Harmonic Progression and AGP

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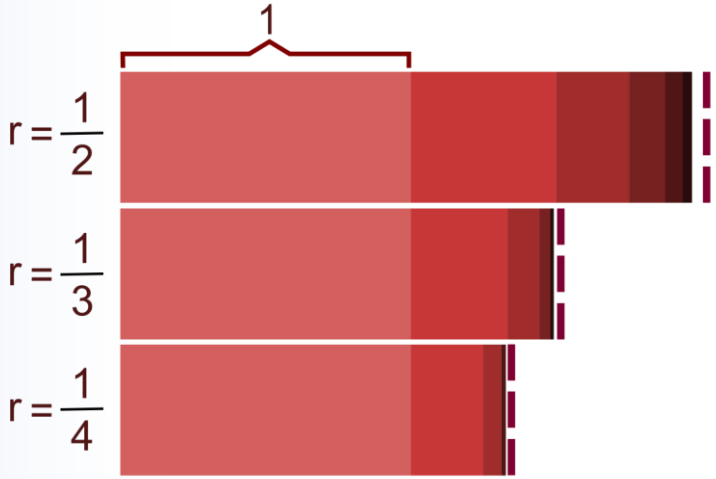
Sequences & Series

5



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Sameer Chincholikar

B.Tech, M.Tech - IIT-Roorkee

- **10+** years Teaching experience
- Taught **1 Million+** Students
- **100+** Aspiring Teachers Mentored

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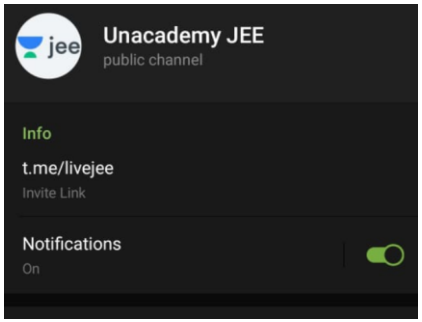

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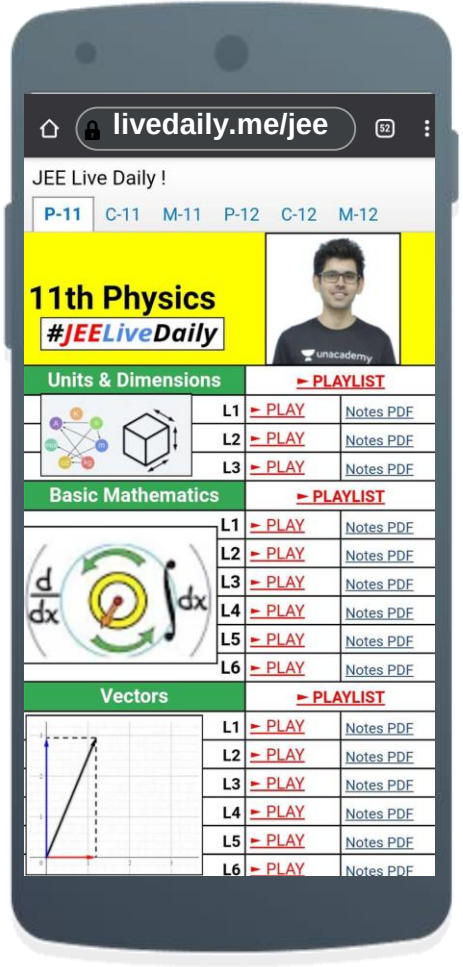

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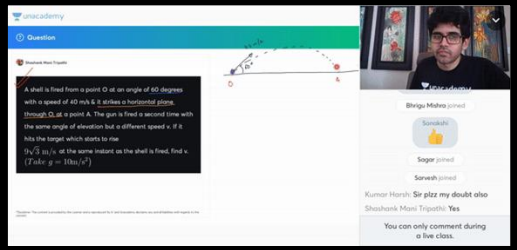

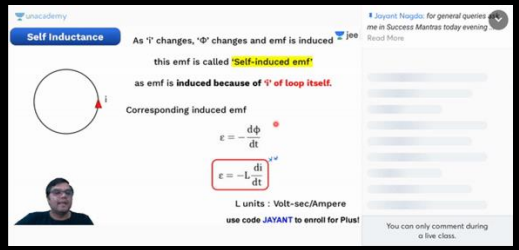

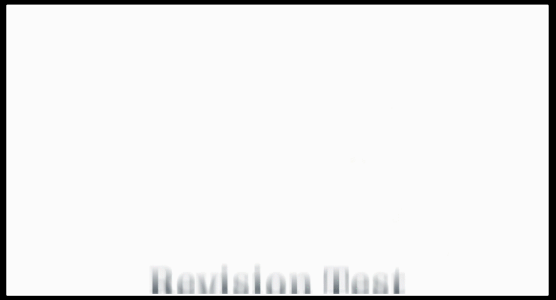

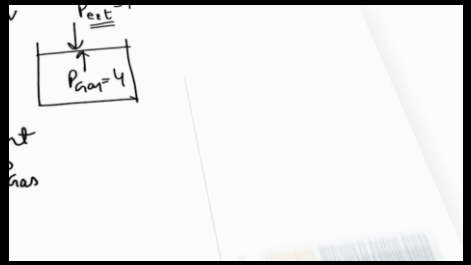

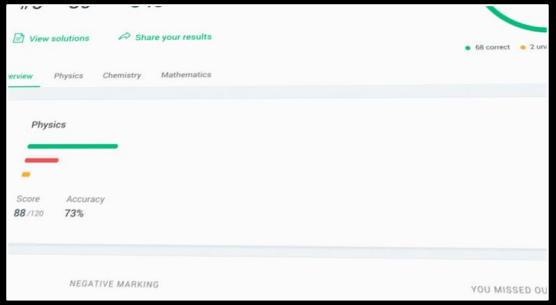

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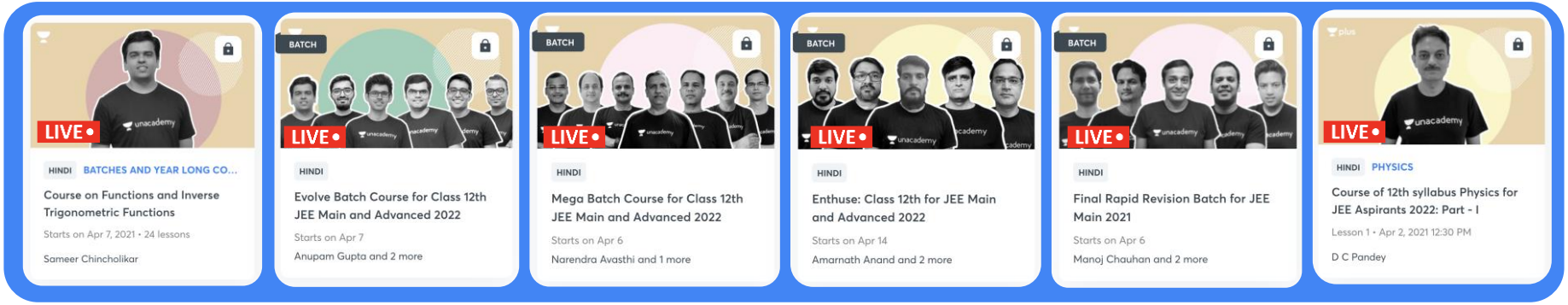

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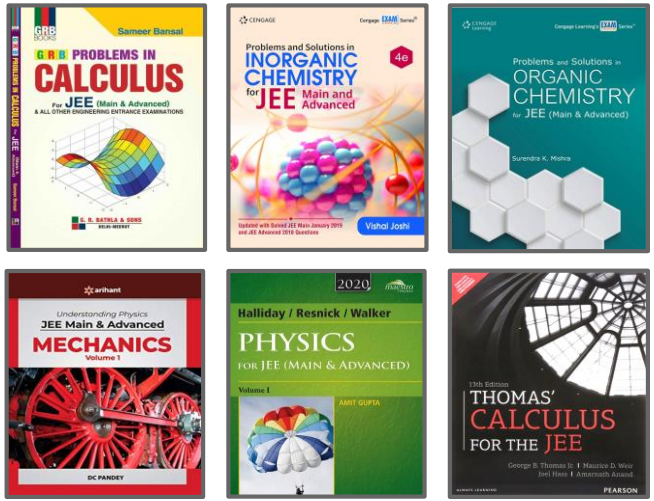

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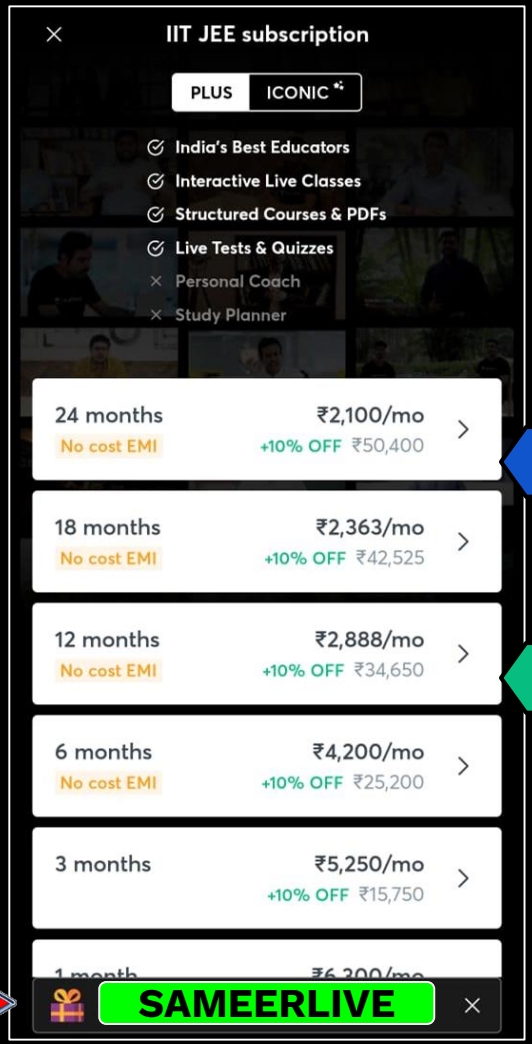

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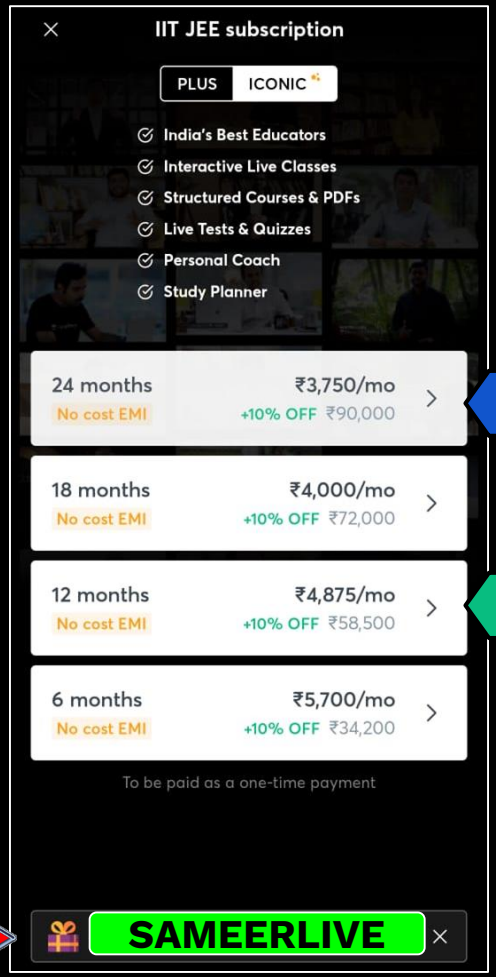
11th / 9, 10

12th / Drop



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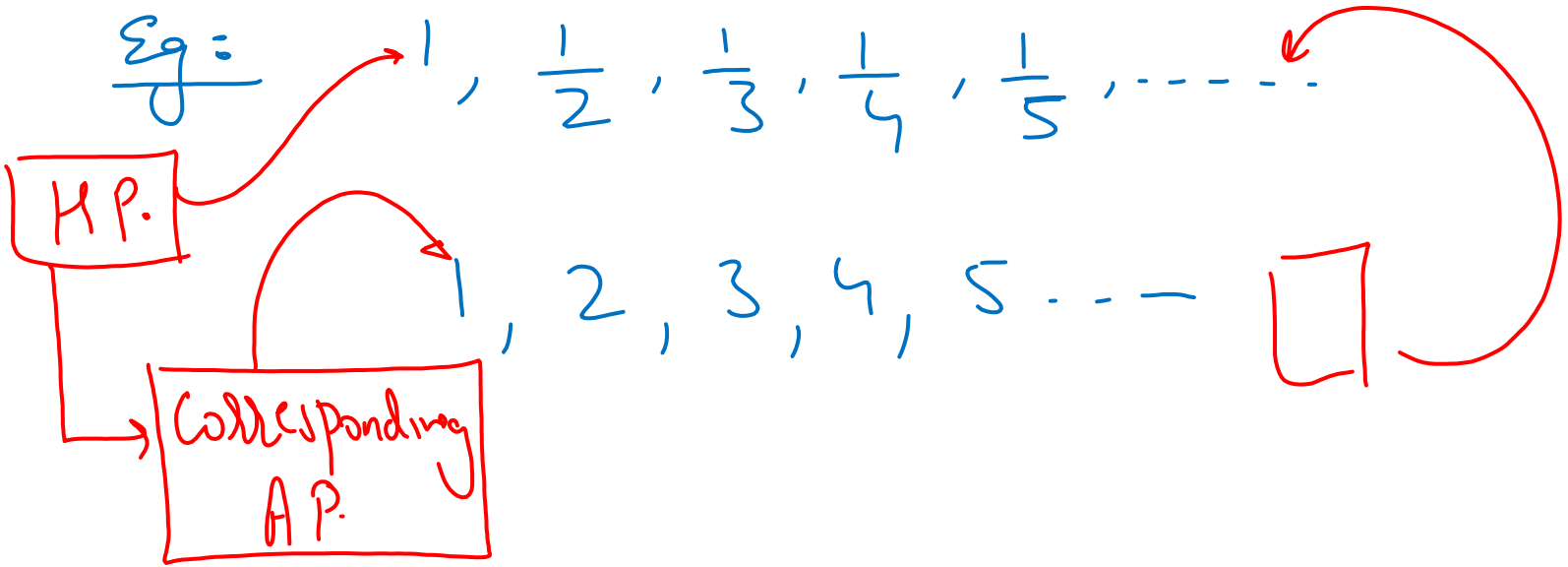
LET'S BEGIN!!

Harmonic Progression

Definition and Understanding

A sequences in which the reciprocal of the terms form an A.P. is called a Harmonic Progression (H.P.).



If $x=\sum_{n=0}^{\infty} a^n, y=\sum_{n=0}^{\infty} b^n, z=\sum_{n=0}^{\infty} c^n$

Where **a, b, c** are in A.P. and **|a| < 1, |b| < 1, |c| < 1,** then **x, y, z** are in

A. G.P **B.** A.P **C.** H.P. **D.** AGP

$$x=\sum_{n=0}^{\infty} a^n = 1+a+a^2+a^3+\cdots\infty$$

$$=\frac{1}{1-a}$$

Similarly; $y=\frac{1}{1-b}$ & $z=\frac{1}{1-c}$

$x = \frac{1}{1-a}$

$y = \frac{1}{1-b}$

$z = \frac{1}{1-c}$

Now:

$a, b, c \longrightarrow$ A.P.

$-a, -b, -c \longrightarrow$ A.P.

$1-a, 1-b, 1-c \longrightarrow$ A.P.

$\left(\frac{1}{1-a}\right); \left(\frac{1}{1-b}\right); \left(\frac{1}{1-c}\right) \longrightarrow$ H.P.

$\Rightarrow x, y; z \longrightarrow$ H.P.

If $a^x = b^y = c^z$ and **a, b, c** are in G.P. then **x, y, z** are in:

A. G.P **B.** A.P **C.** H.P. **D.** AGP

M-1

$a^x = b^y = c^z = k$

$\log a^x = \log b^y = \log c^z = \log k$

$x \log a = y \log b = z \log c = \log k$

$x = \frac{\log K}{\log a}$

$y = \frac{\log K}{\log b}$

$z = \frac{\log K}{\log c}$

Now:

$a, b, c \longrightarrow$ G.P.

$\log a, \log b, \log c \longrightarrow$ A.P.

$\frac{\log a}{\log K}, \frac{\log b}{\log K}, \frac{\log c}{\log K} \longrightarrow$ A.P.

$\frac{\log K}{\log a}; \frac{\log K}{\log b}, \frac{\log K}{\log c} \longrightarrow$ H.P.

M-2 :

Shortcut:

$a^x = b^y = c^z$

$2^x = 4^y = 8^z$

$2^x = 2^{2y} = 2^{3z}$

$x = 2y = 3z = K$

$x = K$

$y = \frac{K}{2}$

$z = \frac{K}{3}$

$\rightarrow \frac{1}{K}, \frac{2}{K}, \frac{3}{K}$

$\rightarrow$ A.P

$\Rightarrow x, y, z \rightarrow$ H.P

NOTE:

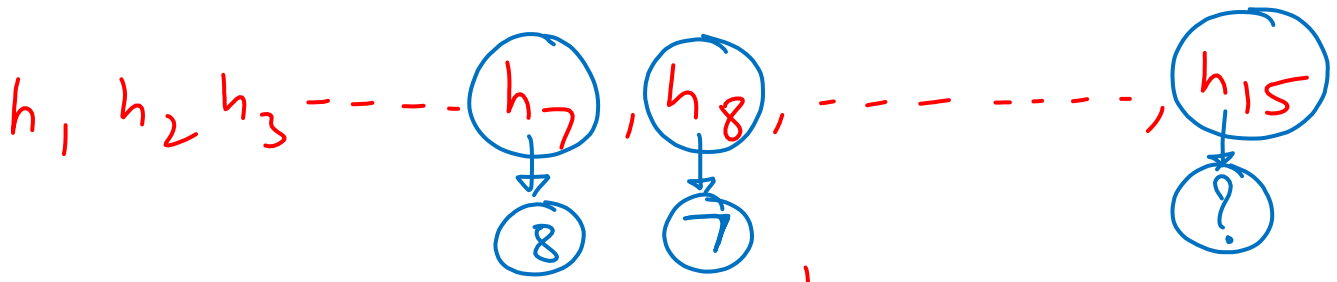
To solve the questions of H.P. we can first solve for the corresponding A.P.

$h_1, h_2, h_3, h_4 \ldots\ldots$ H.P.

$\frac{1}{h_1}, \frac{1}{h_2}, \frac{1}{h_3}, \ldots\ldots$ A.P.

If the 7^{th} **term** of an **H.P.** is 8 and 8^{th} **term** is **7,** then its 15^{th} **term** is

A. 16 **B.** 14 **C.** 27/14 **D.** 56/15



For corresponding A.P.

$T_7 = \frac{1}{8}$; $T_8 = \frac{1}{7}$

$T_7 = a + 6d = \frac{1}{8}$

$T_8 = a + 7d = \frac{1}{7}$

$(-)$: $d = \frac{1}{7} - \frac{1}{8} = \frac{1}{56}$

$$a + 6d = \frac{1}{8}$$

$$a + 6\left(\frac{1}{56}\right) = \frac{1}{8}$$

$$a = \frac{1}{8} - \frac{6}{56}$$

$$\boxed{a = \frac{1}{56}}$$

T_{15} (of corresponding AP)

$$= a + 14d$$

$$= \frac{1}{56} + 14\left(\frac{1}{56}\right)$$

$$= \boxed{\frac{15}{56}}$$

$\Rightarrow$ 15^{th} term of H.P. is $\boxed{\frac{56}{15}}$

Let $a_1, a_2, \ldots\ldots a_{10}$ be in **A.P.**, and $h_1, h_2 \ldots\ldots h_{10}$ be in **H.P.**

If $a_1 = h_1 = 2$ and $a_{10} = h_{10} = 3$, then $a_4 h_7$ is

A. 2 **B.** 3 **C.** 5 **D.** 6

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Part-1 : For A.P.

$a_1\ a_2\ a_3\ a_4 - a_{10}$

2 ? 3

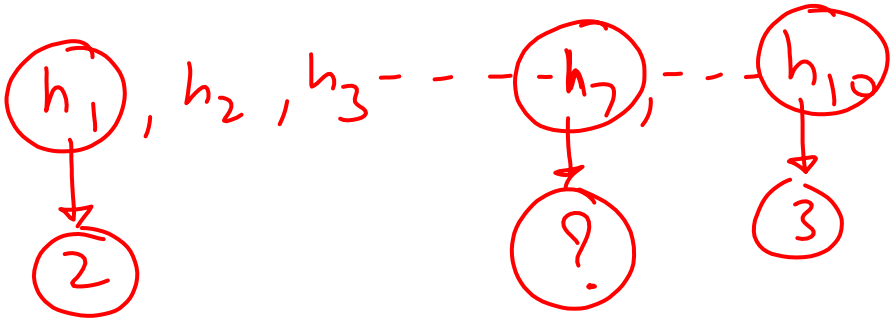
$a_{10} = a_1 + 9d$

$3 = 2 + 9d$

$\boxed{d = \frac{1}{9}}$

$\Rightarrow a_4 = 2 + 3\left(\frac{1}{9}\right) = \boxed{\frac{7}{3}}$

Part-2 : For H.P



Corresponding A.P

$a = \frac{1}{2}$; $T_{10} = \frac{1}{3}$

$T_{10} = a + 9d$

$\frac{1}{3} = \frac{1}{2} + 9d$

$-\frac{1}{6} = 9d$

$$d = -\frac{1}{54}$$

Now.

$$T_7 = a + 6d$$

$$= \frac{1}{2} + 6\left(-\frac{1}{54}\right)$$

$$= \frac{1}{2} - \frac{1}{9}$$

$$= \frac{7}{18}$$

$\Rightarrow$ For H.P.:

$$h_7 = \frac{18}{7}$$

Now.

$$a_7 h_7 = \left(\frac{7}{3}\right)\left(\frac{18}{7}\right)$$

$$= 6$$

NCERT

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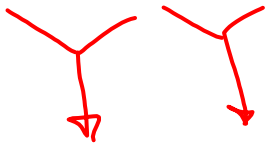
Property of H.P.

If a, b, c are in H.P. then

$a, b, c \rightarrow$ H.P.

$\frac{1}{a}, \frac{1}{b}, \frac{1}{c} \rightarrow$ A.P.



$$\left(\frac{1}{b}-\frac{1}{a}\right)=\left(\frac{1}{c}-\frac{1}{b}\right)$$

$$\frac{2}{b}=\left(\frac{1}{a}+\frac{1}{c}\right)$$

$$b=\left(\frac{2ac}{a+c}\right)$$

If the roots of the equation $a(b-c)x^2 + b(c-a)x + c(a-b) = 0$ are equal then **a, b, c** are in:

A. G.P **B.** A.P **C.** H.P. **D.** AGP

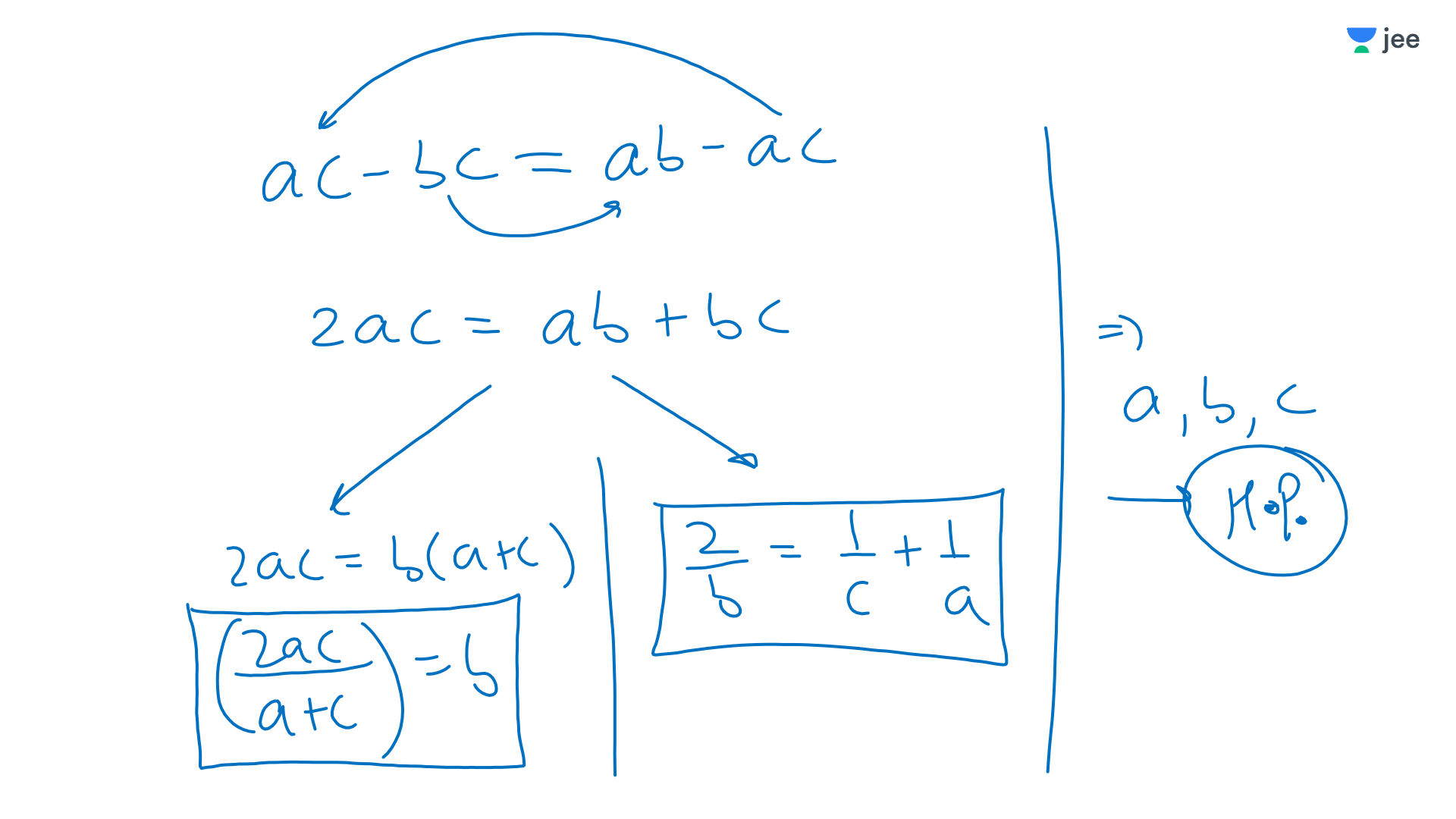
$$a(b-c)x^2 + b(c-a)x + c(a-b) = 0$$

$x = 1$ Satisfies the given equation

$\therefore$ both roots are equal

$$\text{Product of roots} = 1$$

$$\frac{c(a-b)}{a(b-c)} = 1$$


ac - bc = ab - ac
2ac = ab + bc
2ac = b(a+c)
(2ac / a+c) = b
2/b = 1/c + 1/a
⇒
a, b, c
H.P.

If a, b and c are distinct numbers in A.P. and a^2, b^2 and c^2 are in H.P. then which of the following is/are true:

A. a, -b/2 , c are in G.P.

B. a, 2b , c are in G.P.

C. a, b , -c/2 are in G.P.

D. a, b , 2c are in G.P.

Arithmetic Geometric Progression

What is A.G.P. ?

Eg: A.P. : $1, 2, 3, 4, 5, \ldots$

G.P. : $x, x^2, x^3, x^4, x^5, \ldots$

A.G.P. : $x, 2x^2, 3x^3, 4x^4, 5x^5, \ldots$

Sum of A.G.P.

$|x| < 1$

Eg°

$$S = x + 2x^2 + 3x^3 + 4x^4 + \cdots\cdots \infty$$

$$xS = \quad x^2 + 2x^3 + 3x^4 + \cdots\cdots \infty$$

$$(1-x)S = x + x^2 + x^3 + x^4 + \cdots\cdots \infty$$

Pure G.P.

$$(1-x)S = \frac{x}{1-x}$$

$$\boxed{S = \frac{x}{(1-x)^2}}$$

Ans.

The sum to infinite term of the series $1+\frac{2}{3}+\frac{6}{3^2}+\frac{10}{3^3}+\frac{14}{3^4}+\cdots$ is

A. 2 **B.** 3 **C.** 4 **D.** 5

2009 M

$$S = \frac{2}{3} + \frac{6}{3^2} + \frac{10}{3^3} + \frac{14}{3^4} + \cdots$$

$$\frac{1}{3}S = \square + \frac{2}{3^2} + \frac{6}{3^3} + \frac{10}{3^4} + \cdots$$

$$\frac{2S}{3} = \frac{2}{3} + \frac{4}{3^2} + \frac{4}{3^3} + \frac{4}{3^4} + \cdots \infty$$

$$\Rightarrow \frac{2S}{3} = \frac{2}{3} + \frac{4/9}{1-\frac{1}{3}}$$

$$\frac{2S}{3} = \frac{2}{3} + \frac{4}{2 \times 3}$$

$$\frac{2S}{3} = \frac{2}{3} + \frac{2}{3}$$

$$\frac{2S}{3} = \frac{4}{3} \Rightarrow \boxed{S = 2}$$

Answer:

1 + S

1 + 2

$\boxed{3}$

If $(10)^9 + 2(11)^1 (10)^8 + 3(11)^2(10)^7 + \ldots\ldots + 10(11)^9 = k(10)^9$, then **k** is equal to:

2014 M

A. 100 **B.** 110 **C.** 121/10 **D.** 441/100

$$S = (10)^9 + 2(11)^1(10)^8 + 3(11)^2(10)^7 + \cdots + 10(11)^9 \quad \square$$

$$\frac{11}{10}S = \square + (11)^1(10)^8 + 2(11)^2(10)^7 + \cdots + 9(11)^9$$

$$\frac{-S}{10} = (10)^9 + (11)^1(10)^8 + (11)^2(10)^7 + \cdots + (11)^9 - (11)^{10}$$

$$-\frac{S}{10} = \frac{(10)^9\left(\left(\frac{11}{10}\right)^{10}-1\right)}{\left(\frac{11}{10}-1\right)} - (11)^{10} \qquad \left| S_n = \frac{a(r^n-1)}{(r-1)} \right.$$

$$-\frac{S}{10} = 10^{10}\left(\frac{11^{10}}{10^{10}}-1\right) - (11)^{10}$$

$$-\frac{S}{10} = 11^{10} - 10^{10} - (11)^{10}$$

$$\boxed{S = 10^{11}} \rightarrow 10^2 \cdot 10^9 \rightarrow (100)10^9 \rightarrow \boxed{K=100}$$

$2^{1/4} \cdot 4^{1/8} \cdot 8^{1/16} \cdot 16^{1/32}$ ..... is equal to

A. 1 **B.** 2 **C.** 3/2 **D.** 5/2

2014 M

$$2^{1/4} \cdot 2^{2/8} \cdot 2^{3/16} \cdot 2^{4/32} \cdots\cdots$$

$$= (2)^{\left(\frac{1}{4} + \frac{2}{8} + \frac{3}{16} + \frac{4}{32} + \cdots \infty\right)}$$

$$= (2)^{S}$$

Now:

$$S = \frac{1}{4} + \frac{2}{8} + \frac{3}{16} + \frac{4}{32} + \cdots \infty$$

$$\frac{S}{2} = \frac{1}{8} + \frac{2}{16} + \frac{3}{32} + \cdots \infty$$

$$\frac{S}{2} = \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \frac{1}{32} + \cdots \infty$$

$$\frac{S}{2} = \frac{1/4}{1-1/2} \Rightarrow \frac{S}{2} = \frac{1}{2} \Rightarrow \boxed{S = 1}$$

Ans.

2^S

$2^1 \rightarrow (2)$

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11th

Namo Sir | Physics

6:00 - 7:30 PM

Ashwani Sir | Chemistry

7:30 - 9:00 PM

Sameer Sir | Maths

9:00 - 10:30 PM

12th

Jayant Sir | Physics

1:30 - 3:00 PM

Anupam Sir | Chemistry

3:00 - 4:30 PM

Nishant Sir | Maths

4:30 - 6:00 PM

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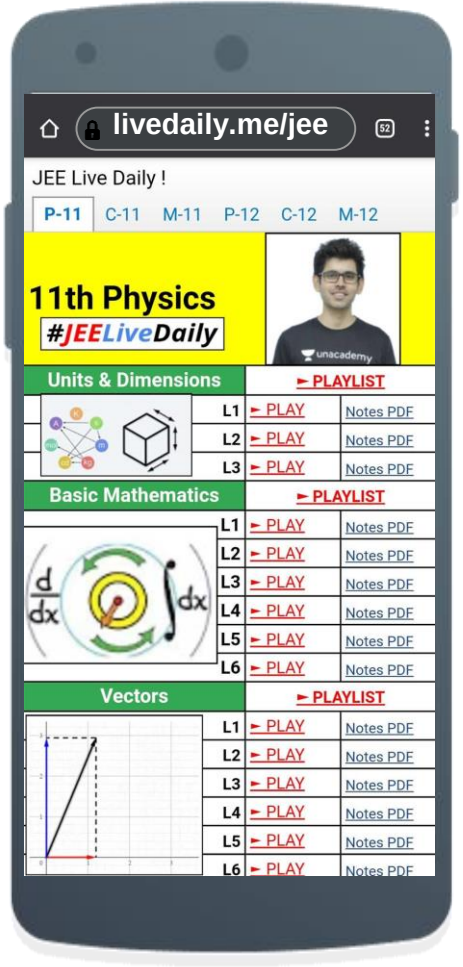

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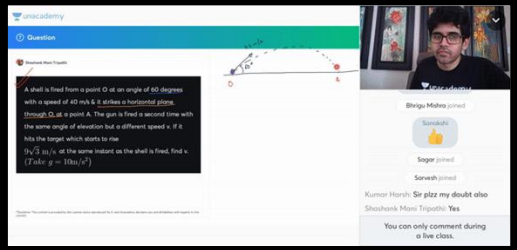

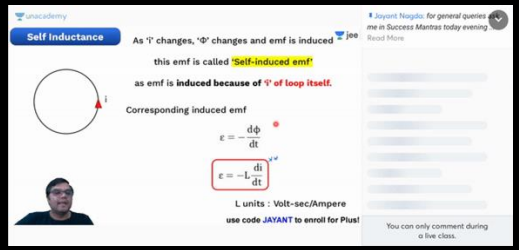

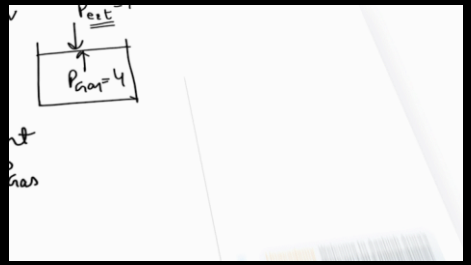

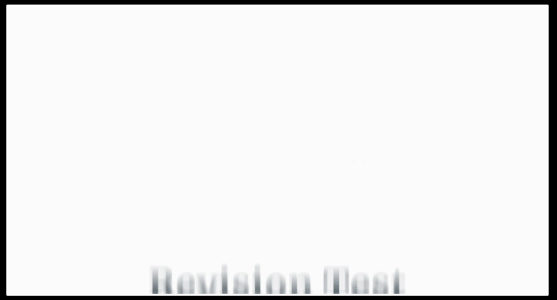

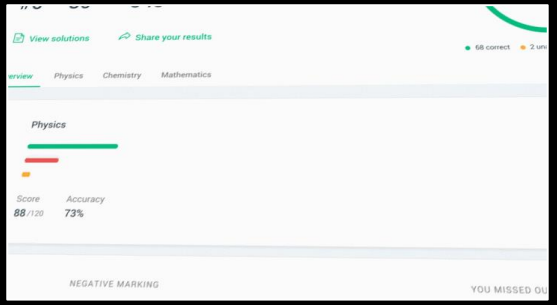

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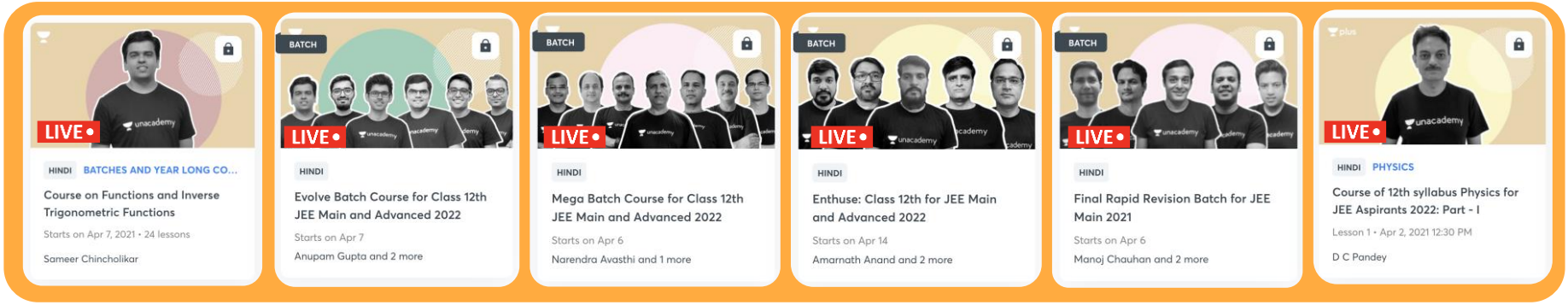

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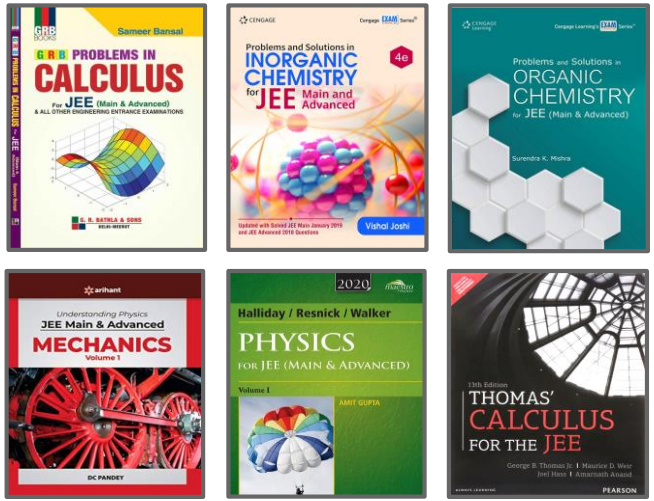

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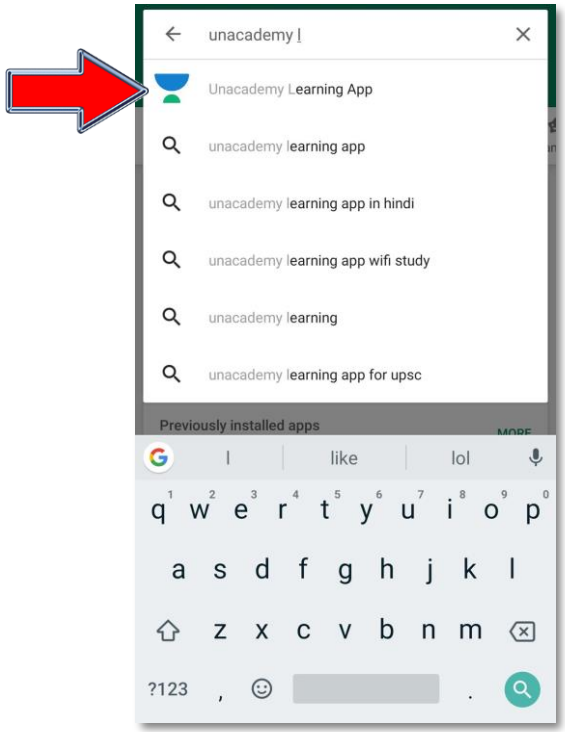
Step 1



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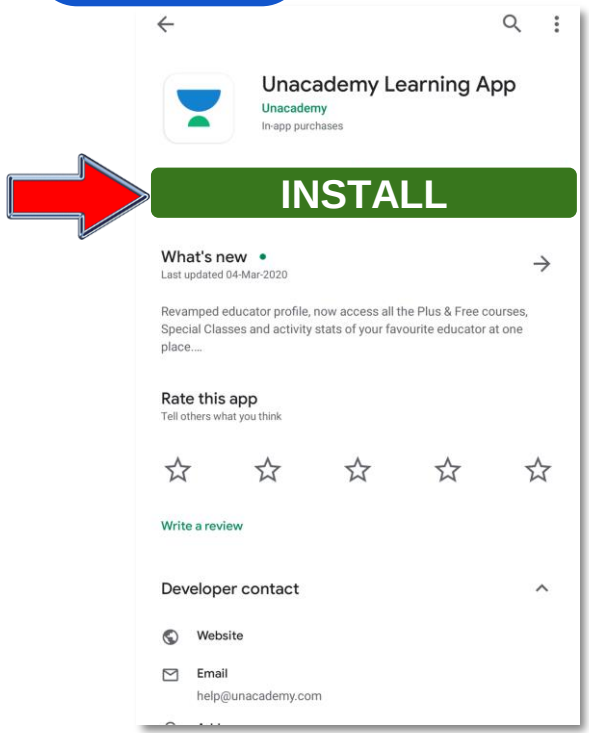

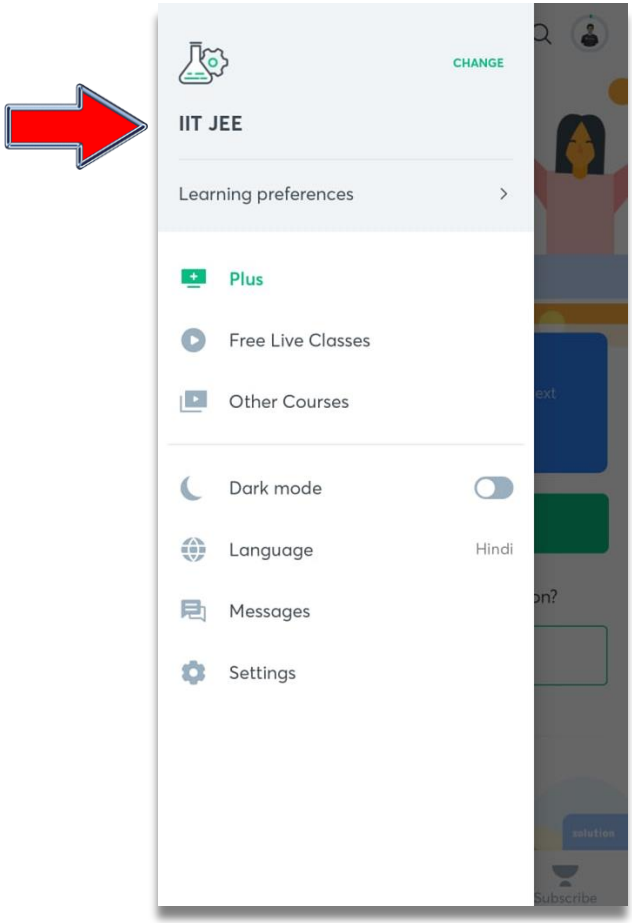
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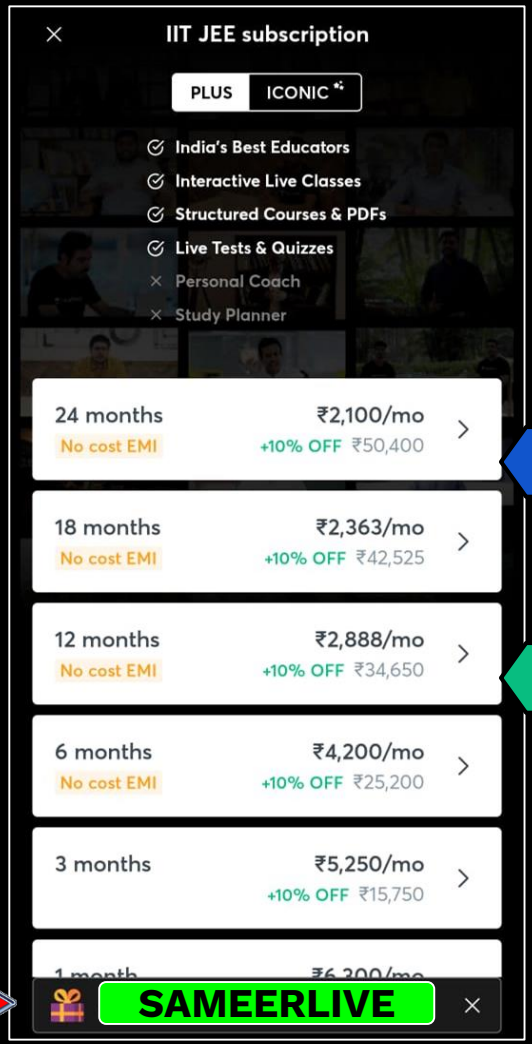

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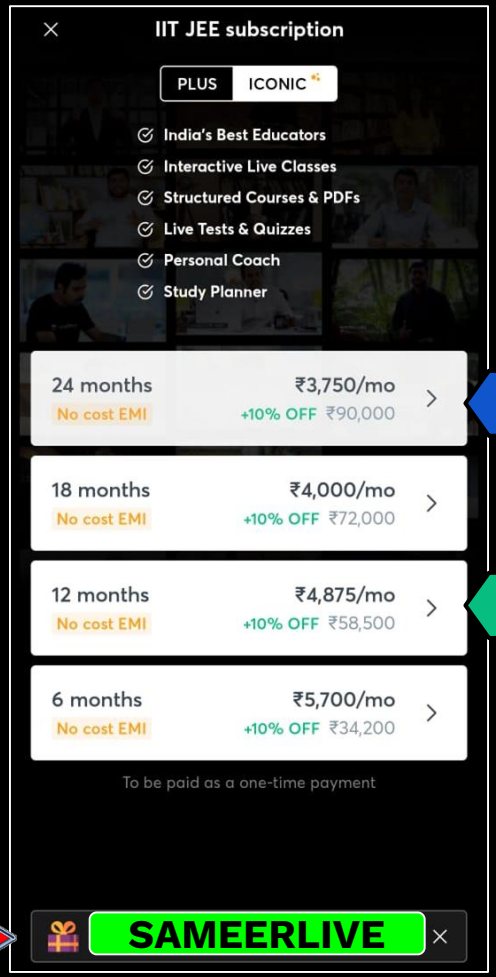

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