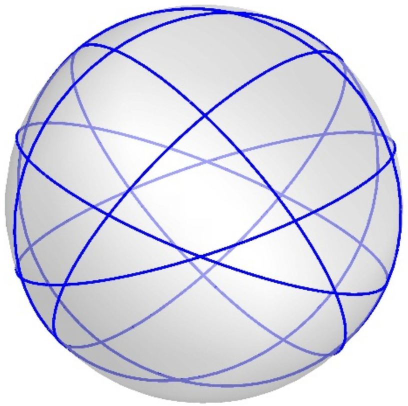

Great Circles Problem

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03/07/2015

Outline

This note mentions some observations obtained from the data that I currently have and some conjectures.

The goal is still to get the chromatic number of the graph as 3.

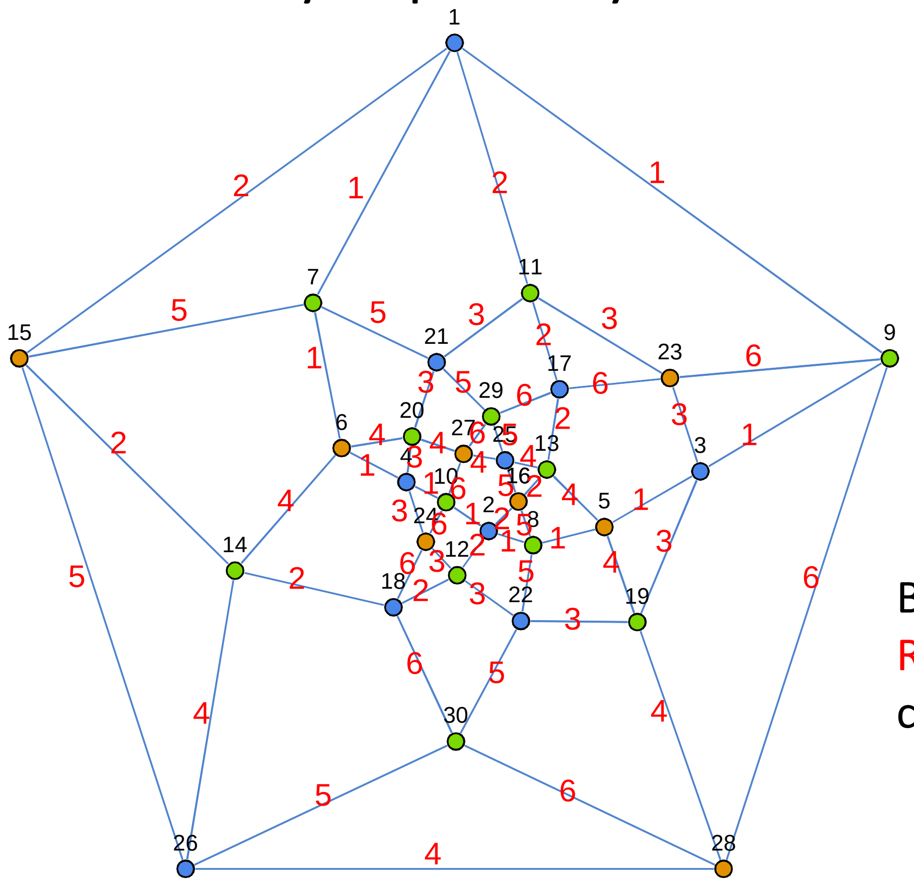
Our problem is actually to color a planar 4-regular graph which is NP Complete [1].

Conjecture 1.

Remove a circle containing all the vertices and edges incident into that vertices will separate the graph into 2 graphs with same colors, number of vertices and shape.

Conjecture 1 - Observation

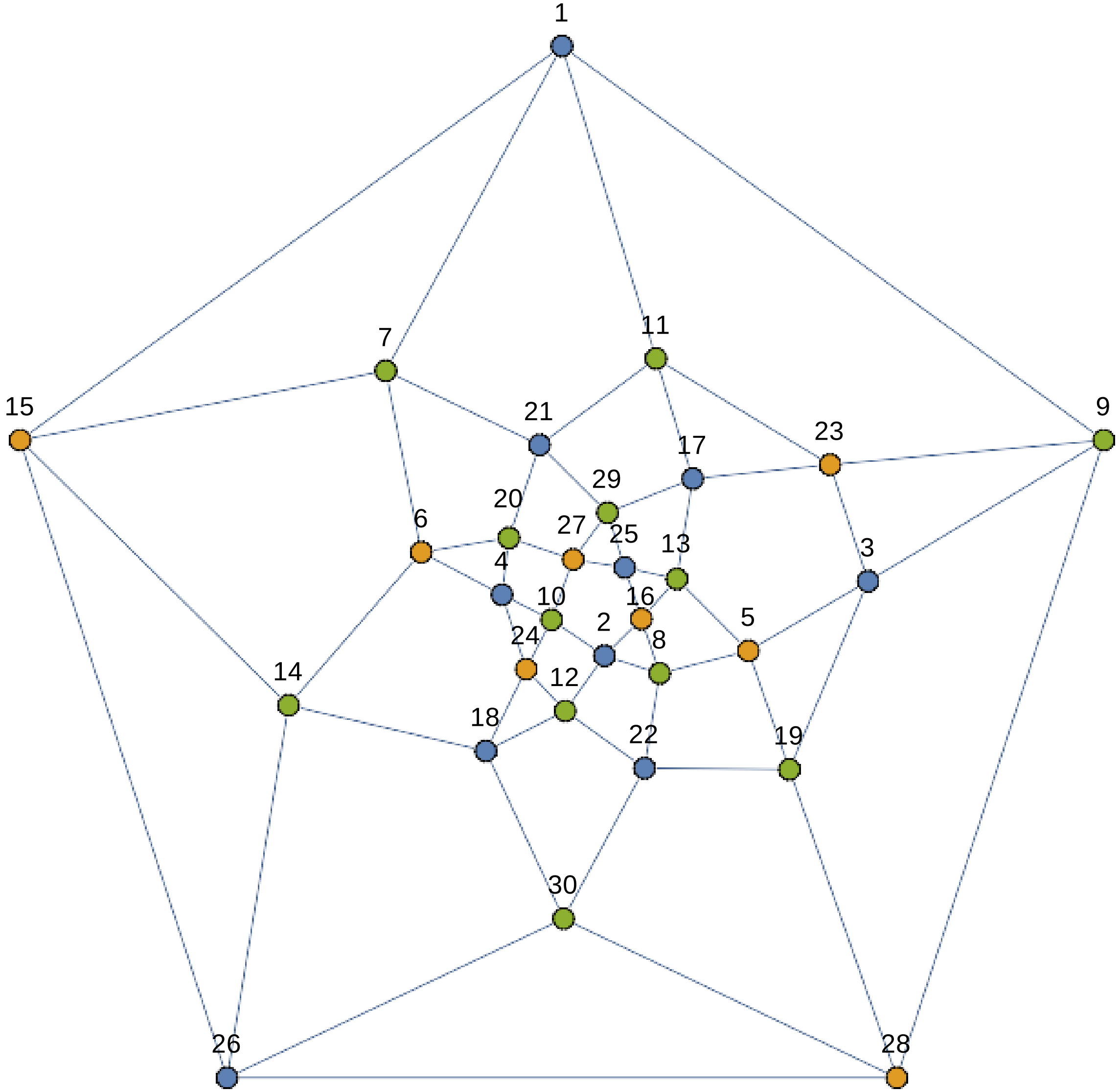
Starting with one non-isomorphic graph of **6** great circles as the graph below, I tried to remove one circles. The reason is to split the problem into sub problems we've already known or to enable a way to prove by induction hypothesis.

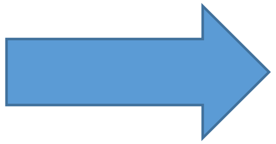


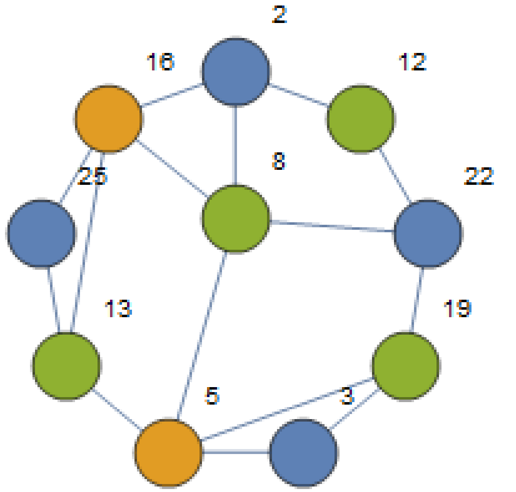
Black numbers: Vertices

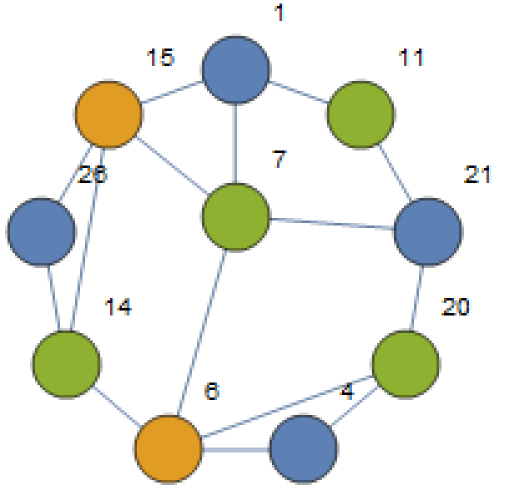
Red numbers: The edges on the circle

Conjecture 1 – Observation - Remove circle 1

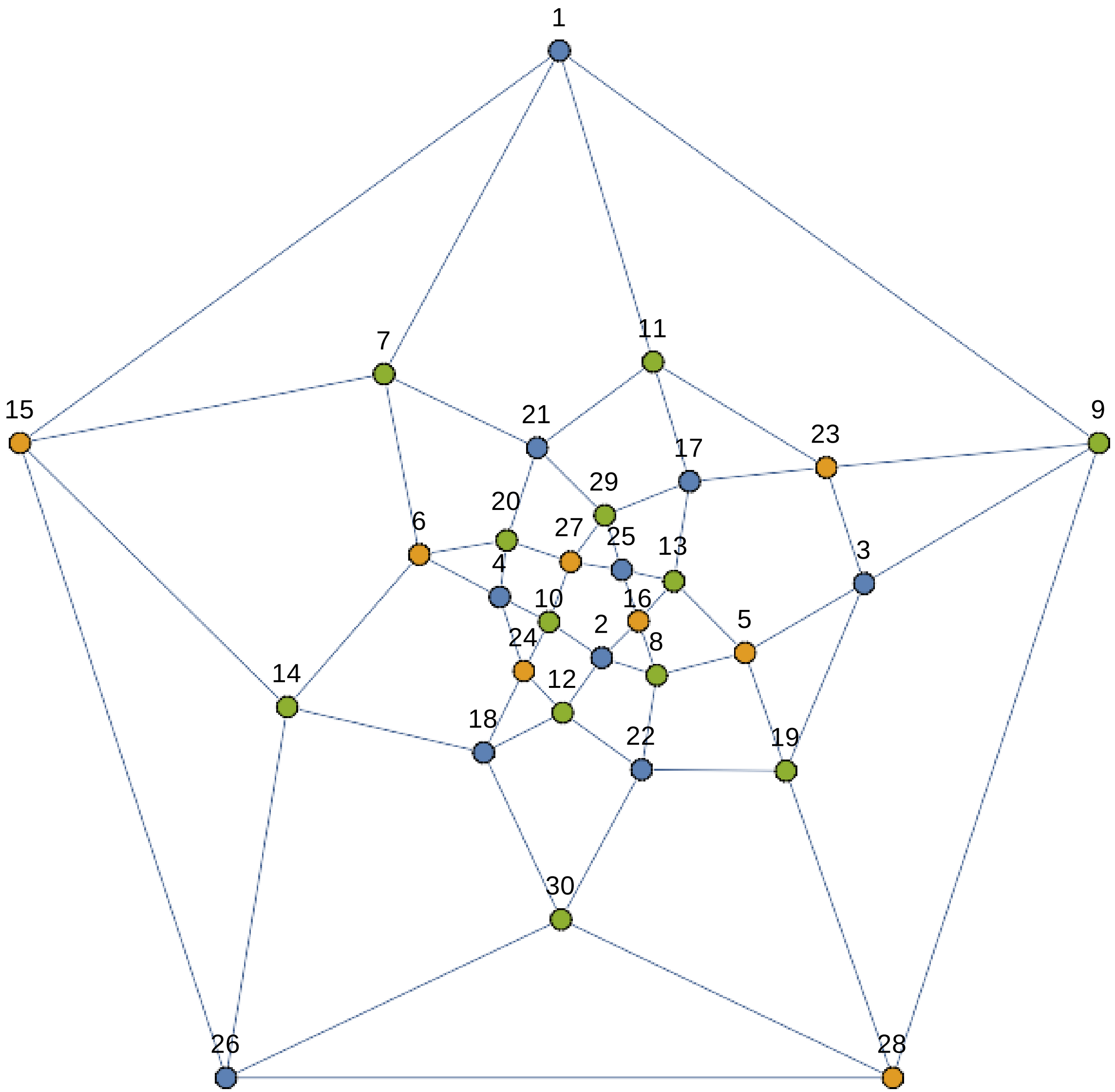


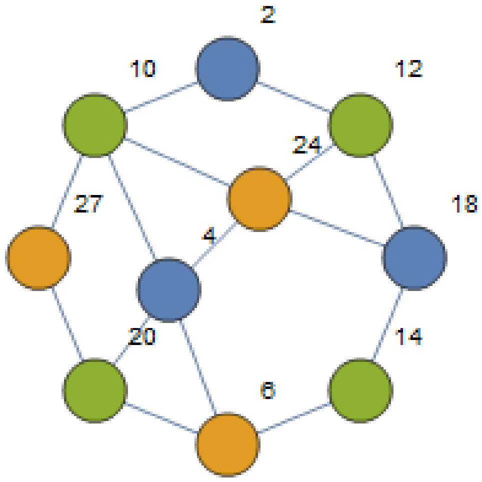


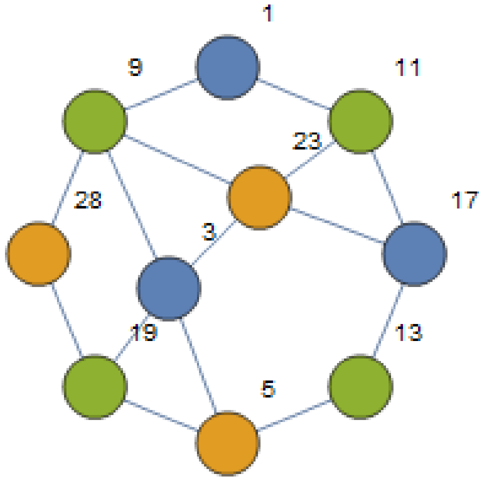




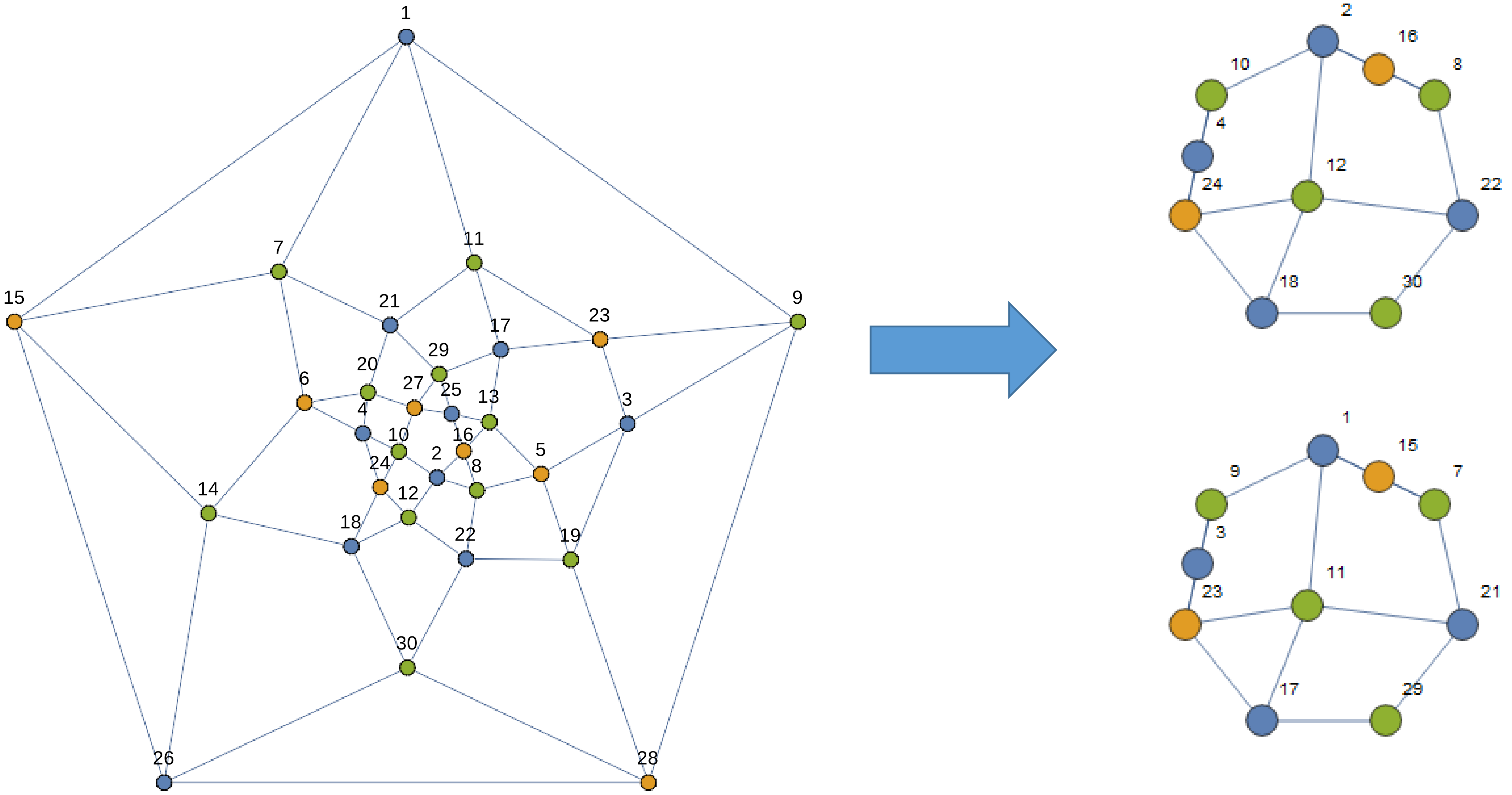
Conjecture 1 – Observation - Remove circle 2



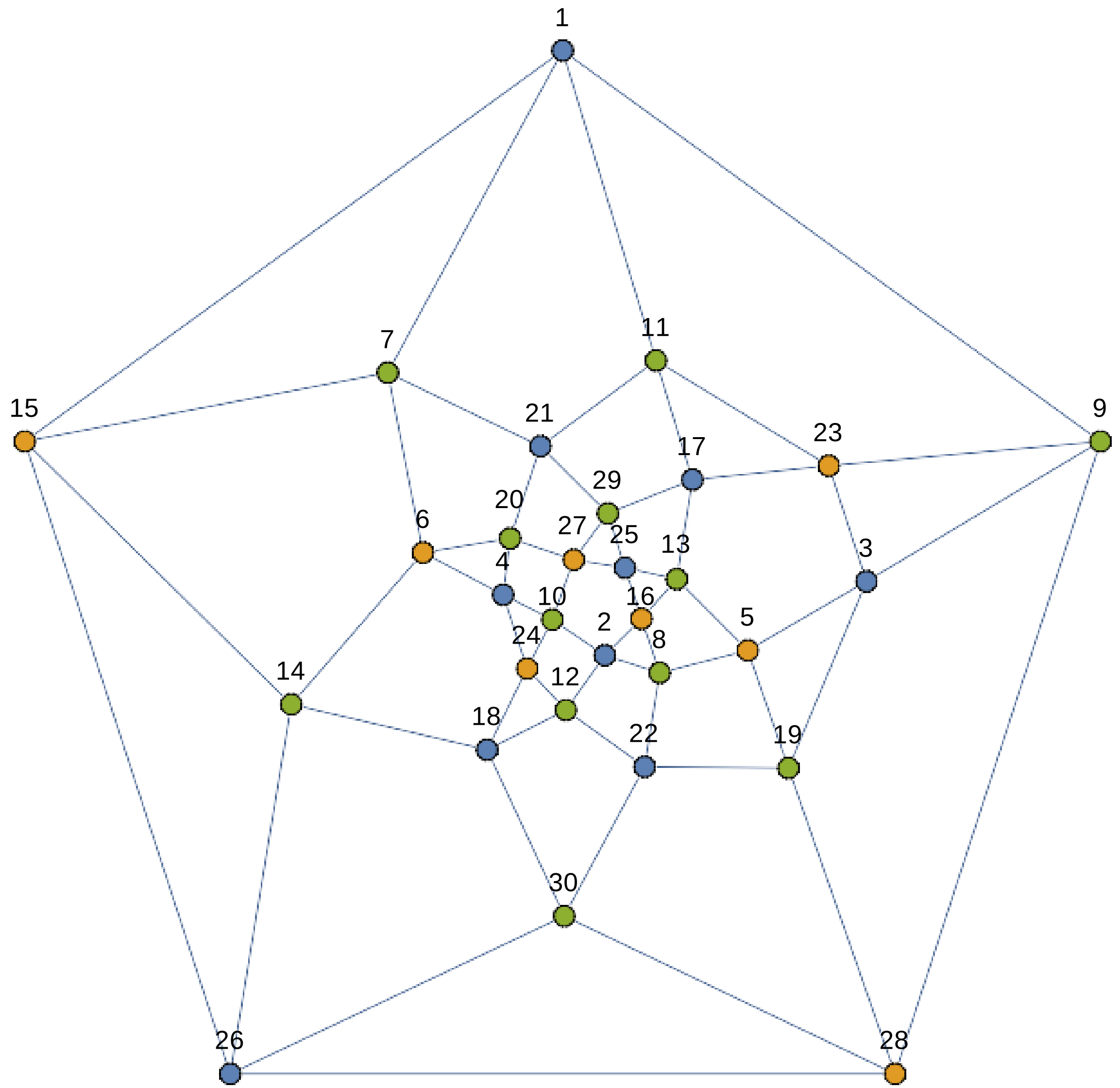


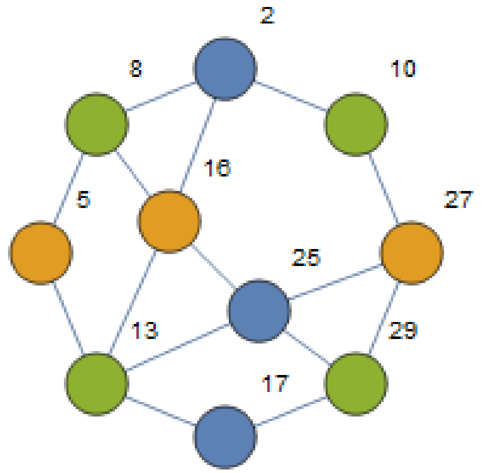


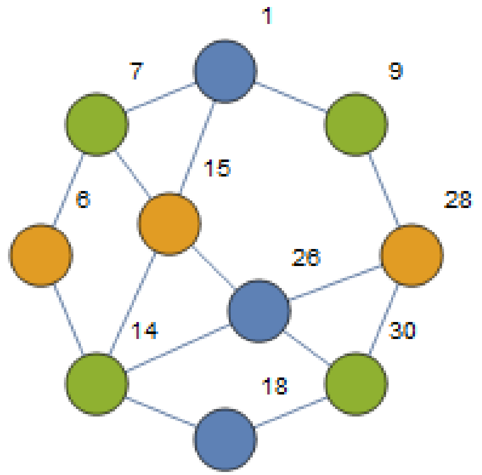
Conjecture 1 – Observation - Remove circle 3



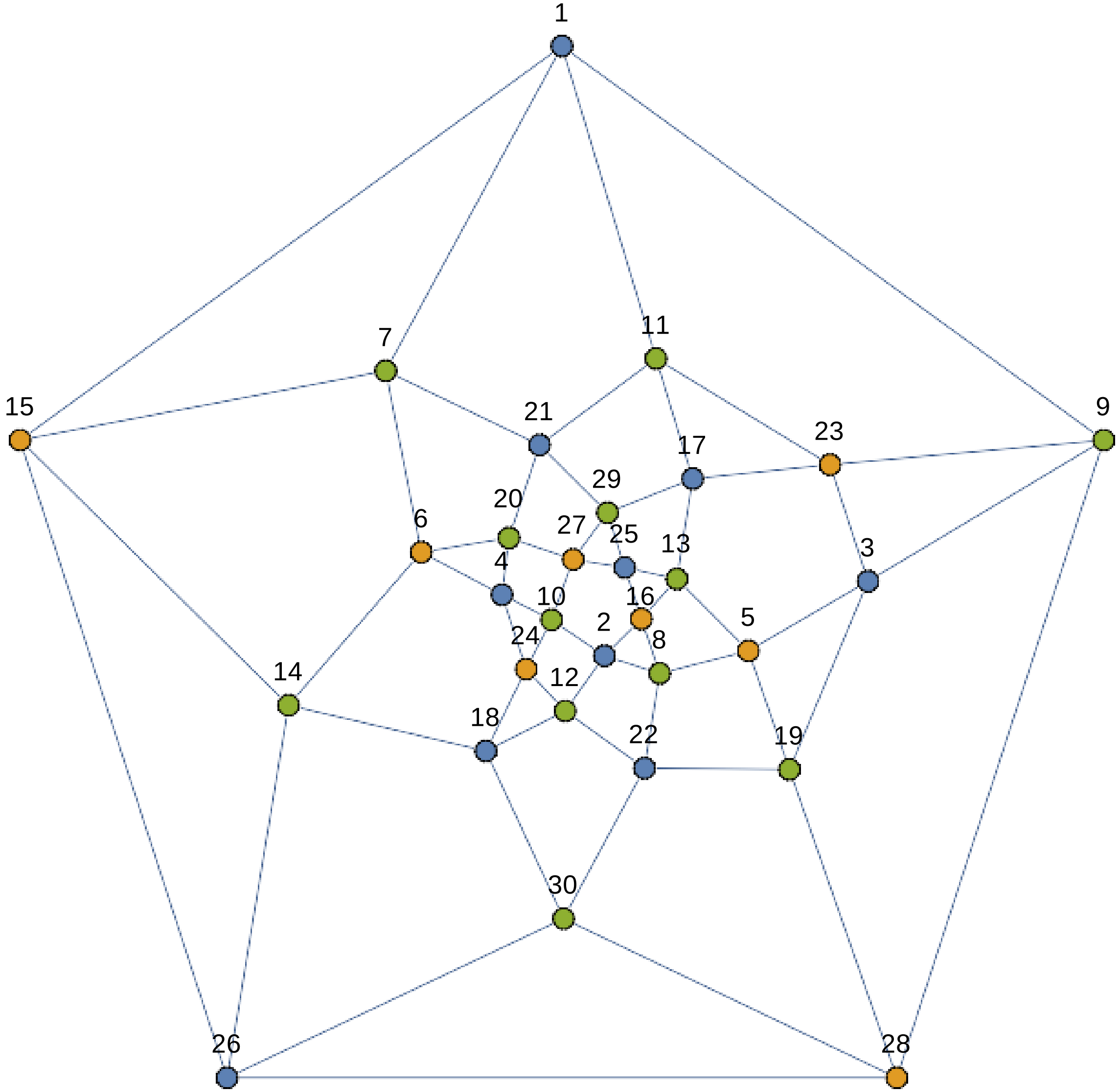
Conjecture 1 – Observation - Remove circle 4

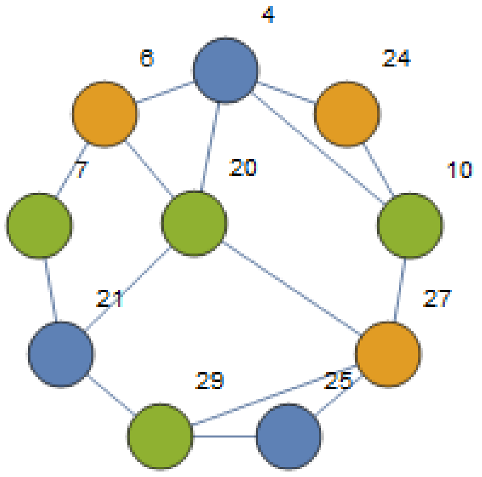


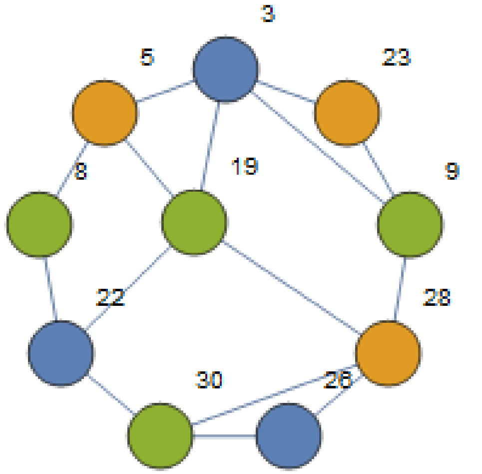




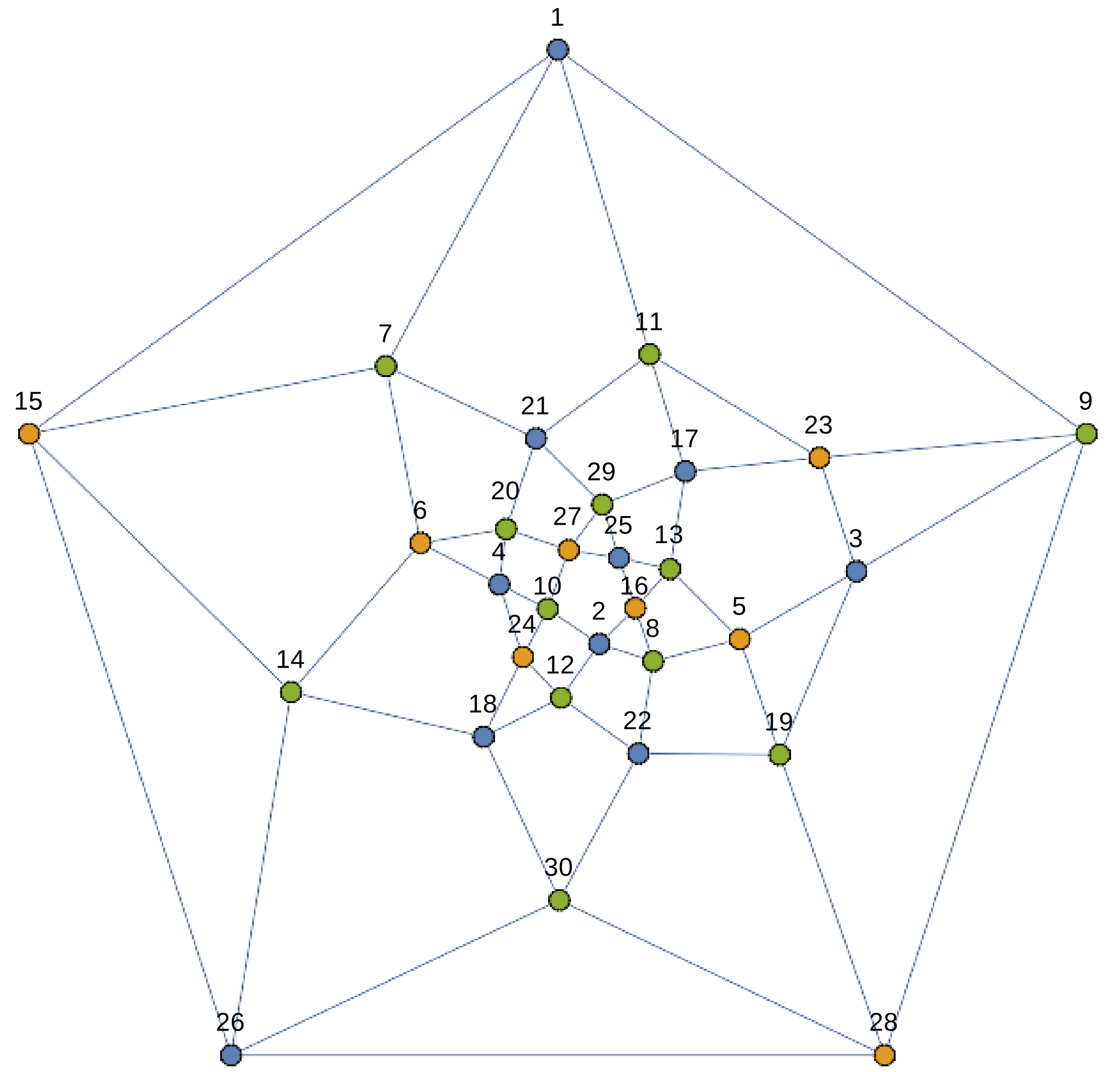
Conjecture 1 – Observation - Remove circle 5

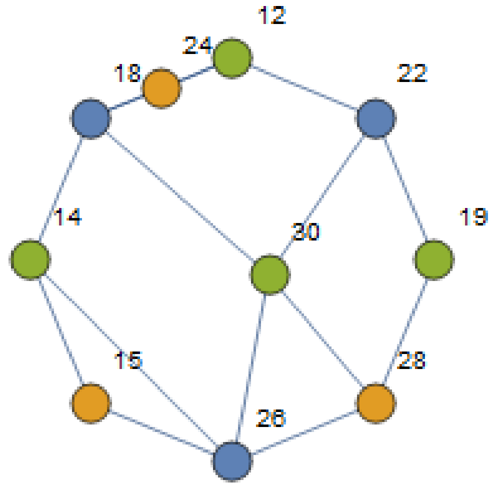


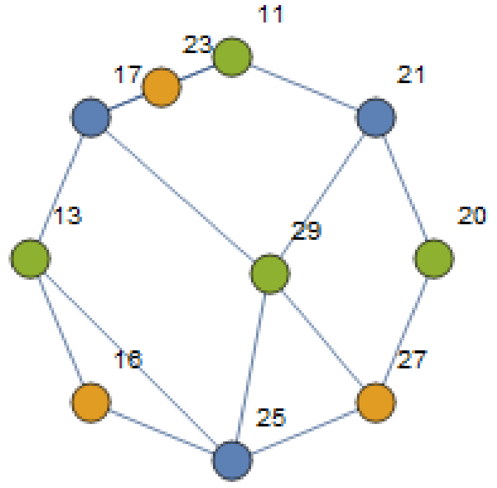




Conjecture 1 – Observation - Remove circle 6







Conjecture 2.

Intersections made by a pair of great circles can have same color.

This one is probably easily proved since we've already known the center is point symmetric, therefore by removing a circle, it will separate the graph into 2 sub-graph equally. Hence, 2 intersections made by a pair of great circles can have same color because they are not in the same sub-graph. What we need more than this conjecture is it still remains 3 colors for the entire graph

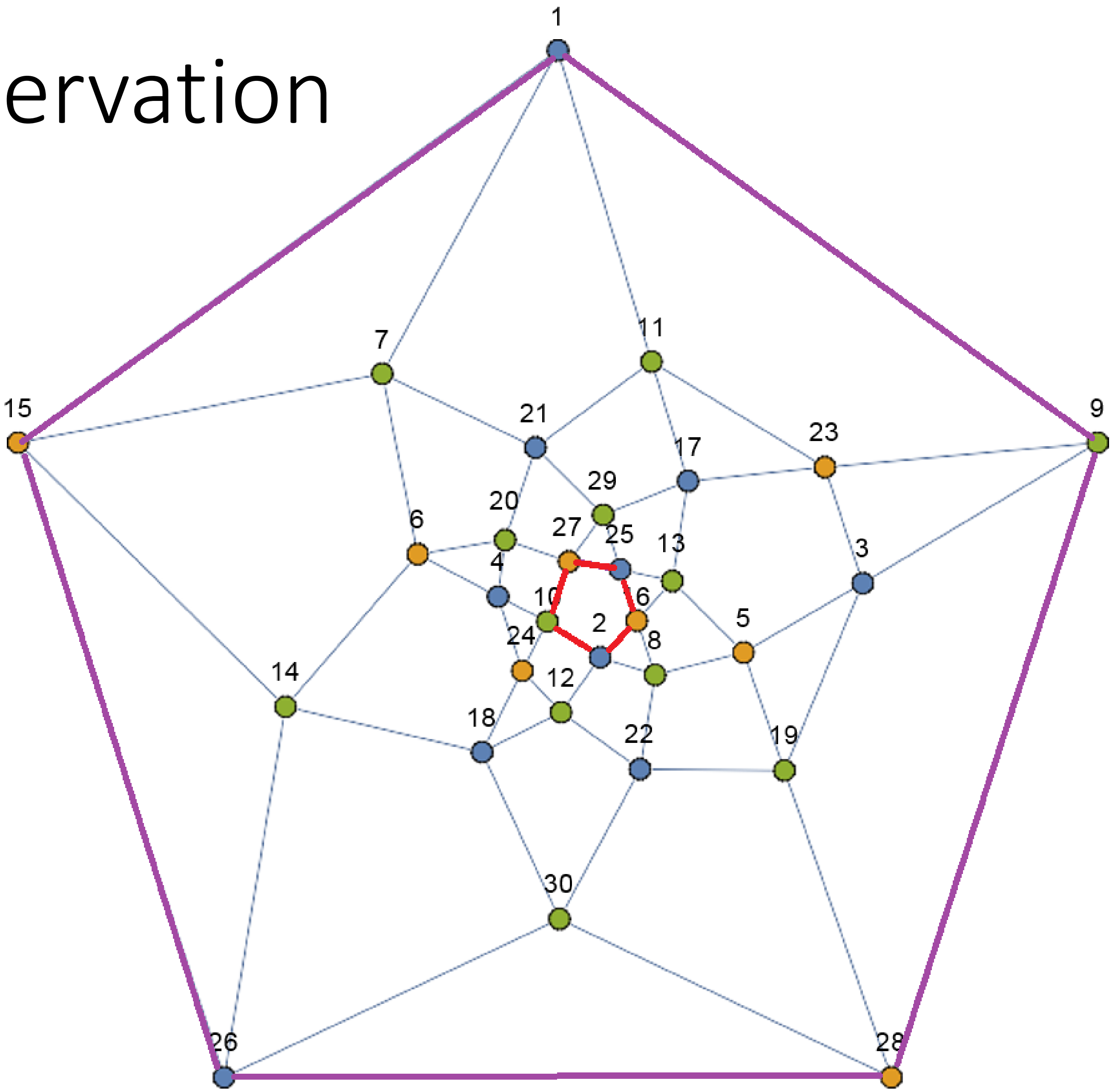
You can check this conjecture on the graphs in the conjecture 1. Check the previous note to find the a point and its duplication (or other intersect made by same pair of circles)

Conjecture 3.

- The graph will have a bounded cycle and a core which are formed by the greatest number of line segments.
- The bounded cycle and the core contain vertices made by different pairs of great circles. In other words, there is no 2 or more vertices can be in a same circle in the bounded cycle and the core.

Conjecture 3 - Observation

Red lines are segments of the core polygon
Purpil lines are segments of the bounded polygon



References

1. David P. Dailey, Uniqueness of colorability and colorability of planar 4-regular graphs are NP-complete, Discrete Math. 30 No.3 289-293 (1980)