Based on the information provided in the "Optym-Take-Home-Exam.pdf" document, we can develop a Mixed Integer Linear Programming (MILP) model for the Aircraft Route Optimization Problem. The model will include complete notations for input parameters, decision variables, constraints, and the objective function.

# Input Parameters:

1. <b>D</b>: Set of demands (customer flying requests).  
2. <b>A</b>: Set of aircraft.  
3. <b>L</b>: Set of flight legs.  
4. <b>T</b>: Time periods for planning (e.g., days).  
5. <b>C<sub>l</sub></b>: Operating cost for each flight leg <i>l ∈ L</i>.  
6. <b>R<sub>d</sub></b>: Revenue from fulfilling demand <i>d ∈ D</i>.  
7. <b>M</b>: Large positive number (for modeling purposes).  
8. <b>S<sub>la</sub></b>: Starting location of aircraft <i>a ∈ A</i> for leg <i>l ∈ L</i>.  
9. <b>E<sub>la</sub></b>: Ending location of aircraft <i>a ∈ A</i> for leg <i>l ∈ L</i>.  
10. <b>HOS<sub>a</sub></b>: Maximum hours of service for aircraft <i>a ∈ A</i>.  
11. <b>Rest<sub>a</sub></b>: Mandatory rest period for aircraft <i>a ∈ A</i> after <i>HOS<sub>a</sub></i> is reached.  
12. <b>P</b>: Penalty cost for operating an empty flight leg.  
13. <b>W<sub>1, 2, 3</sub></b>: objective weights

# Decision Variables:

<b>x<sub>lat</sub></b>: Binary decision variable it's 1 if aircraft <i>a</i> is assigned to fly leg <i>l</i> at time <i>t</i>, and 0 otherwise.   
  
<b>y<sub>dat</sub></b>: Binary decision variable. It takes a value of 1 if demand <i>d</i> is fulfilled by aircraft <i>a</i> at time <i>t</i>, and 0 otherwise.   
  
<b>z<sub>la</sub></b>: Binary decision variable. It takes a value of 1 if leg <i>l</i> is flown empty by aircraft <i>a</i>, and 0 otherwise.   
  
<b>empty\_leg\_vars<sub>alt</sub></b>: Binary decision variable. It takes a value of 1 if aircraft <i>a</i> flies leg <i>l</i> empty (without passengers) at time <i>t</i>, and 0 otherwise.

# Objective Function:

Maximize total profit, which is the revenue from fulfilling demands minus the operating costs:

$$  
\text{Maximize } Z = W\_1 \* \sum\_{d \in D} \sum\_{a \in A} \sum\_{t \in T} R\_d \cdot y\_{dat} - W\_2 \* \sum\_{l \in L} \sum\_{a \in A} \sum\_{t \in T} C\_l \cdot x\_{lat} - W\_3 \* \sum\_{l \in L} \sum\_{a \in A} P \cdot z\_{la}  
$$

## import libraries

## load data

# scenario 1D: Focus on a single day: 24th July

# scenario 2D: Focus on a single day: 24th and 25th July

## preprocess

## parameters

## Model initialization

## Decision Variables

finalized till here

# Constraints

1. - Demand Fulfillment: Each demand must be fulfilled at least once in the planning period.

$$\sum\_{t} y\_{dt} \geq 1 \quad \text{for all } d$$

2. - Aircraft Route Continuity: Ensures that for each aircraft, the number of arrivals at an airport equals the number of departures.

$$\sum\_{l, S\_{la} = k} X\_{lat} = \sum\_{l, E\_{la} = k} X\_{lat} \quad \text{for all } a, t, \text{ and } k \text{ in airports}$$

3- Aircraft Utilization Limit: Each aircraft cannot exceed its maximum hours of service.

$$\sum\_{l} \sum\_{t} x\_{lat} \leq \text{HOS}\_a \quad \text{for all } a$$

4. Mandatory Rest: After reaching the maximum HOS, the aircraft must observe a mandatory rest period.

$$  
\begin{aligned}  
&\text{If } \quad l \in L, \quad \sum x\_{lat} = \text{HOS}\_a, \text{ then} \\  
&\quad l \in L, \quad \sum x\_{la(t+1)} = 0  
\end{aligned}  
$$

Given the complexity of directly implementing this as a linear constraint, a practical approach might be to set a utilization limit slightly less than the maximum HOS for each day, thereby implicitly allowing for rest time. This approach simplifies the model while achieving the intended outcome of ensuring rest periods.

5. Flight Leg Assignment: A flight leg can only be assigned if it is either loaded or flown empty.

$$x\_{lat} \leq M \cdot z\_{la} \quad \text{for all } l, a, \text{ and } t$$

## objective function

## solve the model

Focus of optimization search on maximization of total profit

output csv