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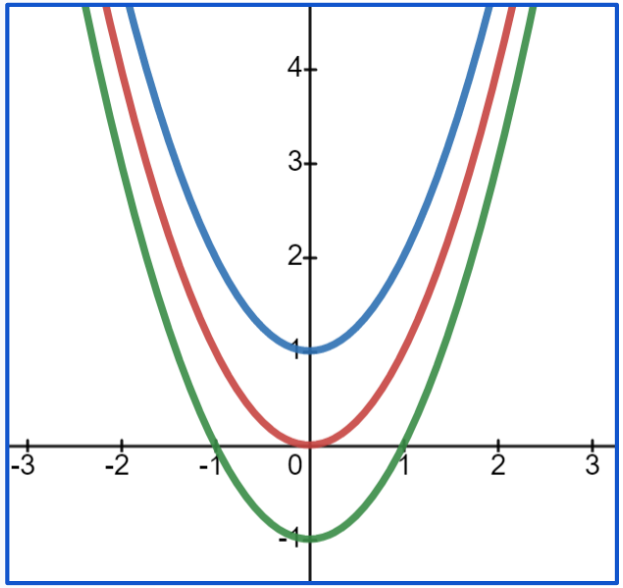
Range of Quadratic

Quadratic Equations

6



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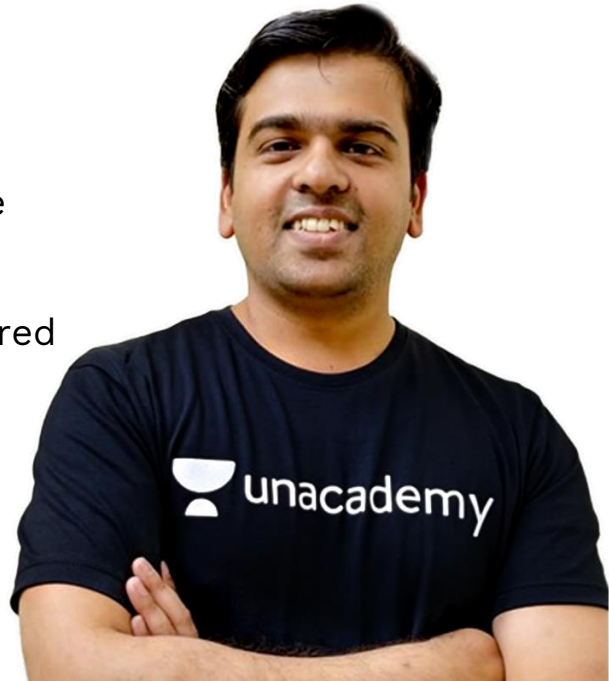
Sameer Chincholikar

B.Tech, M.Tech - IIT-Roorkee

- **10+** years Teaching experience
- Taught **1 Million+** Students
- **100+** Aspiring Teachers Mentored



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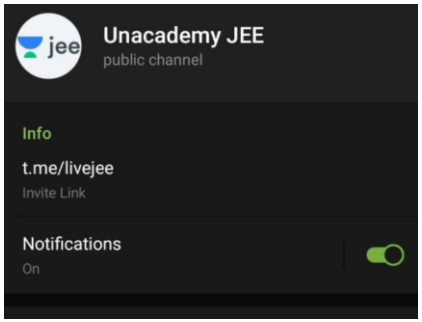

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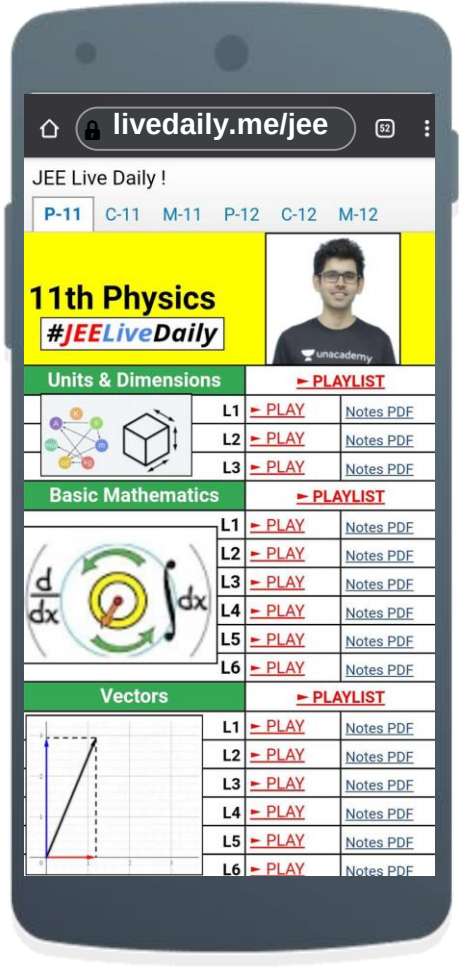

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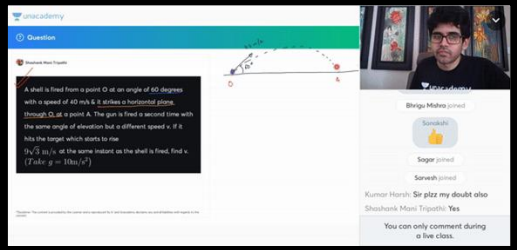

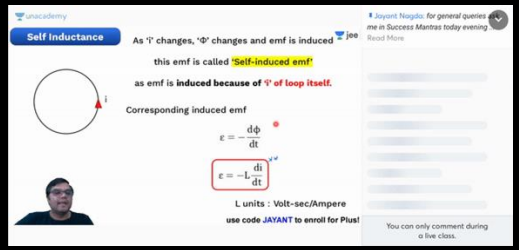

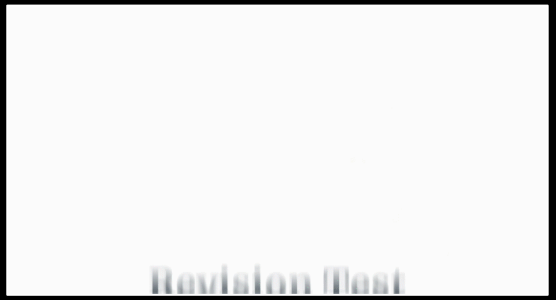

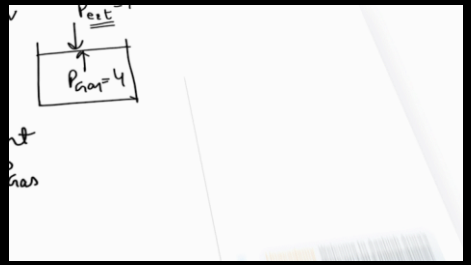

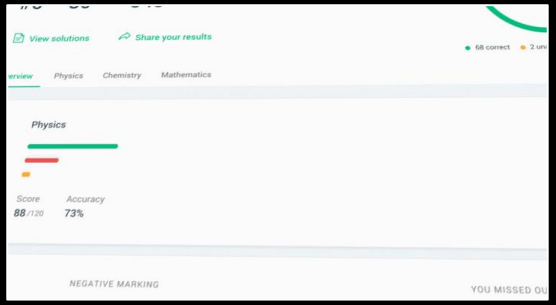

+ LIVE Class Environment

- + **LIVE Polls** & **Leaderboard**
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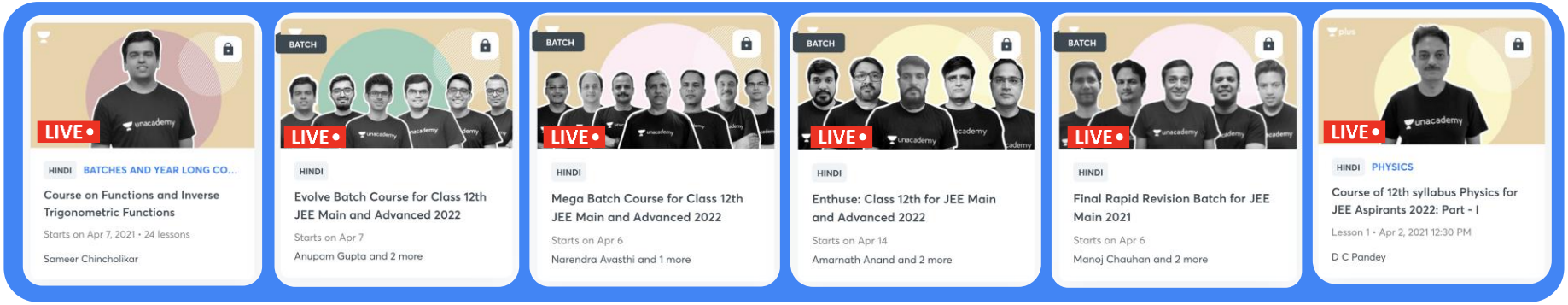

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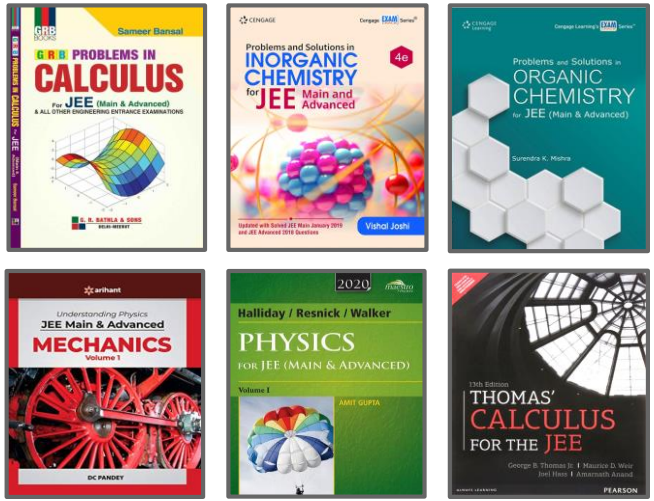

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Top Results

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100 %ile

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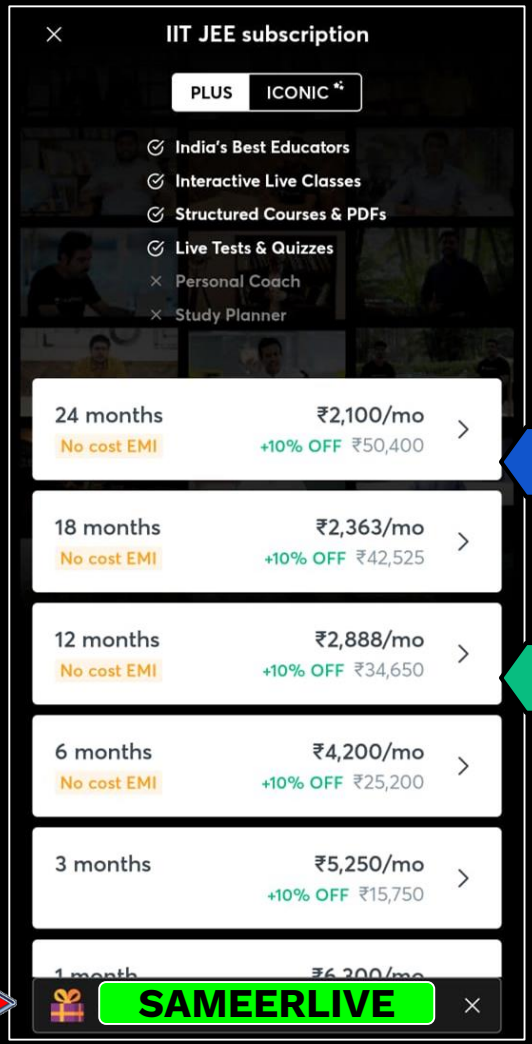

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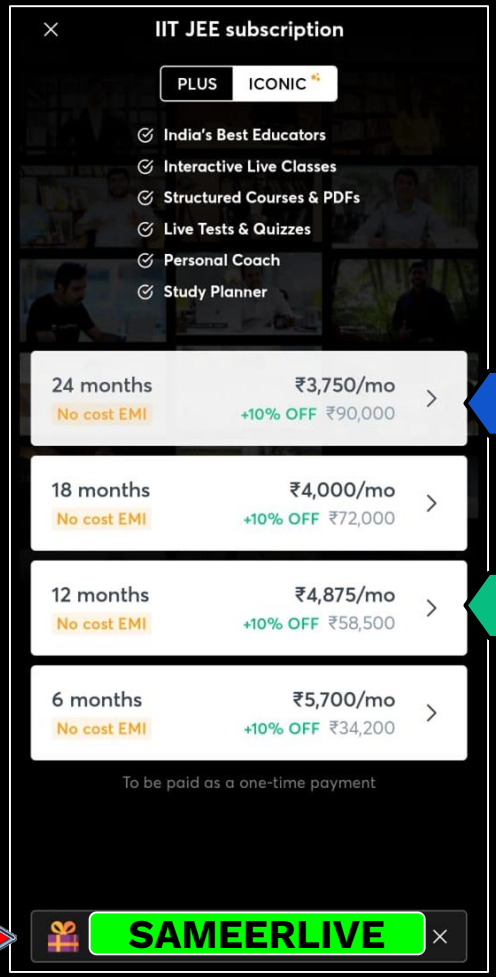
11th / 9, 10

12th / Drop



11th / 9, 10

12th / Drop

jee

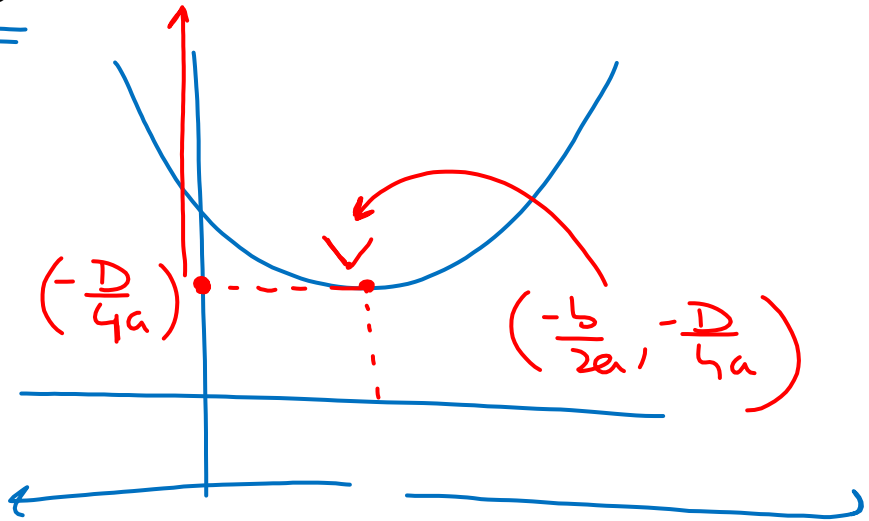
LET'S BEGIN!!

Range of Quadratic Expression

Range of Quadratic Expression (Absolute Range)

For : $y = ax^2 + bx + c$; if $a > 0$

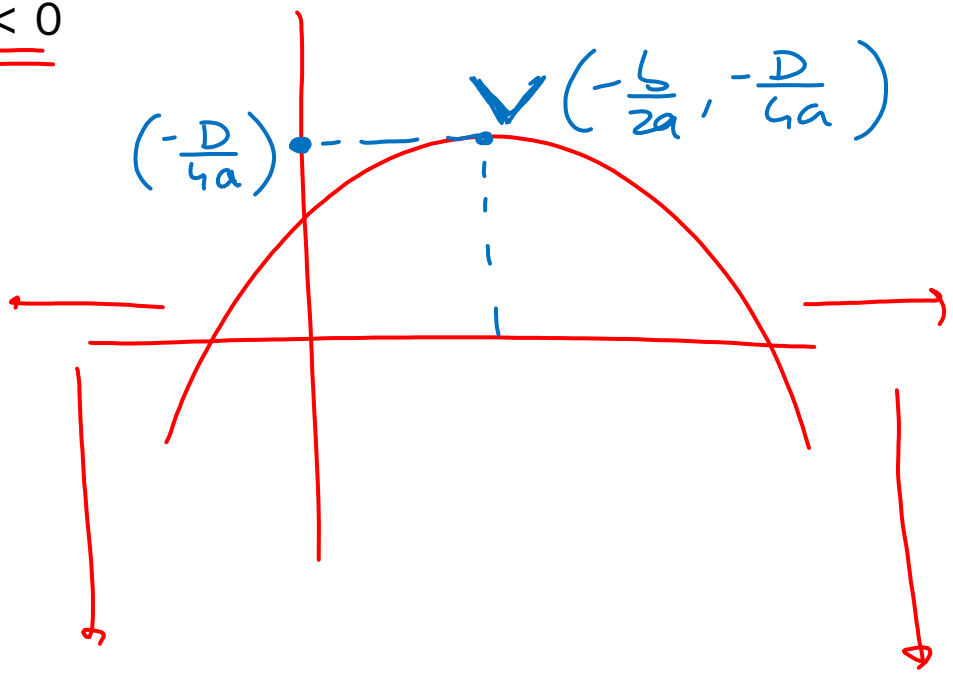
$$f(x) \in \left[-\frac{D}{4a}, \infty\right)$$



Range of Quadratic Expression (Absolute Range)

For : $y = ax^2 + bx + c$; if $a < 0$

$$f(x) \in \left(-\infty, -\frac{D}{4a}\right]$$



Let $P(x) = ax^2 + bx + 8$ is a quadratic polynomial. If the minimum value of $P(x)$ is 6 when $x = 2$, find the values of a and b.

$a > 0$

$P(x) = ax^2 + bx + 8$

$V \equiv \left(\frac{-b}{2a}, -\frac{D}{4a} \right)$

$P(2) = 4a + 2b + 8 = 6$

$\Rightarrow \frac{-b}{2a} = 2$

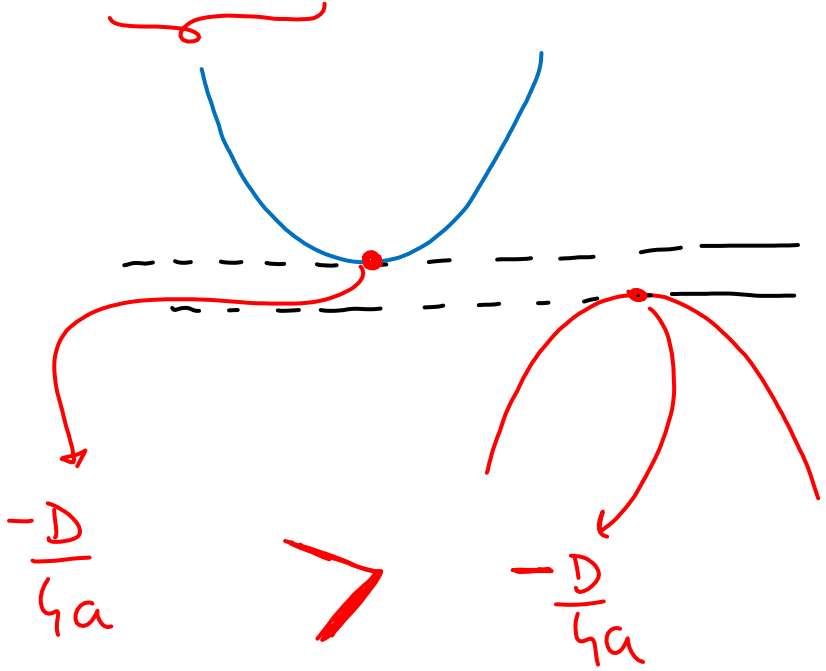
$\Rightarrow b = -4a$

$\Rightarrow 4a + 2(-4a) + 2 = 0$

$\Rightarrow 4a = 2 \Rightarrow a = \frac{1}{2}$

$$\begin{cases} a = \frac{1}{2} \\ \& \\ b = -2 \end{cases}$$

If min $(x^2 + (a - b)x + (1 - a - b))$ > max $(-x^2 + (a + b)x - (1 + a + b))$, prove that $a^2 + b^2 < 4$.



$$\frac{-\left((a-b)^2 - 4(1)(1-a-b)\right)}{4(1)}$$

$$>$$

$$\frac{-\left((a+b)^2 - 4(-1)(-(1+a+b))\right)}{4(-1)}$$

$$(a-b)^2 - 4(1-a-b) < -(a+b)^2 + 4(1+a+b)$$

$$(a^2+b^2-2ab-4+4a+4b) = < -a^2-b^2-2ab+4+4a+4b$$

$$\chi(a^2+b^2) < \chi(4)$$

$$\boxed{a^2+b^2 < 4}$$

Consider the quadratic polynomial $\mathbf{f(x) = x^2 - 4ax + 5a^2 - 6a}$.

Find the largest distance between the roots of the equation f(x) = 0.

$$f(x) = x^2 - 4ax + 5a^2 - 6a$$

$$|\alpha - \beta| = \frac{\sqrt{D}}{|a|}$$

$$= \frac{\sqrt{16a^2 - 4(1)(5a^2 - 6a)}}{1}$$

$$= 2\sqrt{4a^2 - (5a^2 - 6a)}$$

$$= 2\sqrt{(-a^2 + 6a)}$$

Now.

$$g(a) = -a^2 + 6a + 0$$

$$g(a)\Big)_{max} = -\frac{D}{4a}$$

$$= -\frac{(36-0)}{4(-1)}$$

$$= 9$$

$$|\alpha - \beta|_{max} = 2\sqrt{(9)}$$

$$= 6$$

Range in Restricted Domain

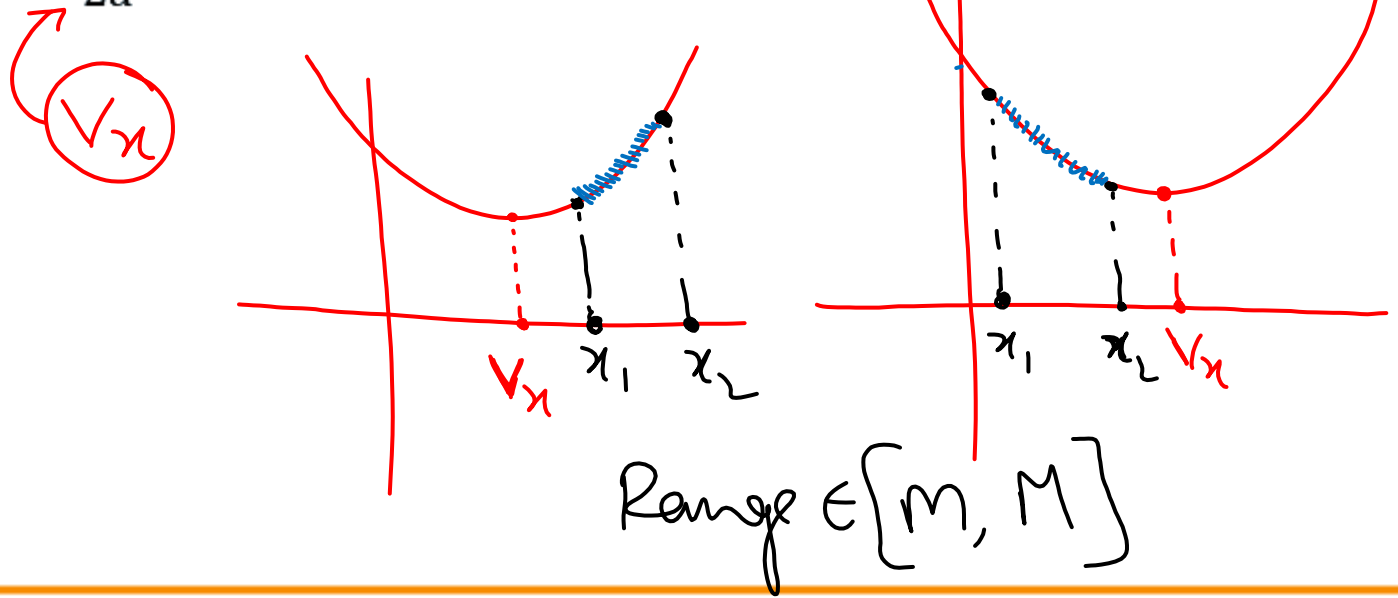
Range in Restricted Domain

For : $y = ax^2 + bx + c$; if $x \in [x_1, x_2]$

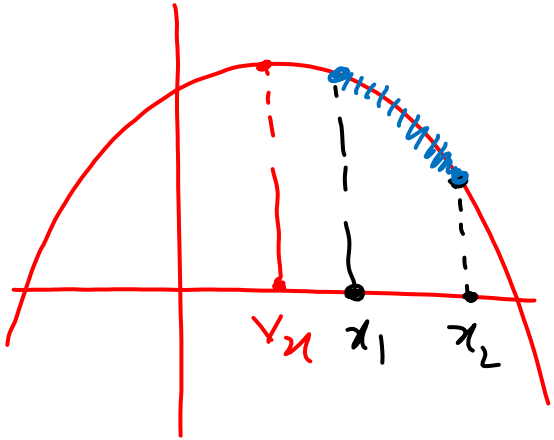
a) If $-\frac{b}{2a} \notin [x_1, x_2]$ **then,**

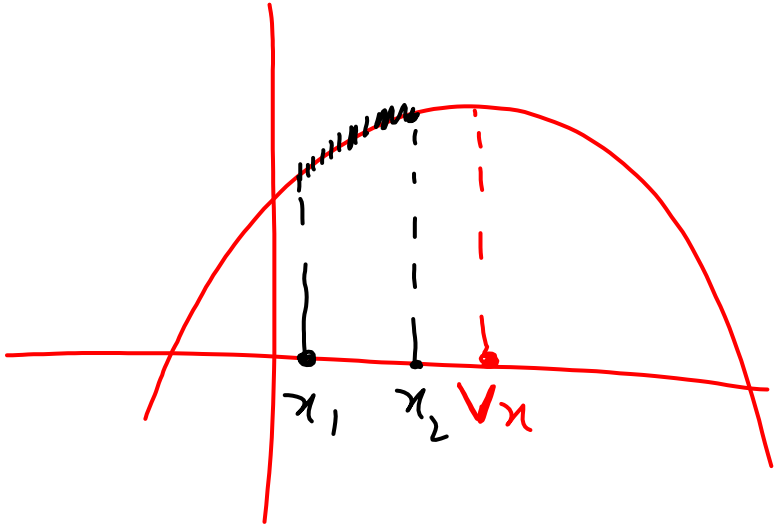
$\max\{f(x_1), f(x_2)\} = M$

$\min\{f(x_1), f(x_2)\} = m$



Range $\in [m, M]$

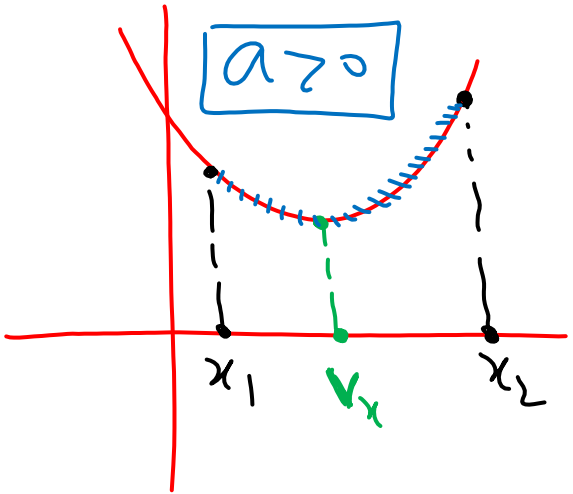

V_x
x_1
x_2

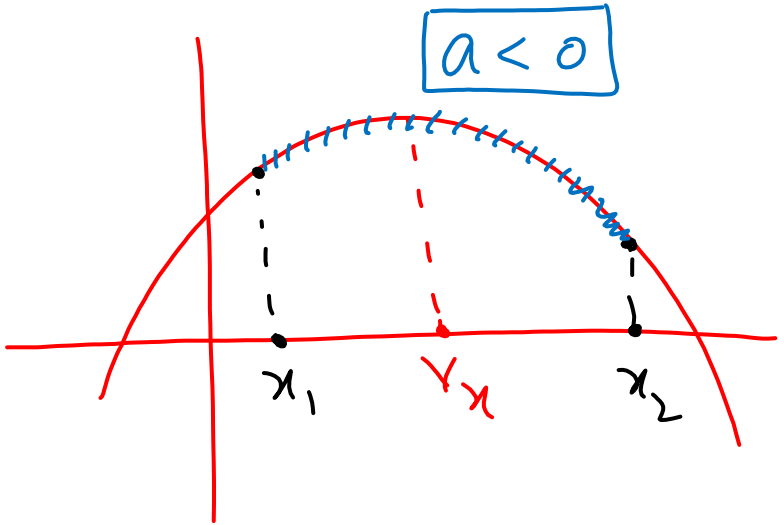

x_1
x_2
V_x

Range in Restricted Domain

For : $y = ax^2 + bx + c$; if $x \in [x_1, x_2]$

b) **If** $-\frac{b}{2a} \in [x_1, x_2]$ **then,**





Find the range of $f(x) = x^2 - 5x + 4$

I. When $x \in [0, 2]$

II. When $x \in [0, 3]$

$f(x) = x^2 - 5x + 4$

$V_x = \frac{-b}{2a} = -\frac{(-5)}{2(1)} = \frac{5}{2}$

(i) $V_x \notin [0, 2]$

$f(0) = 4$

$f(2) = 4 - 10 + 4 = -2$

Range $\in [-2, 4]$

(iii) $\forall x \in [0, 3]$

$f(0) = 4$

$f(3) = 9 - 15 + 4 = -2$

$f\left(\frac{5}{2}\right) = \frac{25}{4} - 5\left(\frac{5}{2}\right) + 4$

$= \frac{25 - 50 + 16}{4} = -\frac{9}{4}$

Range $\in$

$\left[-\frac{9}{4}, 4\right]$

Find the difference between the least and greatest values of

$y = -2x^2 + 3x - 2$ for $x \in [0, 2]$.

$V_x = \frac{-b}{2a} = \frac{-3}{2(-2)}$

$= \boxed{\frac{3}{4}}$

$V_x \in [0, 2]$

$f(0) = -2$

$f(2) = -8 + 6 - 2 = -4$

$f\left(\frac{3}{4}\right) = -2\left(\frac{9}{16}\right) + 3\left(\frac{3}{4}\right) - 2$

$= -\frac{9}{8} + \frac{9}{4} - 2$

$= \frac{-9 + 18 - 16}{8} = -\frac{7}{8}$

Difference $= -2 - \left(-\frac{7}{8}\right) = -2 + \frac{7}{8} = -\frac{9}{8}$

$$f\left(\frac{3}{4}\right) = \frac{-9+18-16}{8}$$

$$= -\frac{7}{8}$$

Range $\in \left[-4, -\frac{7}{8}\right]$

Difference:

$$-\frac{7}{8} - (-4)$$

$$-\frac{7}{8} + 4$$

$$= \frac{-7+32}{8}$$

$$= \boxed{\frac{25}{8}}$$

Find the range of $y = 3^{x+1} + 2.3^{2x} - 2$

$$y = 3^{x+1} + 2 \cdot 3^{2x} - 2$$

$$y = 3 \cdot (3^x) + 2 \cdot (3^x)^2 - 2$$

Let: $3^x = t \Rightarrow t > 0$

$$y = 3t + 2t^2 - 2$$

$$y = 2t^2 + 3t - 2$$

$$V_x = \frac{-b}{2a}$$

$$= \frac{-(3)}{2(2)}$$

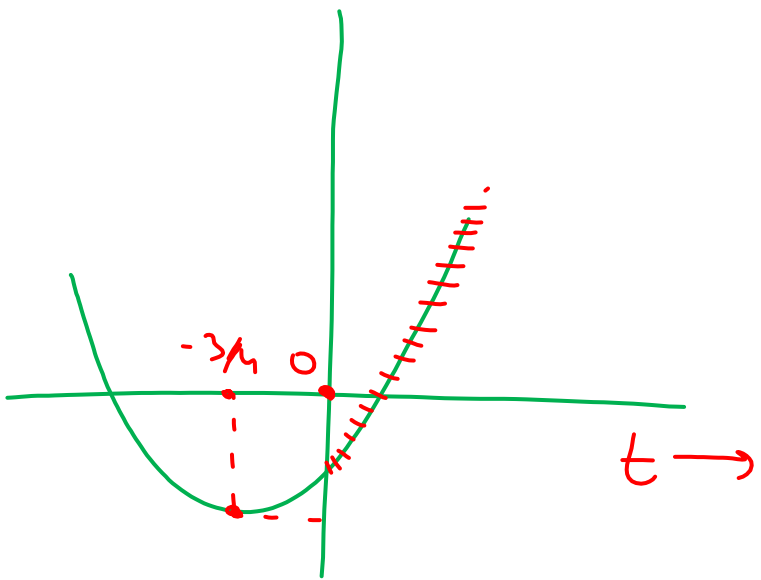
$$= \boxed{\frac{-3}{4}}$$

$t \in (0, \infty)$

$\forall x \notin (0, \infty)$

$\therefore$ Range $\in (-2, \infty)$

$-\frac{25}{8}$



Range of Rational Functions

Range of Rational Function

$$\frac{ax^2 + bx + c}{px^2 + qx + r} = y$$

$$\frac{Q}{Q} \; ; \; \frac{L}{Q} \; ; \; \frac{Q}{L}$$

Find the range of $\frac{2x^2-3x+2}{2x^2+3x+2}$ if x is real.

$$y = \frac{2x^2-3x+2}{2x^2+3x+2}$$

$$2yx^2+3yx+2y = 2x^2-3x+2$$

$$(2y-2)x^2+(3y+3)x+(2y-2)=0$$

$x \in R$

$D \geq 0$

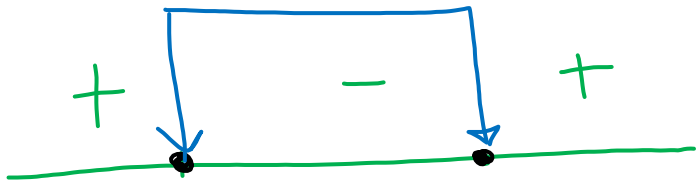
$$(3y+3)^2-4(2y-2)^2 \geq 0$$

$$(3y+3)^2 - (4y-4)^2 \geq 0$$

$$\Rightarrow ((3y+3)+(4y-4))((3y+3)-(4y-4)) \geq 0$$

$$\Rightarrow (7y-1)(7-y) \geq 0$$

$$\Rightarrow (7y-1)(y-7) \leq 0$$



$$\Rightarrow y \in \left[\frac{1}{7}, 7\right]$$

If x is real, the maximum value of $\frac{3x^2+9x+17}{3x^2+9x+7}$ is

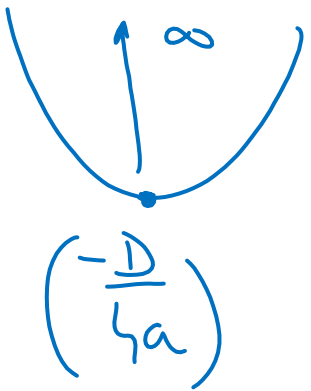
A. $\frac{1}{4}$ **B.** 41 **C.** 1 **D.** $\frac{17}{7}$

$$y = \frac{3x^2+9x+17}{3x^2+9x+7}$$

$$y = \frac{(3x^2+9x+7)}{(3x^2+9x+7)} + \frac{10}{3x^2+9x+7}$$

$$y = 1 + \frac{10}{(3x^2+9x+7)}$$

$$f(x) = 3x^2+9x+7$$



$$f(x)_{min} = \frac{-D}{4a}$$

$$= \frac{-(81-4(3)(7))}{4(3)}$$

$$= \frac{-(81-84)}{12}$$

$$= \frac{3}{12} = \frac{1}{4}$$

$$y_{max} = 1 + \frac{10}{f(x))_{min}}$$

$$= 1 + \frac{10}{(1/4)}$$

$$= 1 + 40$$

$$= \boxed{41}$$

Show that the expression $\frac{mx^2 + 3x - 4}{m + 3x - 4x^2}$ will be capable of all values

when x is real, provided that m has any value between 1 and 7.

H.W

#JEELiveDaily Schedule

jee

11th

Namo Sir | Physics

6:00 - 7:30 PM

Ashwani Sir | Chemistry

7:30 - 9:00 PM

Sameer Sir | Maths

9:00 - 10:30 PM

12th

Jayant Sir | Physics

1:30 - 3:00 PM

Anupam Sir | Chemistry

3:00 - 4:30 PM

Nishant Sir | Maths

4:30 - 6:00 PM

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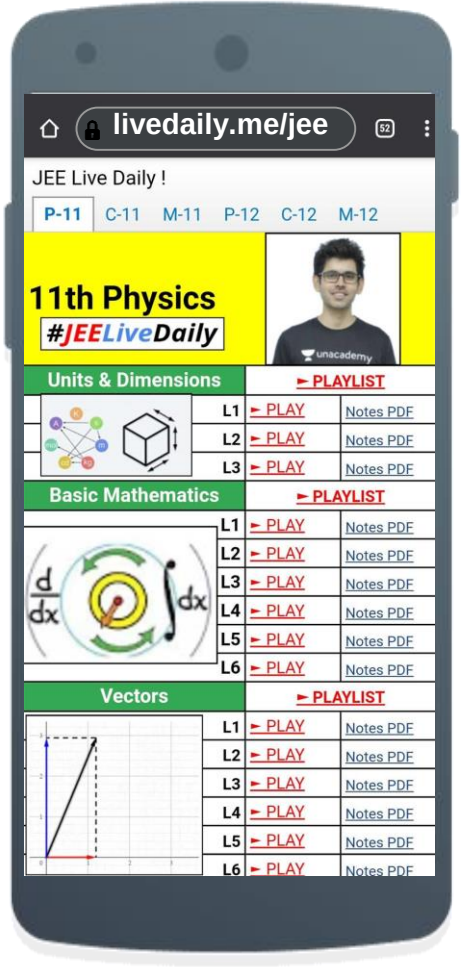

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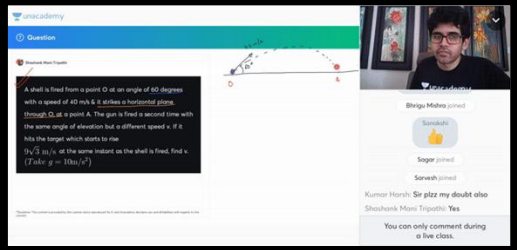

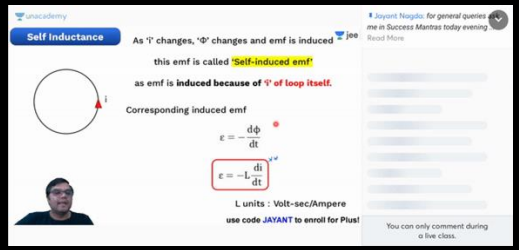

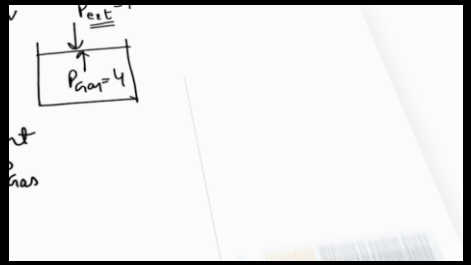

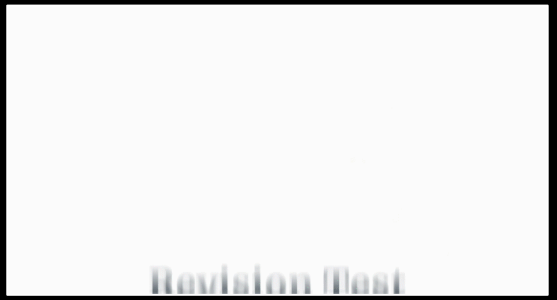

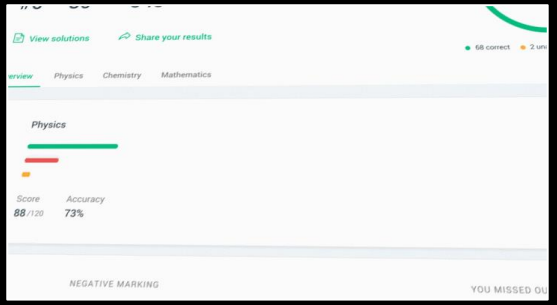

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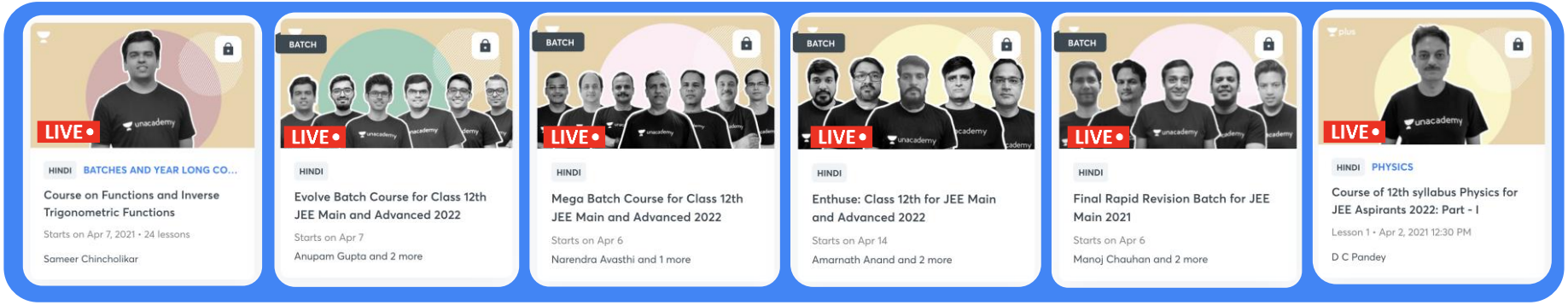

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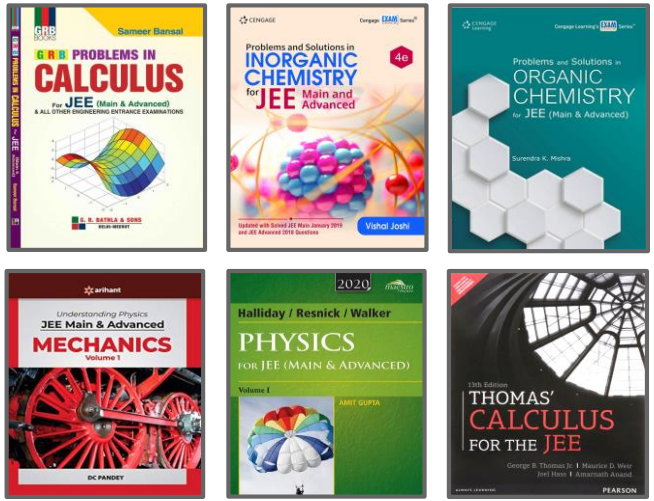

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MIHIR PRAJAPATI
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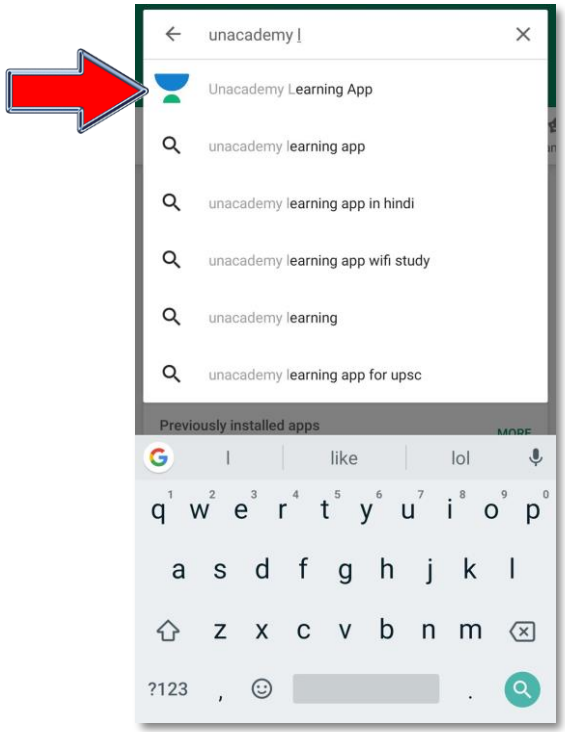
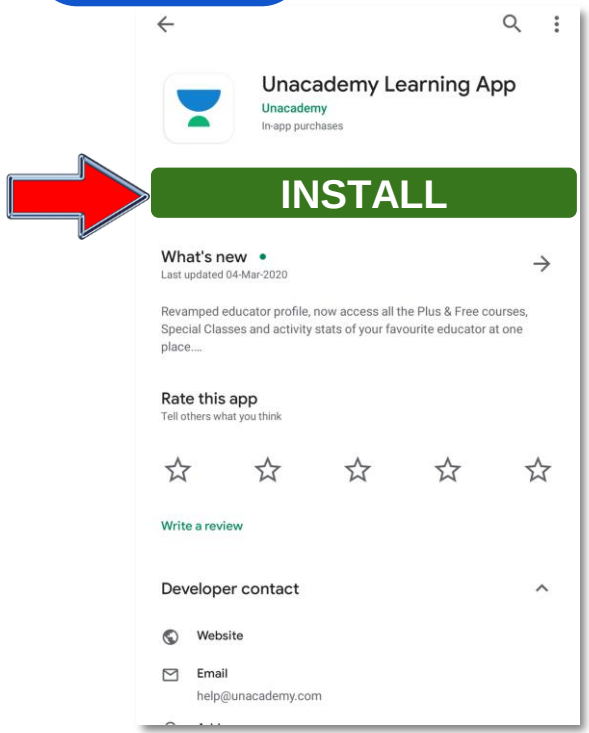
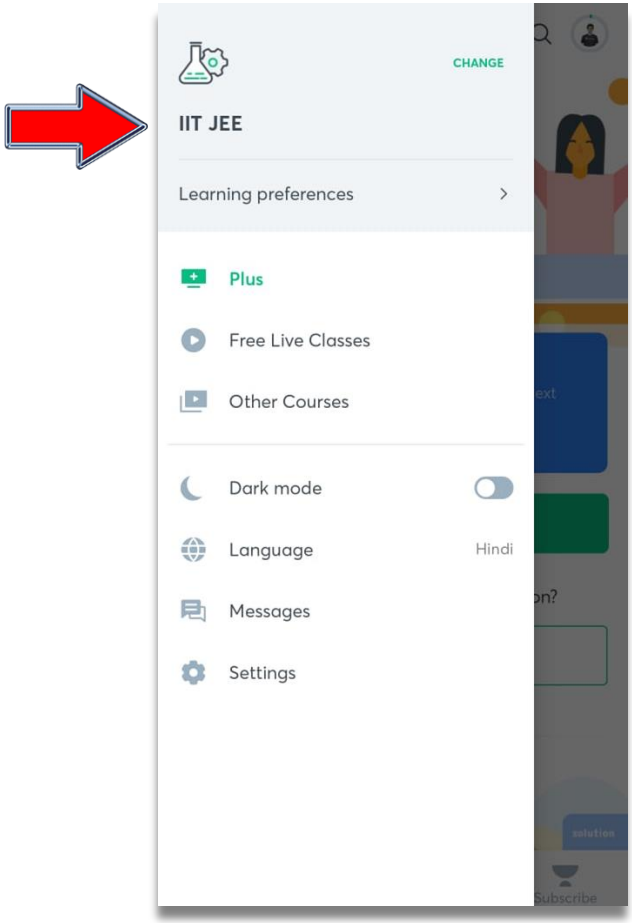
Step 1



Step 2

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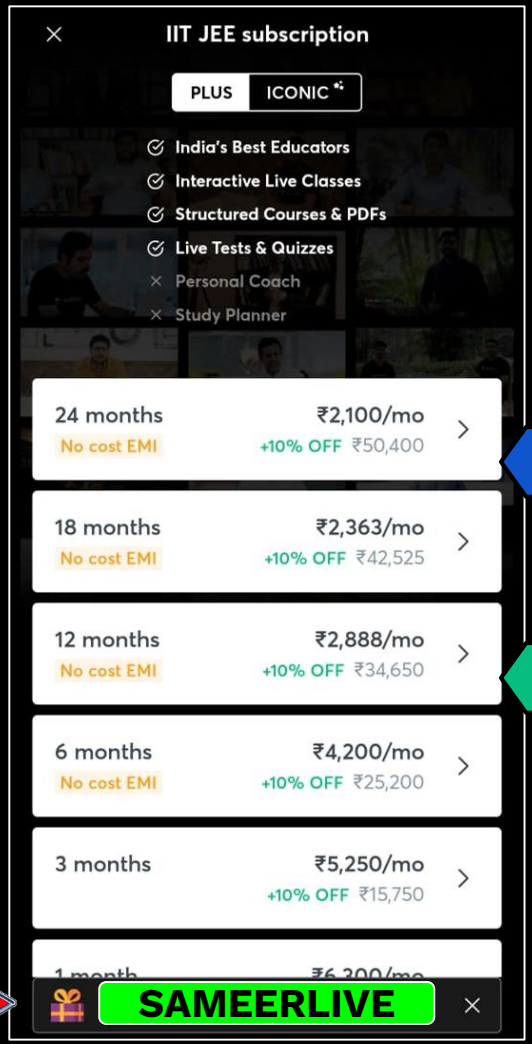

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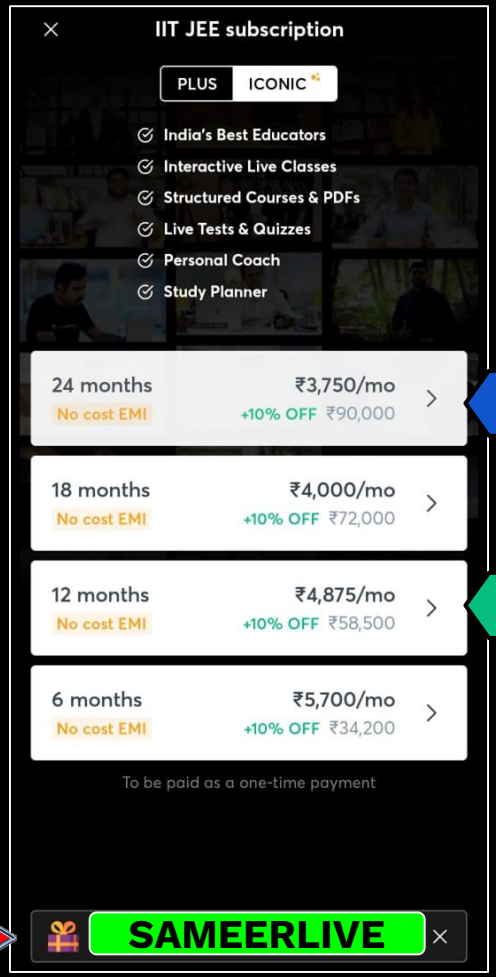

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