

Great Circles Problem

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03/14/2015

Outline

- The transition of a graph by adding a circle
- A base case to color a new graph with a new additional circle
- A case where the coloring rule doesn't work

One approach to solve the problem

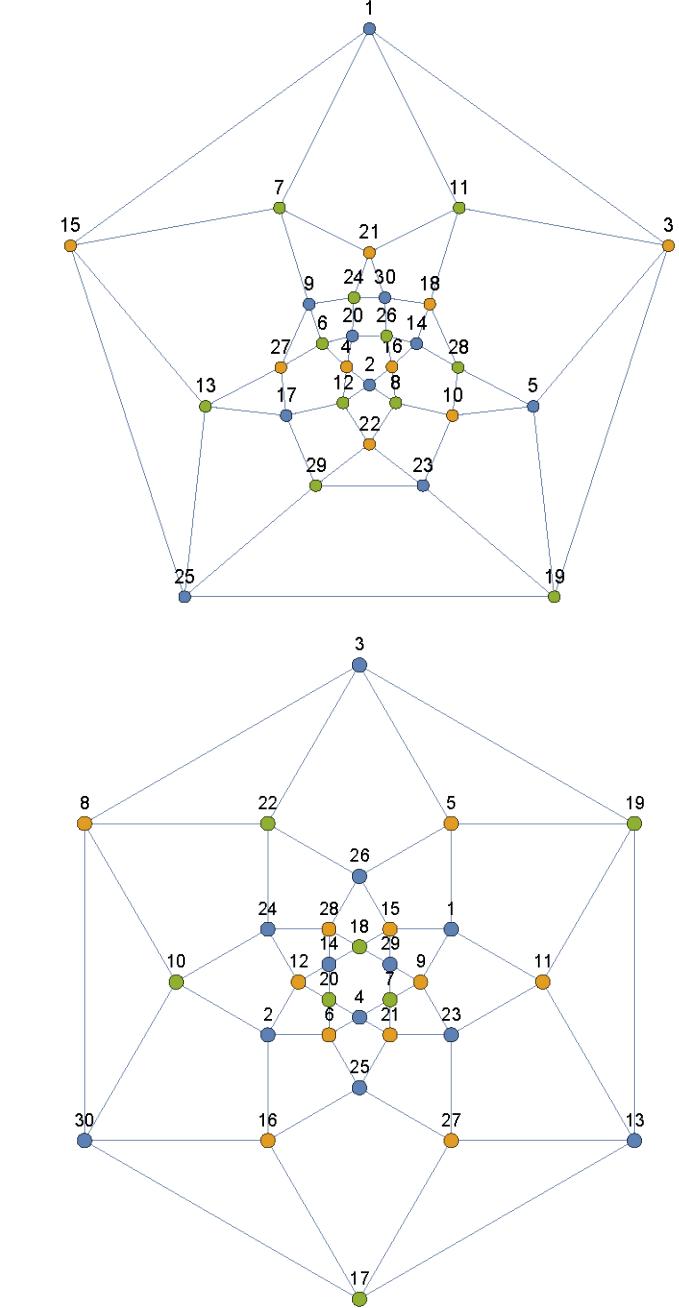
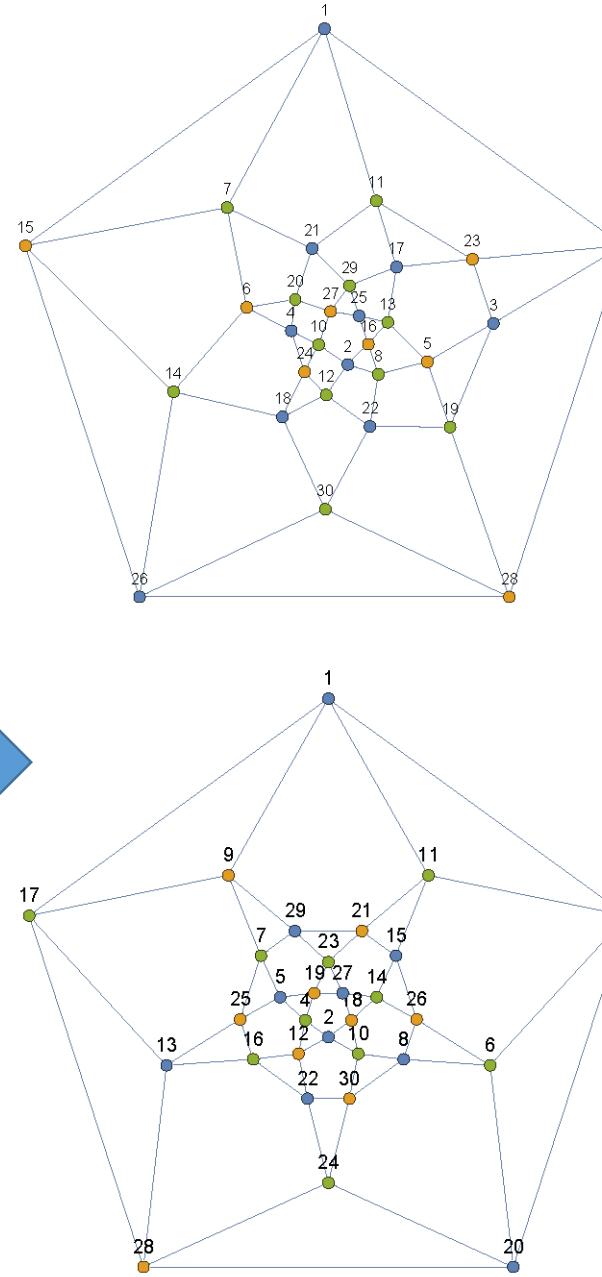
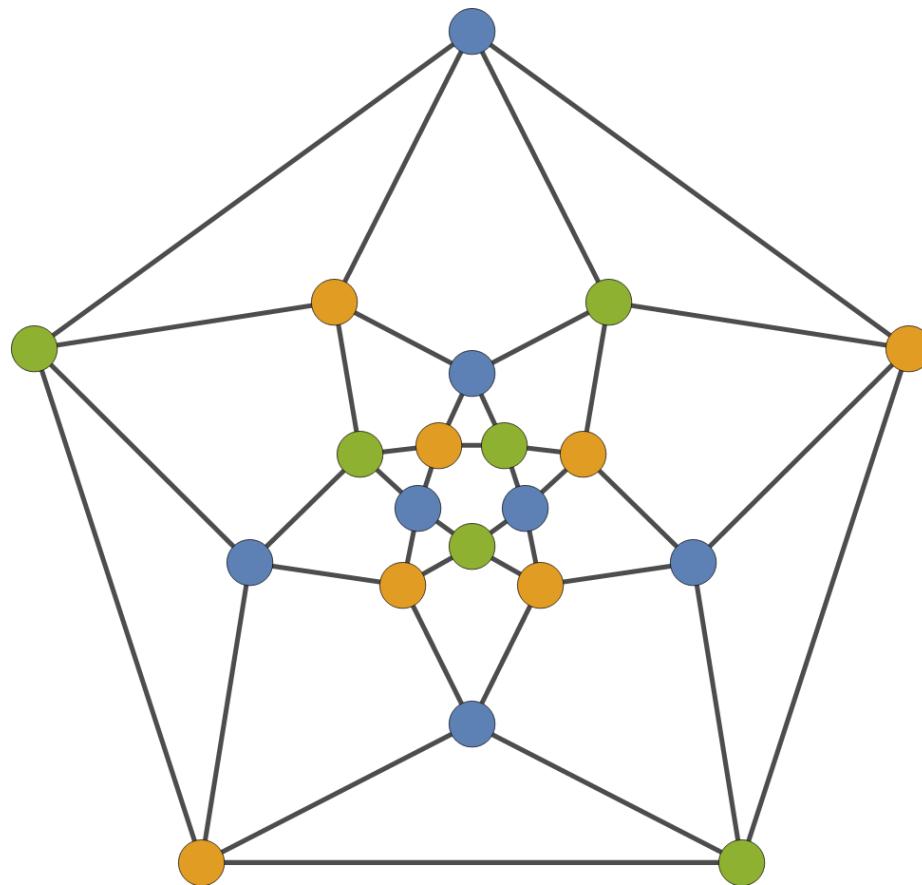
- Since we've already known that coloring 4-regular planar graph is NP-complete and we hope to achieve the chromatic number as 3, it's not a good idea to start coloring a new graph instead of making induction from some graphs we currently have. My idea is to use Kempe chain, so if I color the graph at the beginning, I may not control all the chains with various lengths manually.

Adding a circle

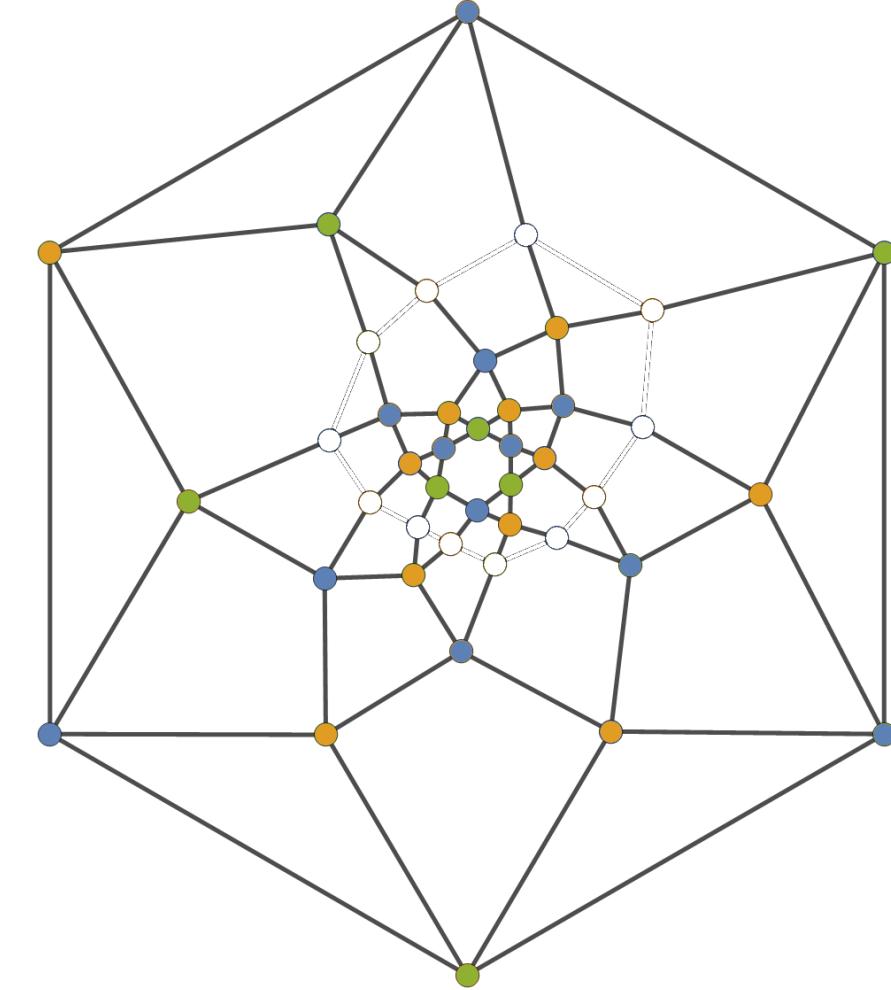
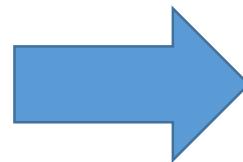
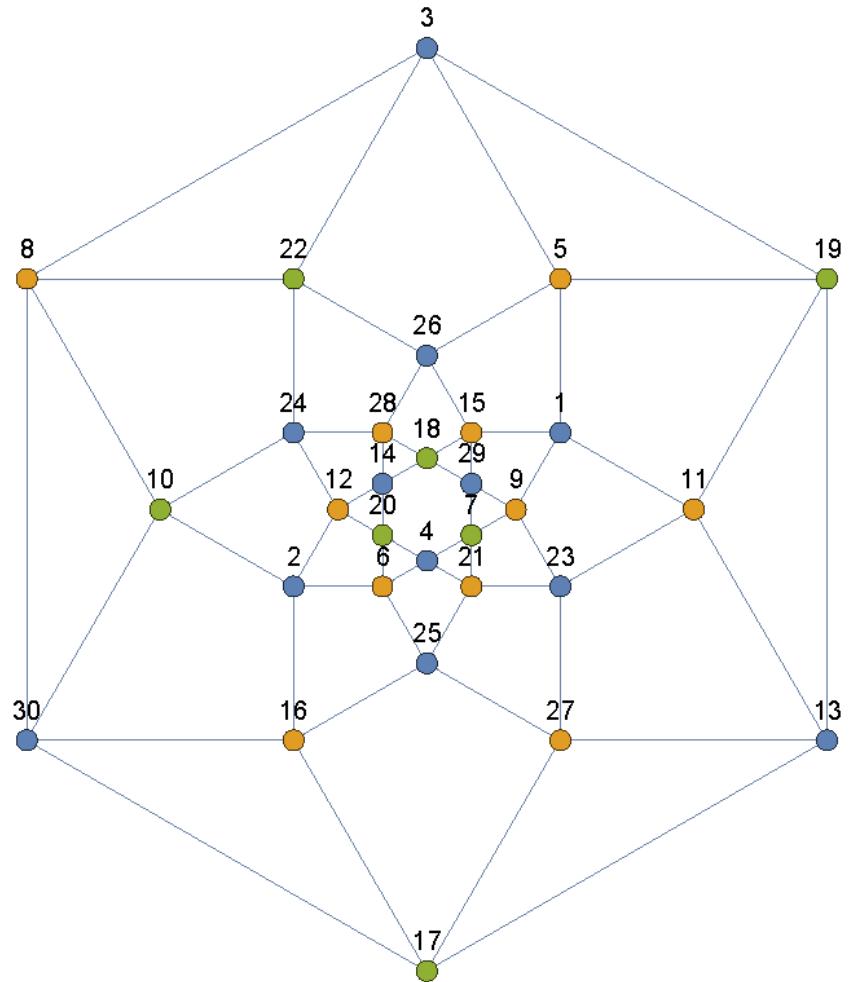
Adding a circle

- This part presents how a graph is changed after adding a random great circle into it
- I will show the transitions from graphs of 5, 6 great circles to 6, 7 great circles respectively

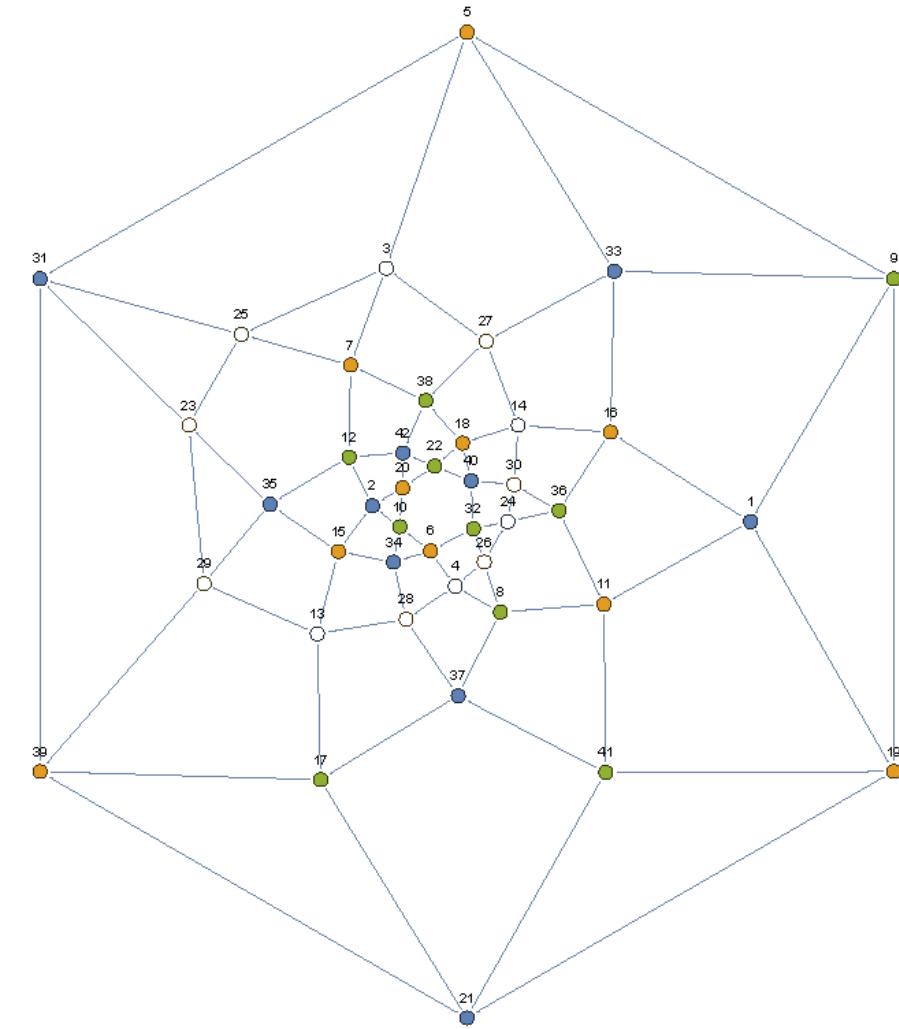
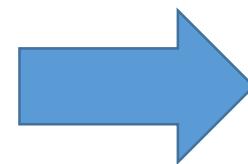
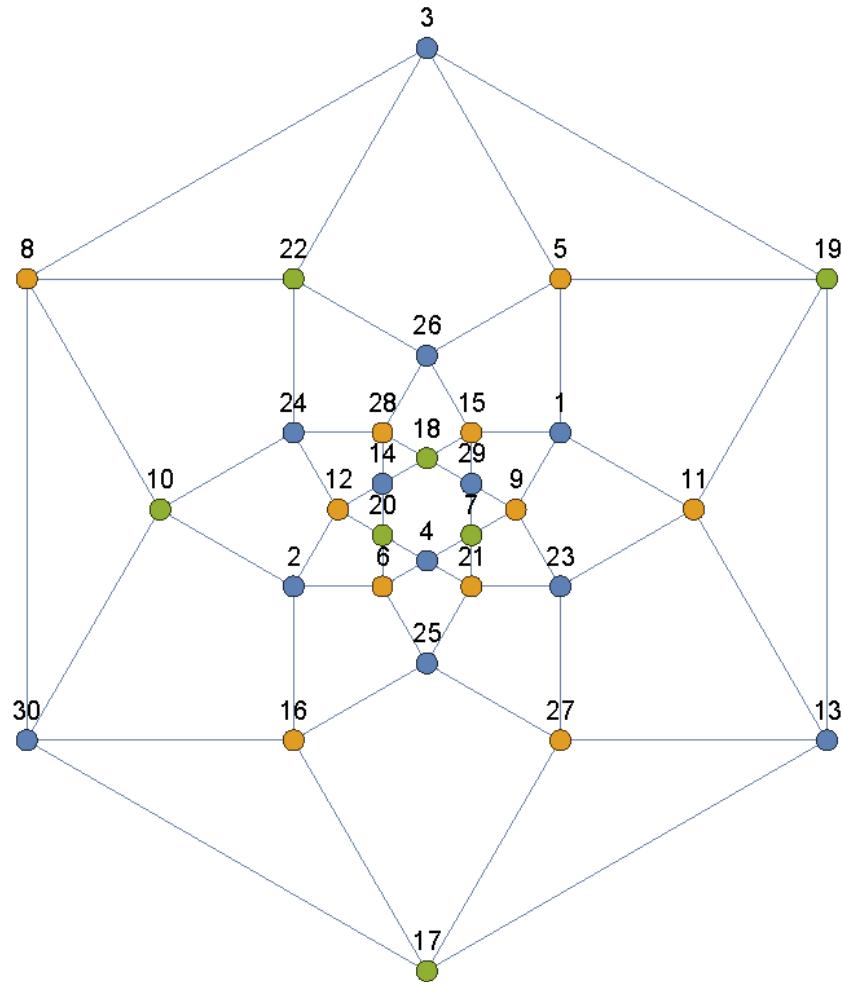
5->6 great circles



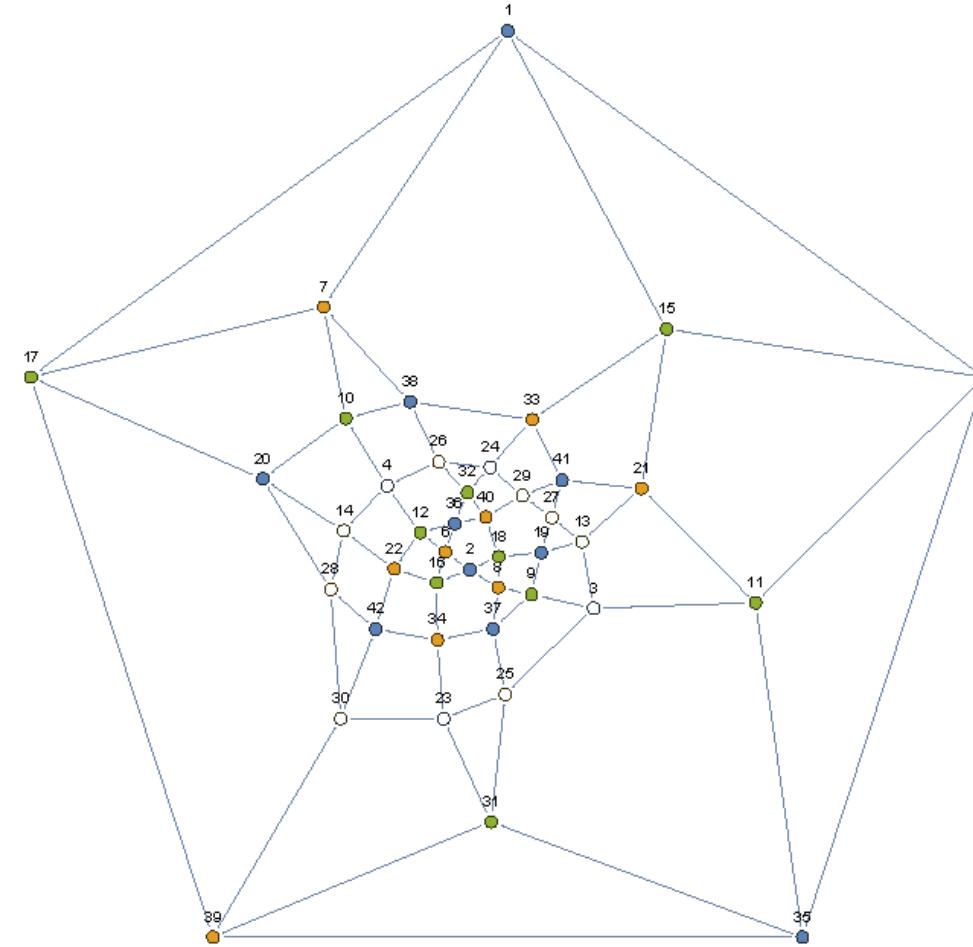
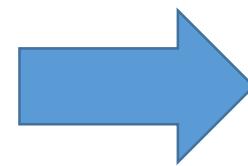
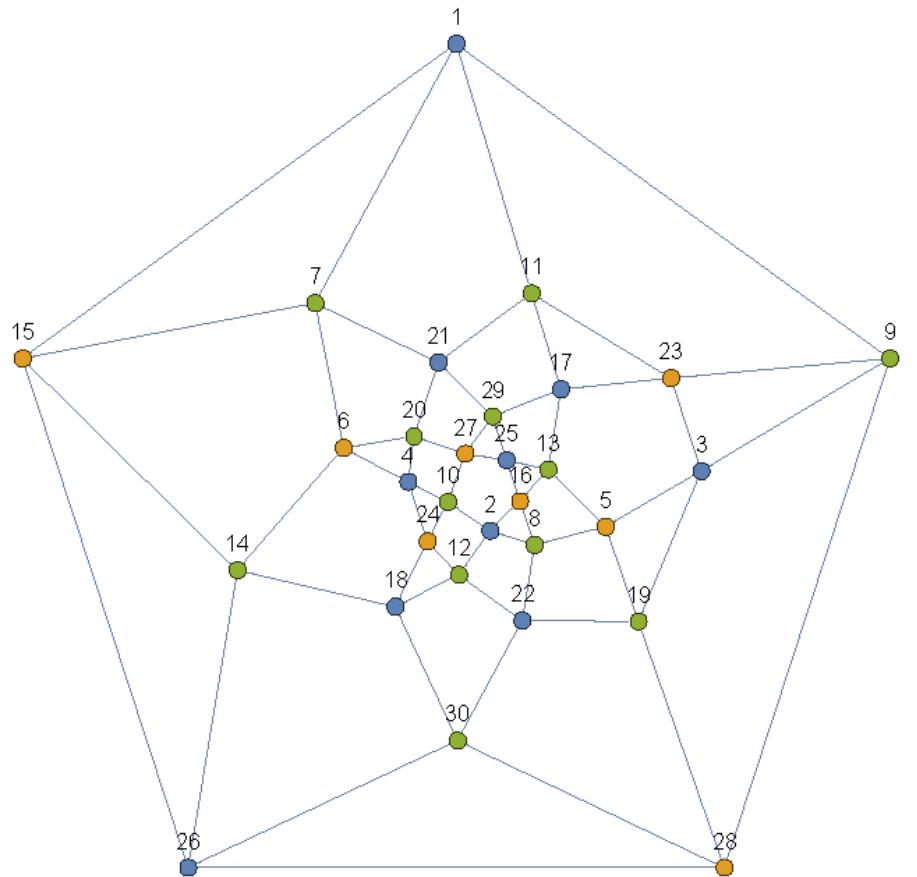
6->7 great circles - 1



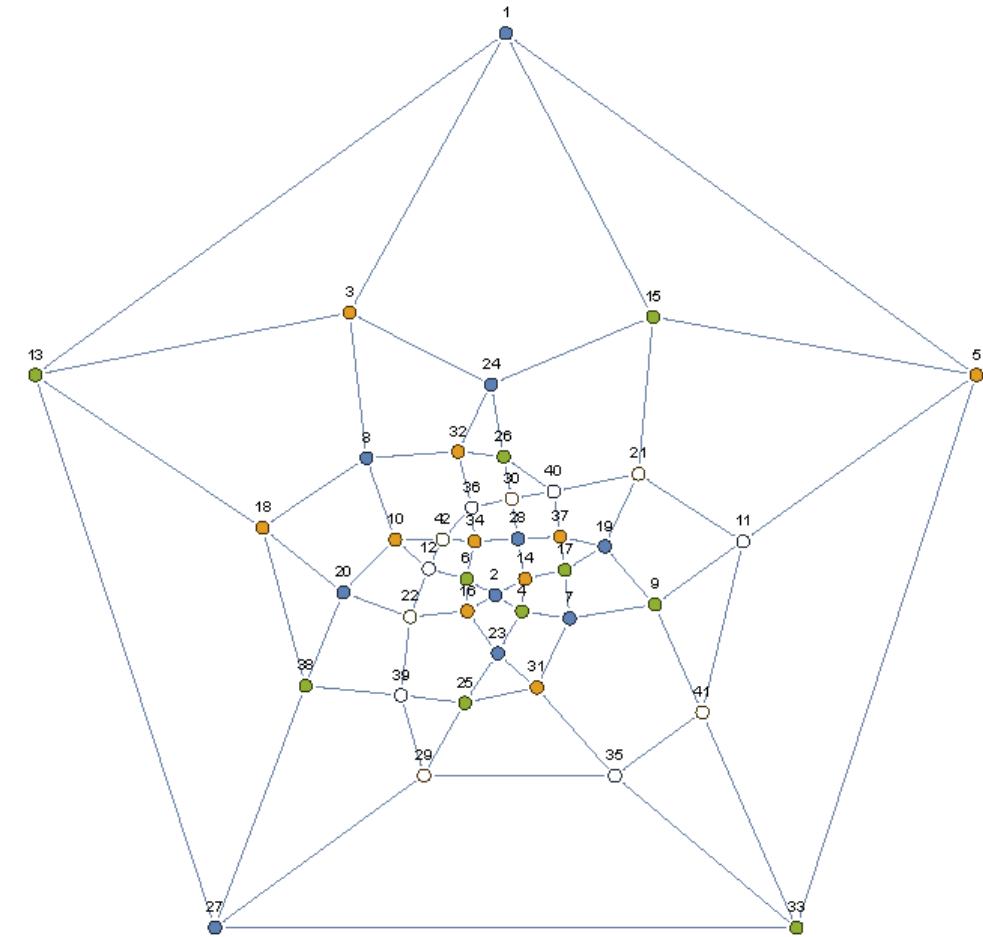
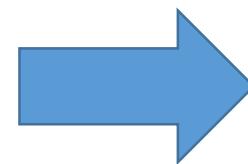
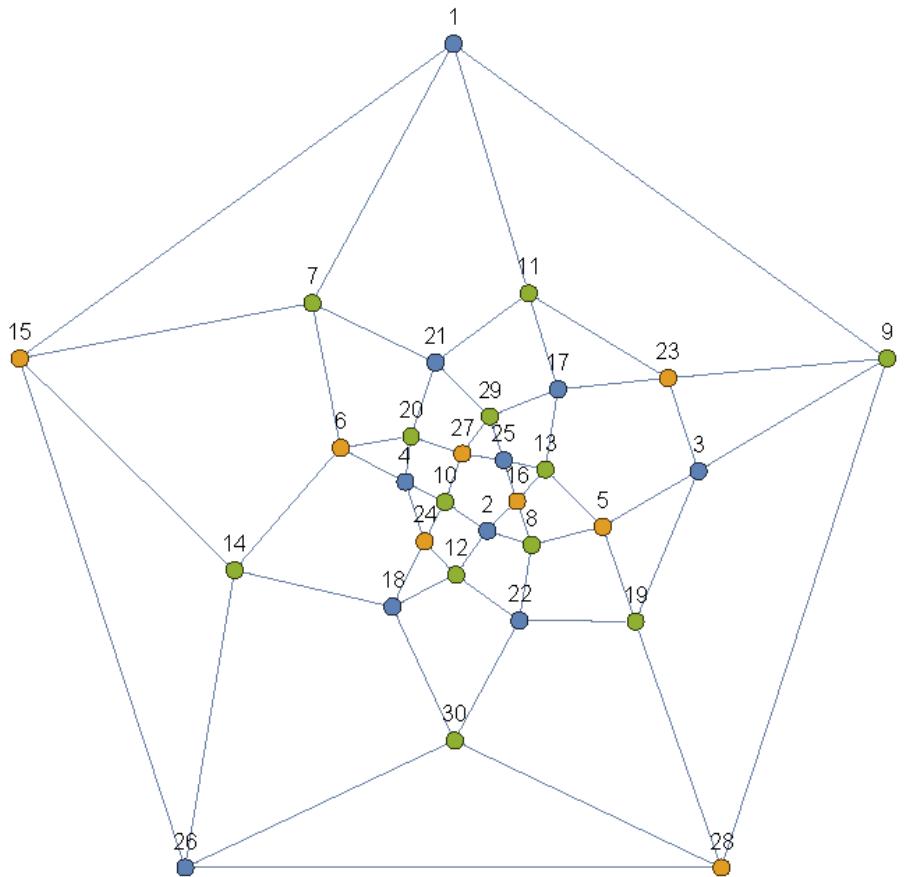
6->7 great circles - 2



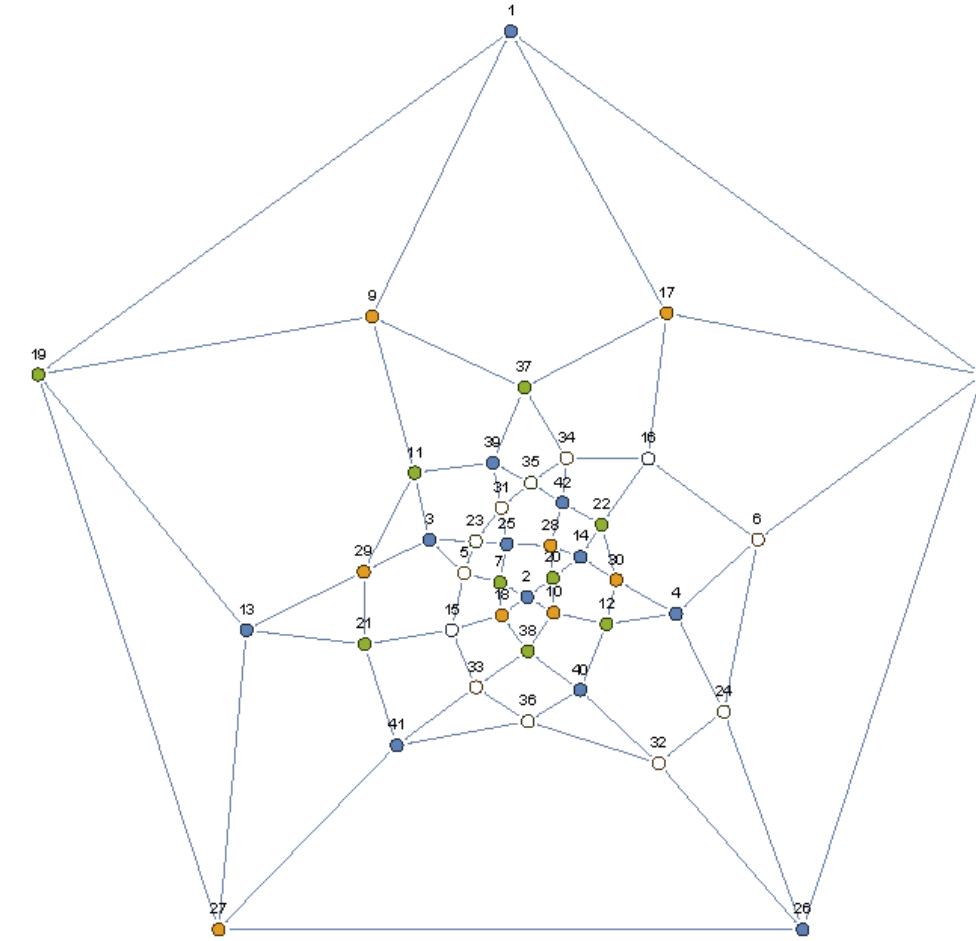
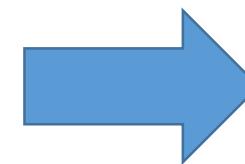
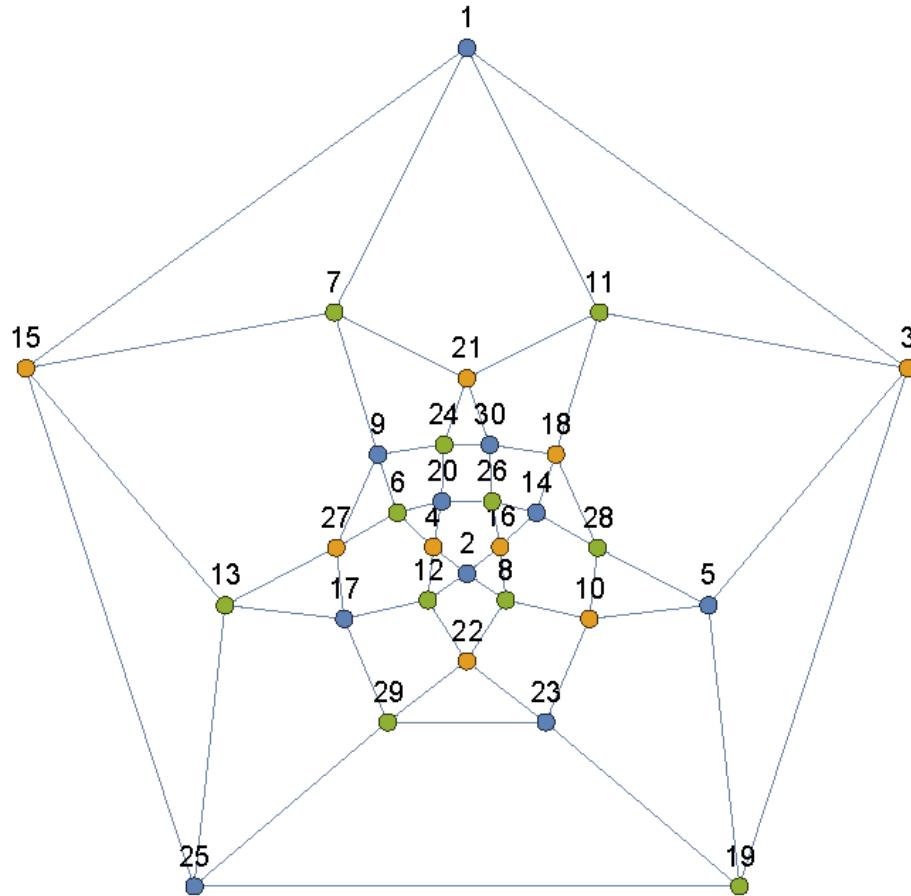
6->7 great circles - 3



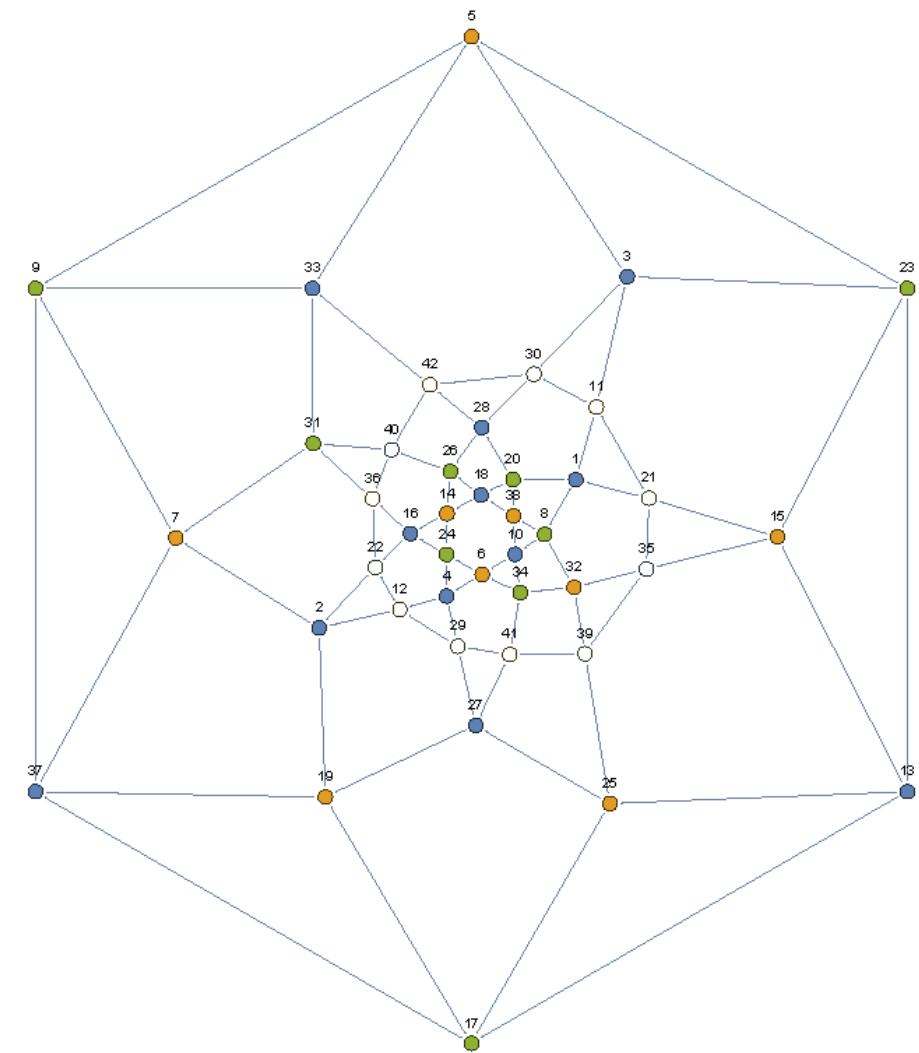
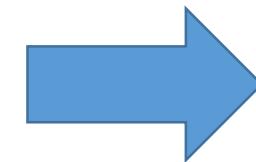
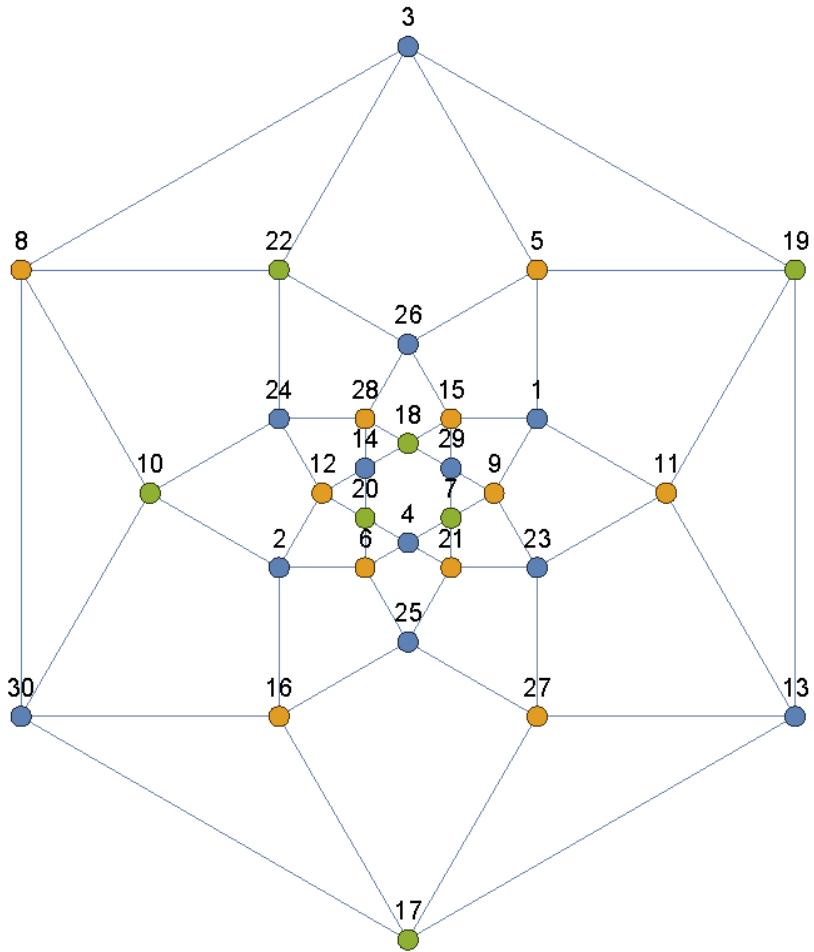
6->7 great circles - 4



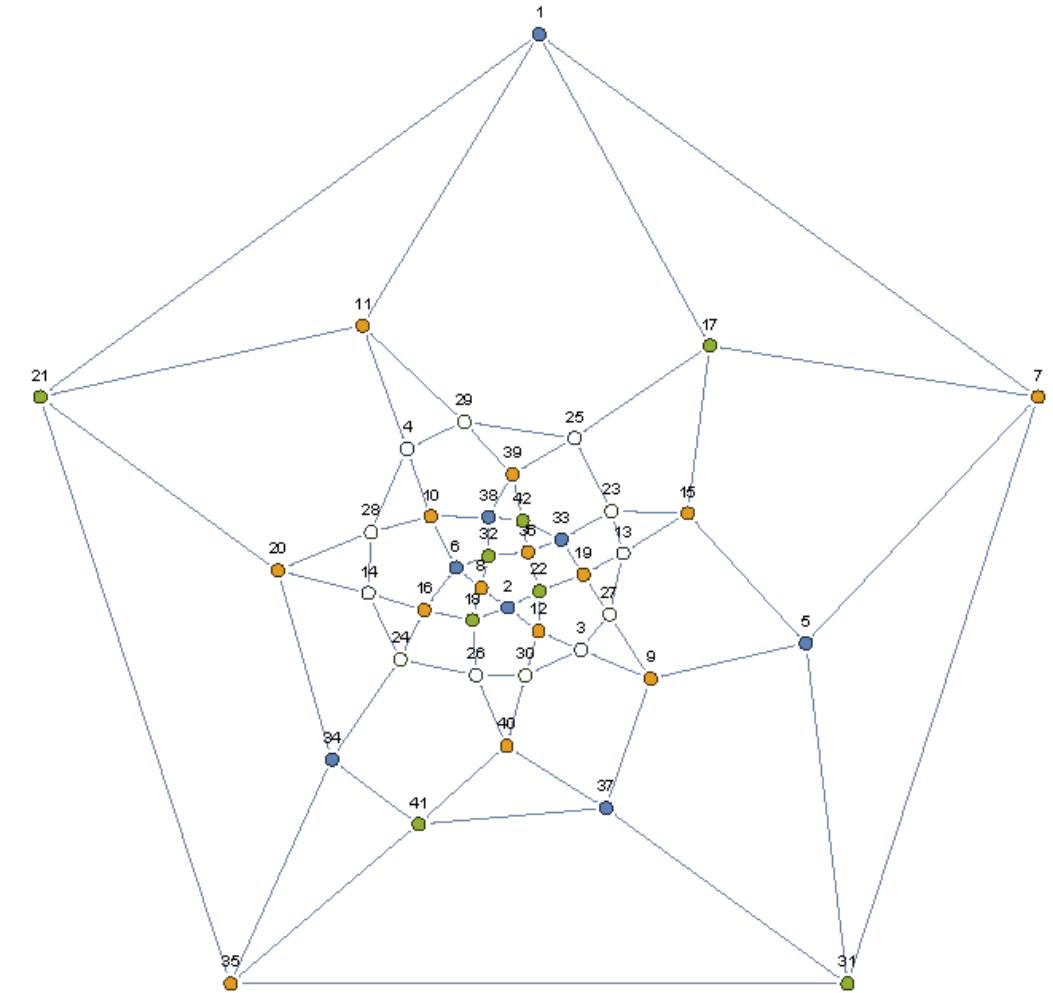
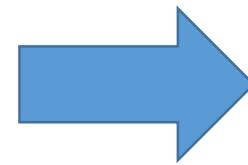
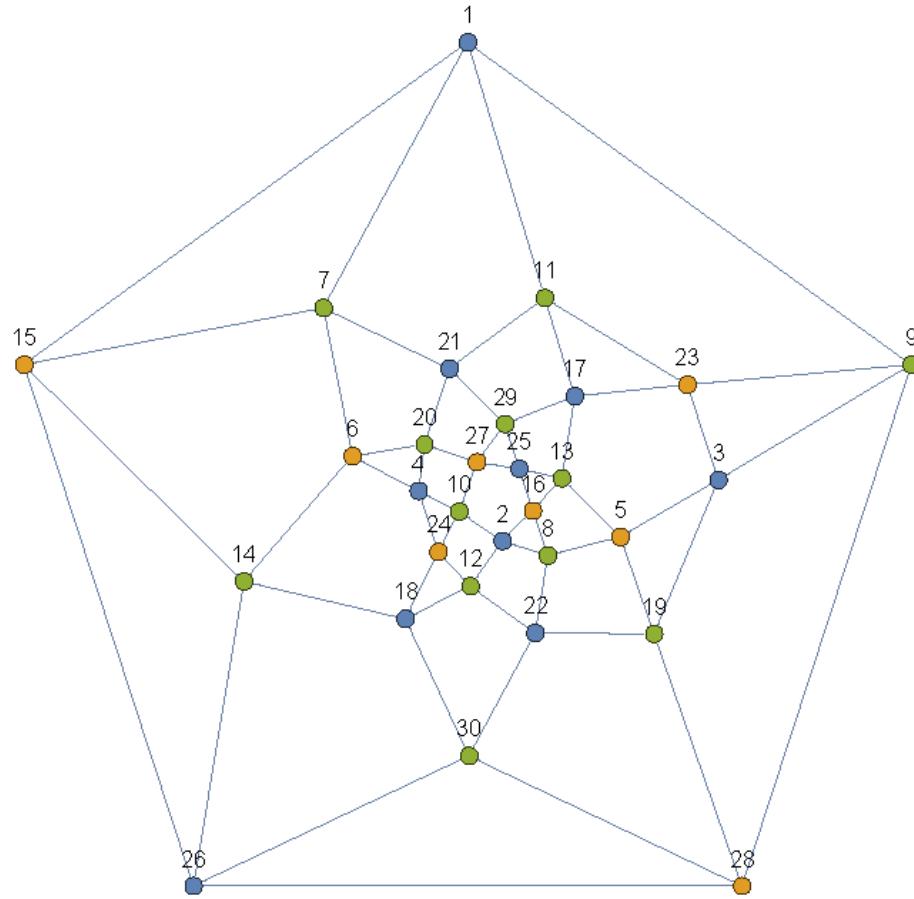
6->7 great circles - 5



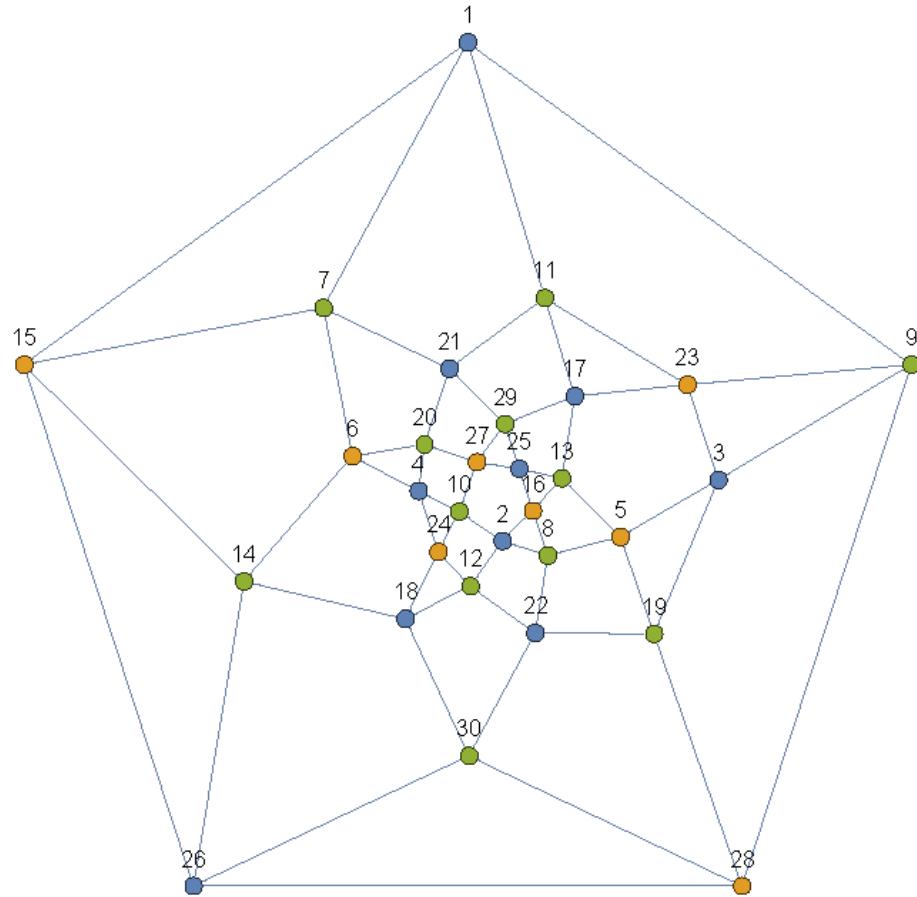
6->7 great circles - 6



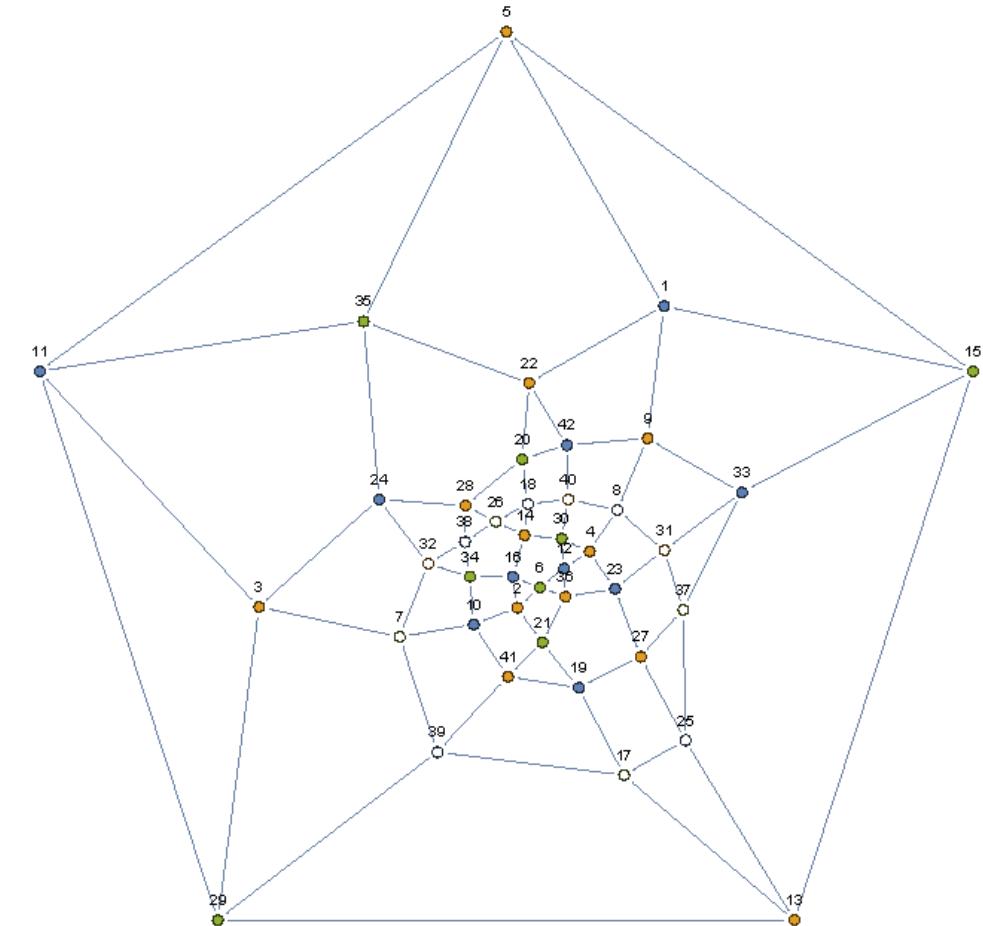
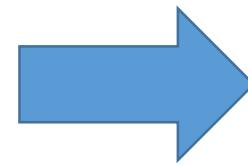
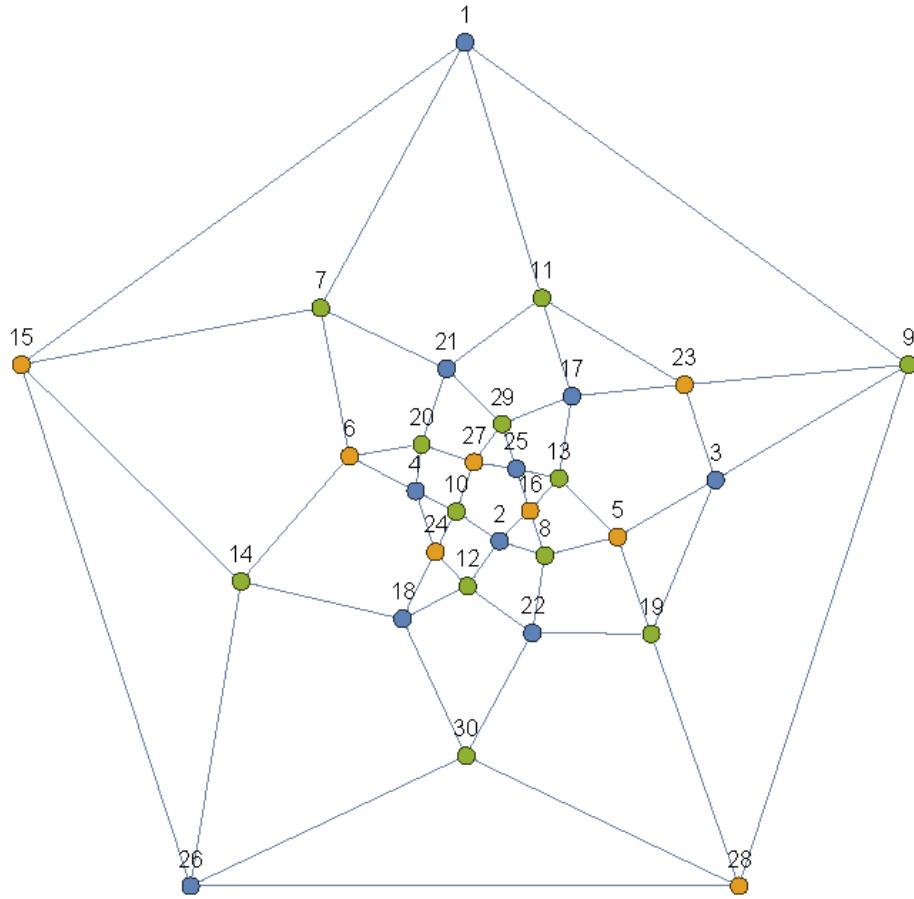
6->7 great circles - 7



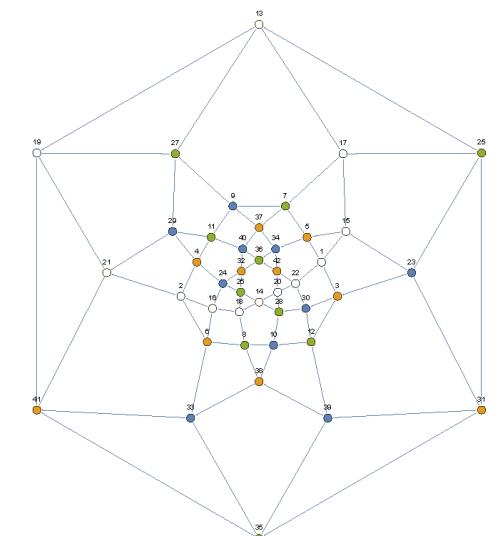
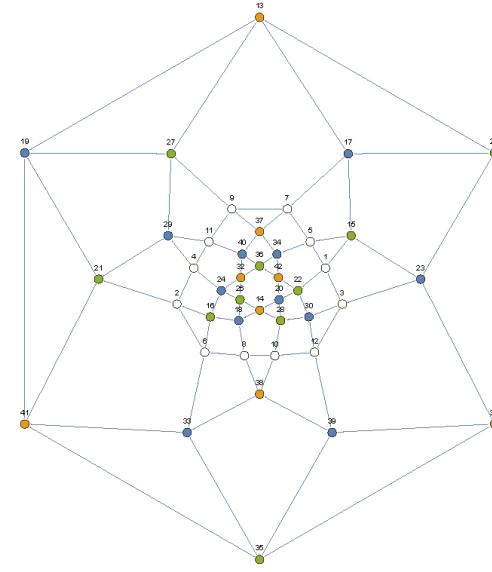
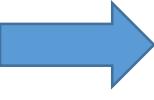
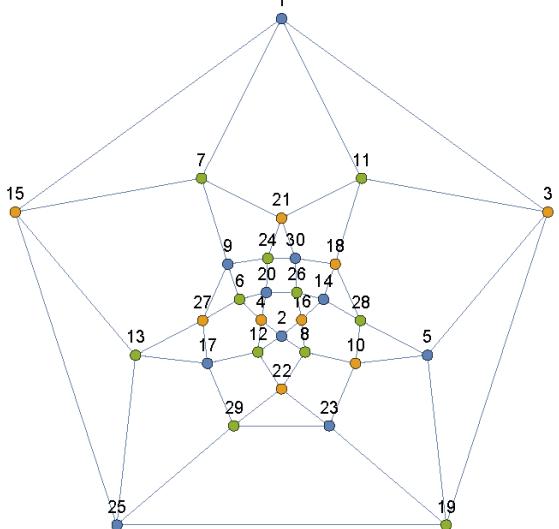
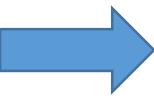
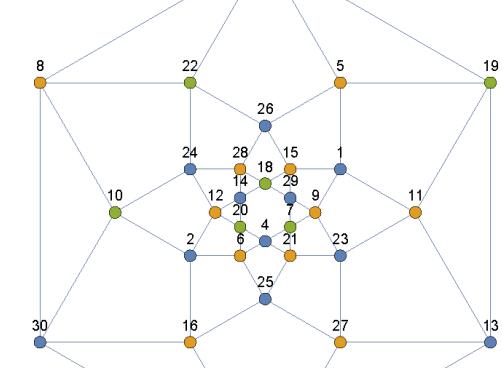
6->7 great circles - 8



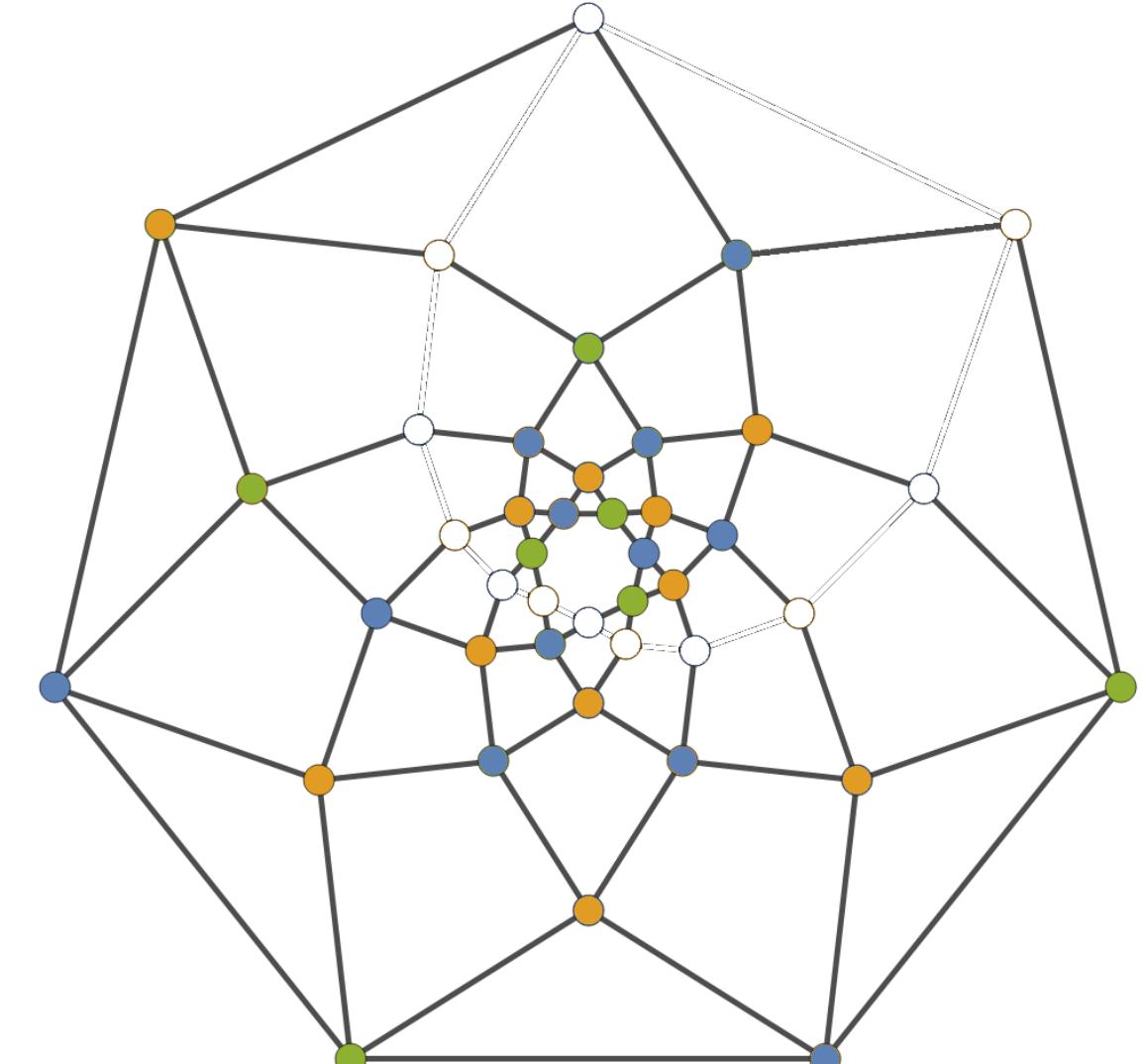
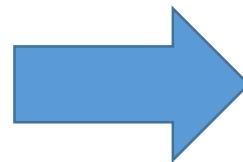
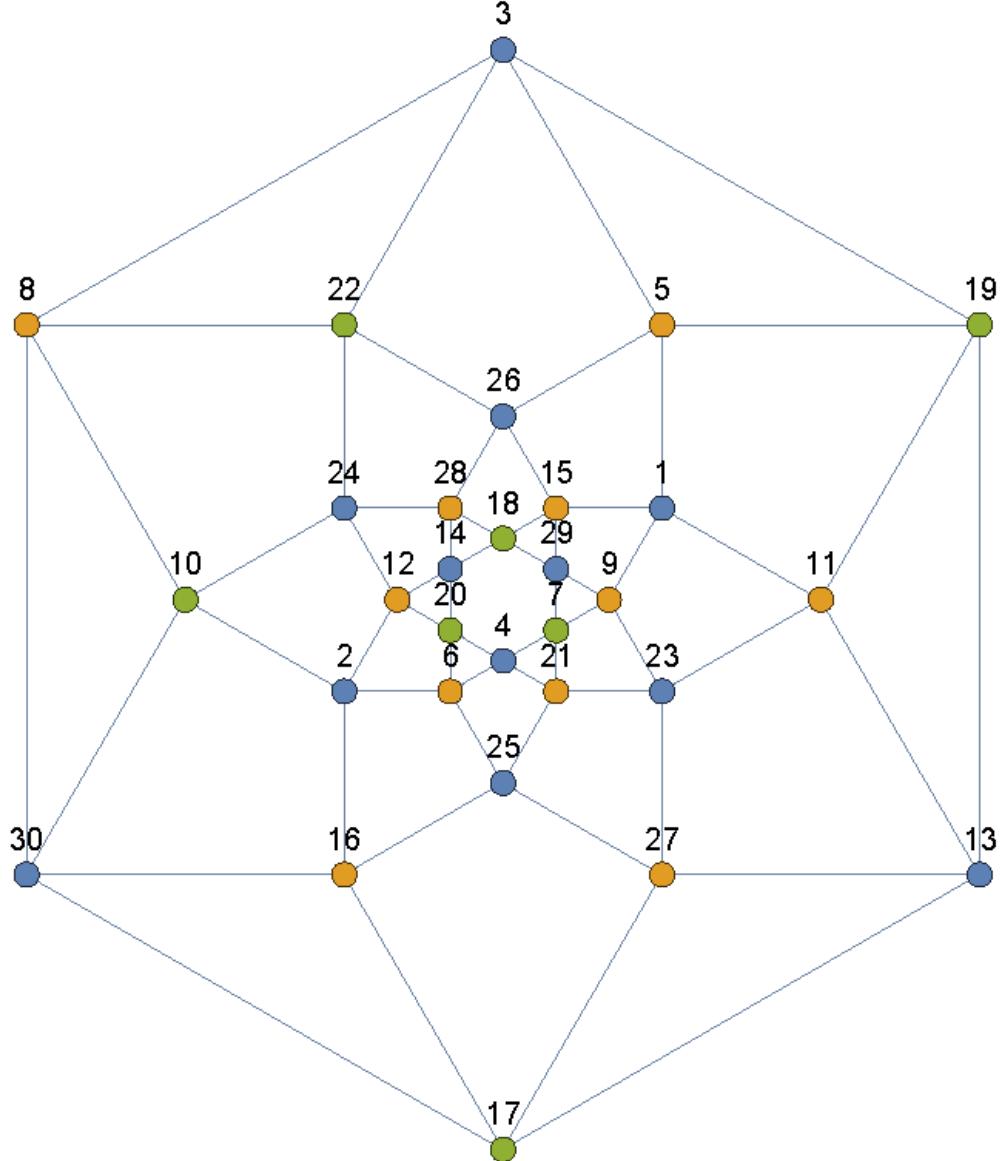
6->7 great circles - 9



6->7 great circles - 10



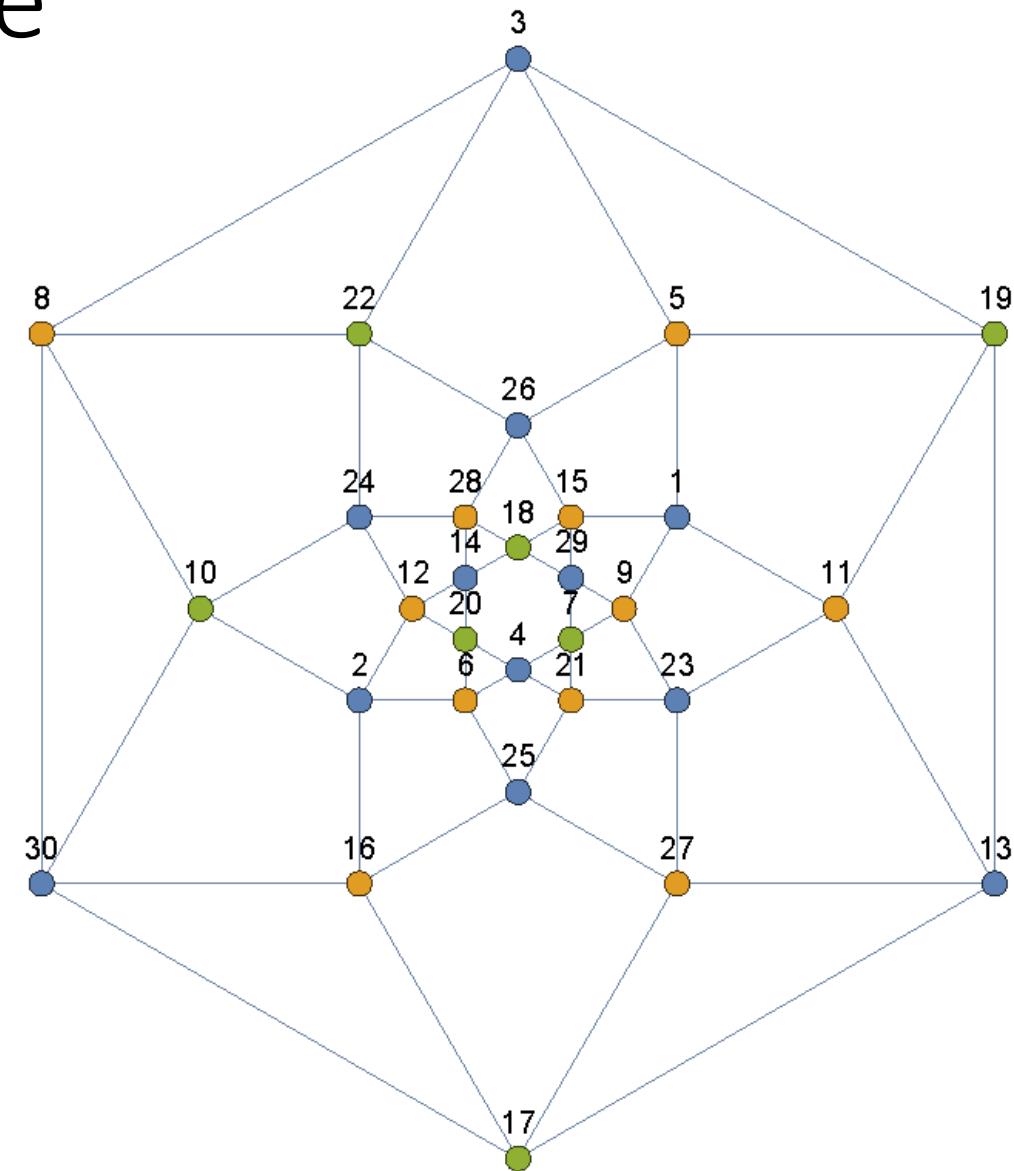
6->7 great circles - 11



A base case

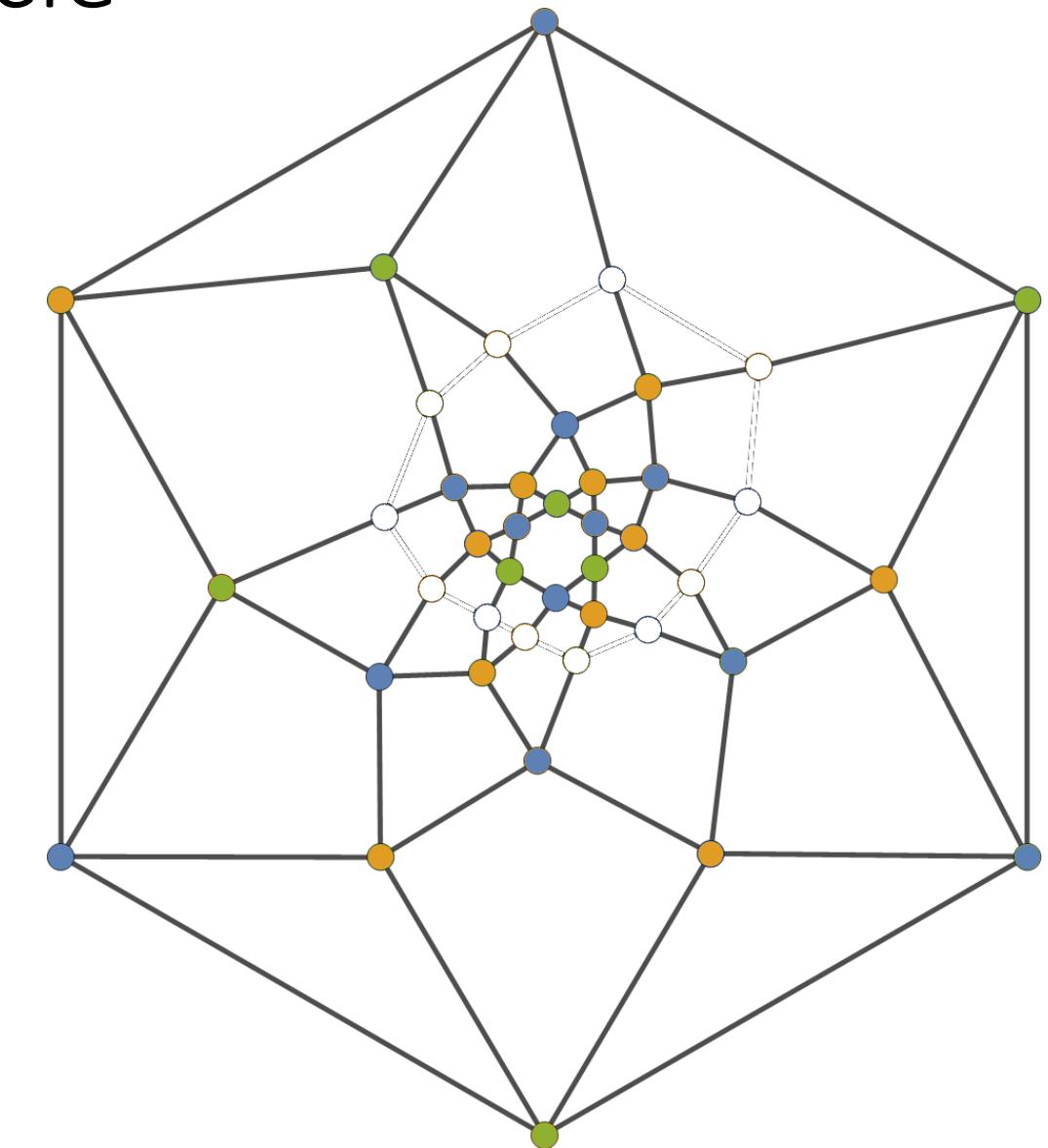
Adding a random great circle

- Before discussing this topic, I want to make an example with a graph of 6 great circles as a base case
- I get this graph in the slide Presentation 02-20-2015, pp. 7. It's 1 of 4 non-isomorphic graphs in 1000 that I generated



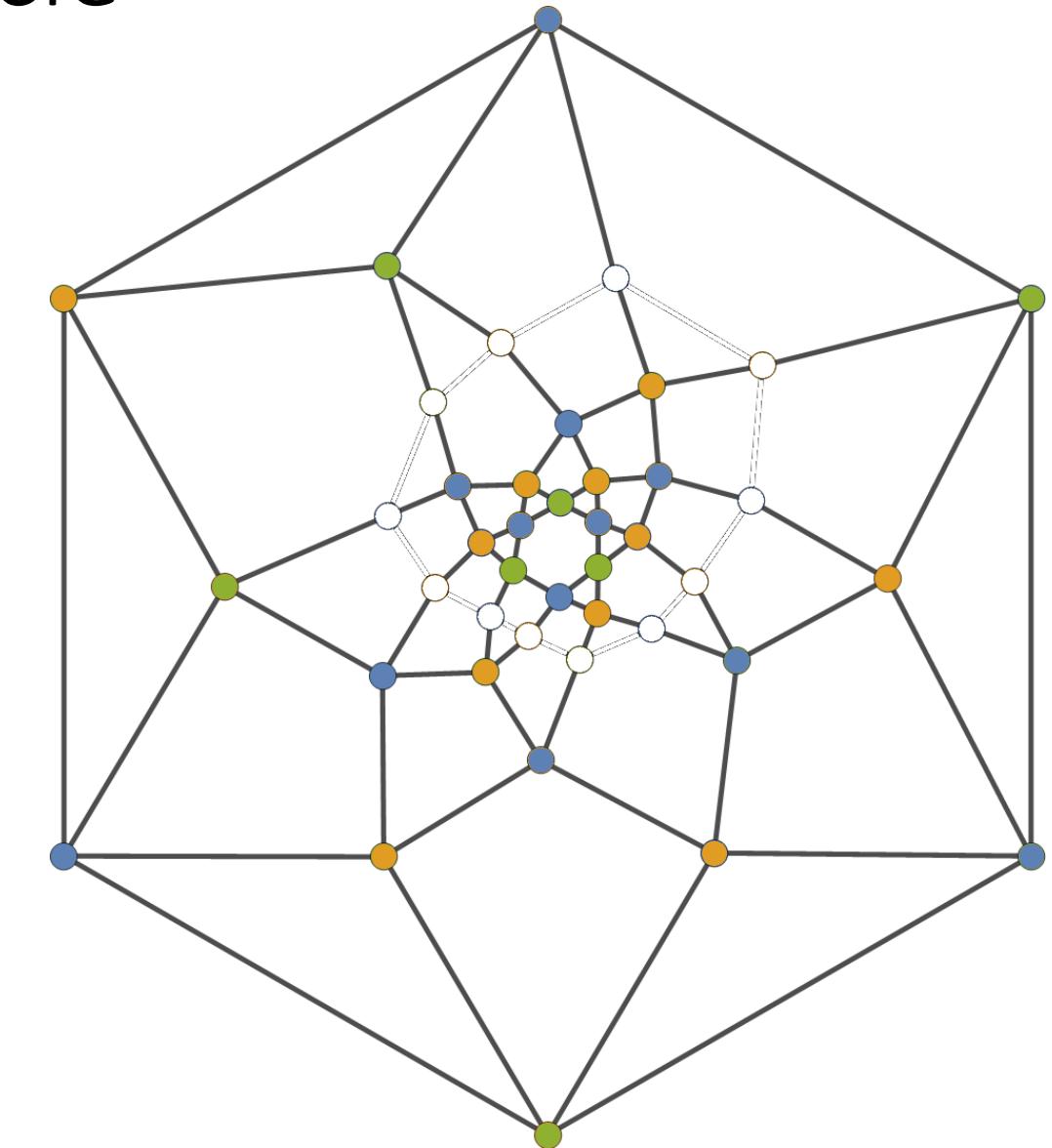
Adding a random great circle

- Now, I try to add one more great circle C_{add} into the graph. The graph on the right is one case of doing that. C_{add} is the circle has white edges and vertices
- In this graph, C_{add} simply intersects other great circles but doesn't change the bounded cycle which is the biggest hexagonal. In other words, there is no edge on C_{add} is in the bounded cycle (the other cases will be mentioned later)



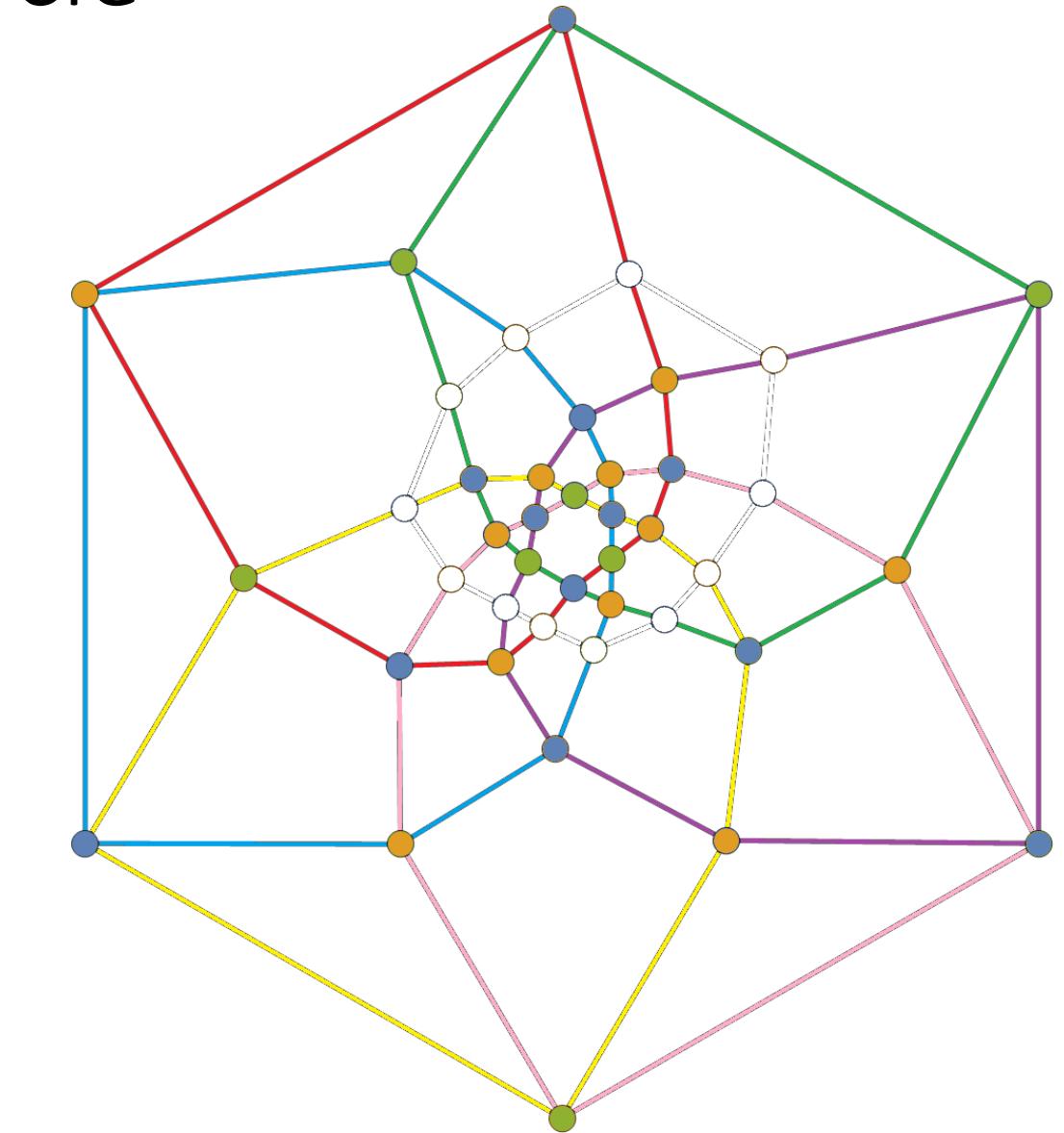
Adding a random great circle

- The graph is having 4 colors: Yellow, Green, Blue and an unknown color \mathfrak{U} (White). \mathfrak{U} will be replaced later so that yields the graph with only 3 colors
- I called the interior region bounded by C_{add} is R_{interior} , while the exterior region is R_{exterior} .



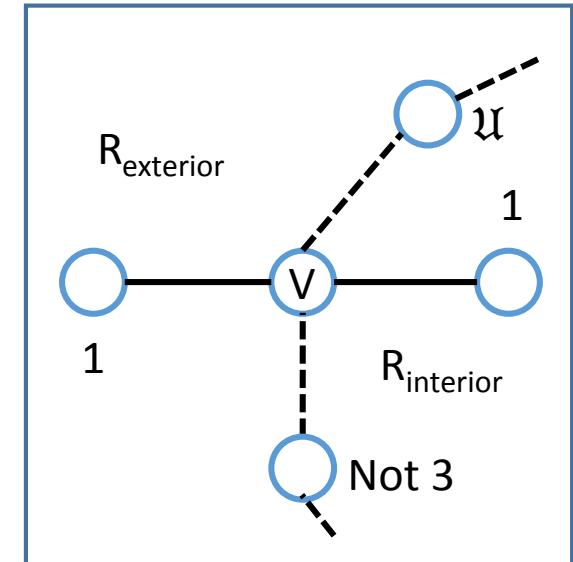
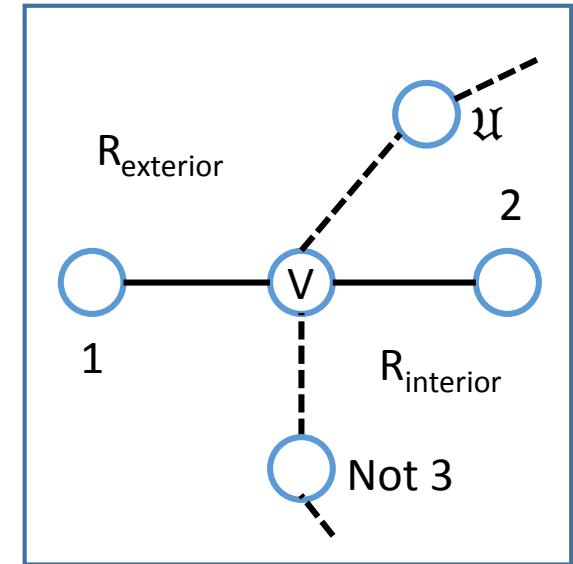
Adding a random great circle

I highlight edges in every great circle
by different colors



Rule to color \mathfrak{U}

1. Call V is the vertex I want to color. If there is a color available in 3 colors for V , use it. If not, go to the step 2.



Rule to color \mathfrak{U}

2. Call V_{ext} is the neighboring vertex of V in R_{exterior} , V_{int} is the neighboring vertex of V in R_{interior}

If 3 colors are used and there is a neighboring vertex isn't colored yet, we have 3 subcases:

2.1. Case 1:

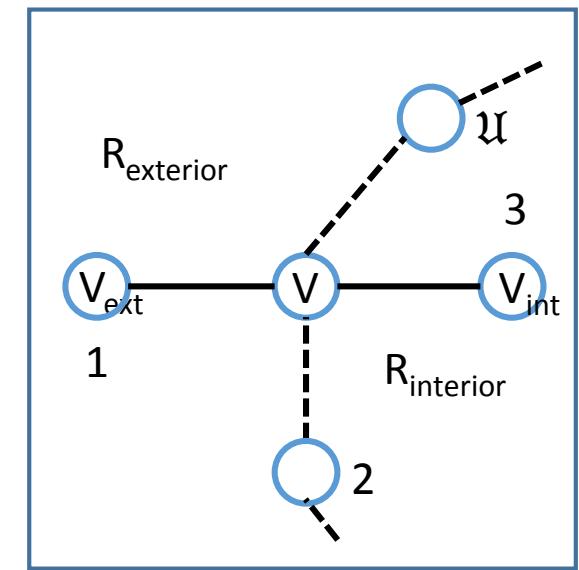
If there is no cycle or an even cycle that include 1-2 Kempe chain starting at V , do 1-2 Kempe chain switch at V

2.2. Case 2:

If there is an odd cycle that include 1-2 Kempe chain starting at V , do 1-3 Kempe chain switch at V

2.2. Case 3:

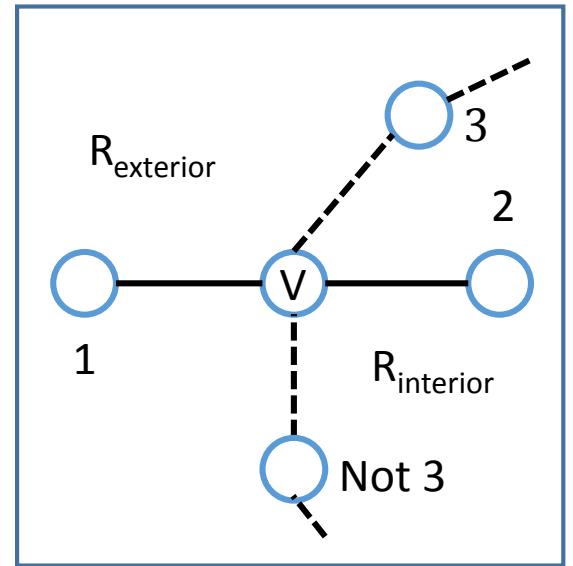
If there is an odd 1-2 chain and 1-3 chain, switch 2-3 at V_{ext}



Rule to color \mathfrak{U}

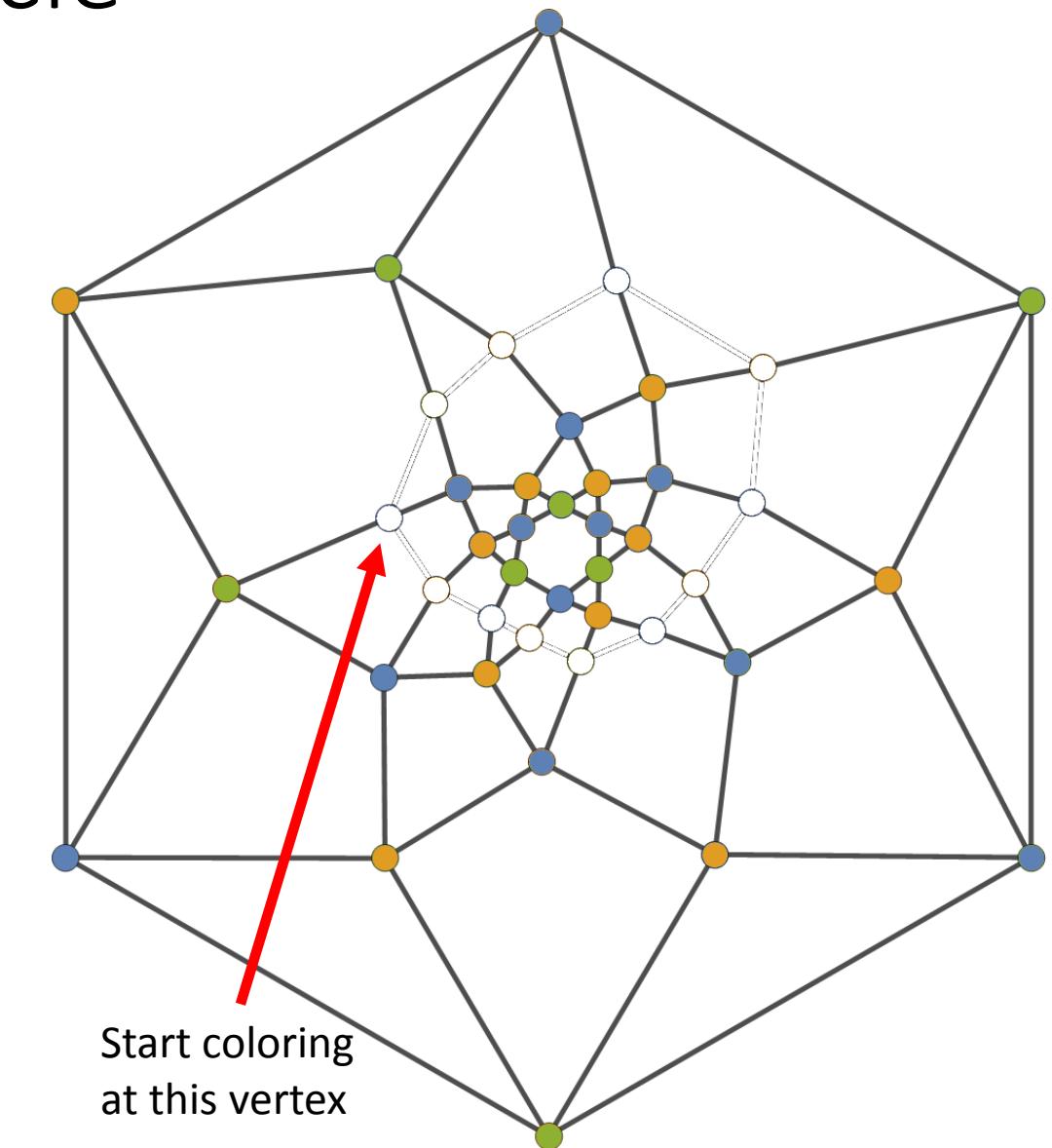
3. Case 3 and other possible cases haven't any clear solutions yet.

I met some problems which I can't use any rule to solve them. Therefore, I may update this rule later and try to prove it.



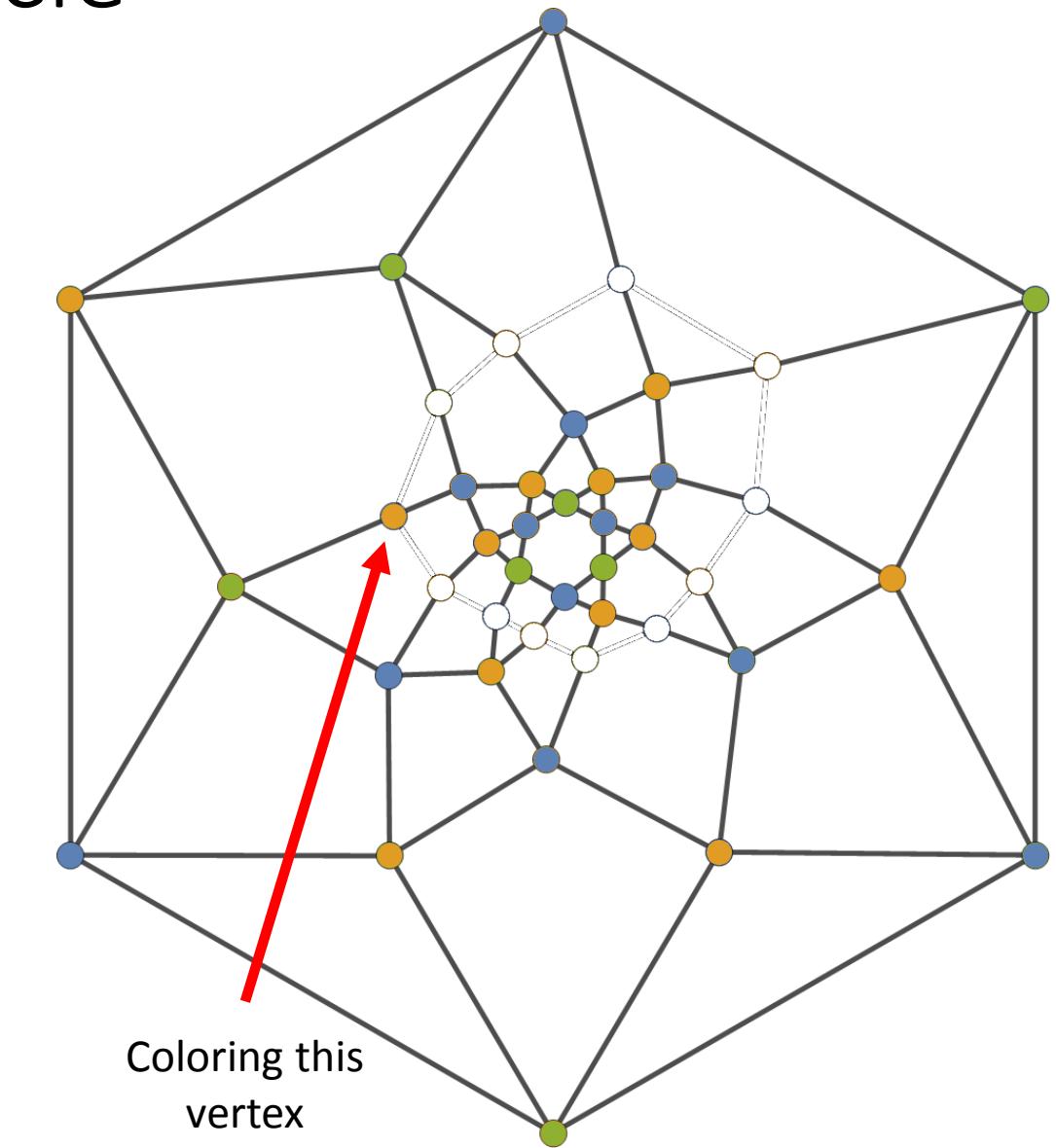
Adding a random great circle

- I'm going to color the graph at a random vertex shown on the right and point to the next vertex in the clockwise order until there is no Υ left.



Adding a random great circle

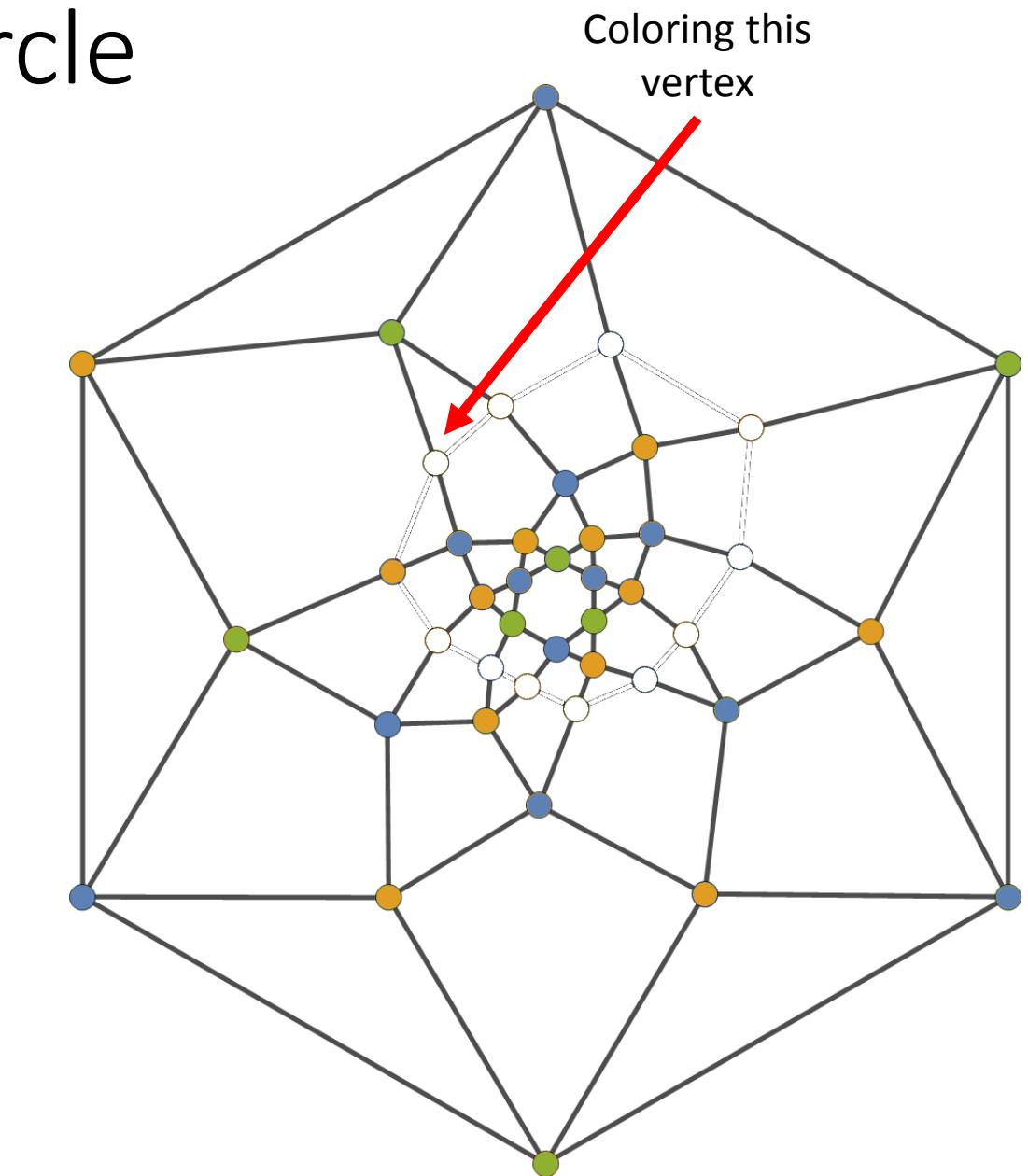
I use Yellow because the neighboring vertices only use 2 colors.



Adding a random great circle

Go to the next vertex, here is the case when 3 colors have been used already.

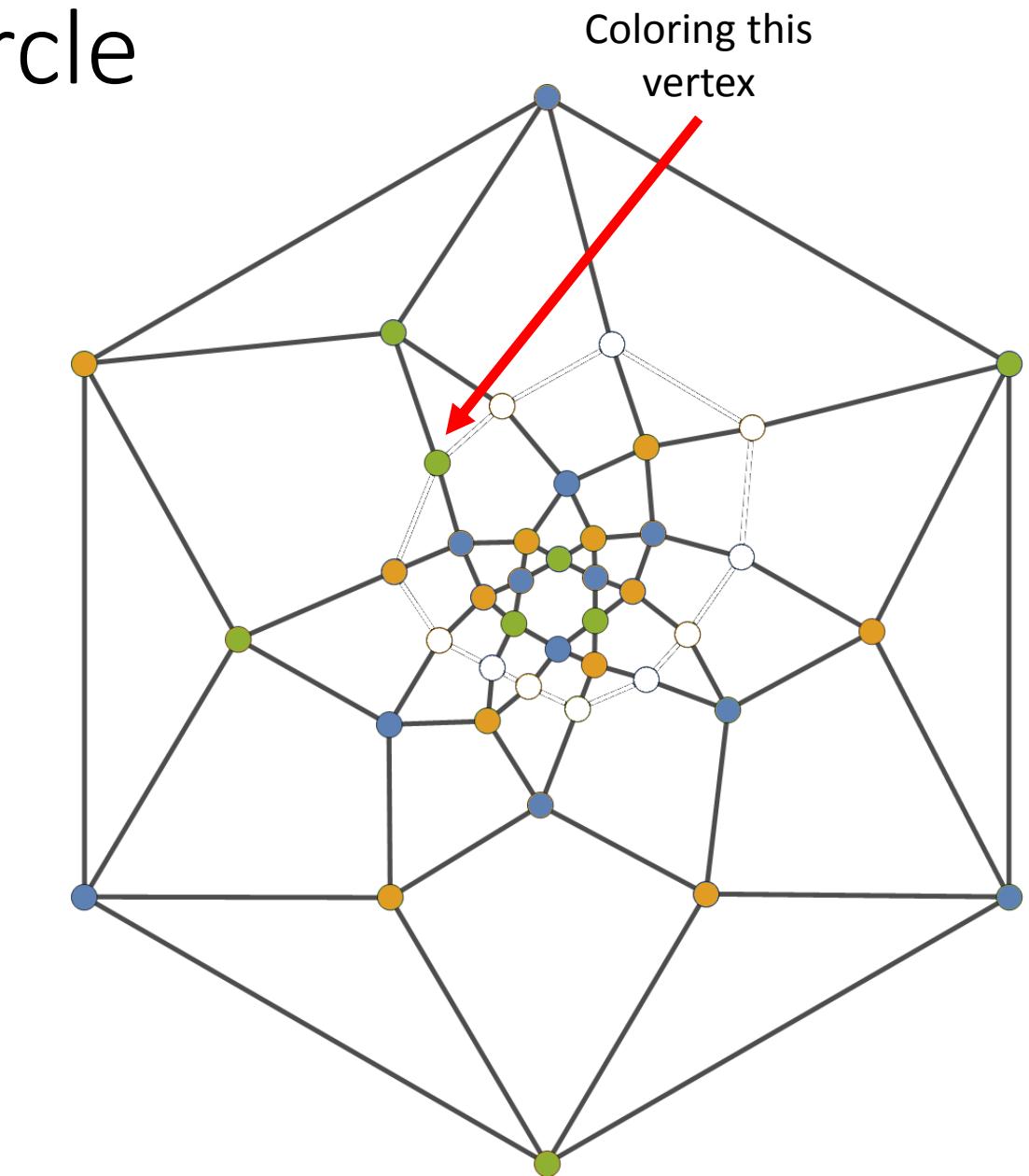
According to the rule, I will use Green to color.



Adding a random great circle

Go to the next vertex, here is the case when 3 colors have been used already.

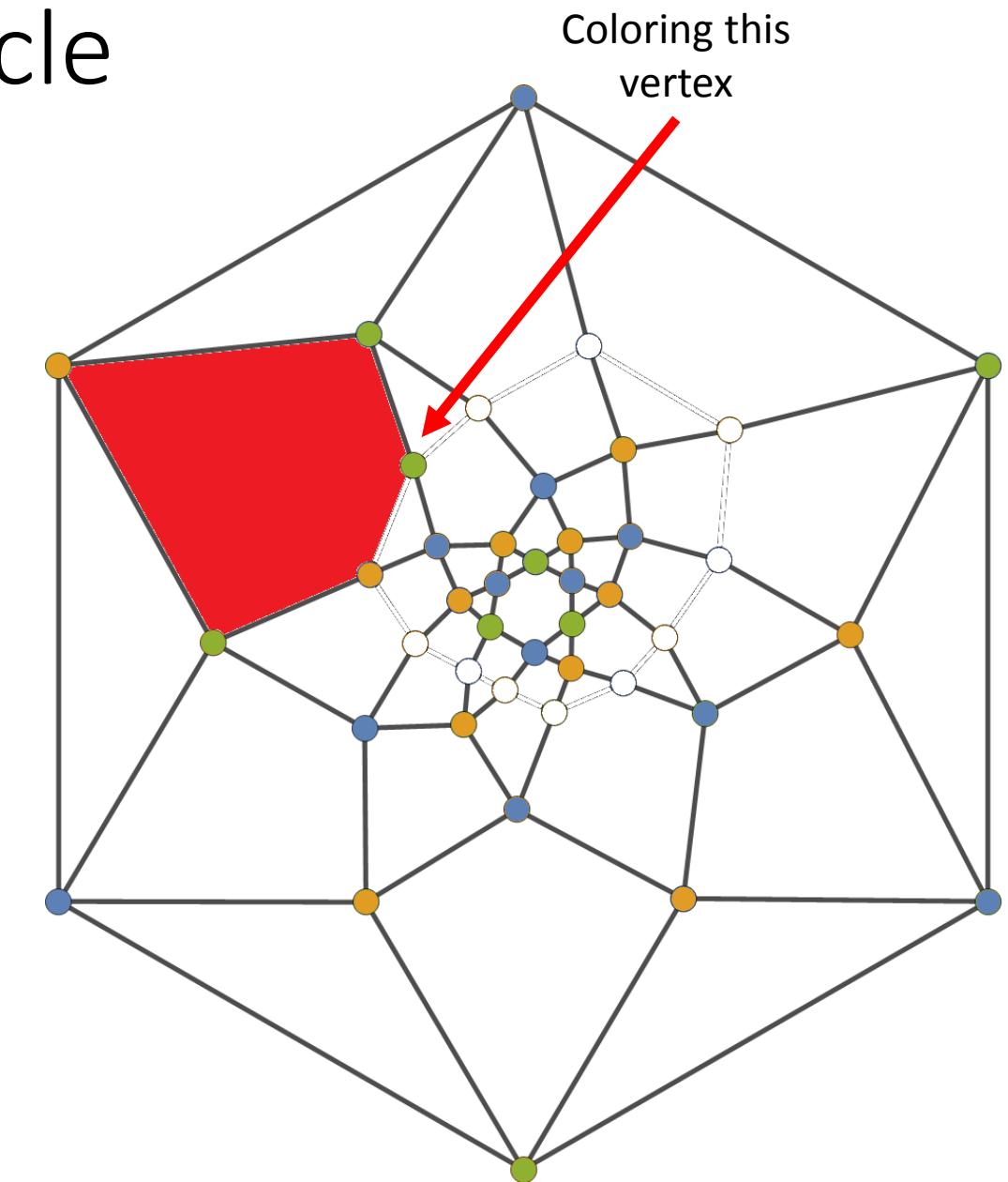
According to the rule, I will use Green to color.



Adding a random great circle

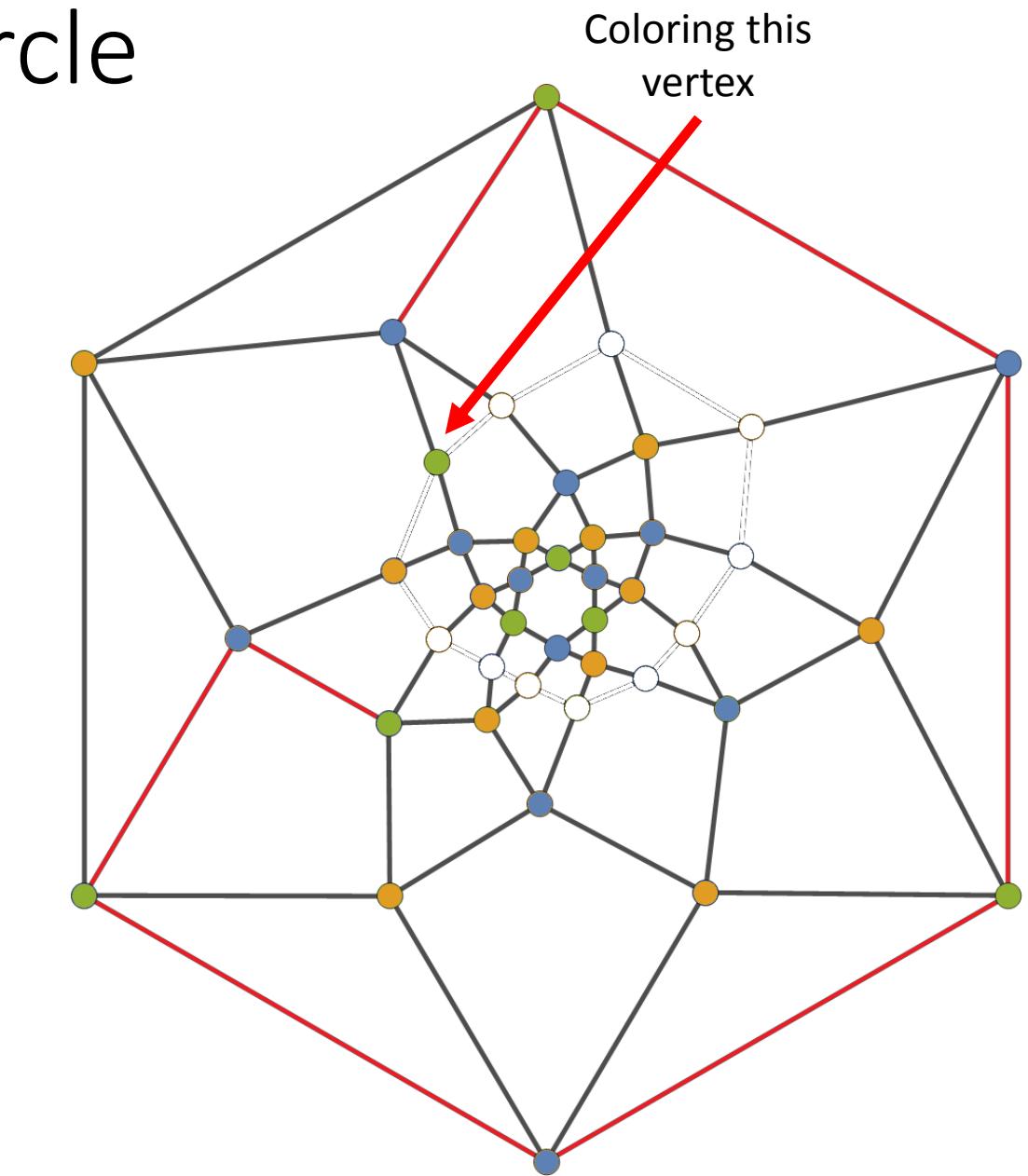
The red polygon shows that I can't do Green-Yellow switch starting at V because there is an odd Green-Yellow cycle there.

Therefore, I will do Green-Blue switch and that switch will only change some vertices on R_{exterior} . Obviously, the Green-Yellow cycle will prevent Green-Blue chain connecting V from R_{exterior}



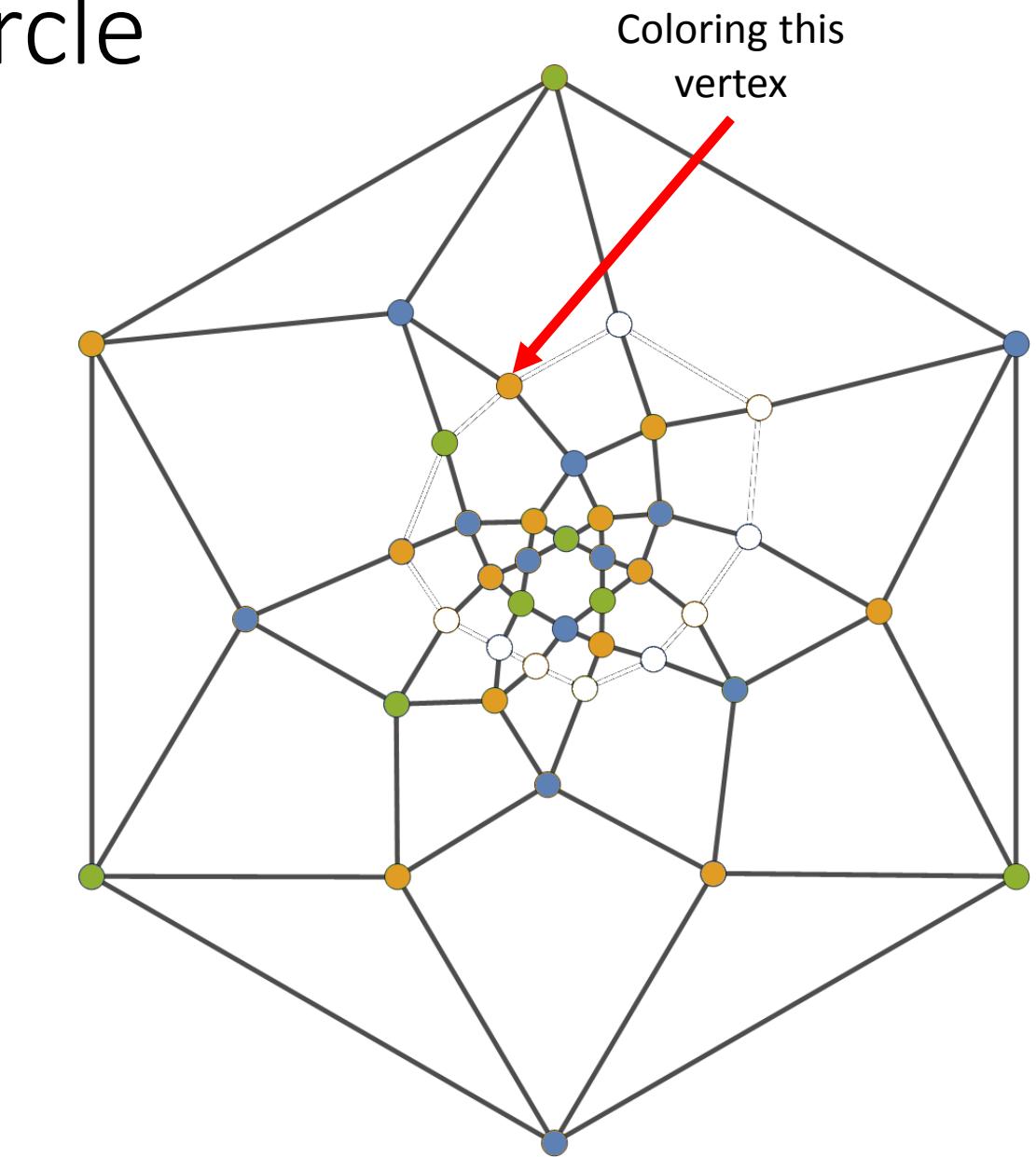
Adding a random great circle

The red edges are in Green-Blue chain
that I follow to switch.



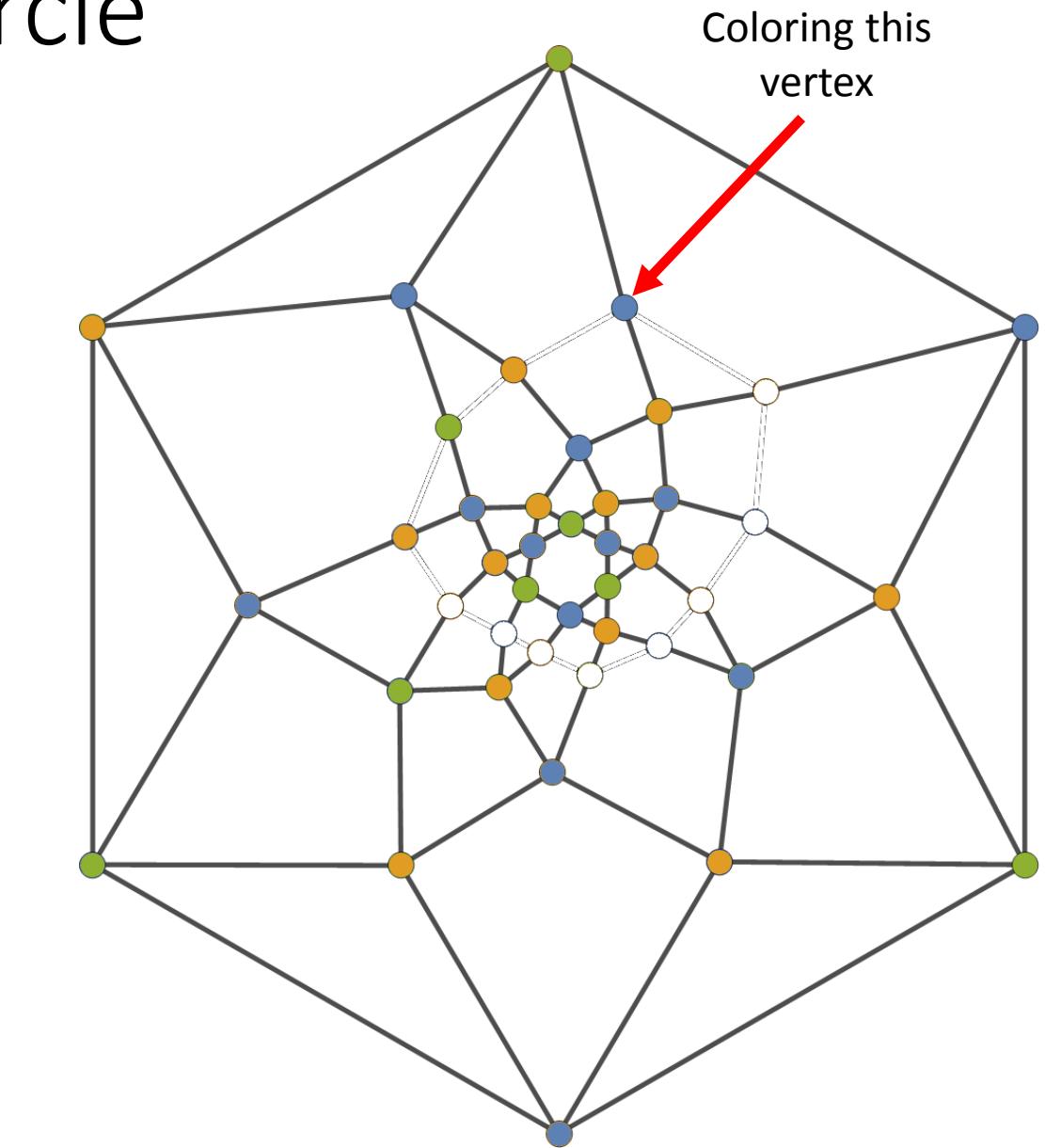
Adding a random great circle

Go to the next vertex, I use Yellow



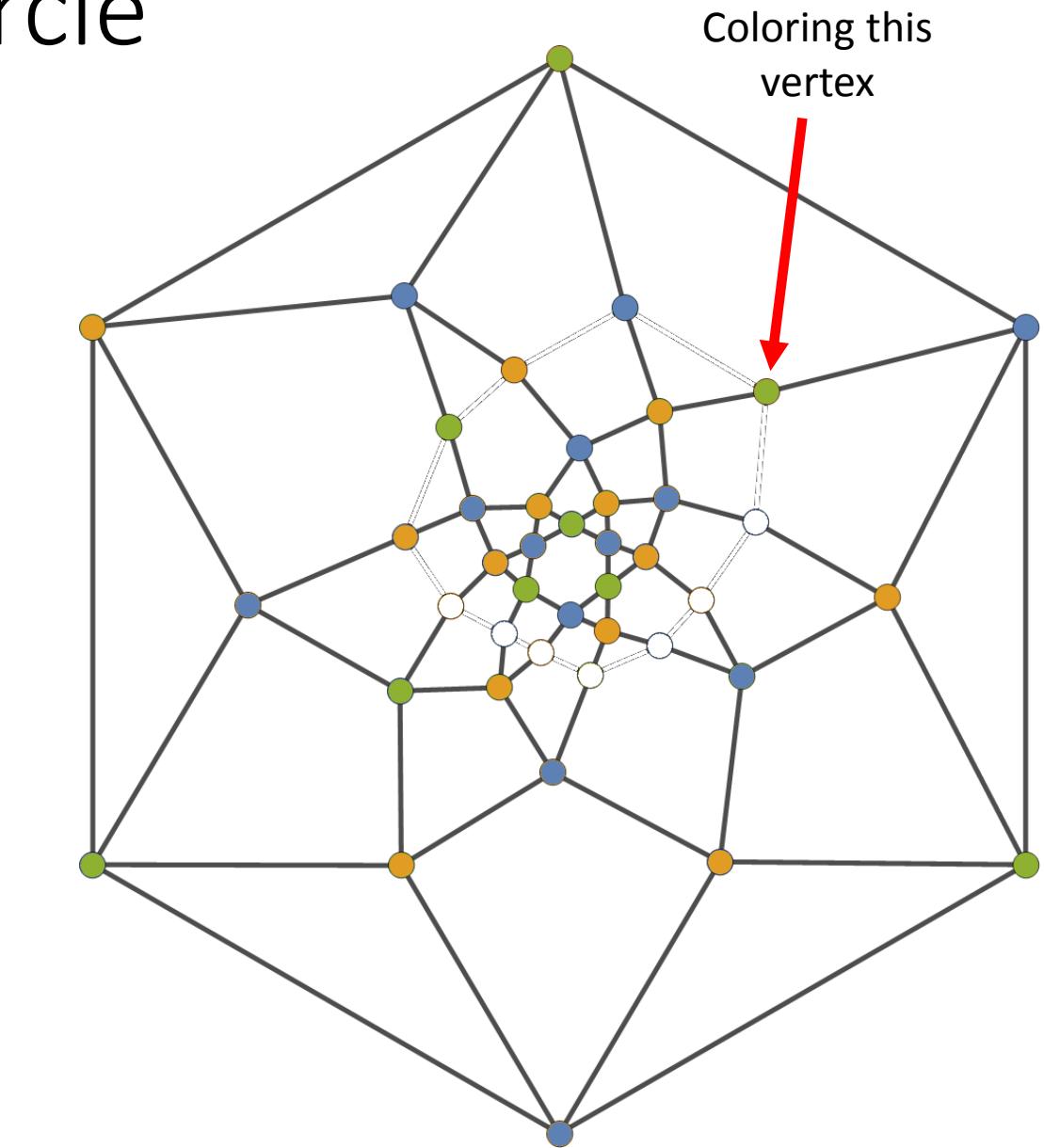
Adding a random great circle

Go to the next vertex, I use Blue



Adding a random great circle

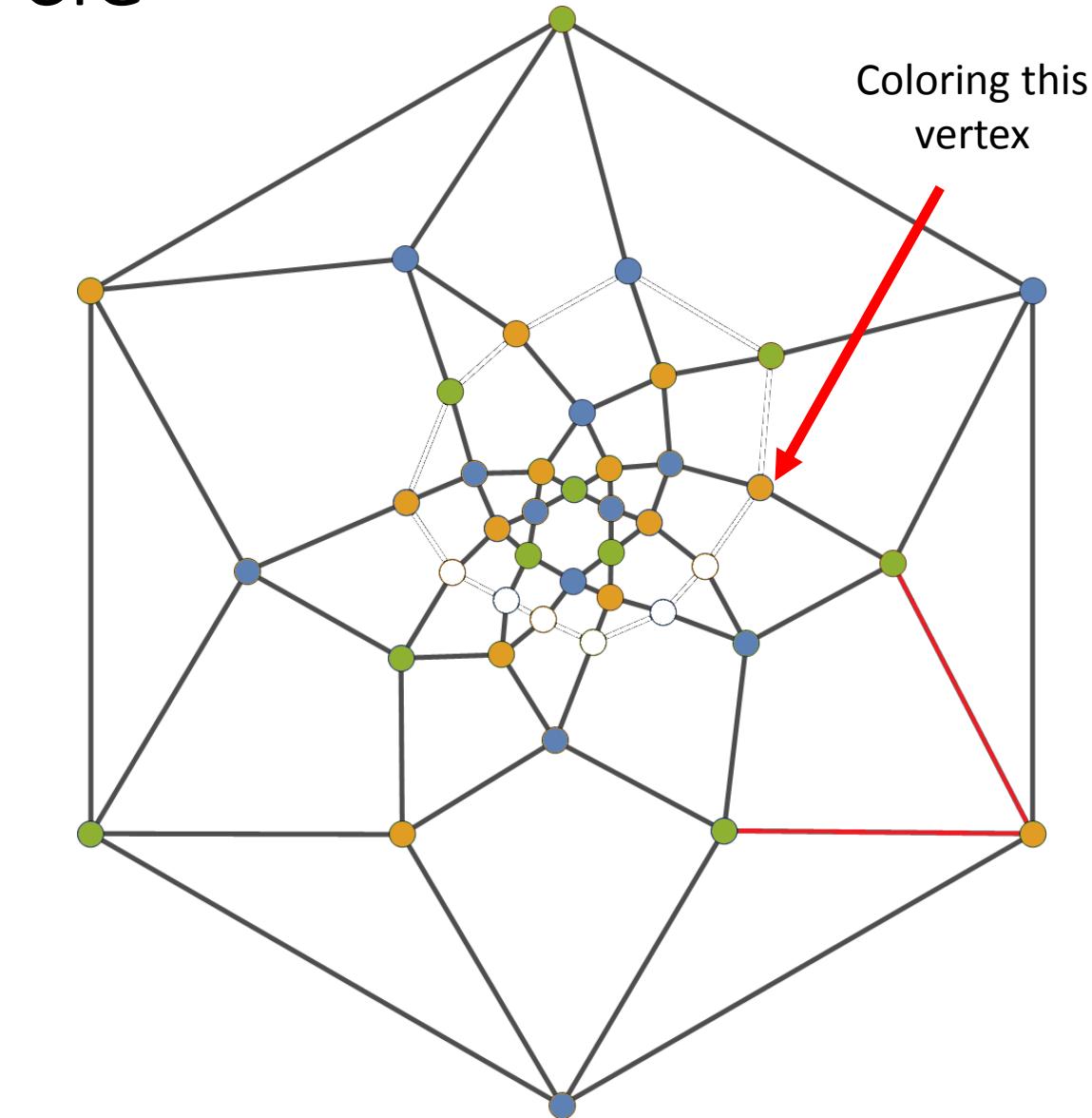
Go to the next vertex, I use Green



Adding a random great circle

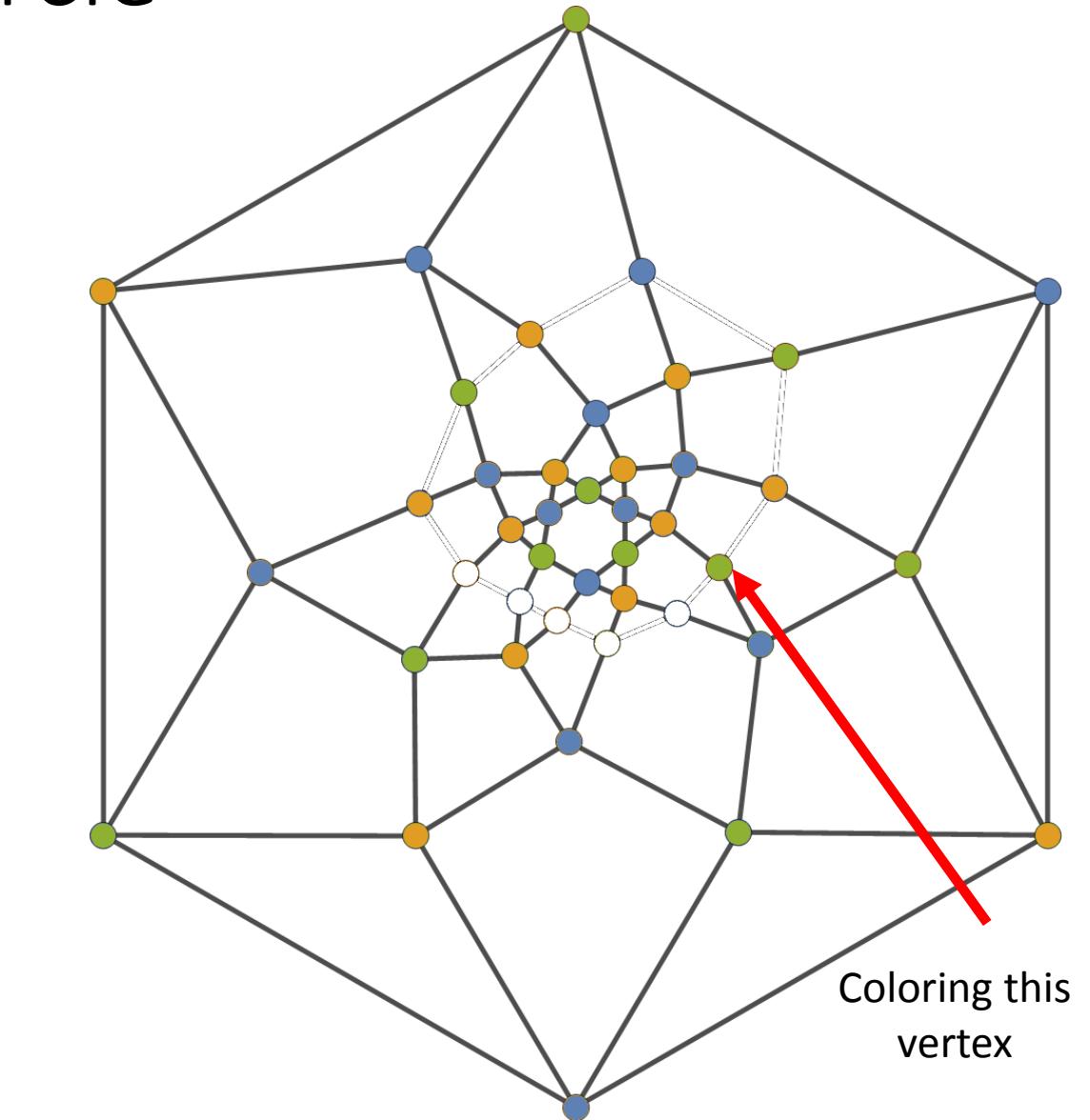
3 colors are used. As usual, I use Yellow from V_{ext} to V

There is no Green-Yellow cycle, so I will do Green-Yellow switch starting at V



Adding a random great circle

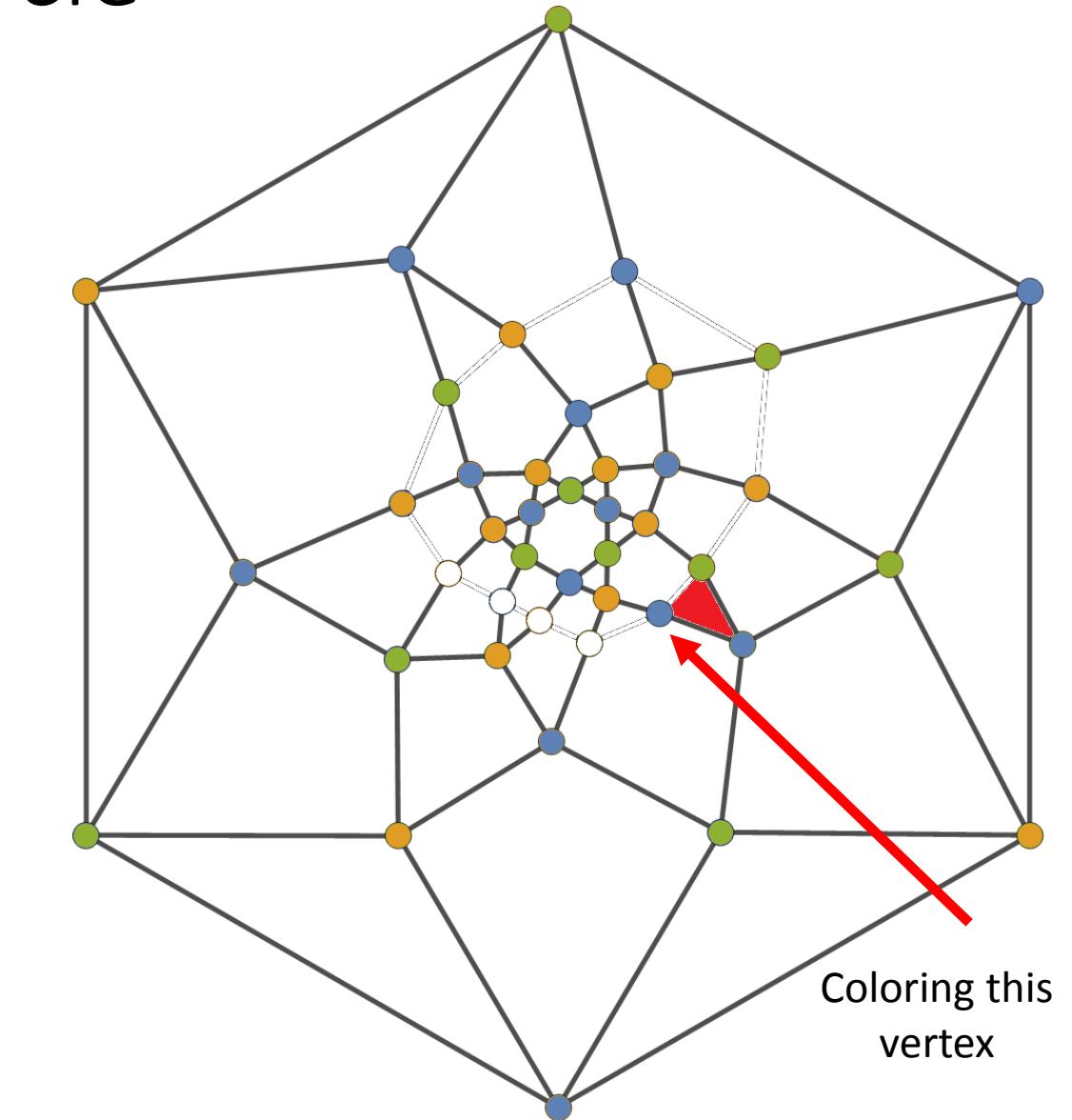
Go to the next vertex, I use Green



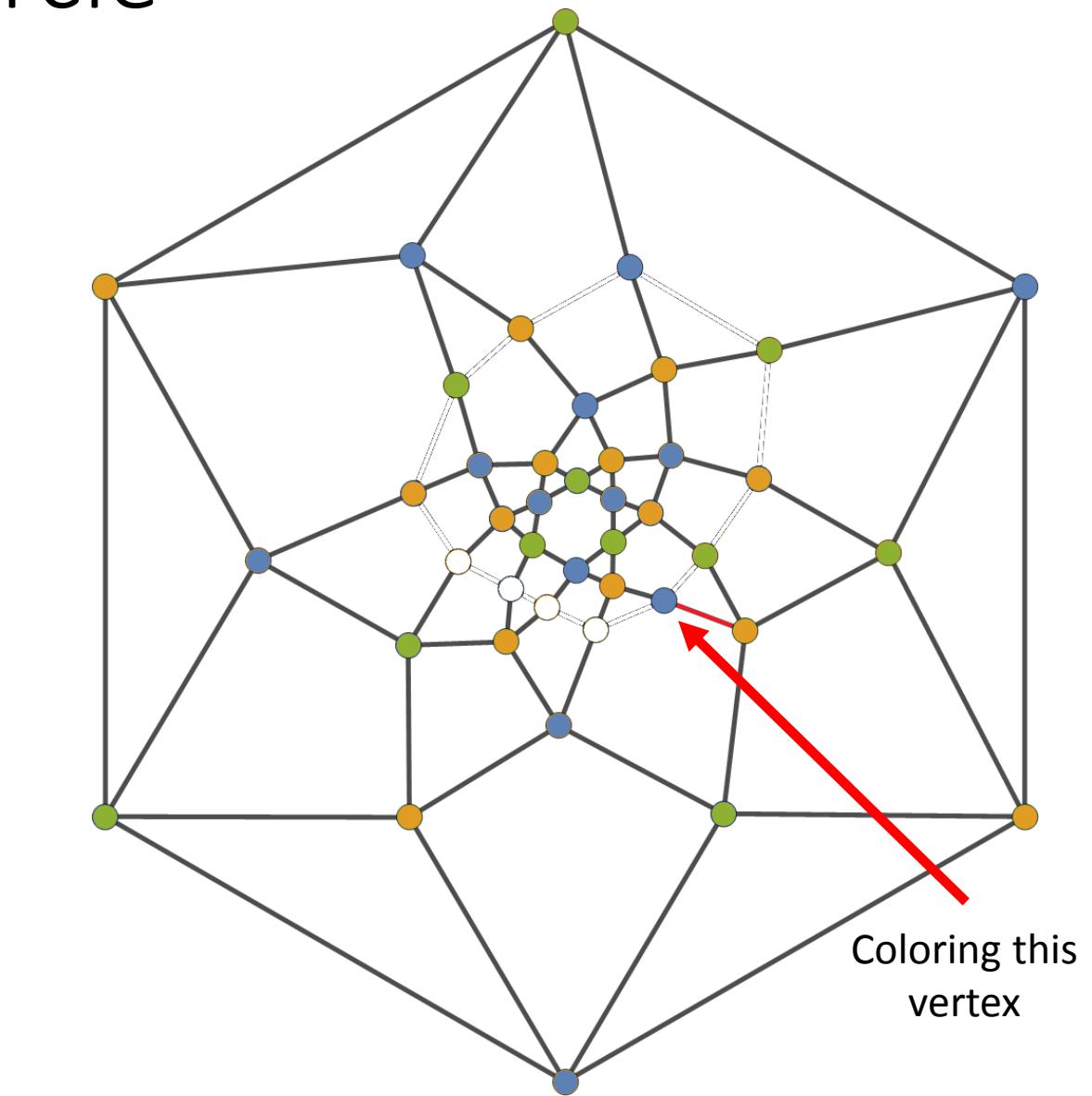
Adding a random great circle

3 colors are used. As usual, I use Blue from V_{ext} to V

There is an odd Blue-Green cycle, so I have to switch Blue-Yellow chain starting at V

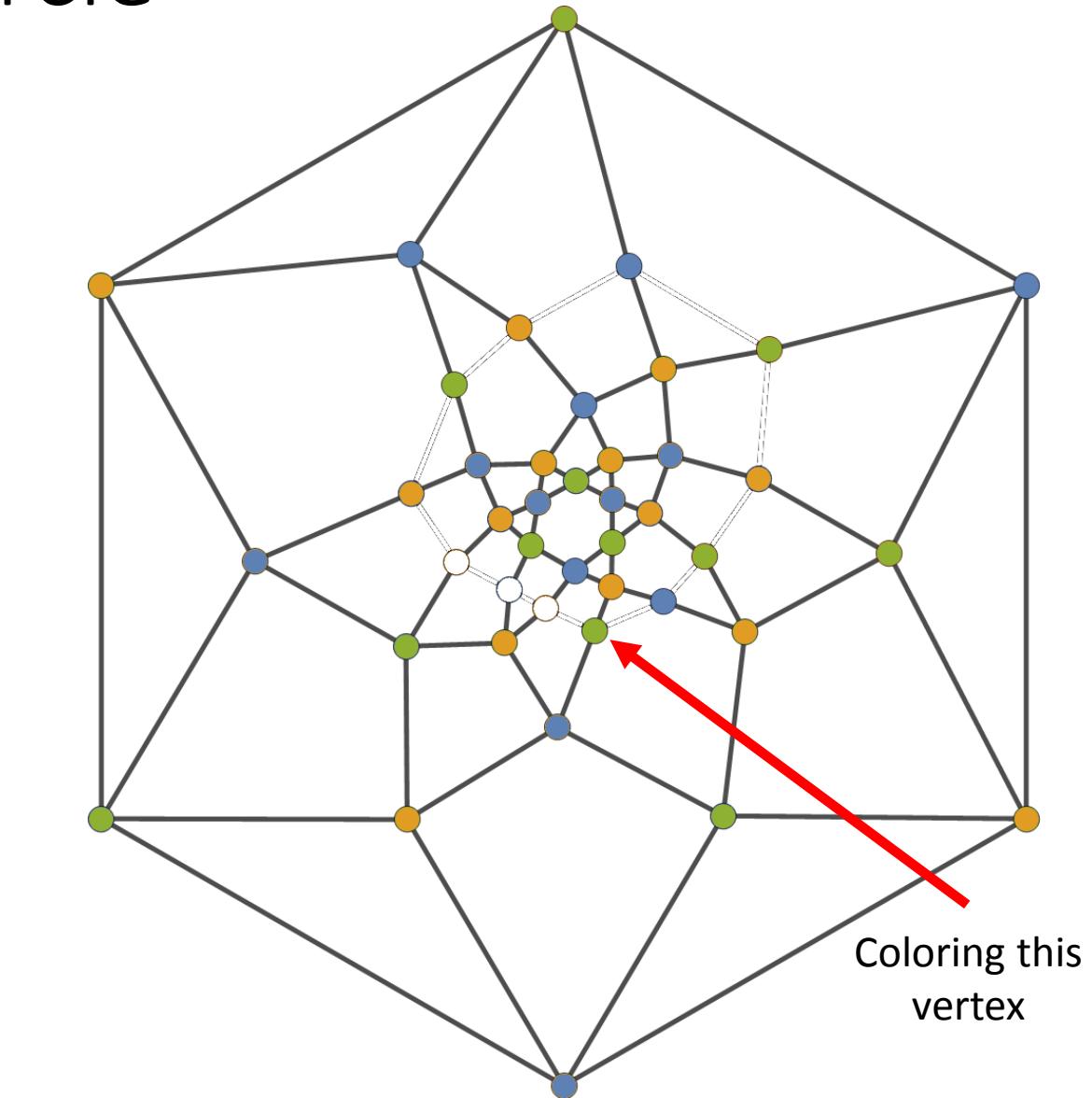


Adding a random great circle



Adding a random great circle

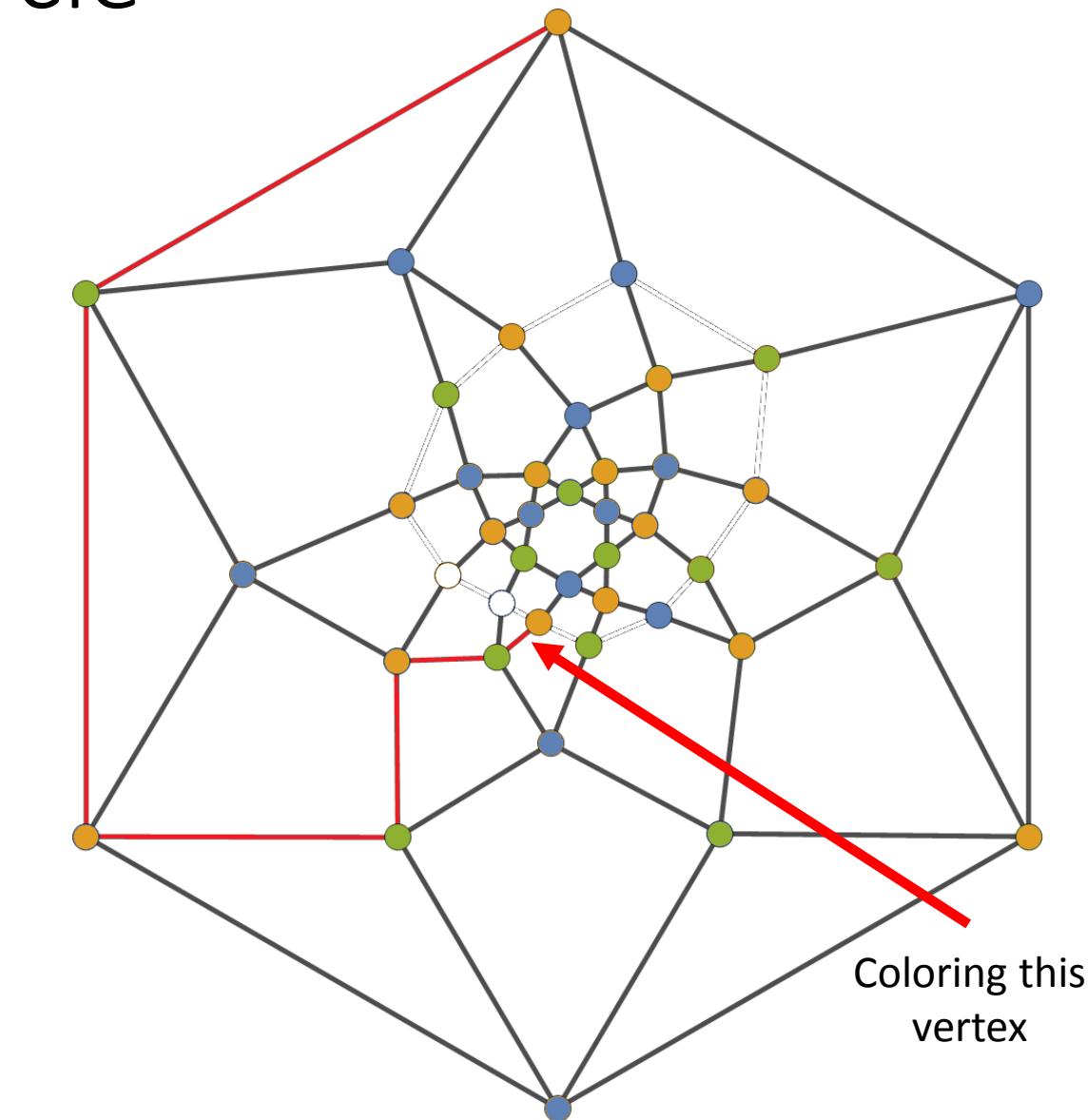
Go to the next vertex, I use Green



Adding a random great circle

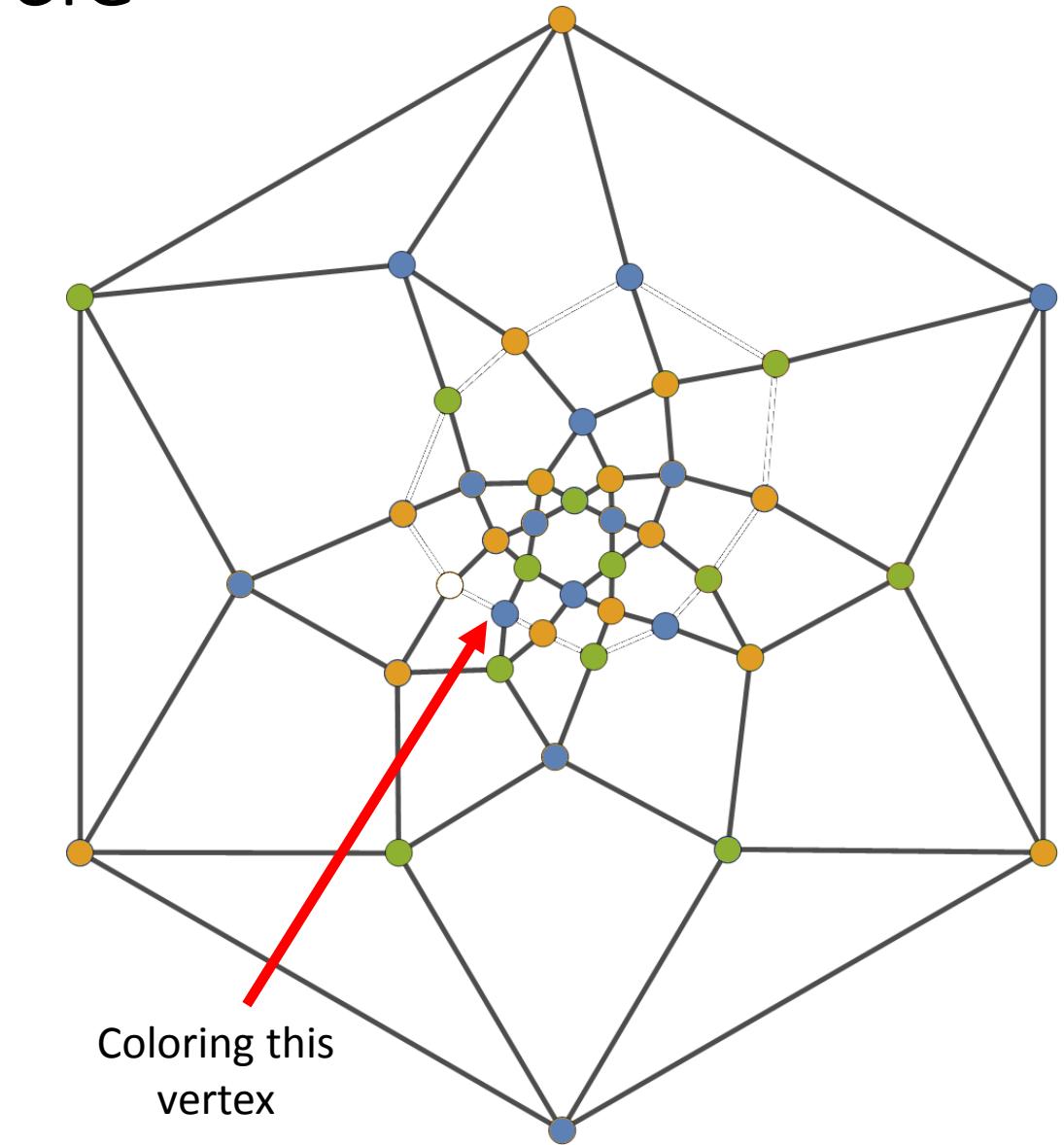
3 colors are used. As usual, I use Yellow from V_{ext} to V

There is an odd cycle Blue-Green, so I have to switch Blue-Yellow chain starting at V



Adding a random great circle

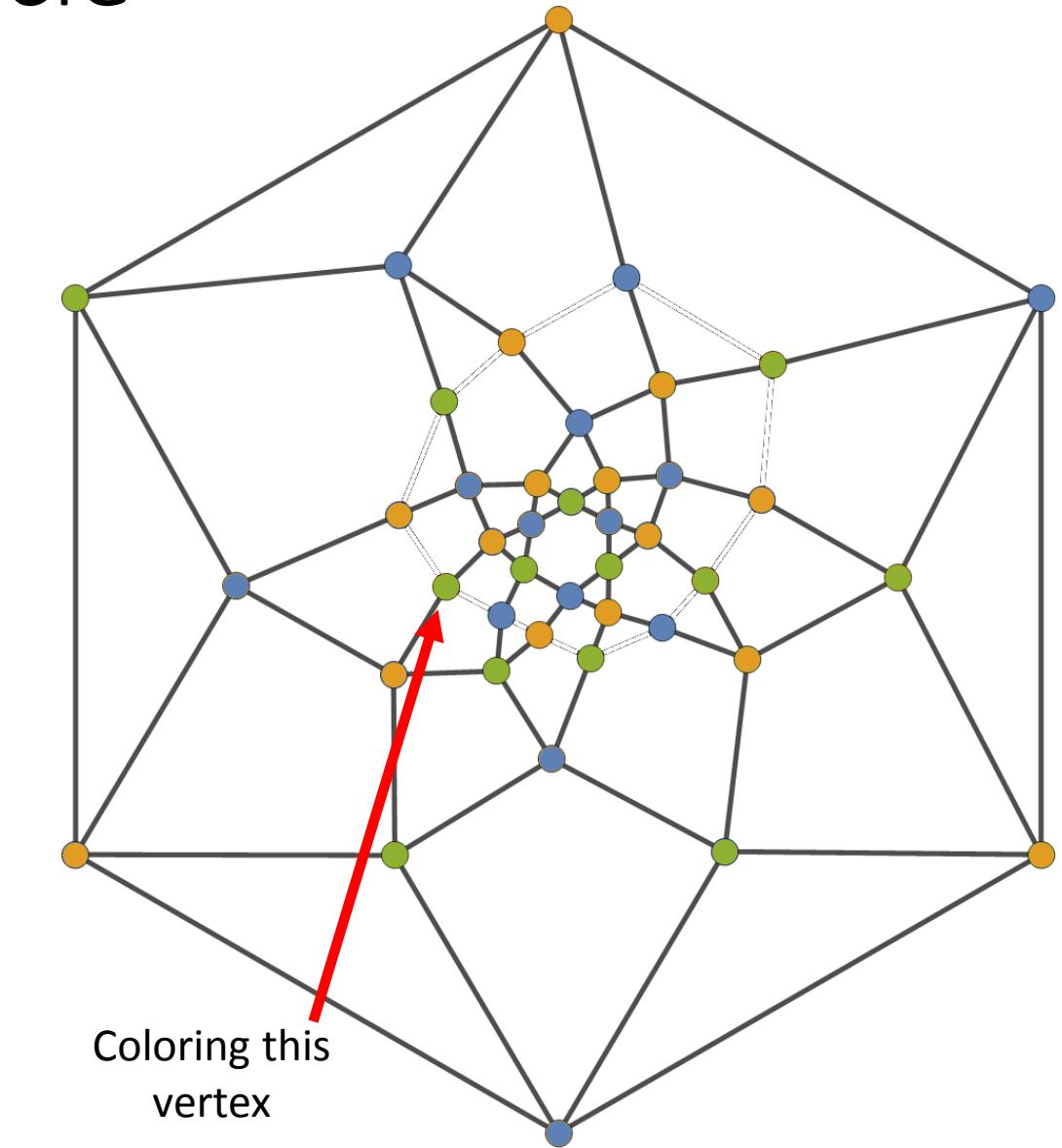
Go to the next vertex, I use Blue



Adding a random great circle

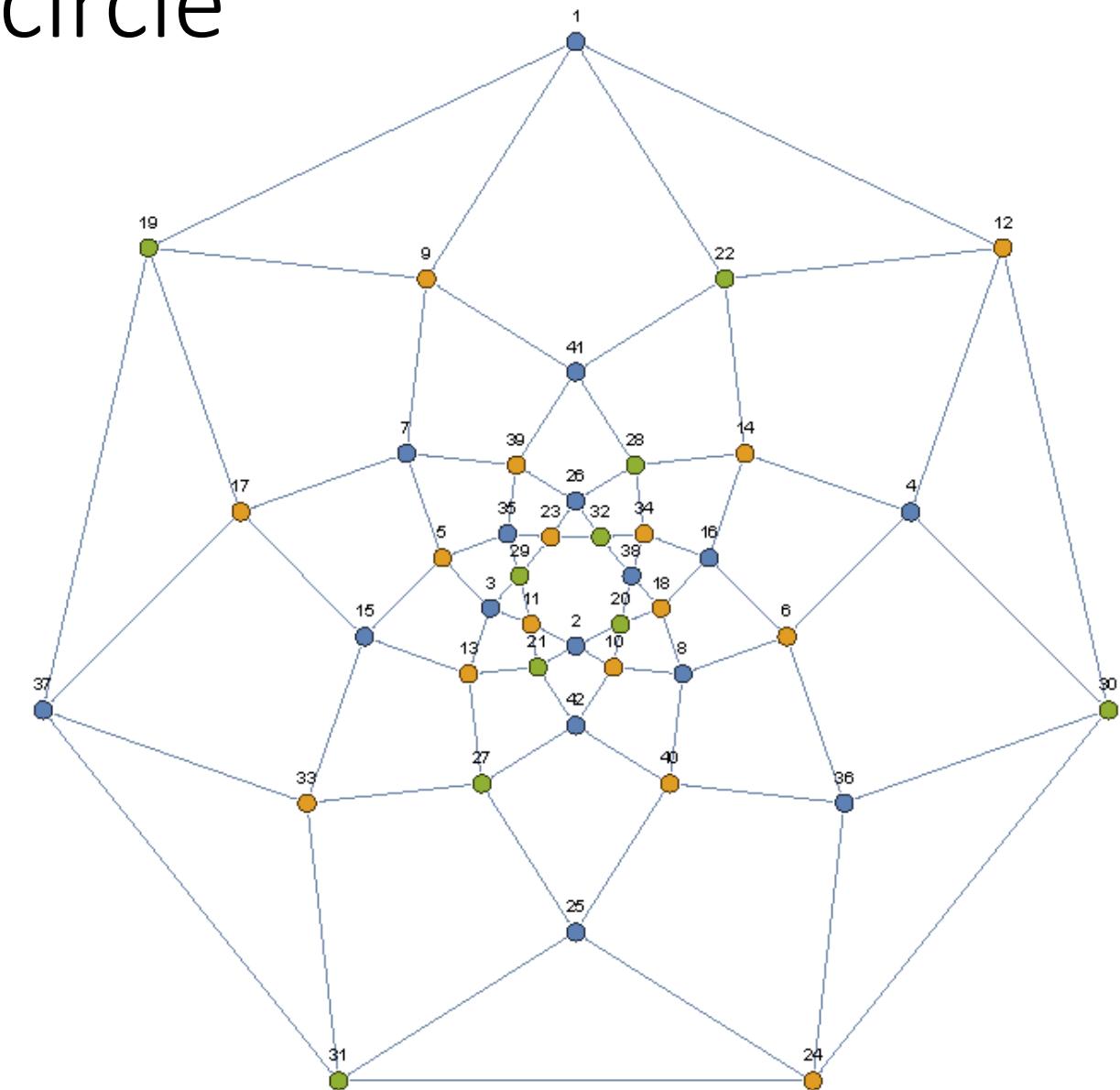
Go to the next vertex, I use Green

DONE!!!



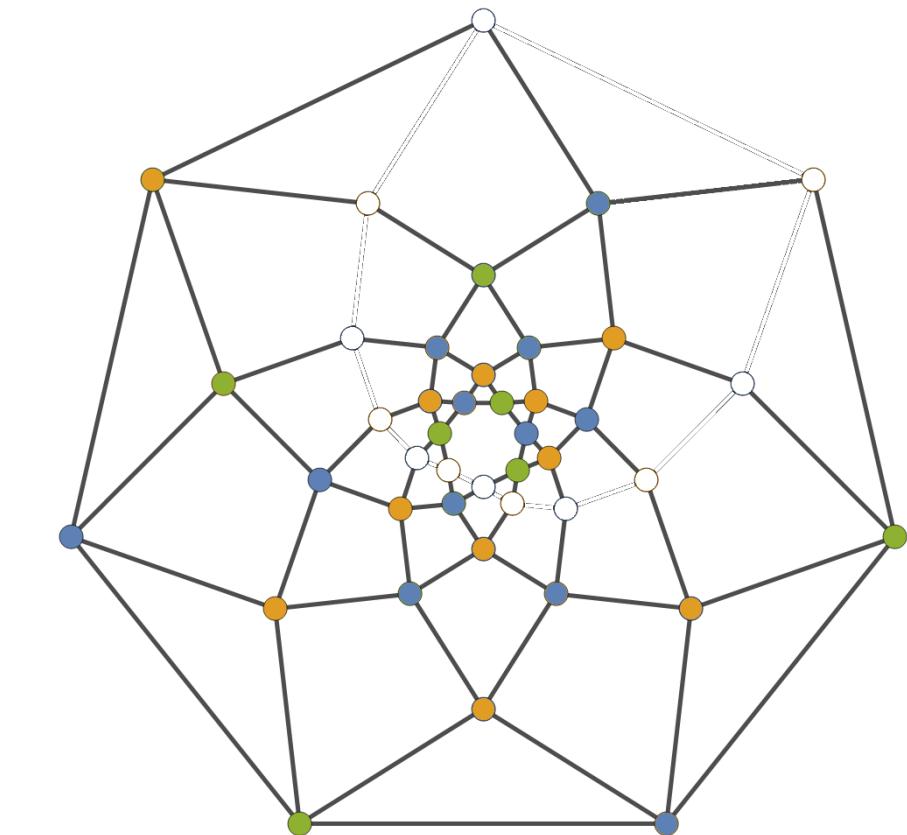
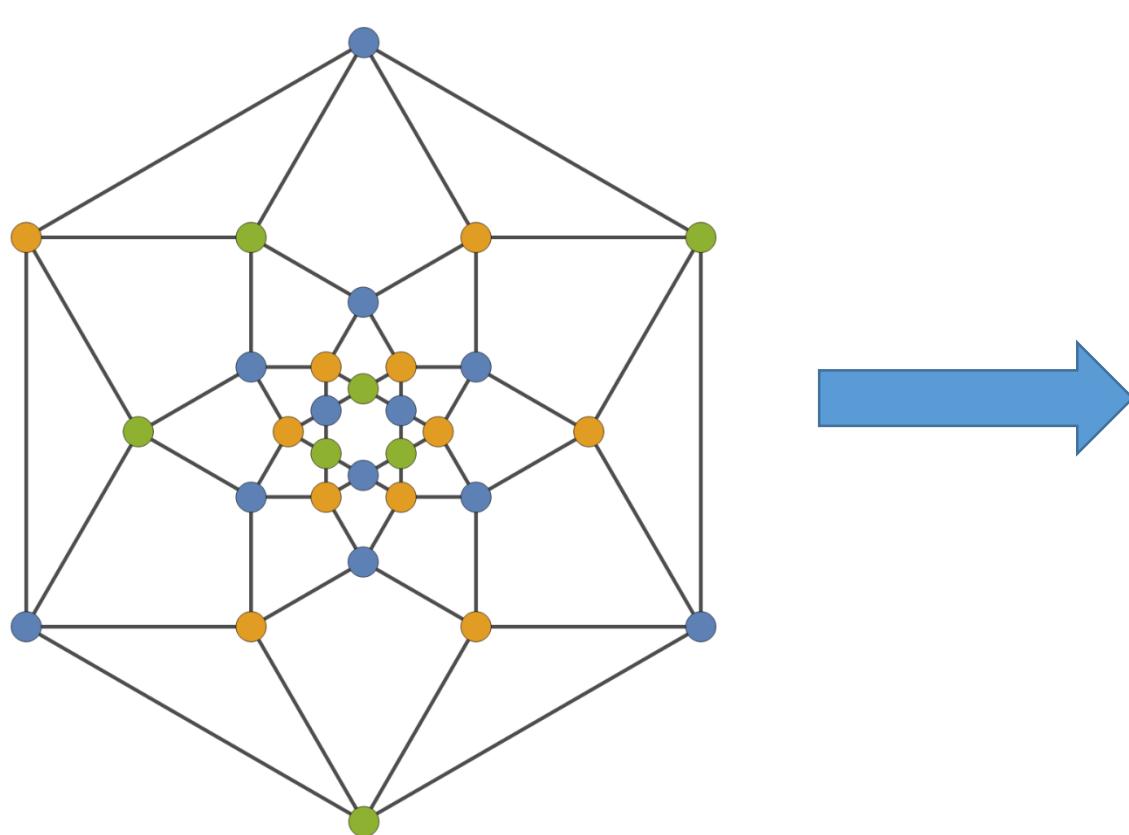
Adding a random great circle

- Another case is when C_{add} has an edge on the bounded cycle. Moreover, C_{add} increases 1 more edge on the bounded cycle
 - Here is the graph supposed to be



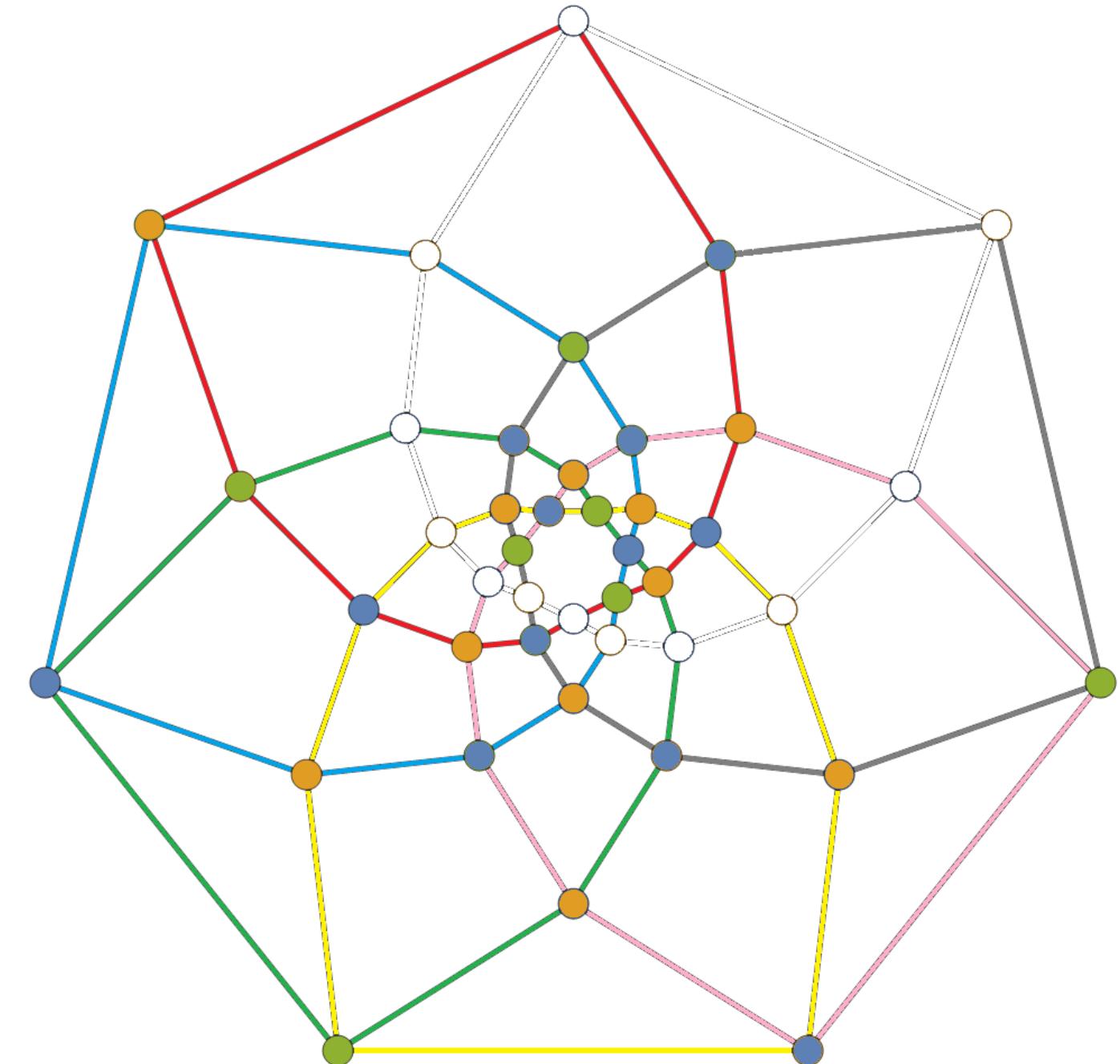
Adding a random great circle

- Now I'm trying to build the desired graph based on the known graph of 6 circles



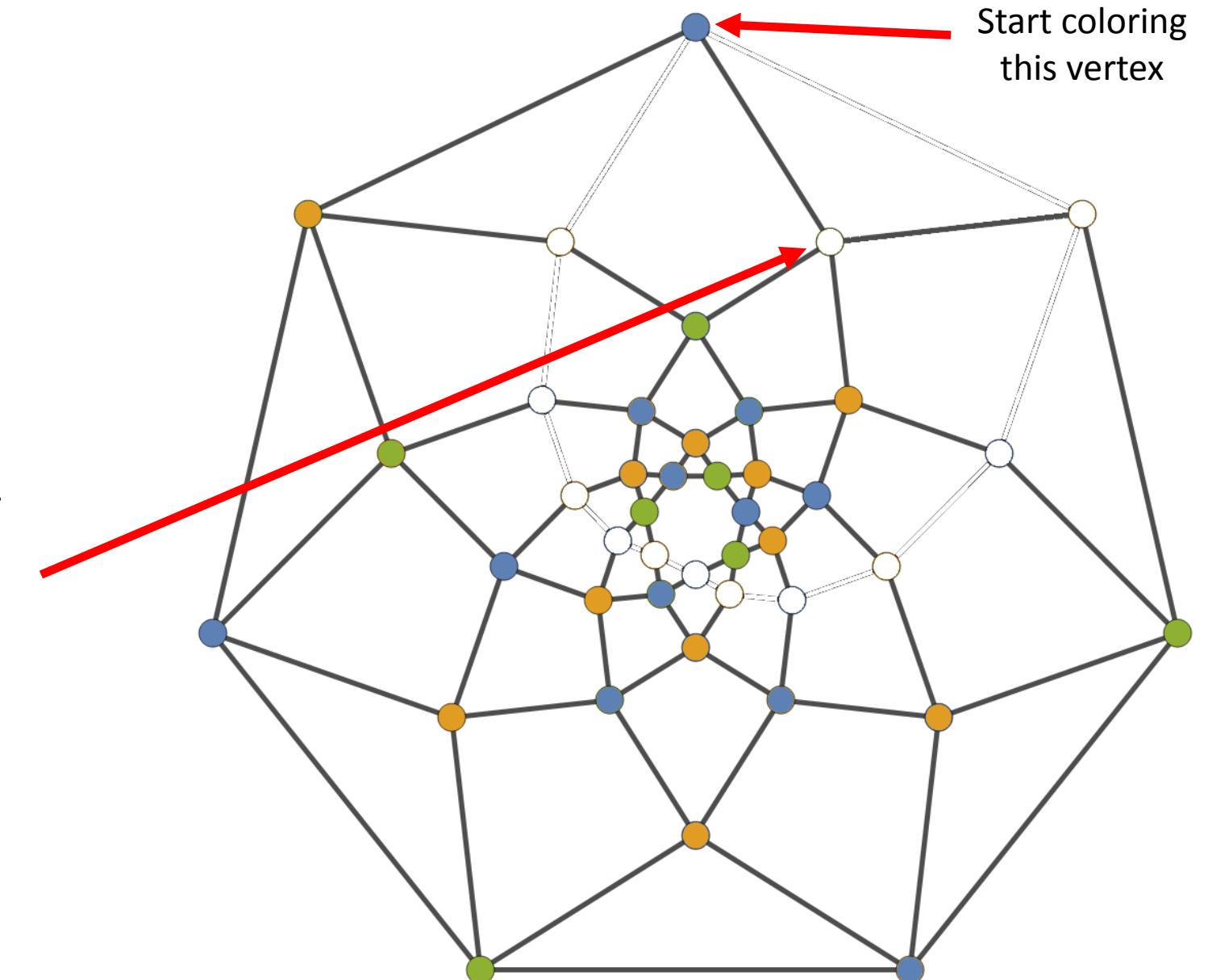
Adding a random great circle

I highlight edges in every great circle by different colors



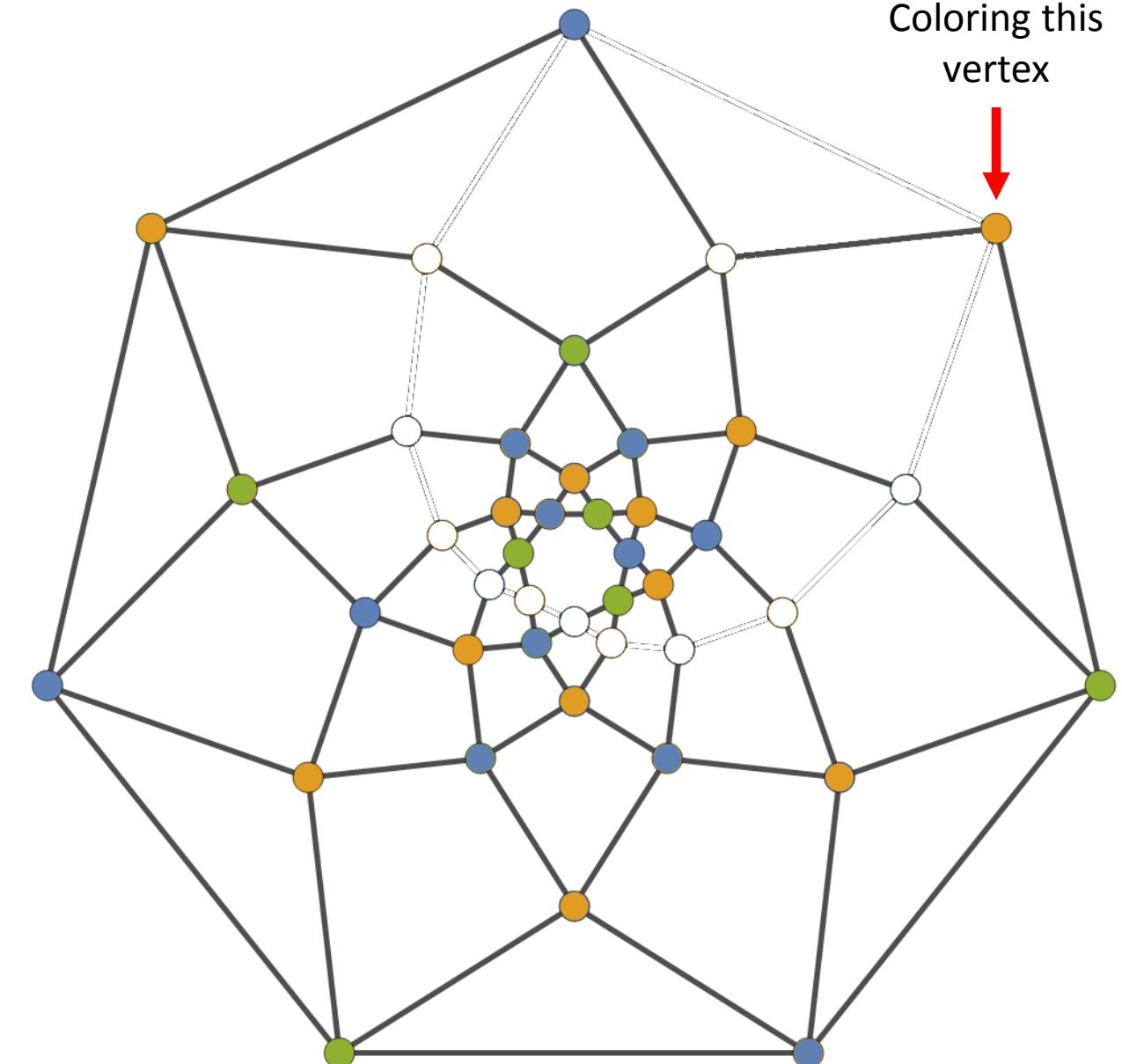
Adding a random great circle

Remove color of this vertex. If not, I can't use the rule to complete. You can check this case in the next part where the rule doesn't work out



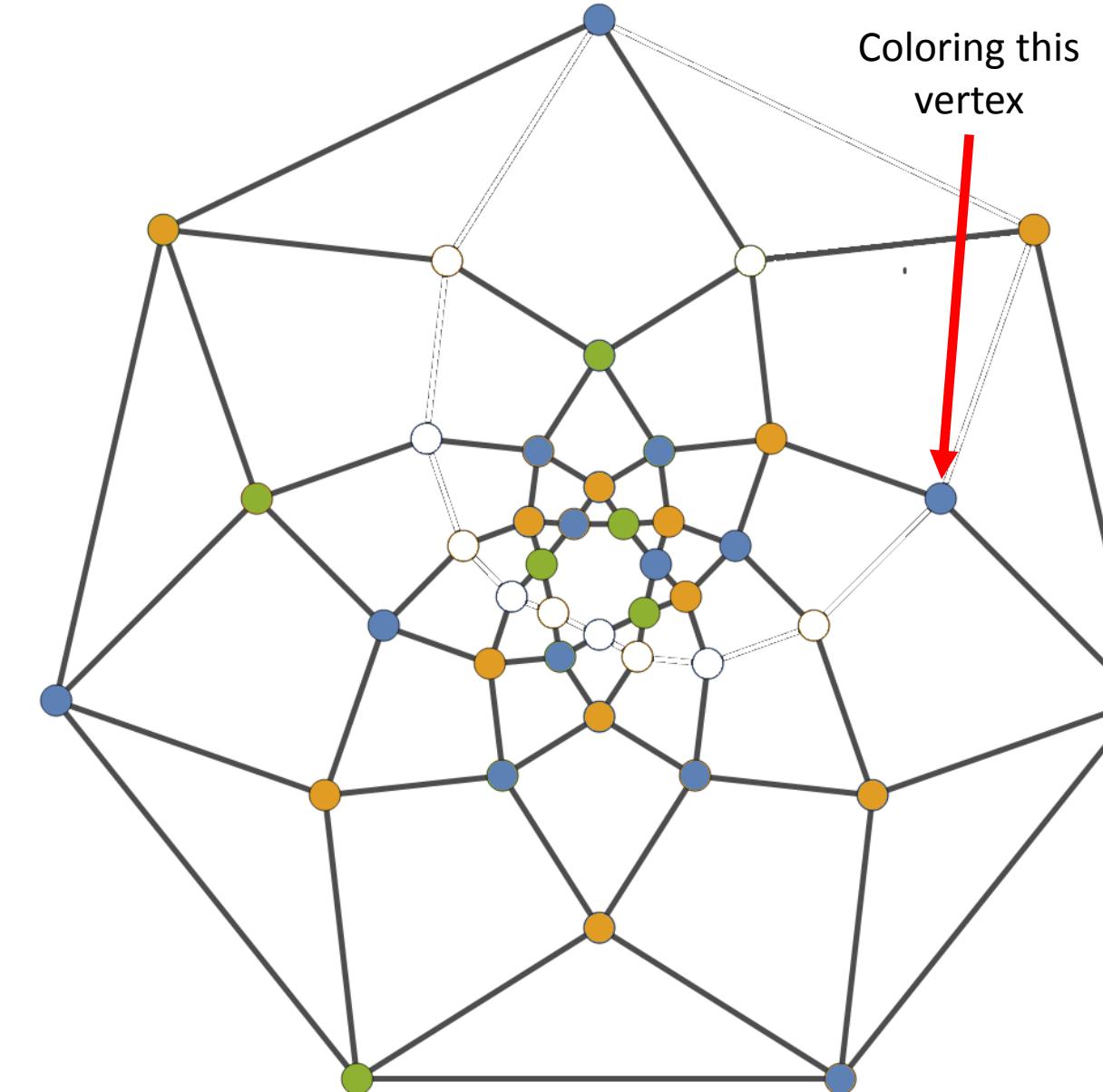
Adding a random great circle

Go to the next vertex, I use
Yellow



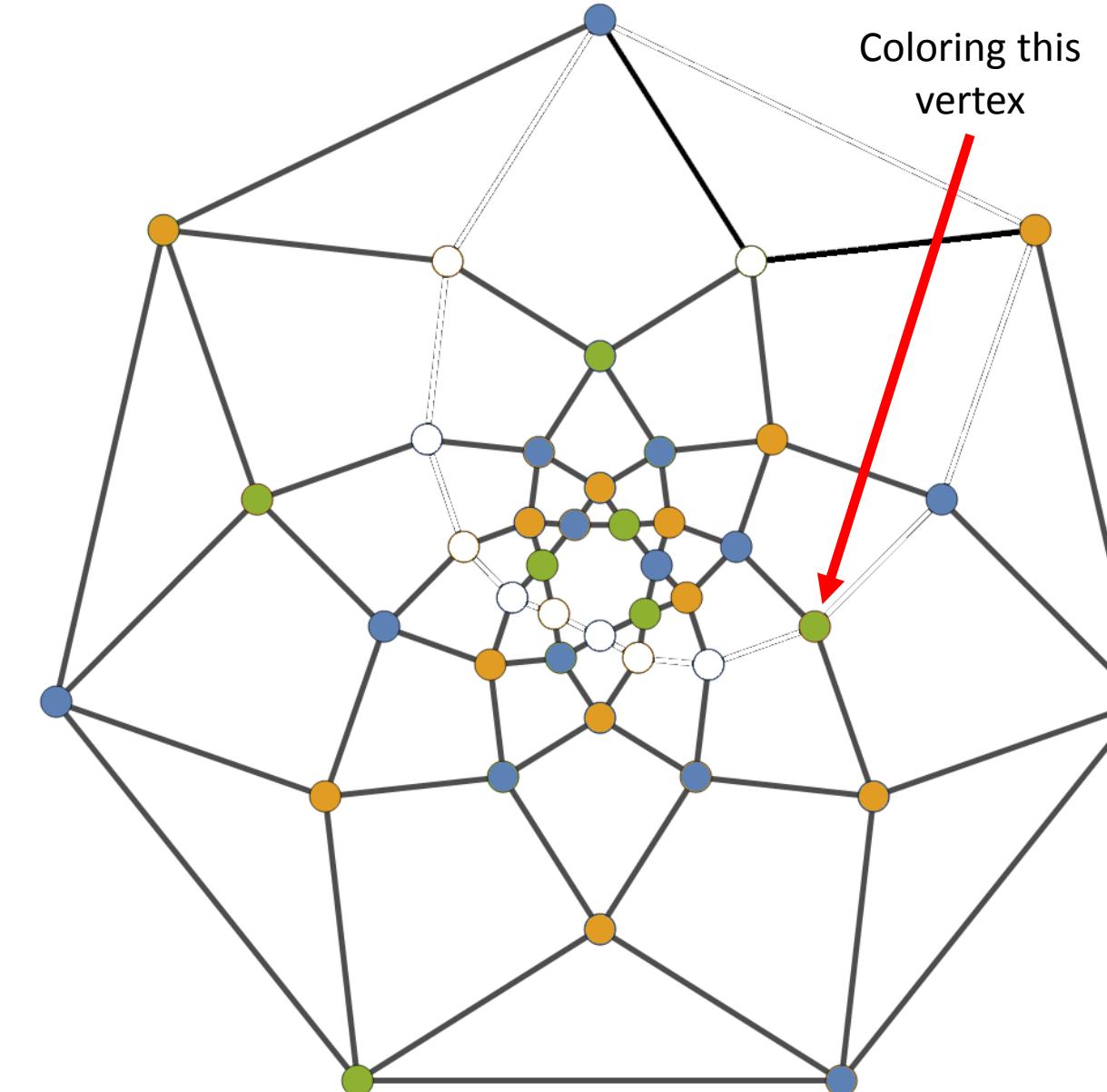
Adding a random great circle

Go to the next vertex, I use Blue



Adding a random great circle

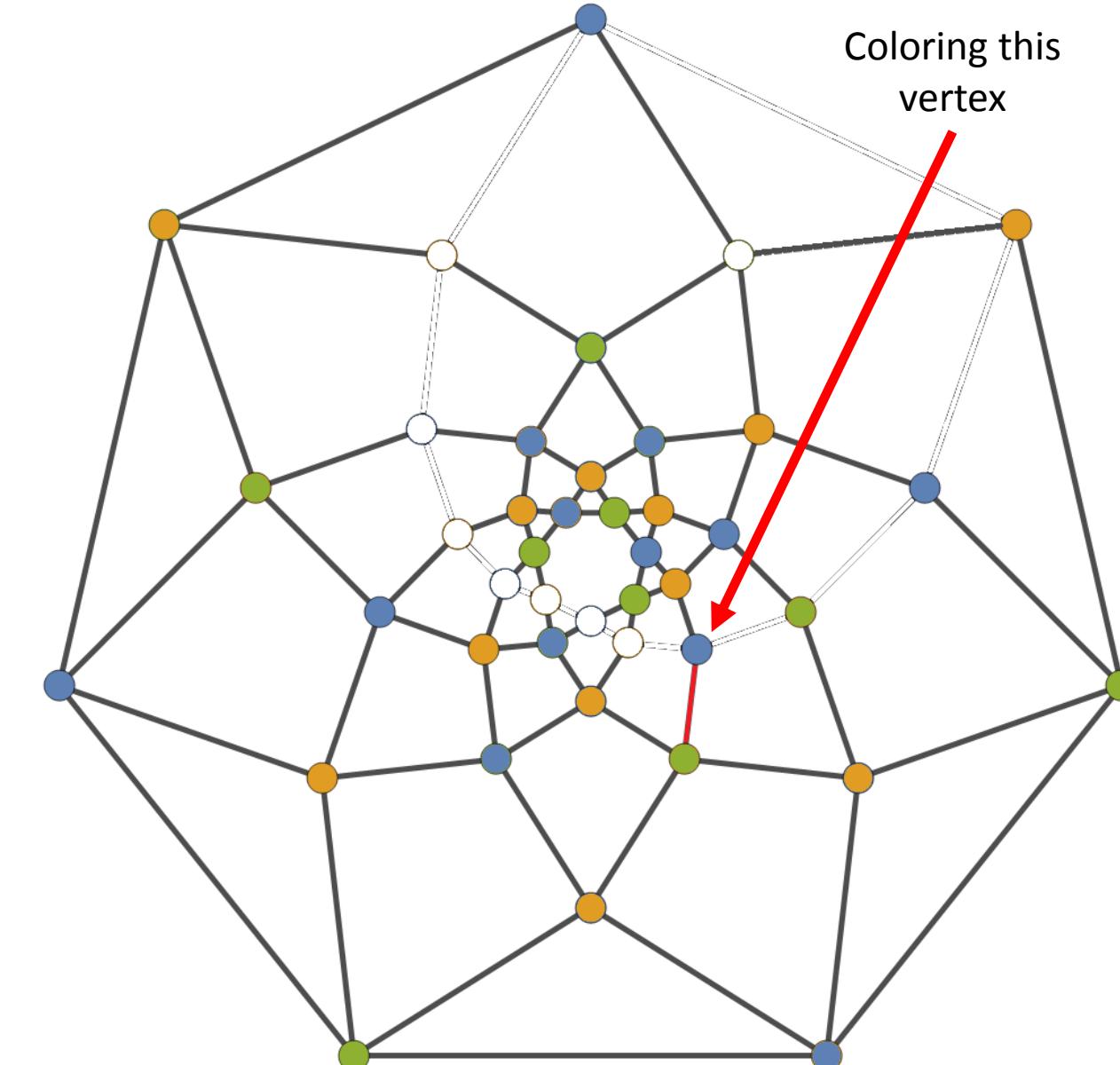
Go to the next vertex, I use
Green



Adding a random great circle

3 colors are used, so I will use Blue of V_{ext} for V

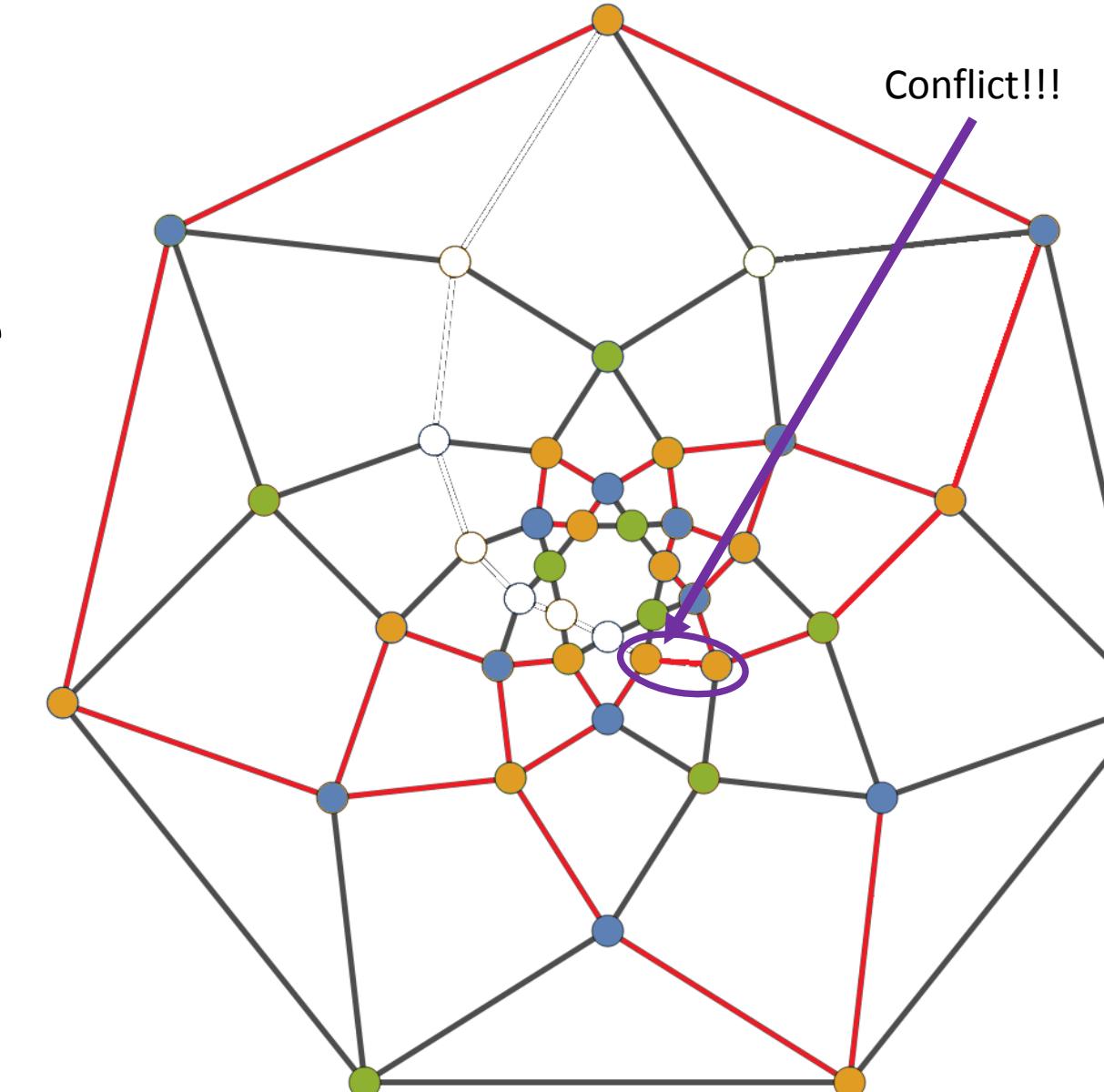
Because there is no Blue-Green cycle that starts at V and then returns back to V, so I will do switch Blue-Green chain at V



Adding a random great circle

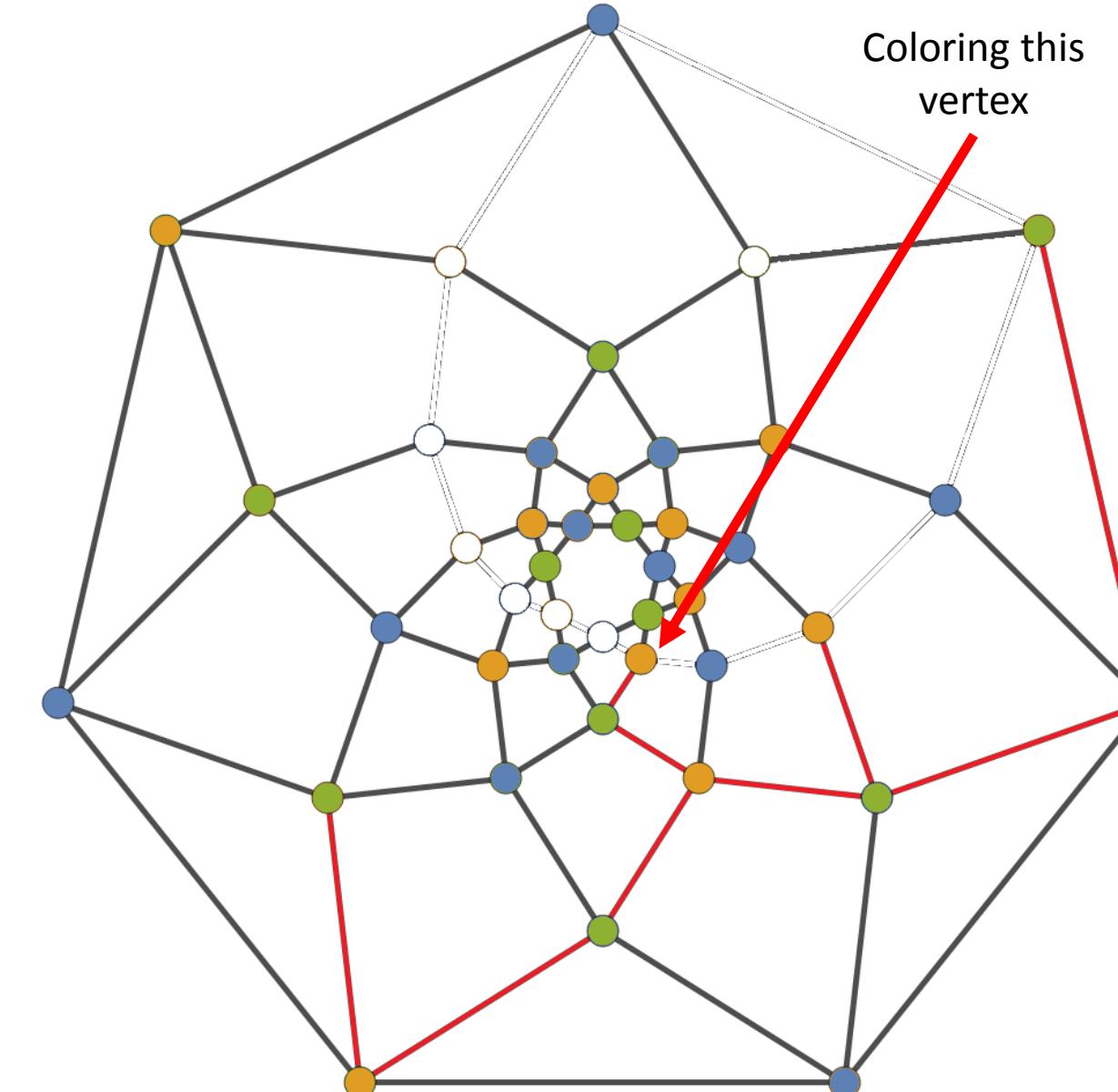
3 colors are used, so I will use Yellow on V_{ext} for V

There is a Yellow-Blue cycle in the graph starting at V if I colored V on Yellow



Adding a random great circle

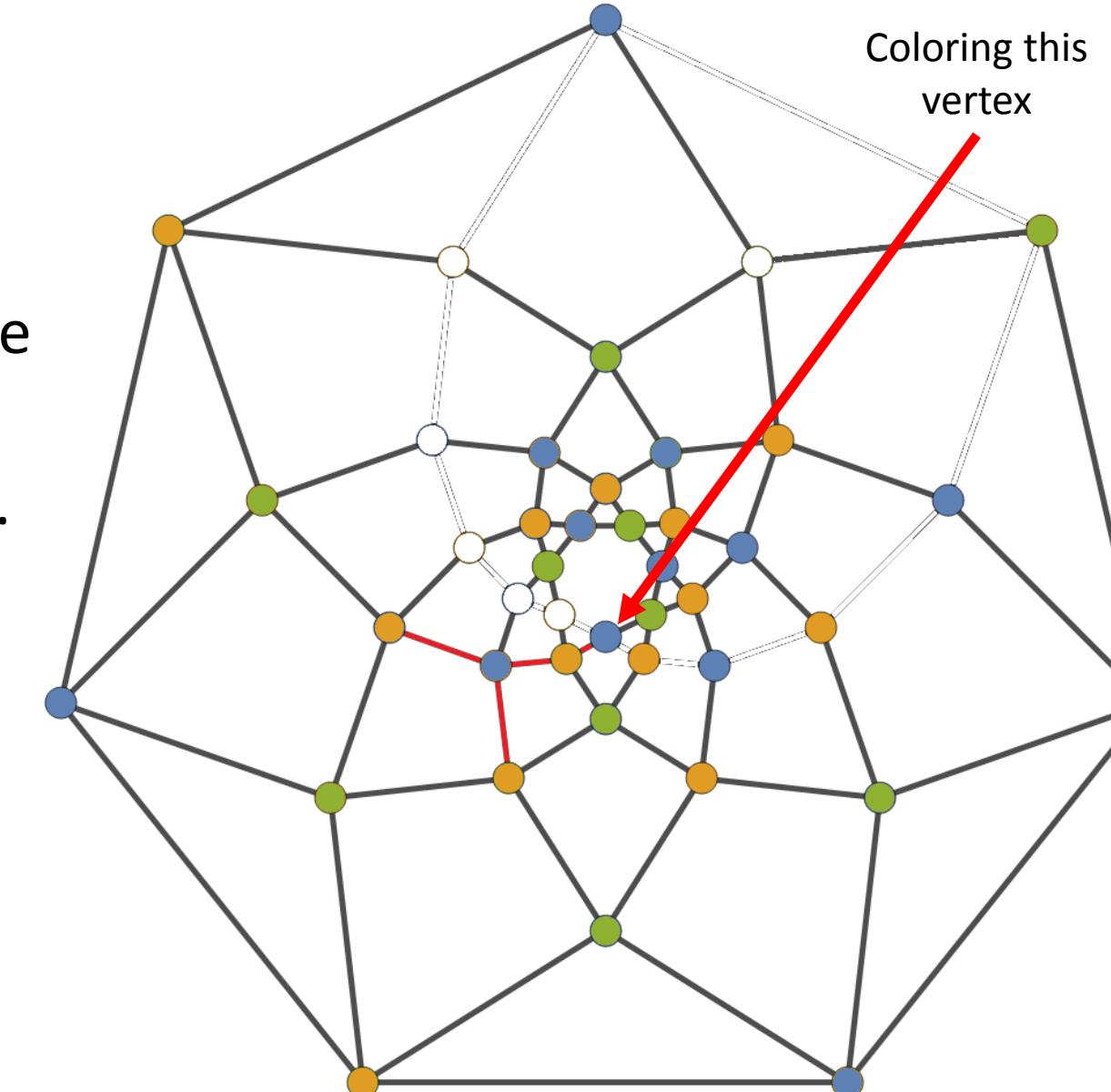
Therefore, I switch Yellow-Green on V and there is no Yellow-Green cycle at V



Adding a random great circle

3 colors are used, so I will use Blue on V_{ext} for V.

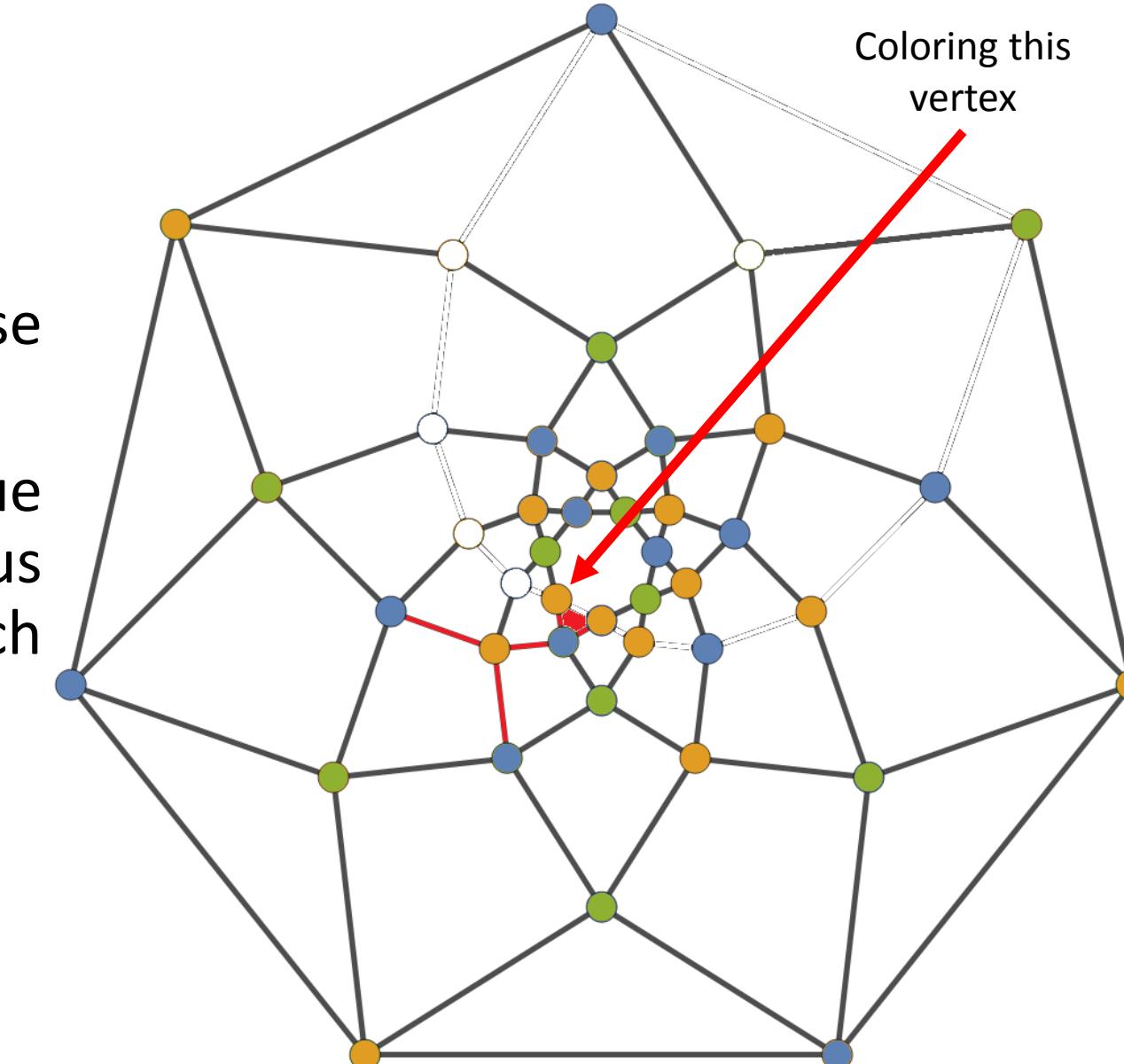
There is no Blue-Yellow cycle at V.



Adding a random great circle

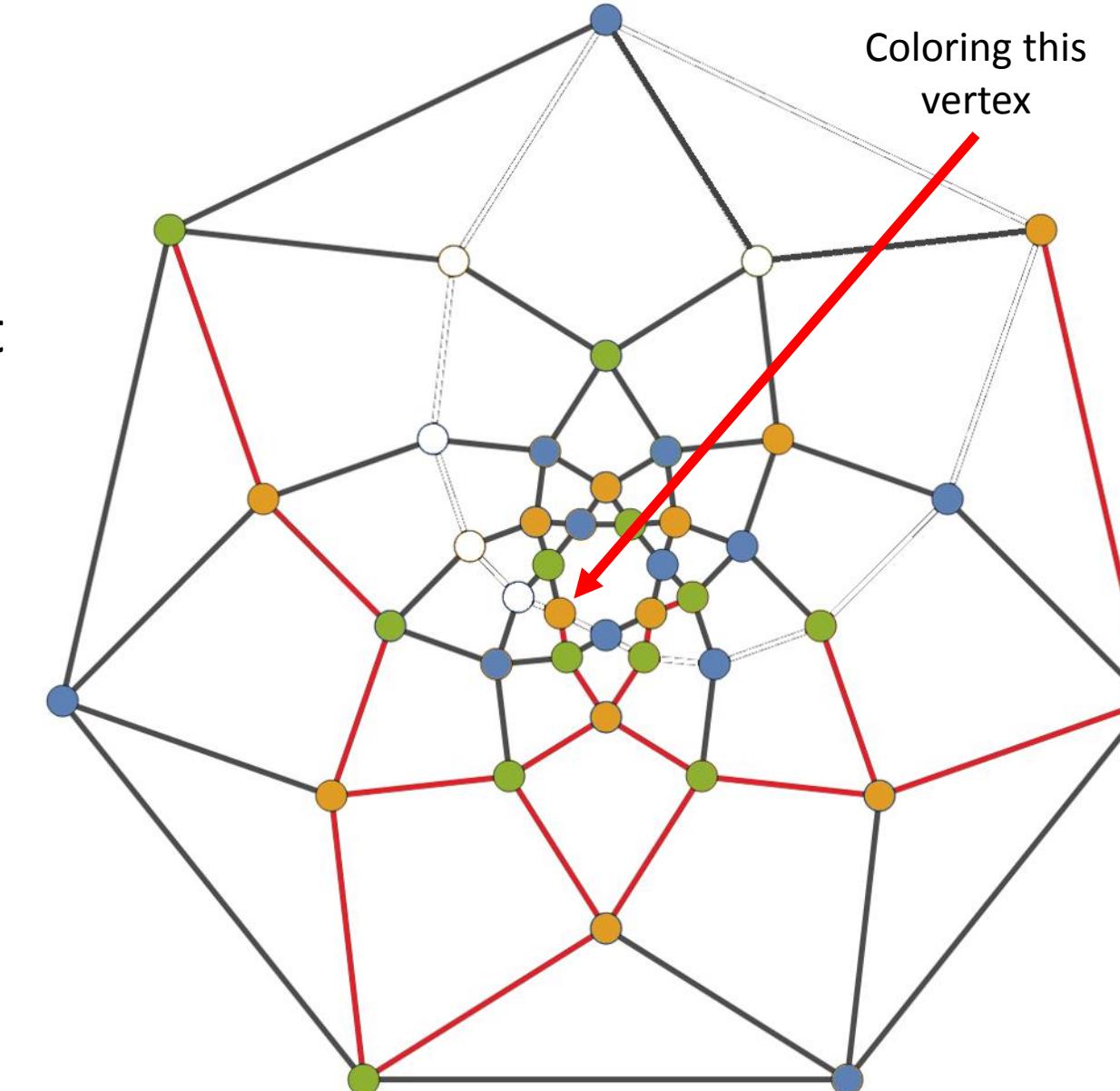
3 colors are used, so I will use Yellow on V_{ext} for V .

However, there is a Yellow-Blue starting at V – the previous pointed vertex – V , so I will switch Yellow-Green on V



Adding a random great circle

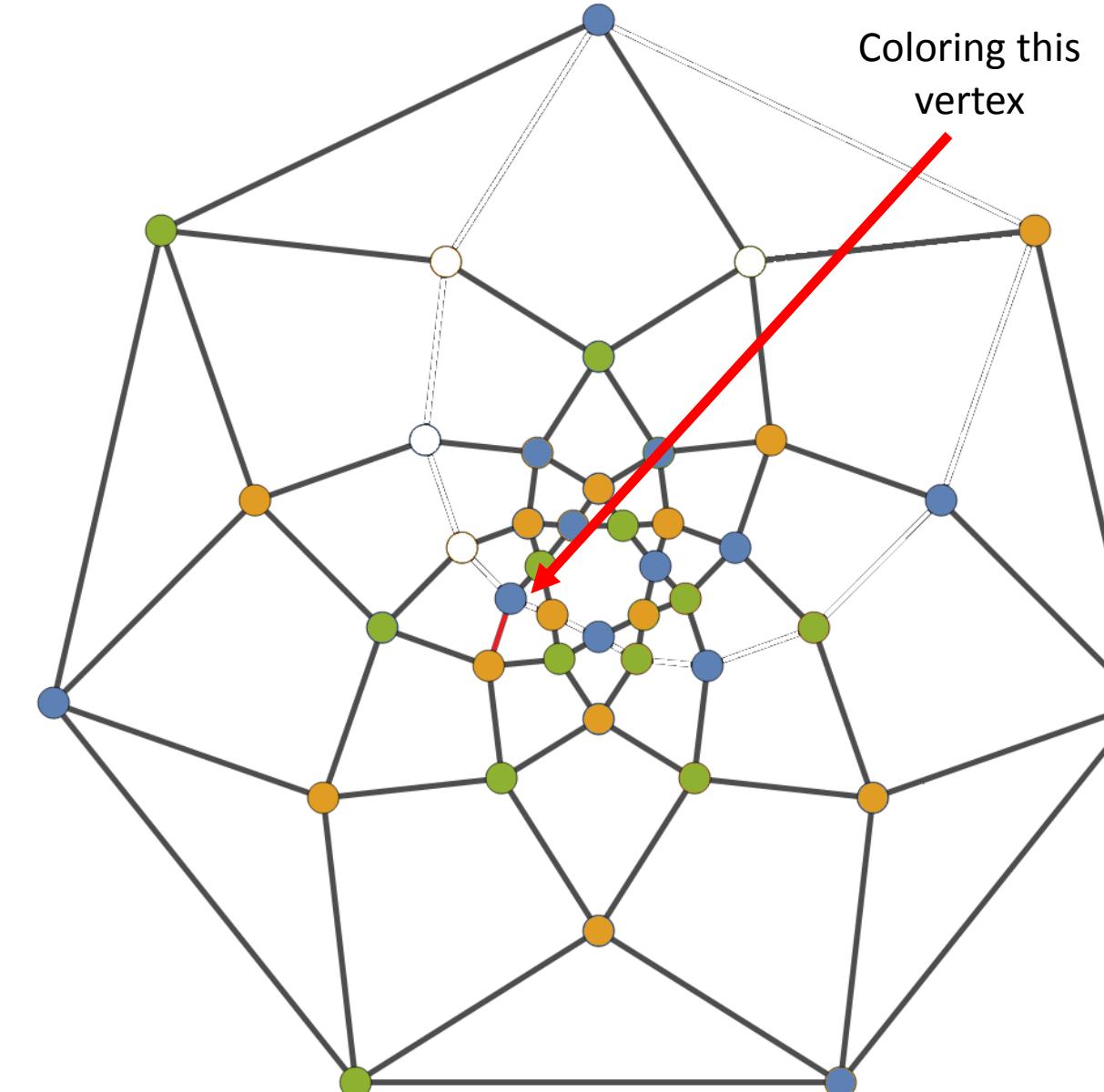
There is no Yellow-Green cycle at v



Adding a random great circle

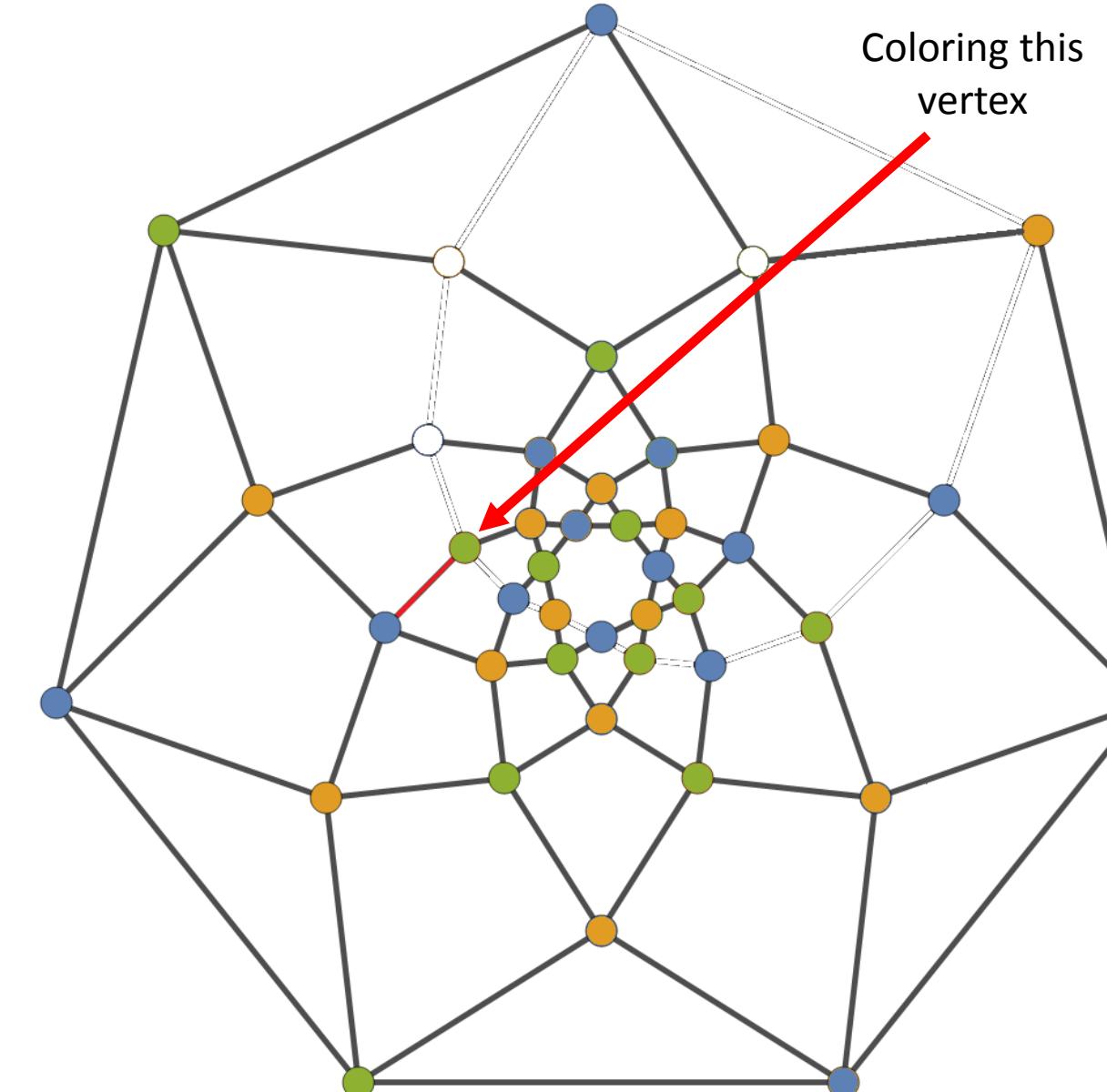
Go to the next vertex, there is no available color for V also.

I use Blue on V_{ext} for V and there is no Blue-Yellow cycle at V



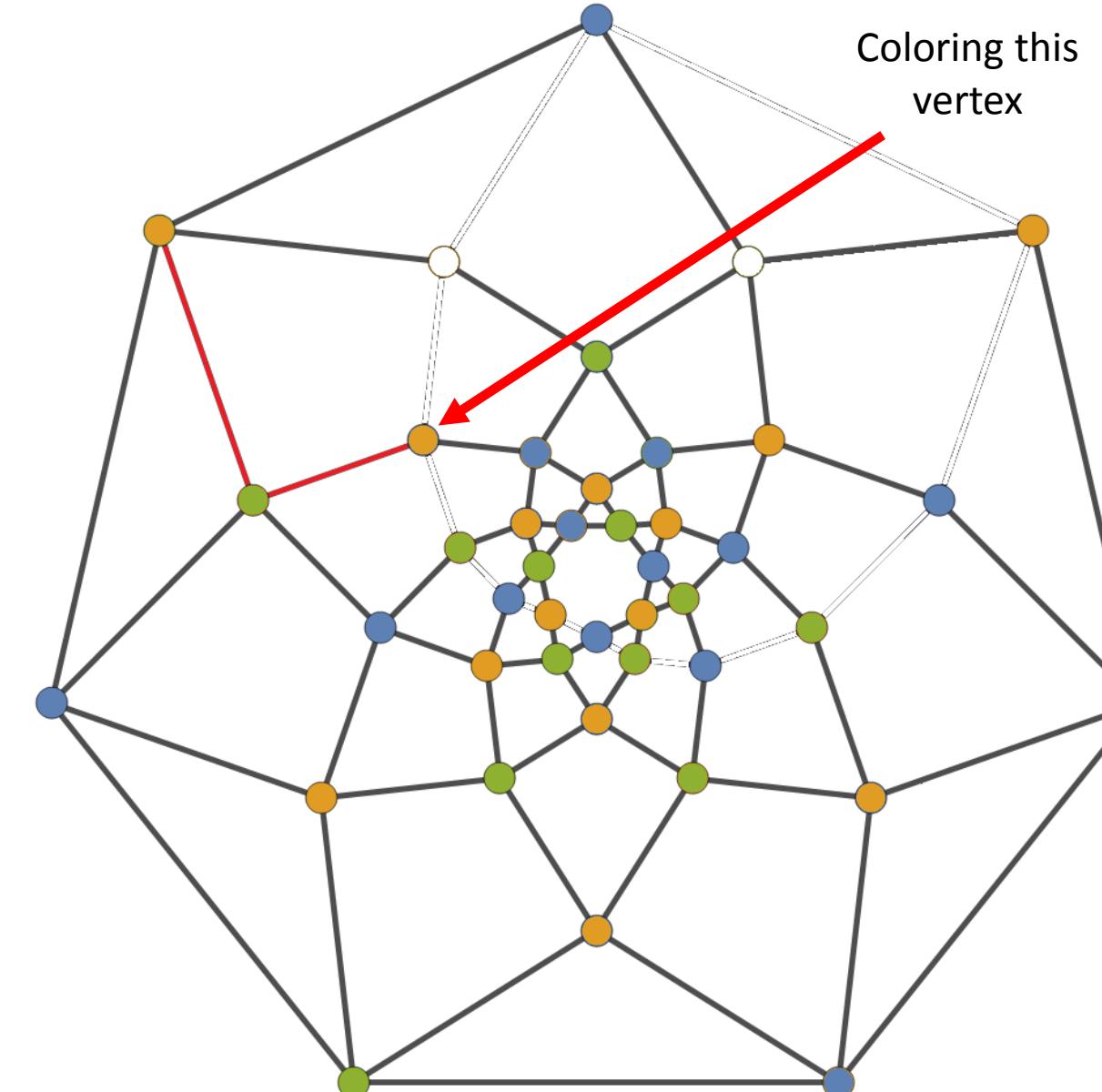
Adding a random great circle

Go to the next vertex, V uses Green on V_{ext} and there is no Green-Blue cycle at V



Adding a random great circle

Go to the next vertex, V uses Yellow on V_{ext} and there is no Yellow-Green cycle at V

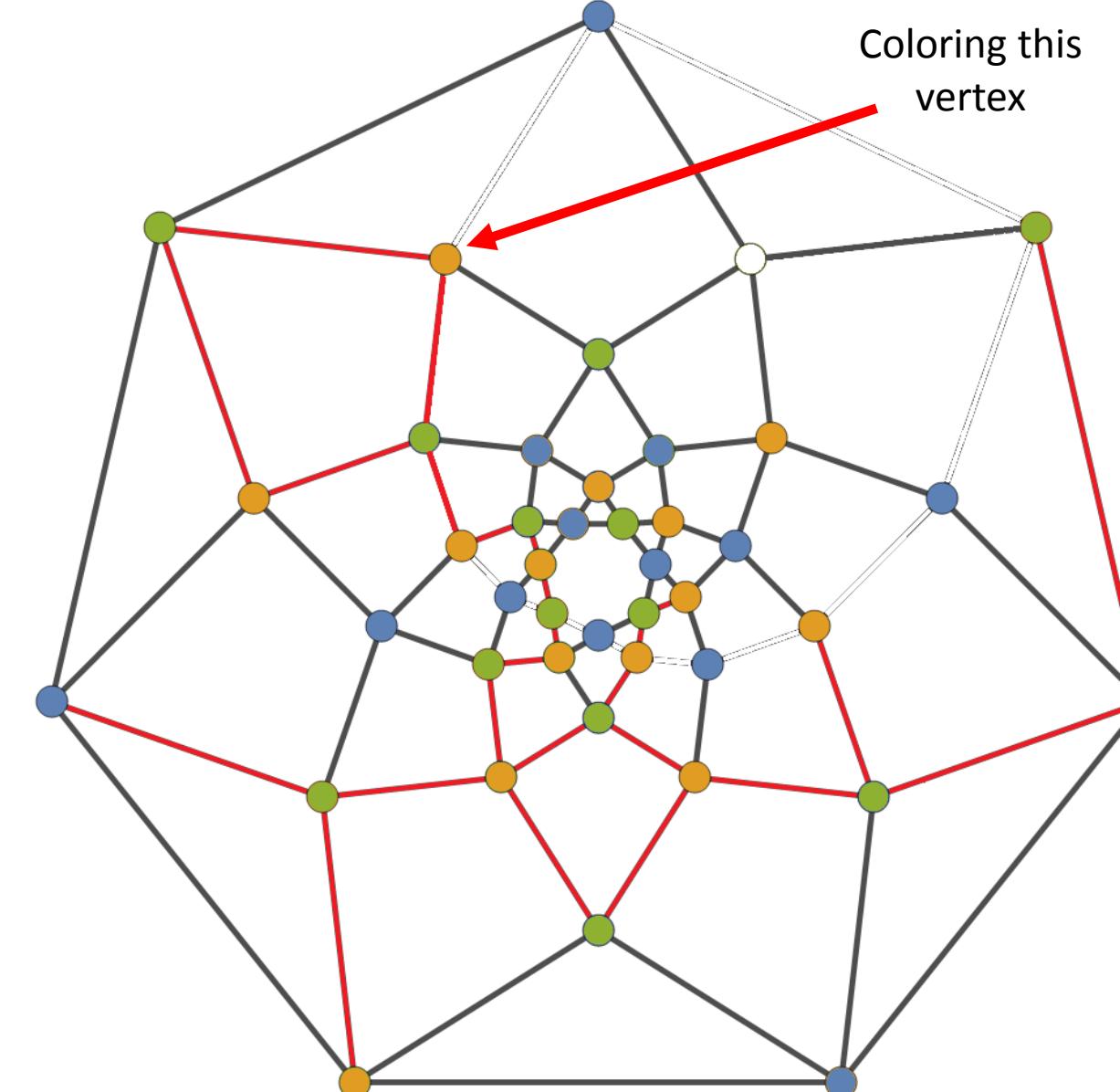


Adding a random great circle

Go to the next vertex, **we have a new case**. V_{ext} and the previous pointed vertex have the same color, so we can't use Yellow-Yellow because this chain only contain 1 color.

Therefore, I will do switch Yellow-Green on V and there is even cycle Yellow-Green at V.

If there is such a case that this Yellow-Green cycle has odd length, so I will use Blue for V and switch Blue-Yellow chain on V. There must be no Blue-Yellow cycle at V because Yellow-Green cycle has blocked 1 possible way to go back to V

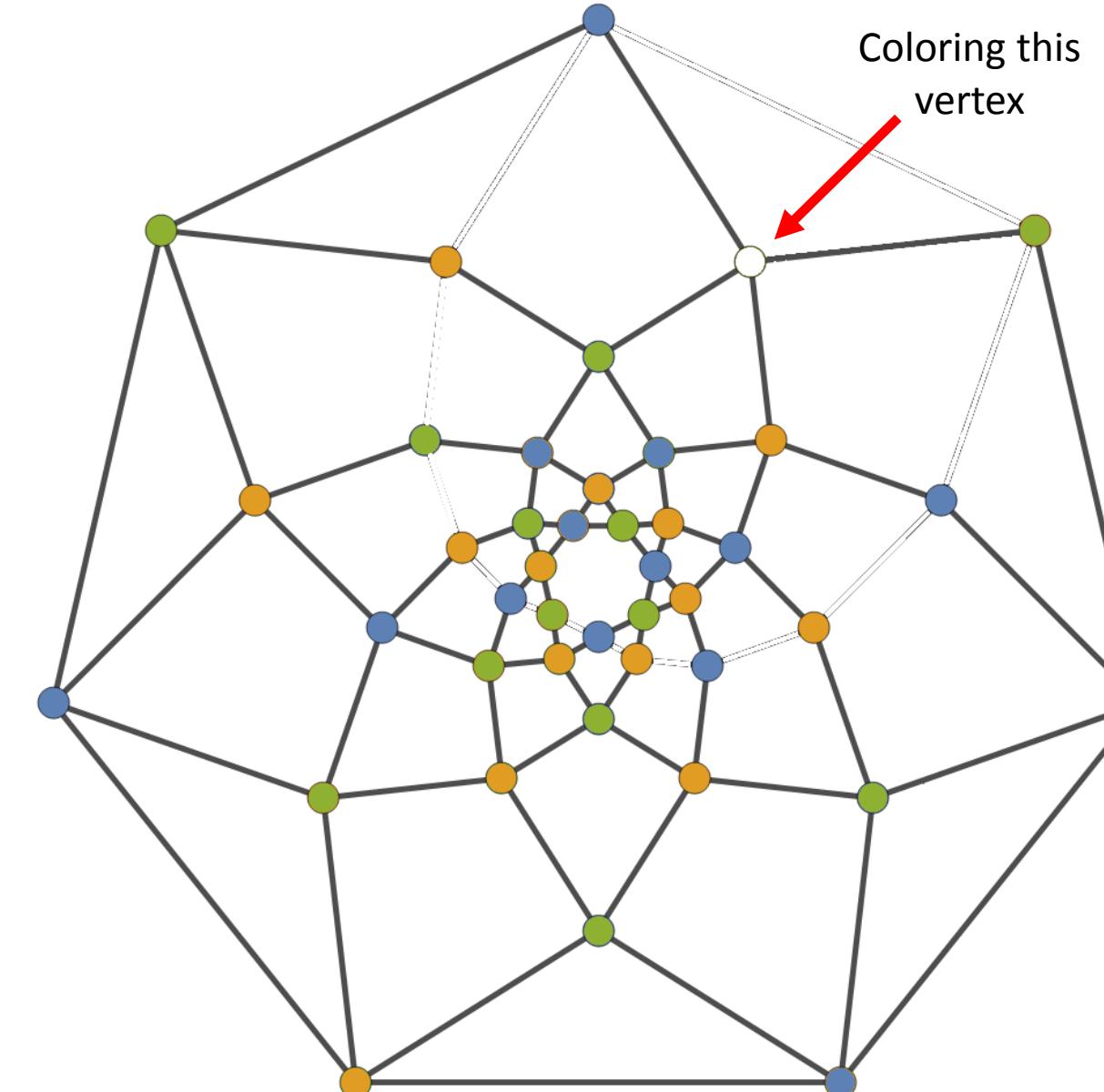


Adding a random great circle

Go to the next vertex, we have the case where:

- 3 colors are used
- There are many paths to V
- No V_{ext}

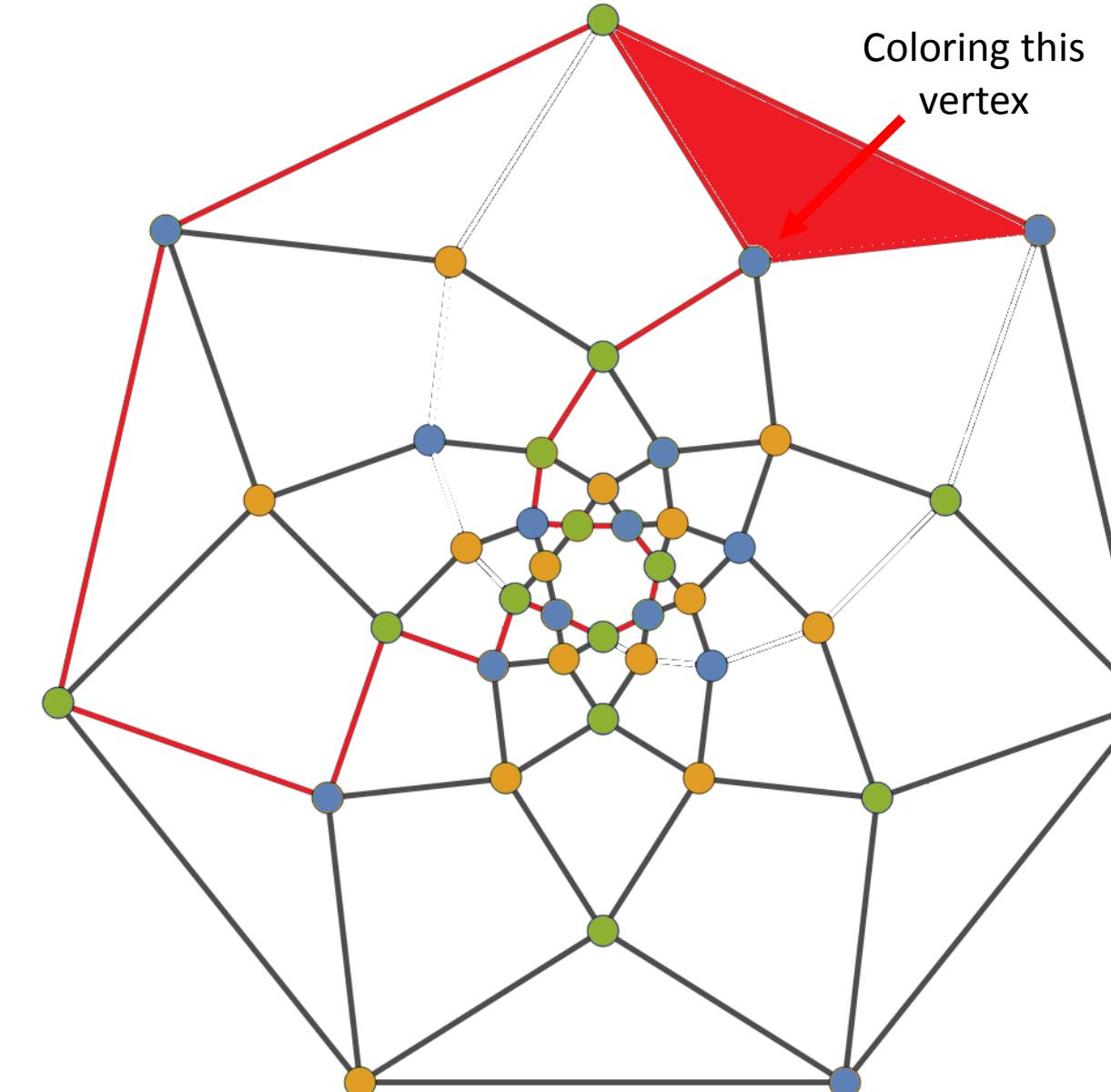
The only way to possibly color this vertex is to switch 1 of $C_3^2 = 3$ pairs of colors Yellow-Blue, Yellow-Green and Blue-Green on V



Adding a random great circle

Try Yellow-Blue chain first. I will use Blue for V and do switch Yellow-Blue chain at V

Unfortunately, there are 2 Green-Blue cycles with odd lengths at V. Therefore, I need to use another chain

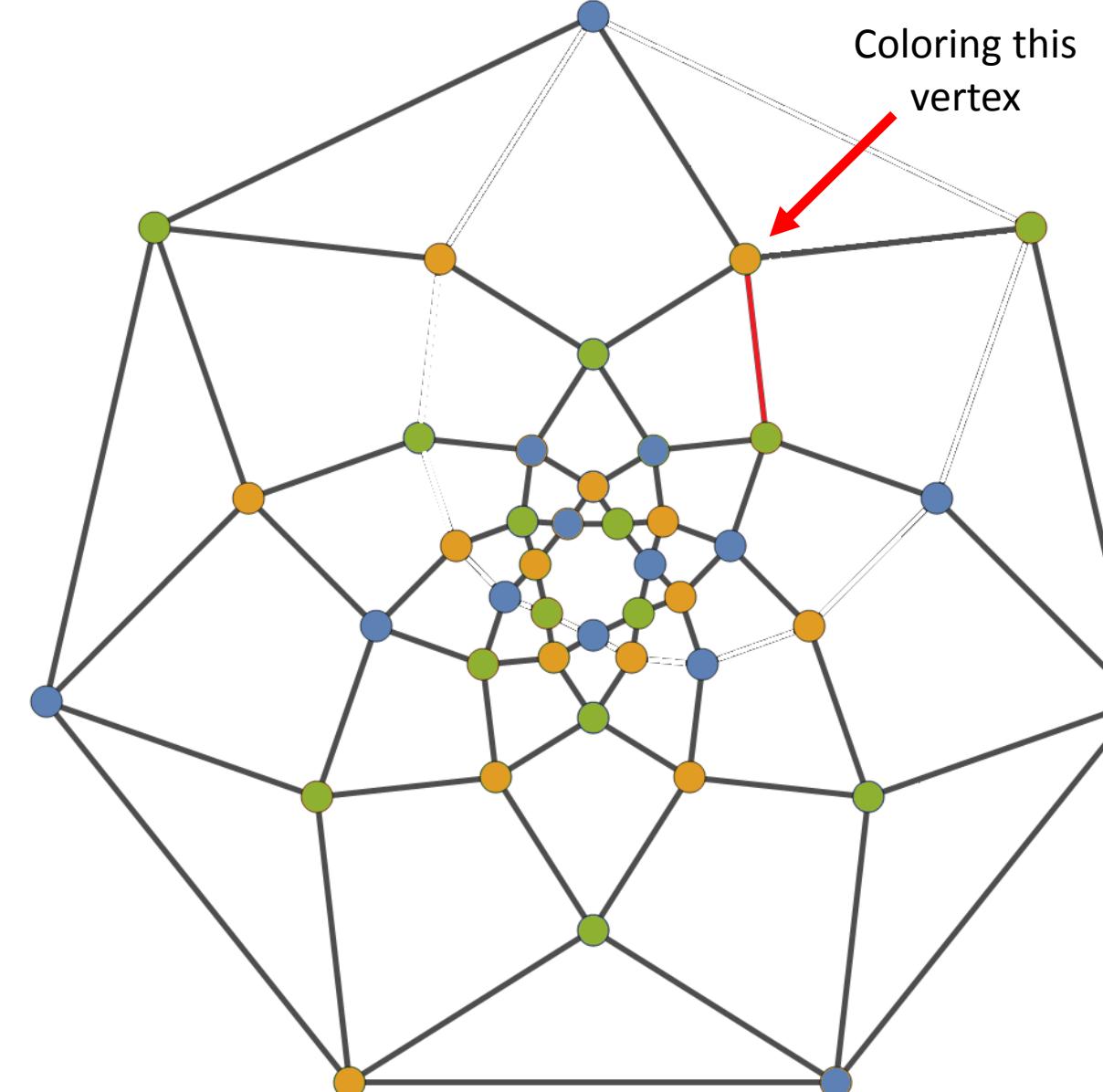


Adding a random great circle

However, switching chain on V can be Yellow-Blue blocked

Yellow-Green done because out all the

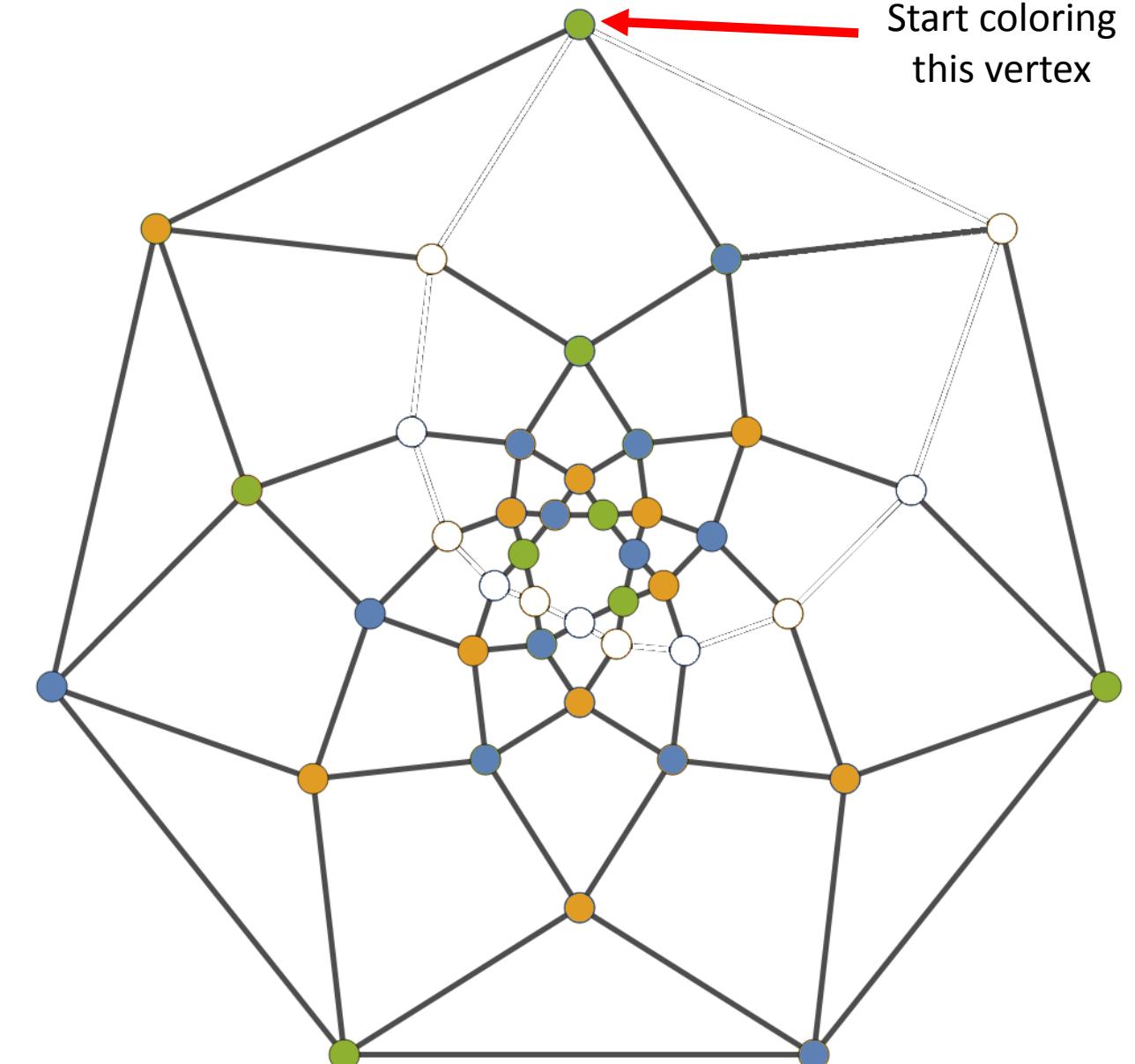
DONE!!!



A case where the coloring
rule doesn't work

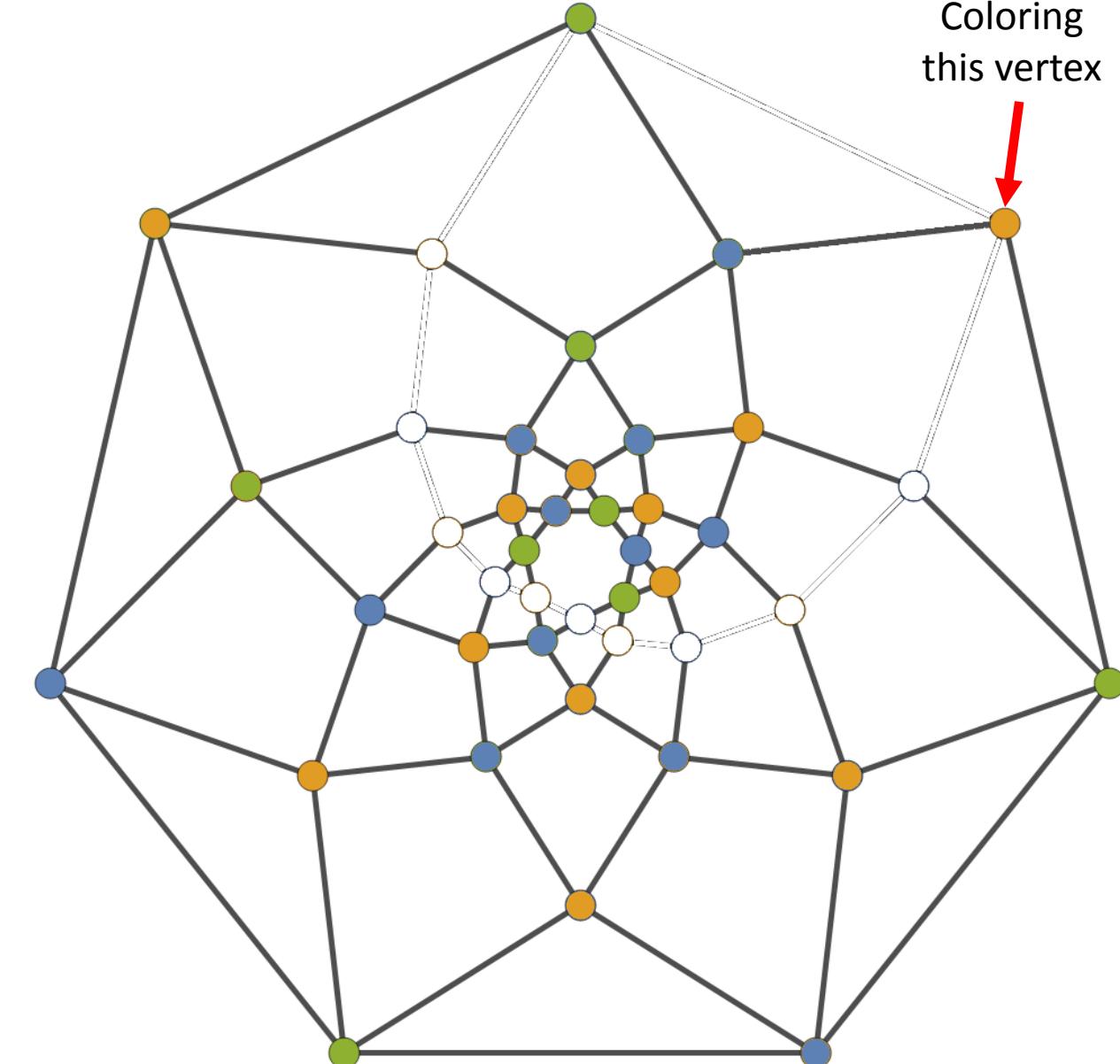
Adding a random great circle

I will start coloring at the vertex like the figure. Green is used because the neighboring vertices only use Yellow and Blue



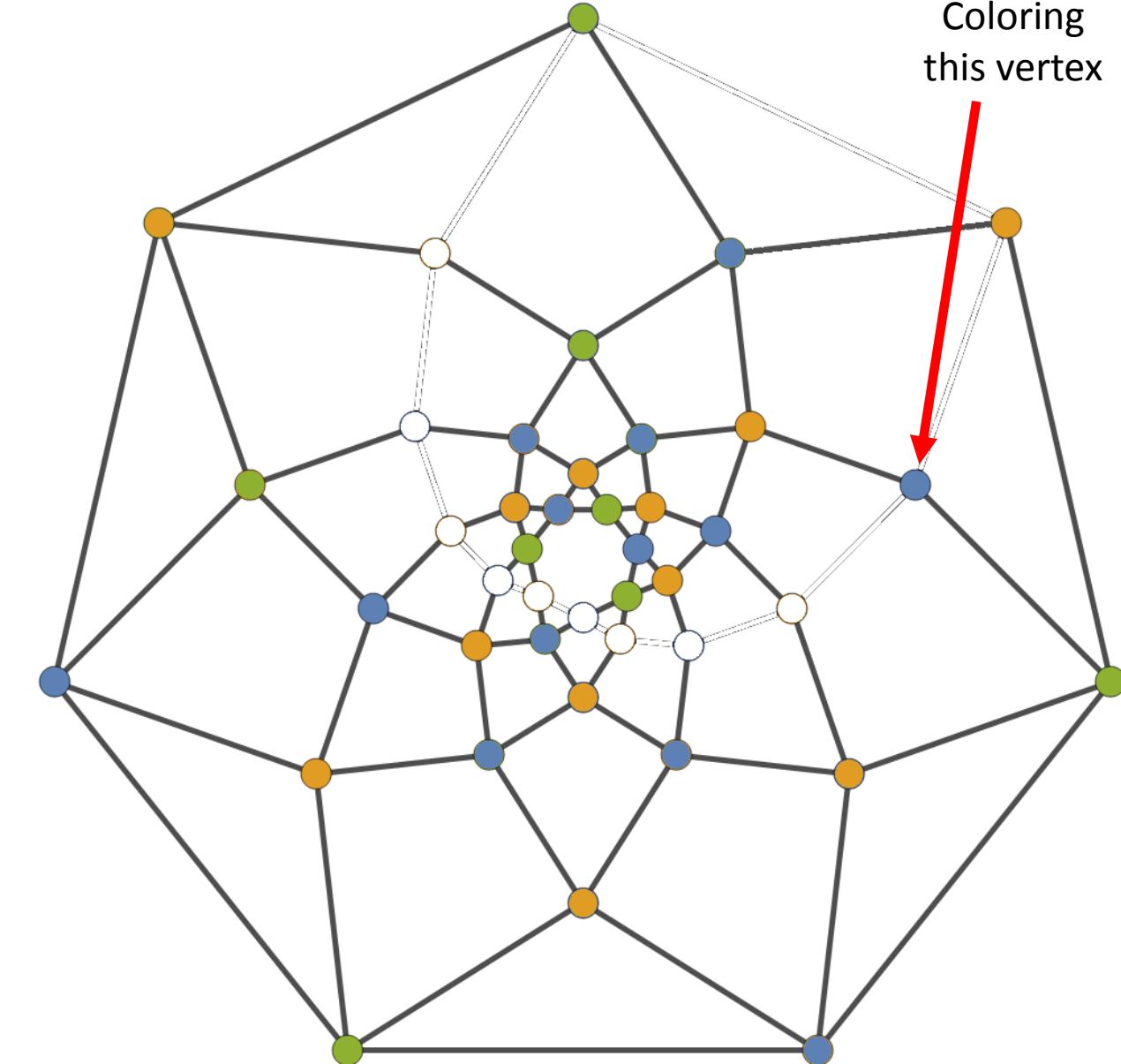
Adding a random great circle

Go to the next vertex, I use
Yellow



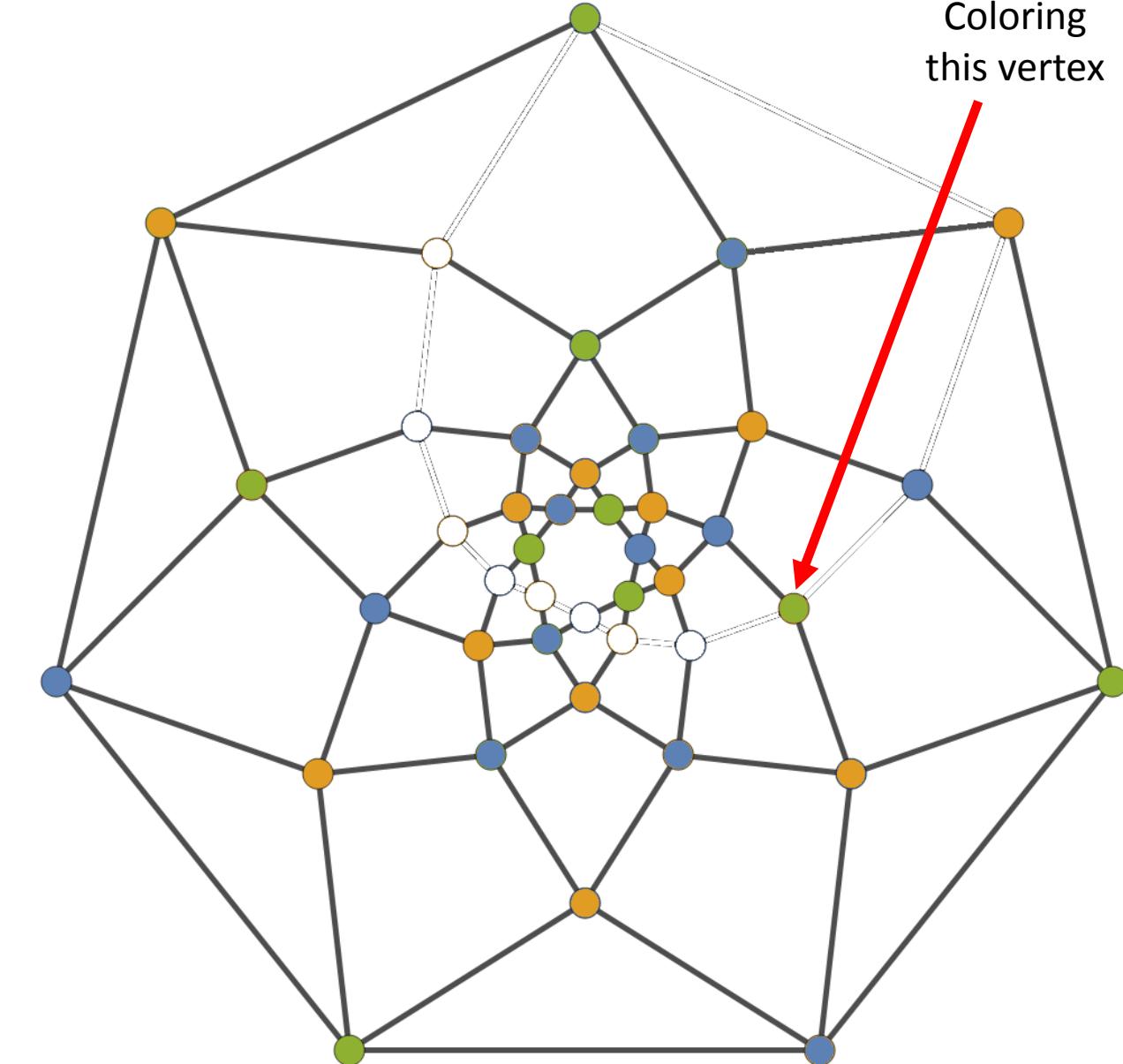
Adding a random great circle

Go to the next vertex, I use Blue



Adding a random great circle

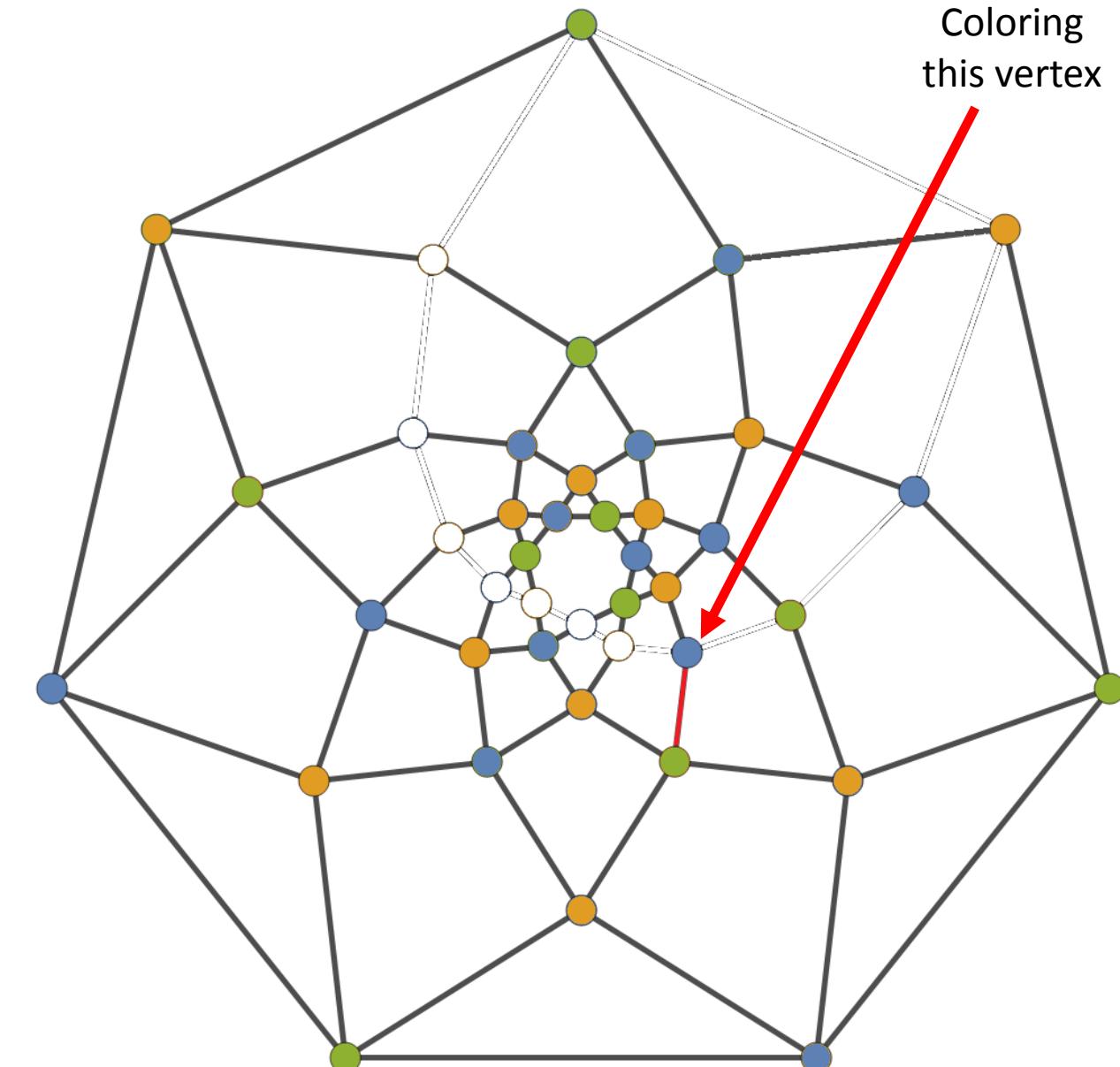
Go to the next vertex, I use Blue



Adding a random great circle

3 colors are used, so I will use Blue of V_{ext} for V and do switch Blue-Green chain on V

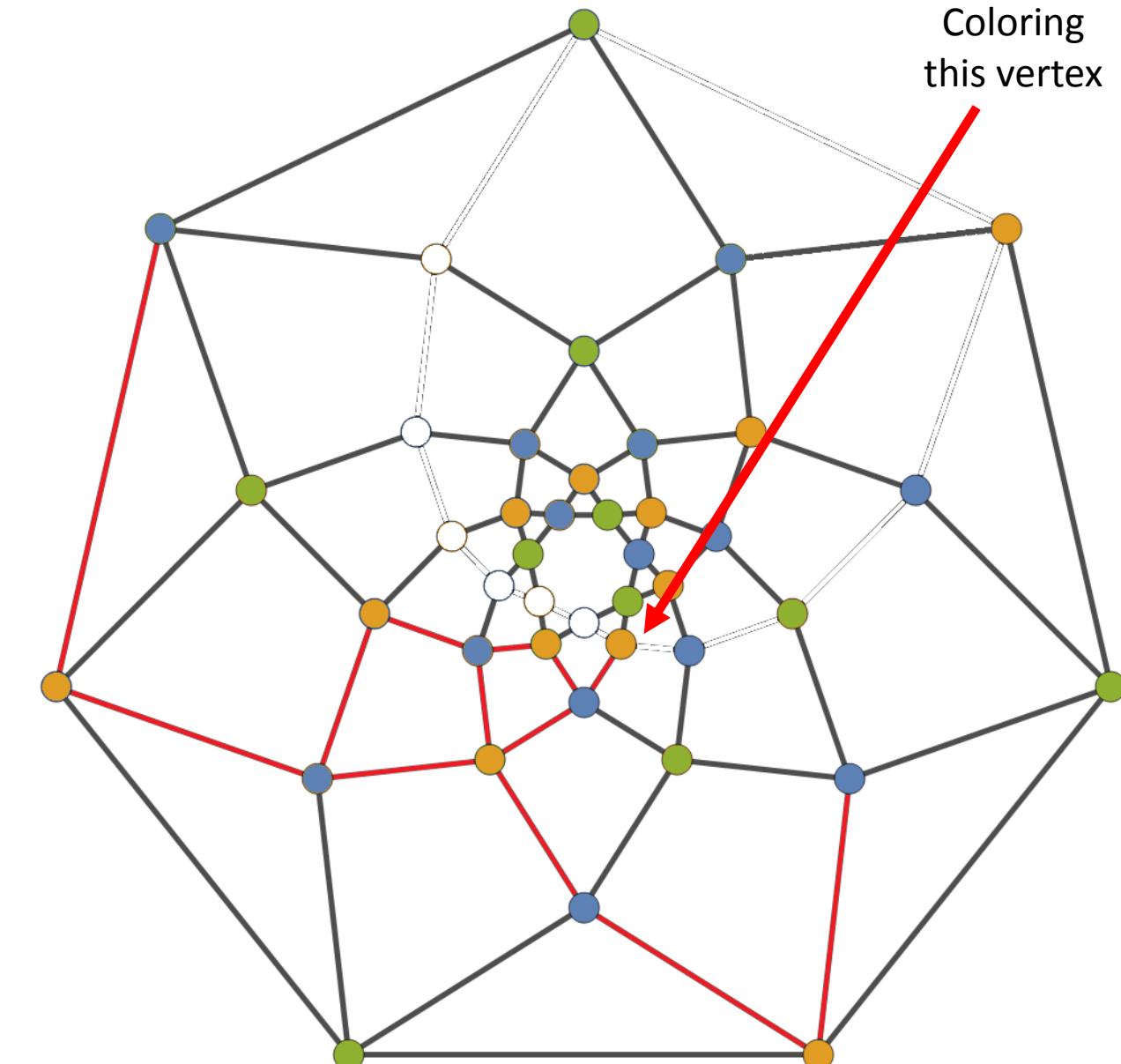
There is no Blue-Green cycle at V



Adding a random great circle

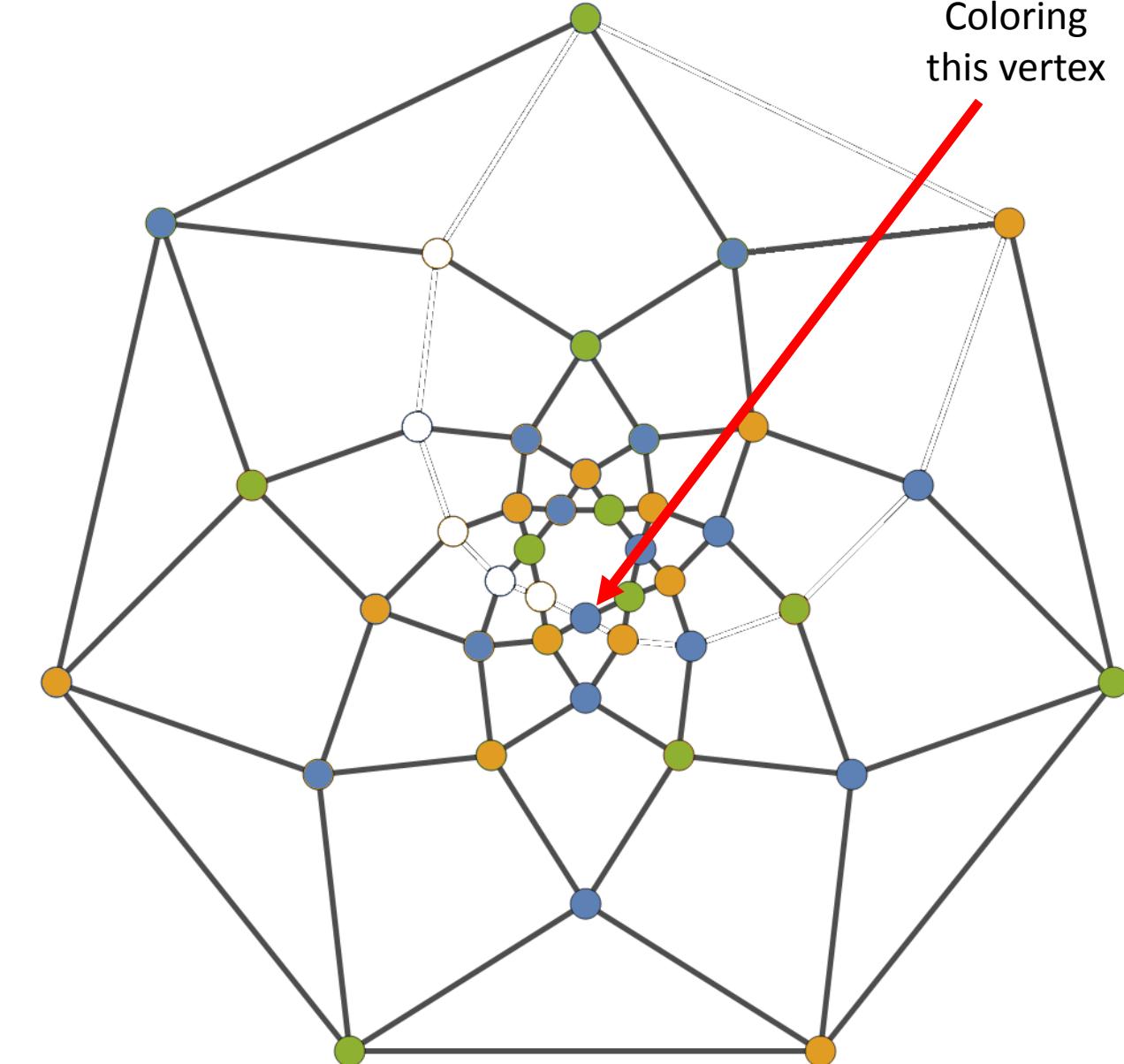
3 colors are used, so I will use Yellow of V_{ext} for V and do switch Yellow-Blue chain on V

There is no Yellow-Blue cycle at V



Adding a random great circle

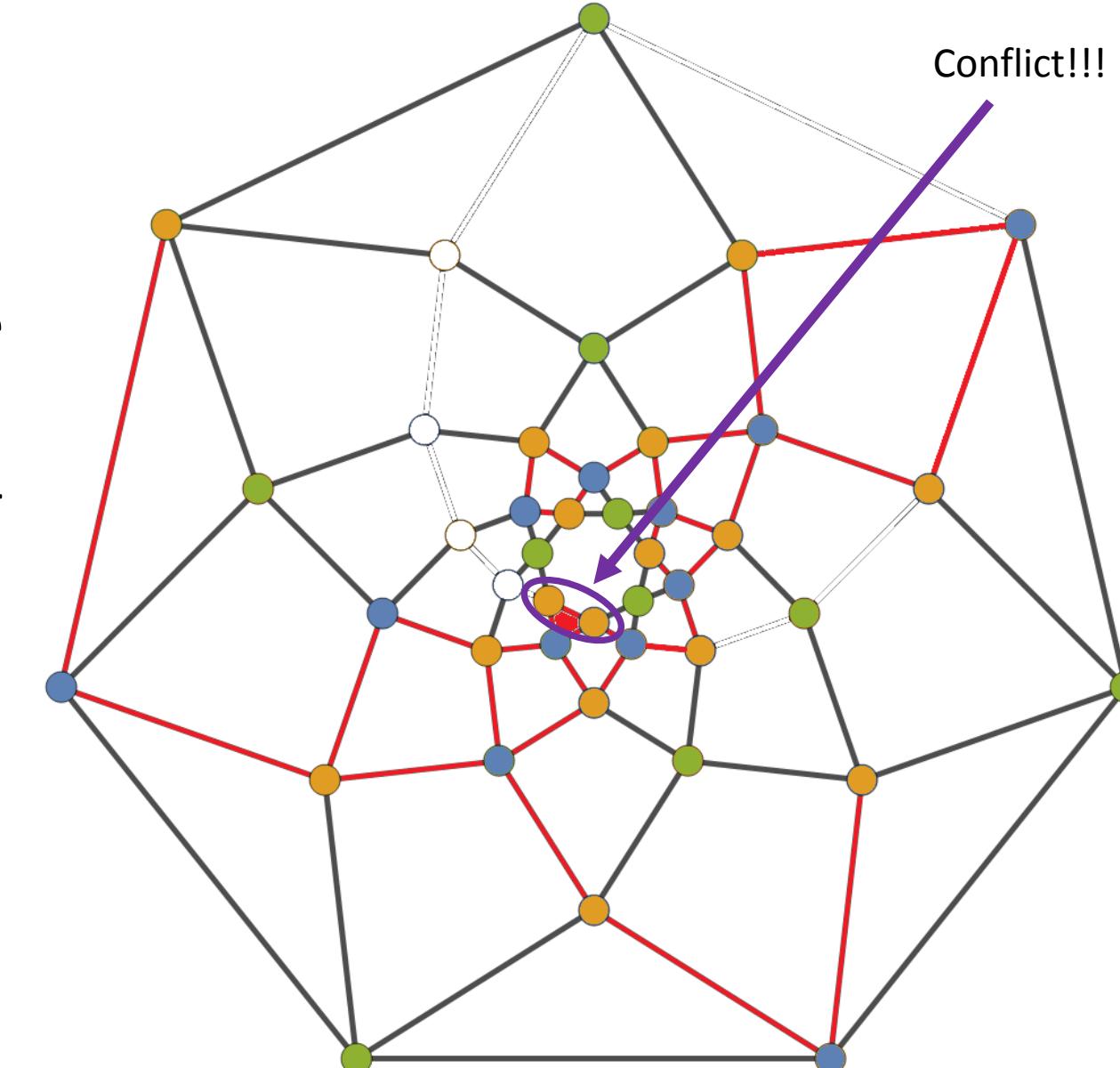
Go to the next vertex, I use Blue



Adding a random great circle

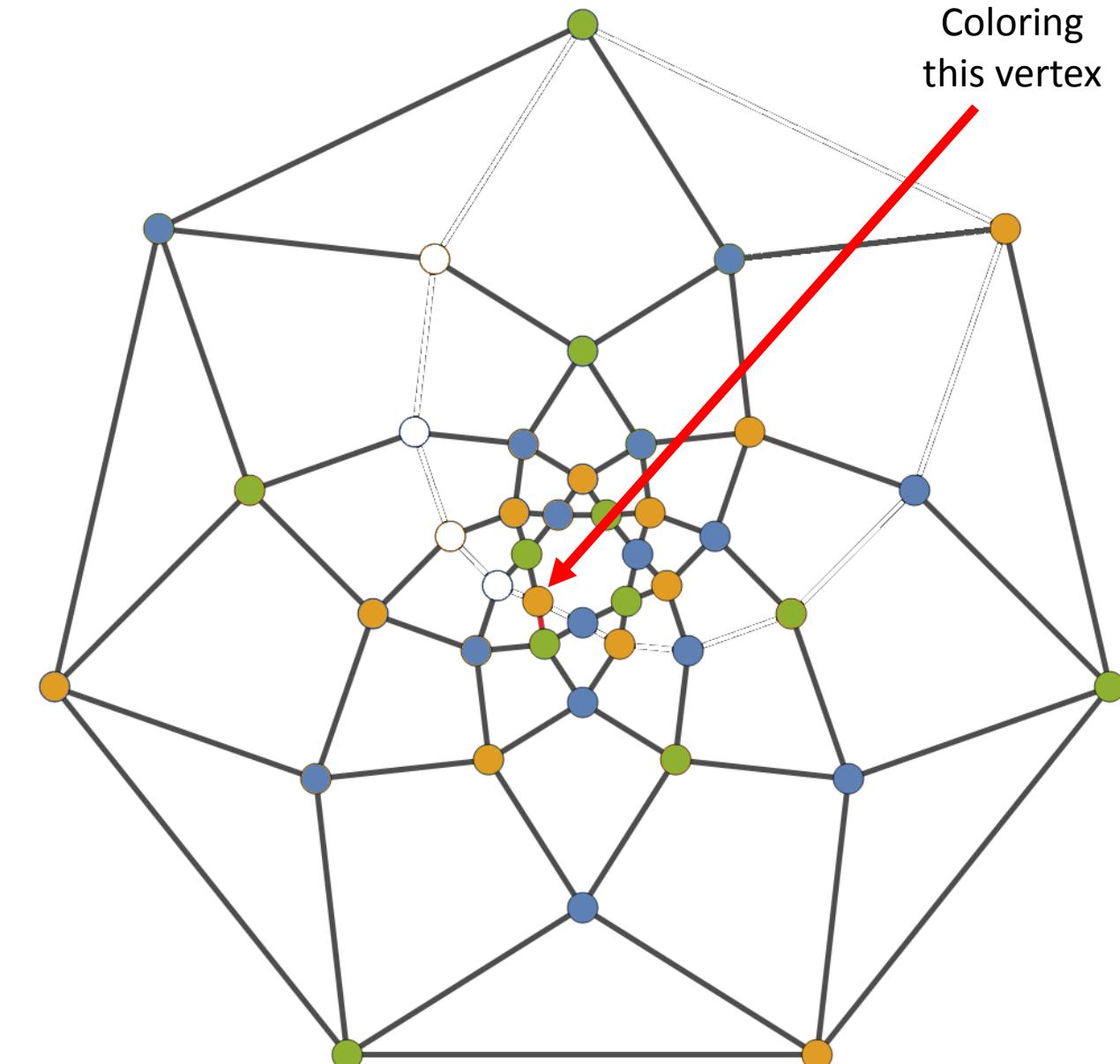
3 colors are used, so I will use Yellow of V_{ext} for V

However, there is an odd Yellow-Blue cycle at V .



Adding a random great circle

Therefore, I will switch Yellow-Green chain at V.

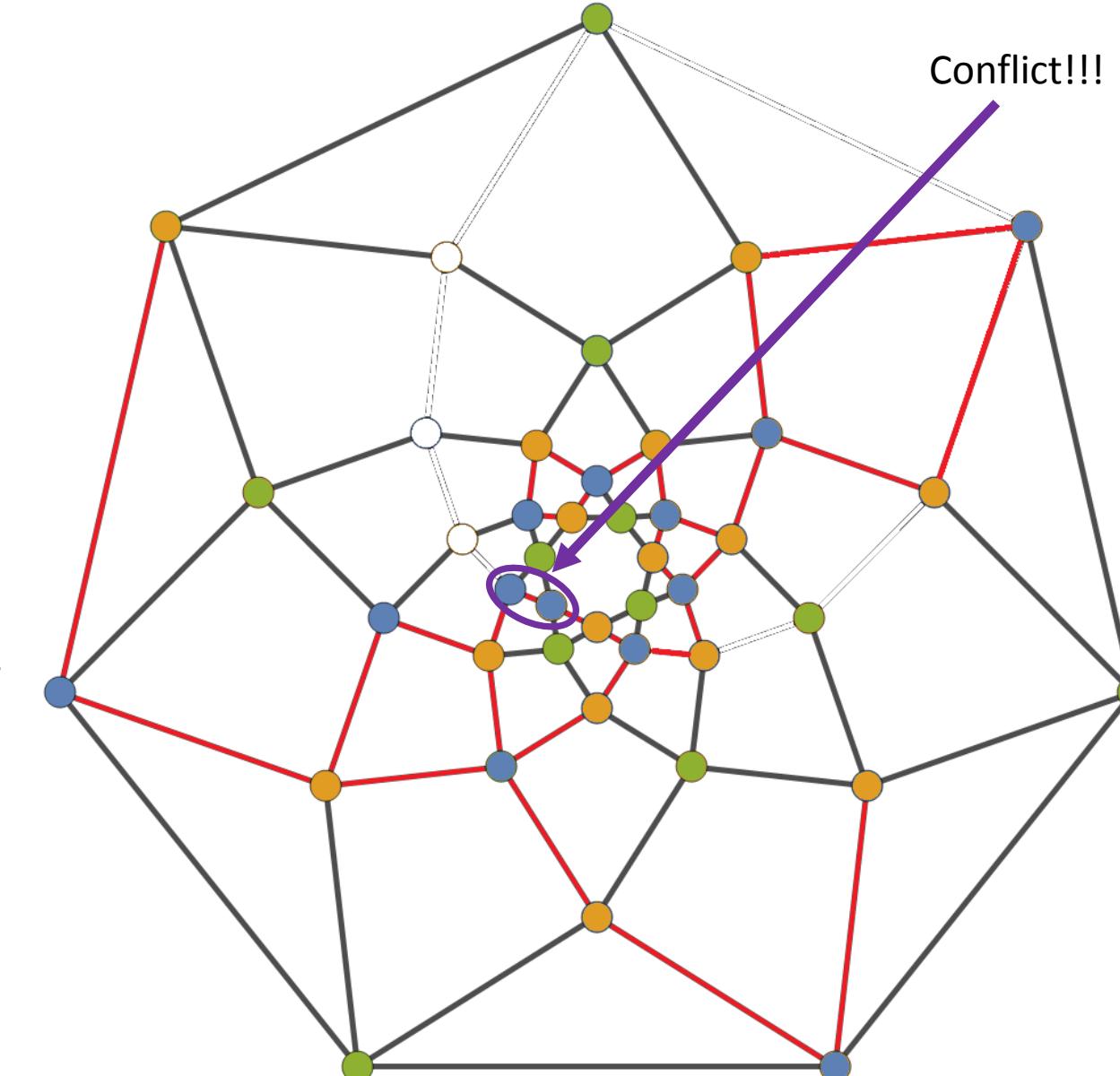


Adding a random great circle

Go to the next vertex, here is the most difficult case!!!

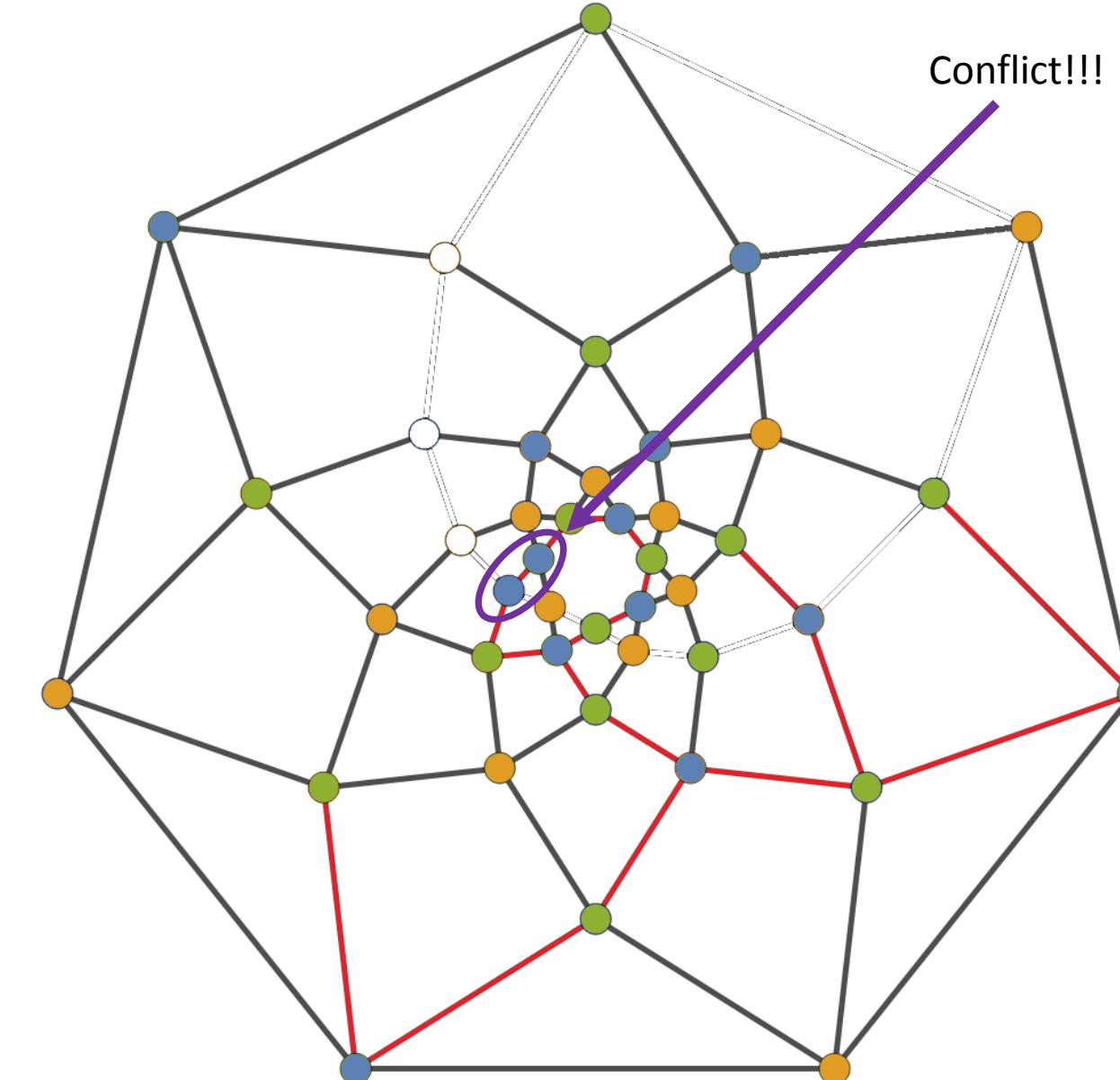
3 colors are used, so I will use Blue of V_{ext} for V

However, there is an odd Yellow-Blue cycle at V.



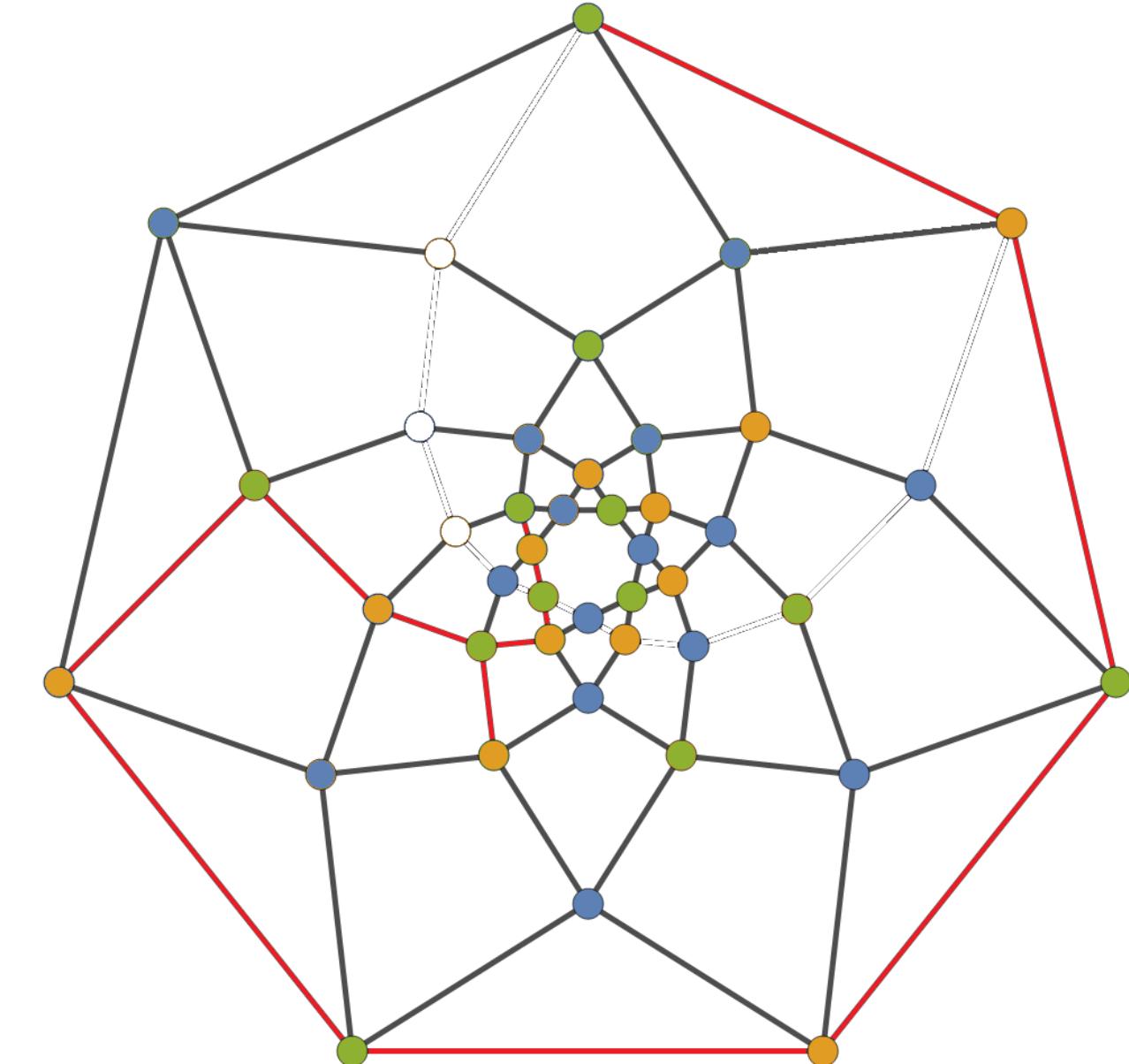
Adding a random great circle

Try Blue-Green on V but there is
an odd Blue-Green cycle at V



Adding a random great circle

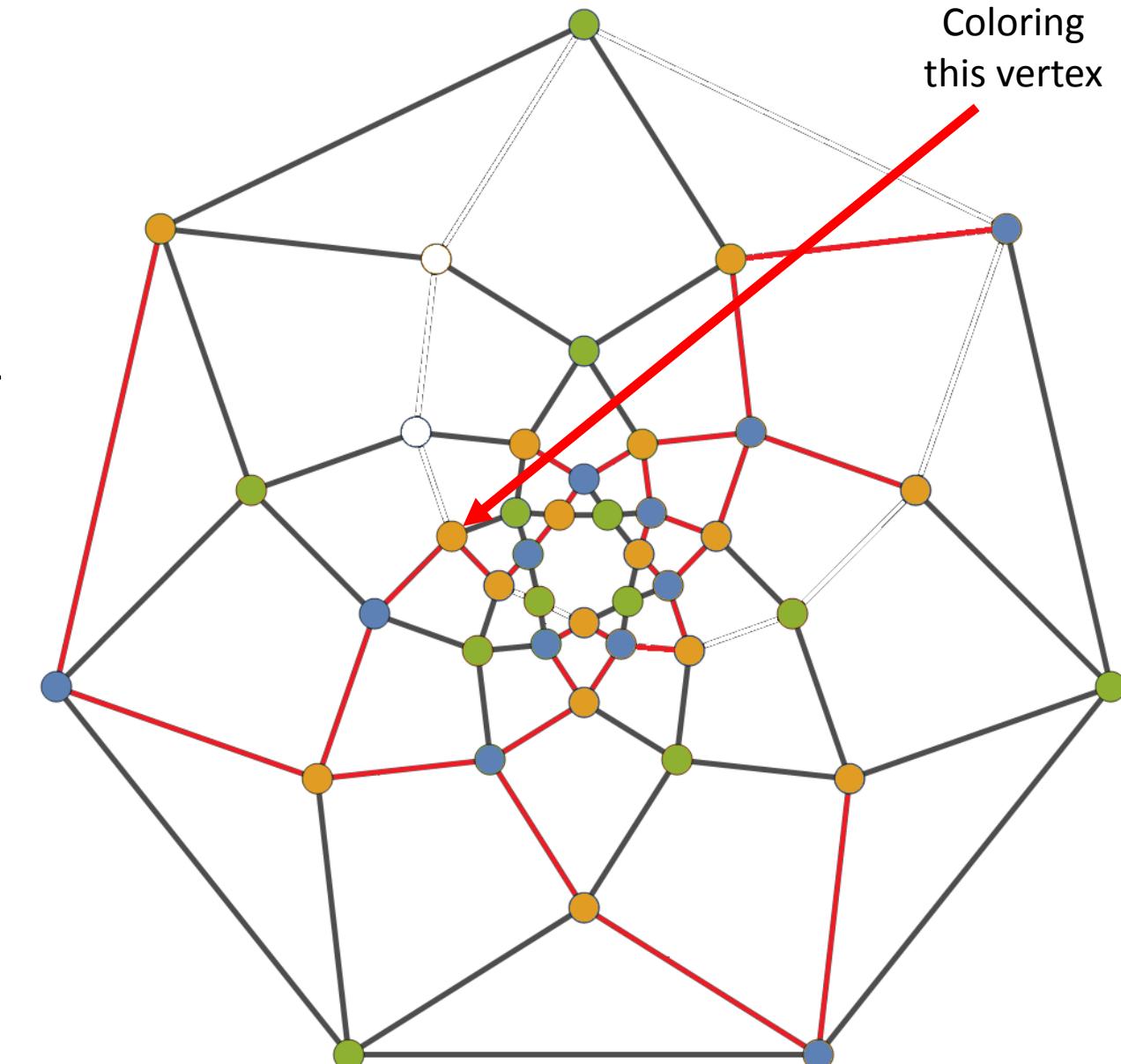
Therefore, I keep Blue on V ,
replace V_{ext} to Yellow and switch
Yellow-Green at V_{ext}



Adding a random great circle

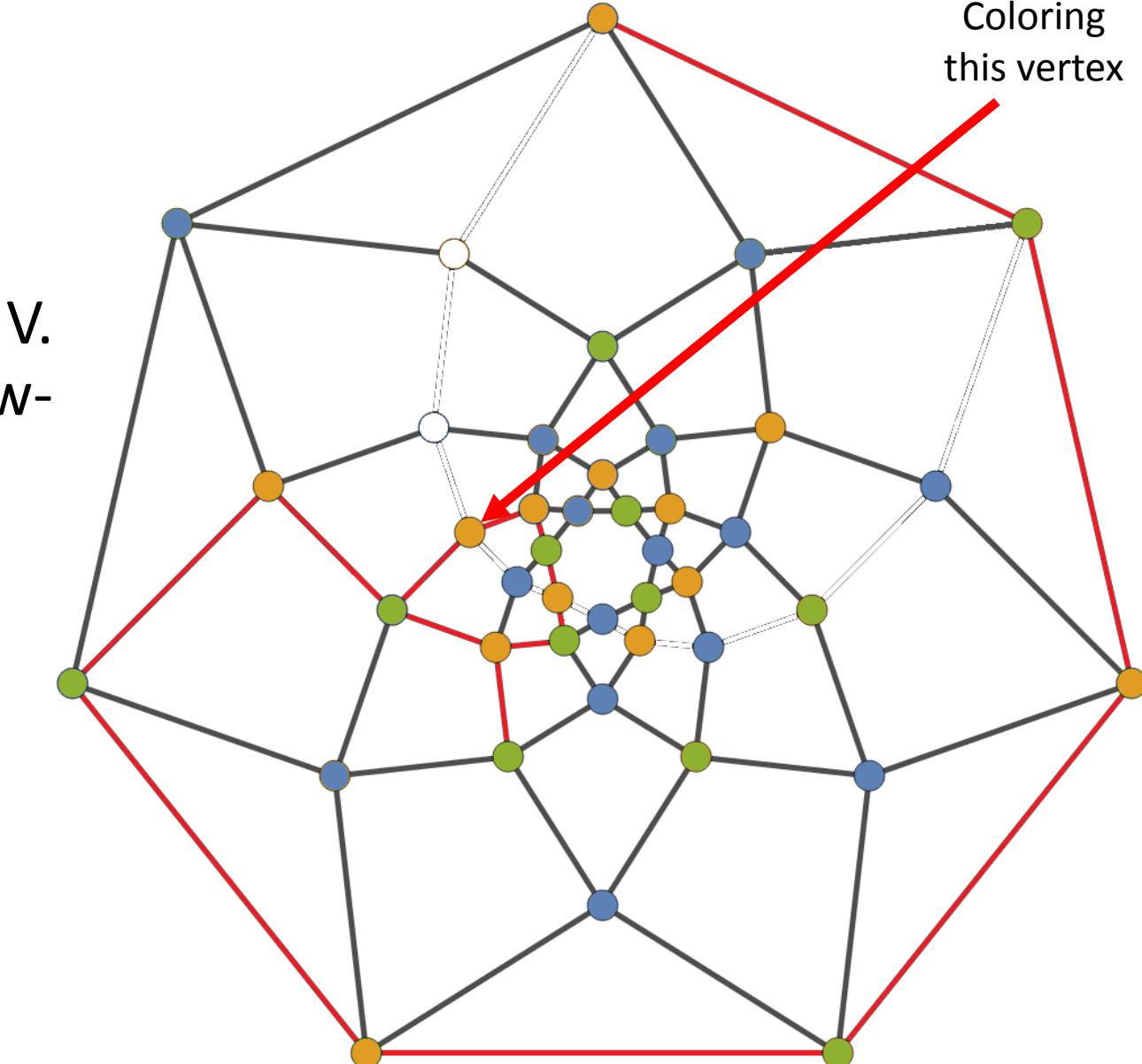
Go to the next vertex, 3 colors are used, so I will use Yellow of V_{ext} for V and do switch Yellow-Blue at V

However, there is a odd Yellow-Blue cycle at V



Adding a random great circle

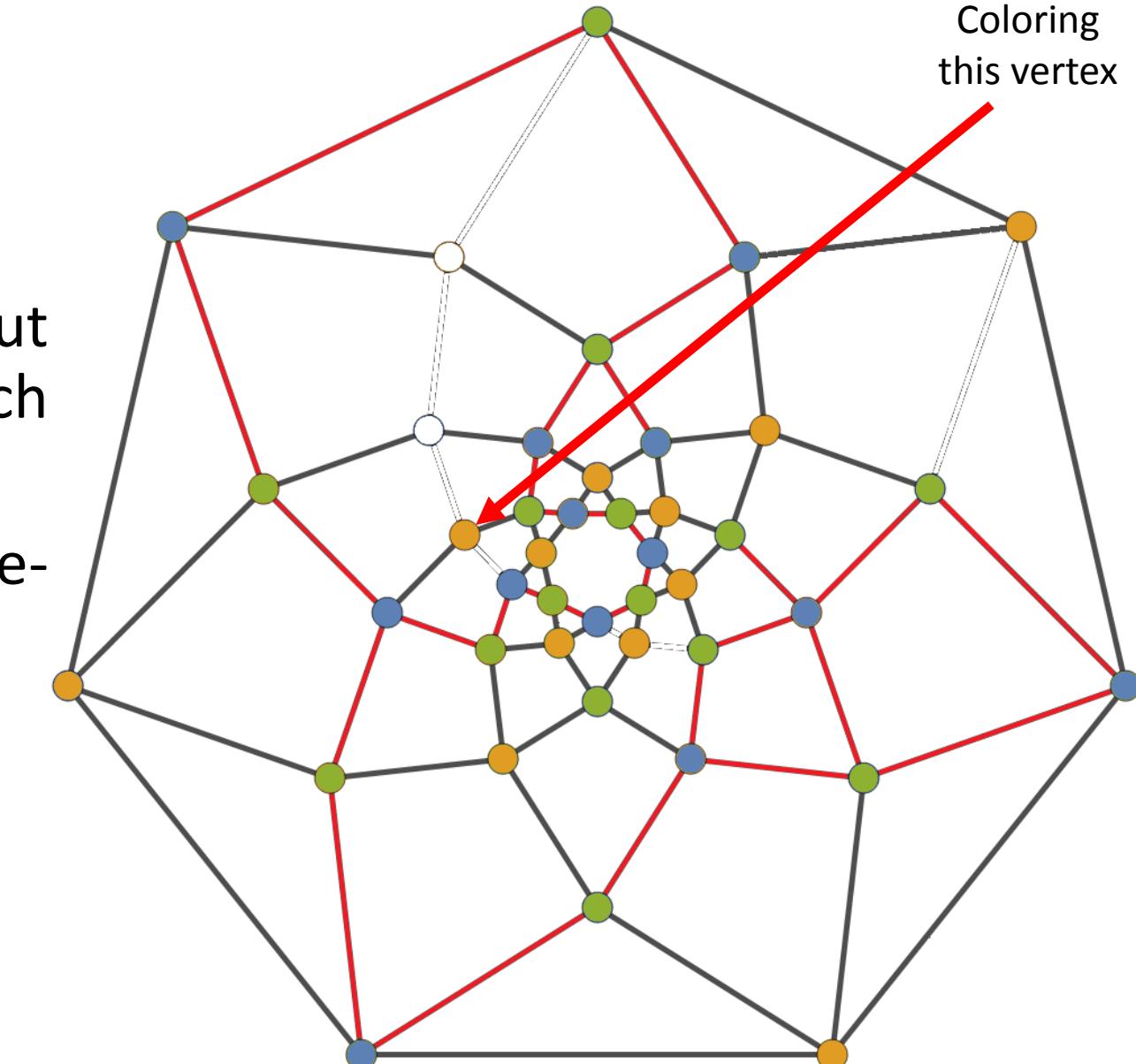
Hence, I switch Yellow-Green at V.
However, there is a odd Yellow-Green cycle at V



Adding a random great circle

So, I still keep Yellow on V but replace V_{ext} to Blue and switch Blue-Green at V_{ext}

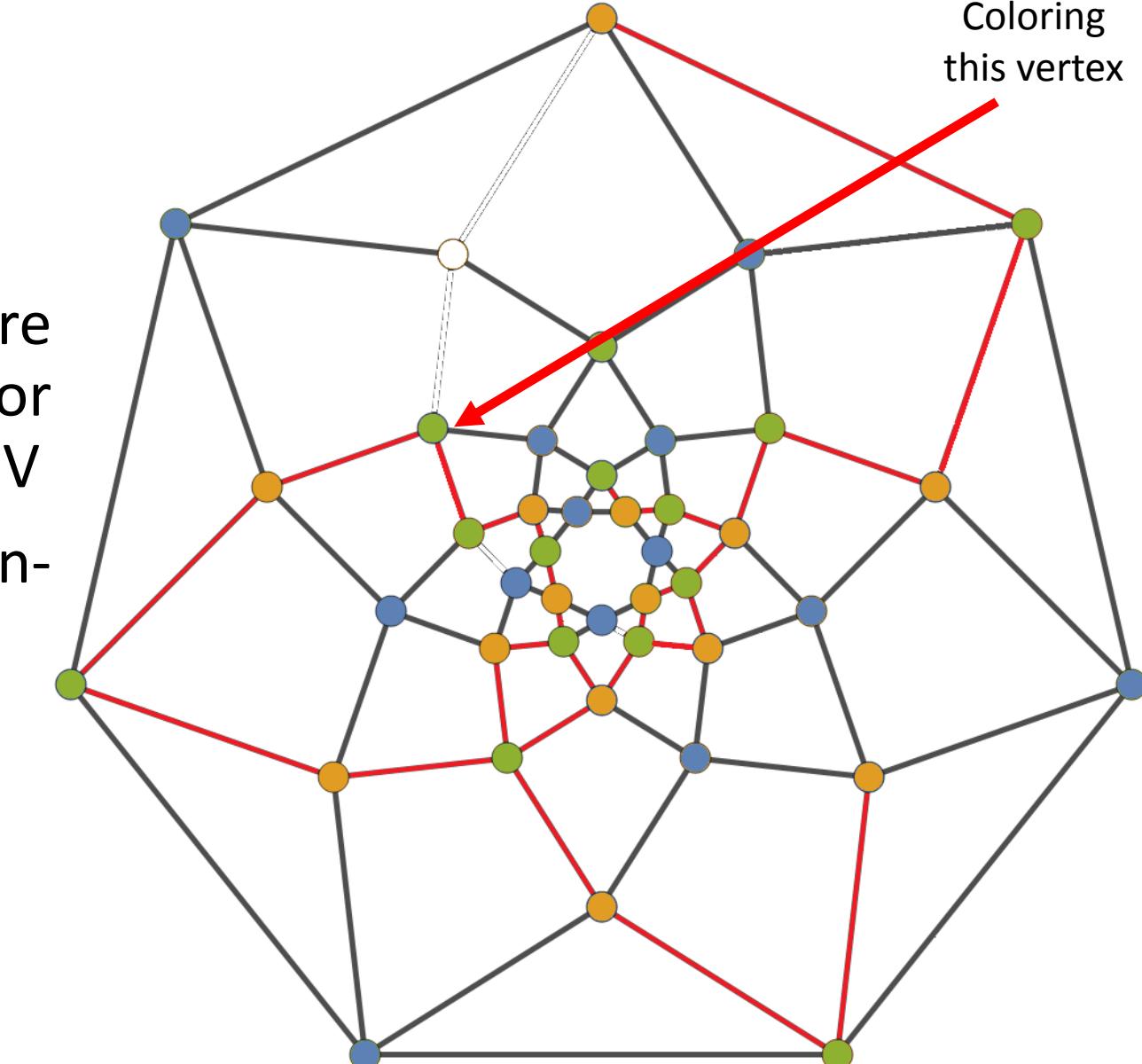
Luckily, there is an even Blue-Green cycle at V_{ext}



Adding a random great circle

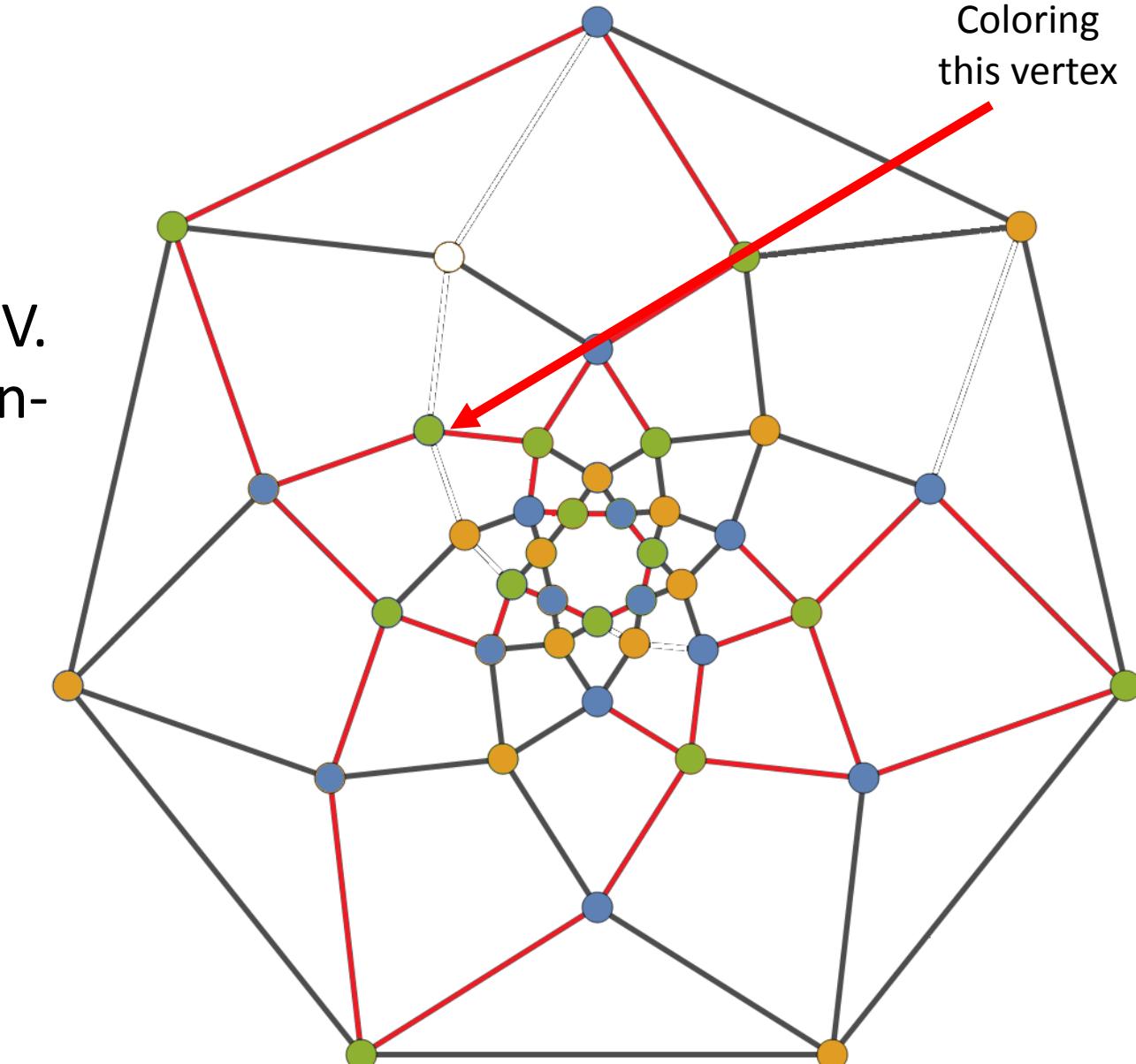
Go to the next vertex, 3 colors are used, so I will use Green of V_{ext} for V and do switch Green-Yellow at V

However, there is a odd Green-Yellow cycle at V



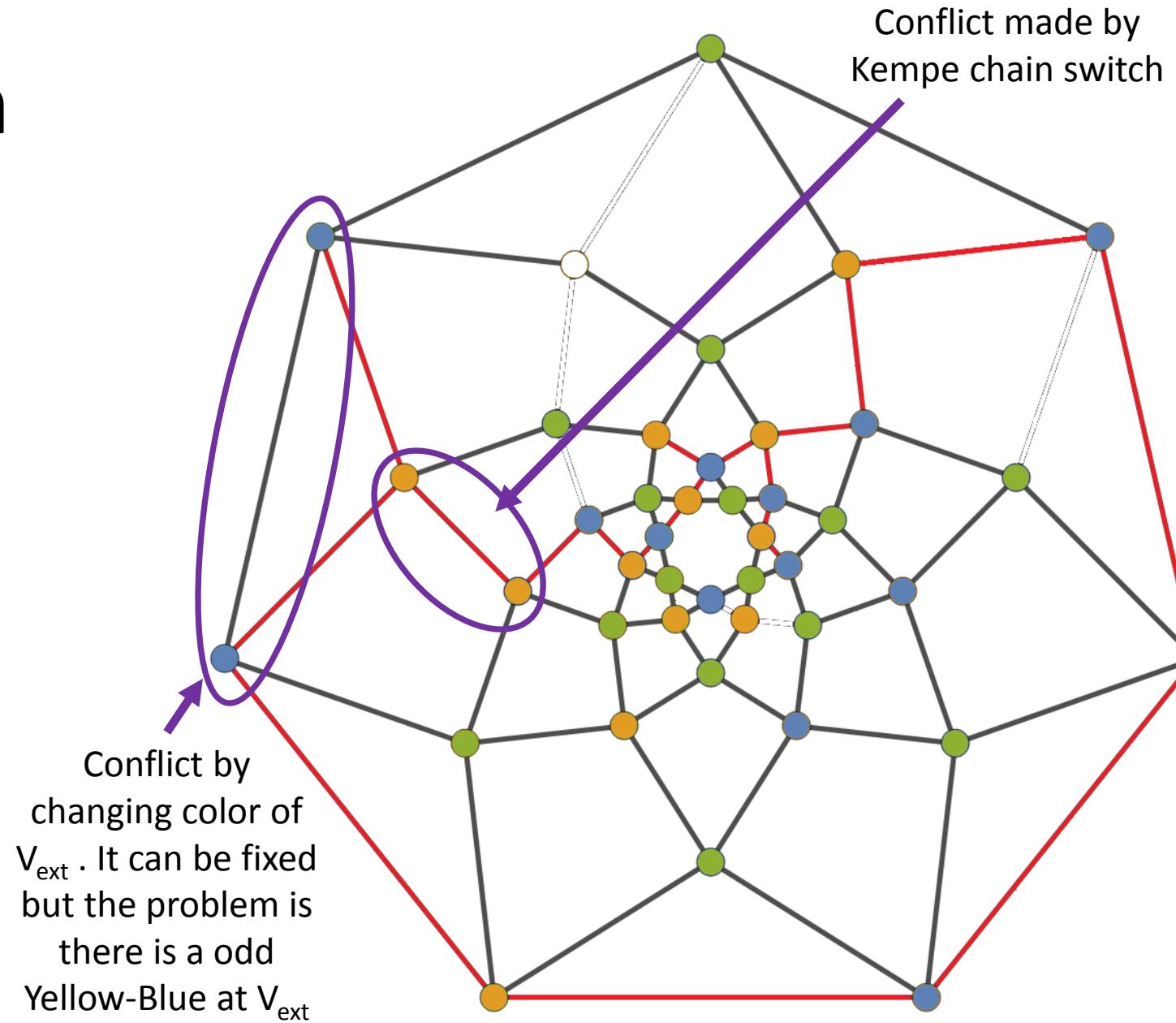
Adding a random great circle

Hence, I switch Green-Blue at V.
However, there is a odd Green-
Blue at V



Adding a random great circle

I still keep Green on V ,
but replace V_{ext} to
Yellow and switch
Yellow-Blue at V_{ext}



Adding a random great circle

I decided to stop coloring this case after few switches. I may solve that problem easily but it's not convincing for a the entire proof