

Double Sigma Problems

Sequences & Series

10

$$\sum_{n=1}^{10} \sum_{i=1}^n i$$



Sameer Chincholikar
B.Tech, M.Tech - IIT-Roorkee

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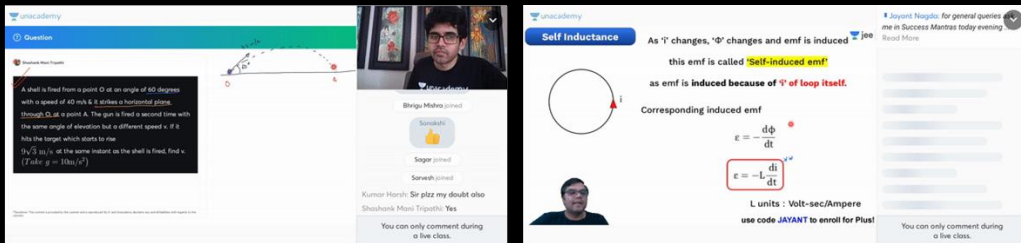
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Questions

A shell is fired from a point O at an angle of 60 degrees with a speed of 40 m/s. It strikes a horizontal plane through O at a point A. The gun is fired a second time with the same angle of elevation but a different speed v . If it hits the target which starts to rise $(\sqrt{3}/2) \sin t$ at the same instant as the shell is fired, find v . (Take $g = 10 \text{ m/s}^2$)

Shreyas Mishra joined

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Saravali joined

Kumar Harsh: Sir plz my doubt also

Shashank Masi Tripathi: Yes

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Self Inductance

As \vec{I} changes, $\vec{\Phi}$ changes and emf is induced

this emf is called **Self-induced emf**

as emf is induced because of \vec{I} of loop itself.

Corresponding induced emf

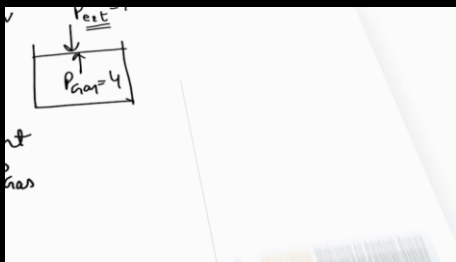
$$\mathcal{E} = -\frac{d\Phi}{dt}$$

$$\mathcal{E} = -L \frac{dI}{dt}$$

L units: Volt-sec/Ampere

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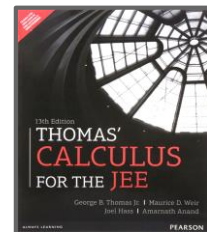
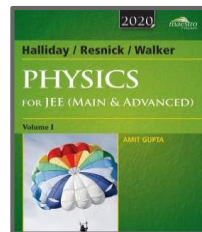
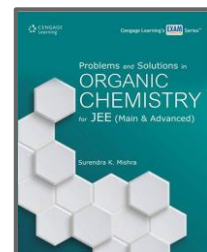
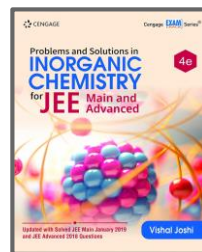
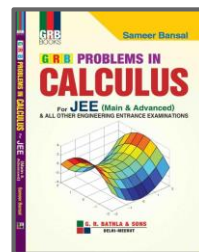
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Double Sigma Problems

Sequences & Series

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$$\sum_{n=1}^{10} \sum_{i=1}^n i$$



Variables independent of each other





Evaluate:

$$\sum_{n=1}^{10} \sum_{i=1}^n i$$

$$\sum_{n=1}^{10} (1 + 2 + 3 + \dots + n)$$

$$\sum_{n=1}^{10} \frac{n(n+1)}{2} = \frac{1}{2} \left[\sum_{n=1}^{10} n^2 + \sum_{n=1}^{10} n \right]$$

$$= \frac{1}{2} \left[\frac{\overset{5}{\cancel{10}} \times 11 \times \overset{7}{\cancel{21}}}{\underset{2 \times 7}{\cancel{14}}} + \frac{10 \times 11}{2} \right]$$

$$= \frac{1}{2} [385 + 55]$$

$$= \boxed{220}$$



Evaluate:

$$\sum_{j=1}^n \sum_{i=1}^n i \cdot j$$

$$\sum_{j=1}^n \sum_{i=1}^n i \cdot j$$

Handwritten red annotations: A red arrow points from the inner sum $\sum_{i=1}^n i$ to the outer sum $\sum_{j=1}^n$. The inner sum $\sum_{i=1}^n i$ is circled in red. The variable j in the outer sum is circled in red.

$$\sum_{j=1}^n j \cdot \frac{n(n+1)}{2}$$

Handwritten blue annotations: A blue arrow points from the circled term $\frac{n(n+1)}{2}$ to the sum $\sum_{j=1}^n$.

$$= \frac{n(n+1)}{2} \left(\sum_{j=1}^n j \right)$$

$$= \left(\frac{n(n+1)}{2} \right) \left(\frac{n(n+1)}{2} \right)$$



Evaluate:

$$\sum_{j=1}^n \sum_{i=1}^n i \cdot 3^j$$

$$= \left(\sum_{j=1}^n 3^j \right) \left(\sum_{i=1}^n i \right)$$

$$= \left(3^1 + 3^2 + 3^3 + \dots + 3^n \right) \left(\frac{n(n+1)}{2} \right)$$

$$\frac{3(3^n - 1)}{(3 - 1)} \quad \frac{n(n+1)}{2}$$

$$\boxed{\frac{3}{4} n(n+1)(3^n - 1)}$$



Evaluate:

$$\sum_{i=1}^3 \sum_{j=2}^4 (i+j)$$

$$\sum_{i=1}^3 \left(\sum_{j=2}^4 i + \sum_{j=2}^4 j \right)$$

$$\sum_{i=1}^3 \left(i \cdot \sum_{j=2}^4 1 + (2+3+4) \right)$$

$\rightarrow (1+1+1)$

$$\sum_{i=1}^3 (3i+9)$$

$$3 \sum_{i=1}^3 i + 9 \sum_{i=1}^3 1$$

\downarrow \downarrow

$(1+2+3)$ (3)

$$3(6) + 9(3)$$

$$= 18 + 27$$

$$= 45$$



Evaluate:

$$\sum_{k=1}^{\infty} \sum_{n=1}^{\infty} \frac{k}{2^{n+k}}$$

Diagram illustrating the evaluation of the double sum:

The sum is split into two parts, S_1 and S_2 , based on the order of summation:

$$\left(\sum_{k=1}^{\infty} \frac{k}{2^k} \right) \cdot \left(\sum_{n=1}^{\infty} \frac{1}{2^n} \right)$$

Where $S_1 = \sum_{k=1}^{\infty} \frac{k}{2^k}$ and $S_2 = \sum_{n=1}^{\infty} \frac{1}{2^n}$.

A diagram shows the summation indices $k=1$ and $n=1$ with arrows pointing to the respective sums. A box highlights the term $\frac{k}{2^n \cdot 2^k}$.

$$S_2 = \sum_{n=1}^{\infty} \frac{1}{2^n}$$

$$= \frac{1}{2} + \frac{1}{2^2} + \frac{1}{2^3} + \dots \infty$$

$$= \frac{\frac{1}{2}}{1 - \frac{1}{2}} = 1$$

$$S_1 = \sum_{k=1}^{\infty} \frac{k}{2^k}$$

$$S_1 = \frac{1}{2^1} + \frac{2}{2^2} + \frac{3}{2^3} + \dots \infty$$

$$\frac{S_1}{2} = \frac{1}{2^2} + \frac{2}{2^3} + \dots \infty$$

$$\frac{S_1}{2} = \frac{1}{2} + \frac{1}{2^2} + \frac{1}{2^3} + \dots \infty$$

$$\frac{S_1}{2} = 1 \Rightarrow \boxed{S_1 = 2}$$

... Ans.

$$(S_1)(S_2)$$

$$= 2$$



$\sum_{r=1}^n r^2 - \sum_{m=1}^n \sum_{r=1}^m r$ is **equal** to

A. 0

B. $\frac{1}{2} \left(\sum_{r=1}^n r^2 + \sum_{r=1}^n r \right)$

D. None of these

✓ C. $\frac{1}{2} \left(\sum_{r=1}^n r^2 - \sum_{r=1}^n r \right)$

$$\begin{aligned} \sum_{m=1}^n \sum_{r=1}^m r &= \sum_{m=1}^n \left(\frac{m(m+1)}{2} \right) \\ &= \frac{1}{2} \sum_{m=1}^n (m^2 + m) \end{aligned}$$

$$\begin{aligned} \sum_{k=1}^n k &= \frac{n(n+1)}{2} \\ \sum_{t=1}^n t &= \frac{n(n+1)}{2} \\ \sum_{l=1}^n l &= \frac{n(n+1)}{2} \end{aligned}$$

$$Q_{ul} \equiv \sum_{k=1}^n k^2 - \sum_{m=1}^n \sum_{k=1}^m k$$

$$= \sum_{k=1}^n k^2 - \frac{1}{2} \left[\sum_{m=1}^n m^2 + \sum_{m=1}^n m \right]$$

$$= \sum_{k=1}^n k^2 - \frac{1}{2} \left[\sum_{k=1}^n k^2 + \sum_{k=1}^n k \right]$$

$$= \frac{1}{2} \left[\sum_{k=1}^n k^2 - \sum_{k=1}^n k \right]$$



Find the **value of the expression**

$$\sum_{i=1}^n \sum_{j=1}^i \sum_{k=1}^j 1$$

$$\sum_{i=1}^n \sum_{j=1}^i \left(\sum_{k=1}^j 1 \right)$$

$$\sum_{i=1}^n \sum_{j=1}^i (j)$$

$$= \sum_{i=1}^n \frac{i(i+1)}{2}$$

$$= \frac{1}{2} \left[\sum_{i=1}^n i^2 + \sum_{i=1}^n i \right]$$

$$= \frac{1}{2} \left[\frac{n(n+1)(2n+1)}{6} + \frac{n(n+1)}{2} \right]$$

Variables dependent of each other

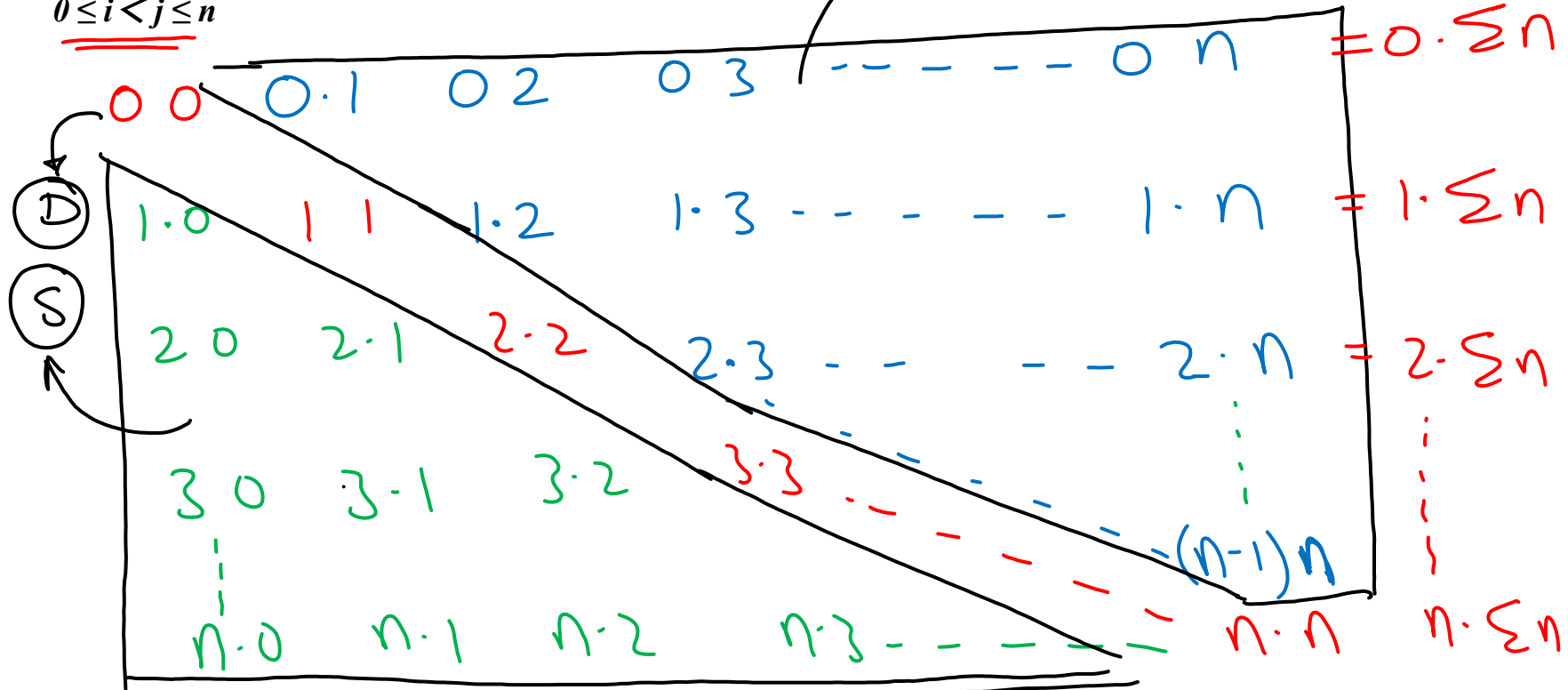




Evaluate:

$$\sum \sum \sum (i,j) = S$$

$$0 \leq i < j \leq n$$



$$\text{Total}(T) = 2S + D$$

$$(\sum n)^2 = 2S + \sum(n^2)$$

$$S = \frac{(\sum n)^2 - \sum(n^2)}{2}$$



Evaluate:

$$\sum \sum_{i \neq j} (i, j)$$

$$= T - D$$

$$= (\sum n)^2 - (\sum n^2)$$

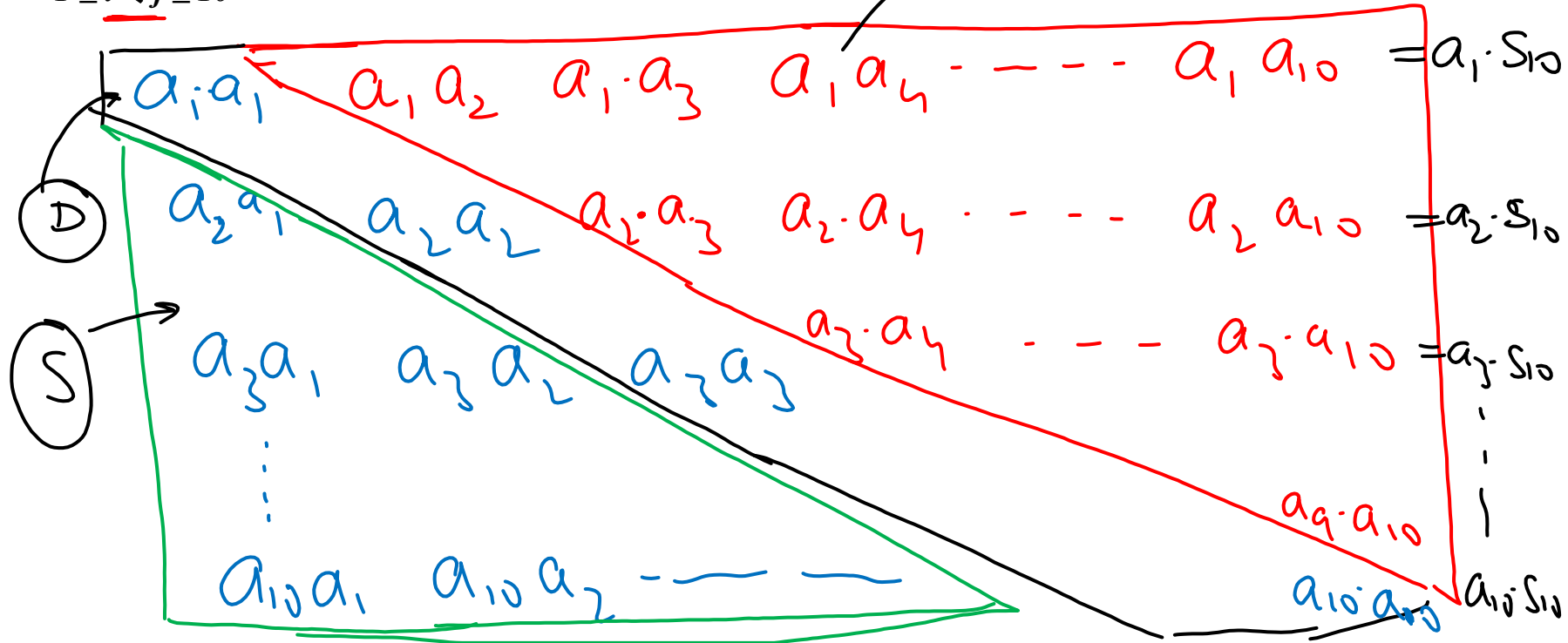


If for the sequence $\langle a_n \rangle$, sum of n terms is given by $\underline{\underline{S_n = 2n^2 + 3n}}$,
then find the sum

$$\sum \sum_{1 \leq i < j \leq 10} a_i \cdot a_j = S$$

$$1 \leq i < j \leq 10$$

(S)



$$T = 2S + D$$

$$S = \frac{T - D}{2}$$

$$S = \frac{(S_{10})^2 - \sum_{i=1}^{10} a_i^2}{2}$$

①

Now.

$$S_n = 2n^2 + 3n$$

$$S_{10} = 2(100) + 3(10)$$

$$= \boxed{230}$$

$$- \textcircled{2}$$

$$\begin{aligned}T_n &= S_n - S_{n-1} \\&= (2n^2 + 3n) - (2(n-1)^2 + 3(n-1)) \\&= 2(n^2 - (n-1)^2) + 3(n - (n-1)) \\&= 2(2n-1) + 3 \\&= 4n+1\end{aligned}$$

$$\Rightarrow a_n = 4n+1$$

Now.

$$\sum_{i=1}^{10} a_i^2 = \sum_{i=1}^{10} (4i+1)^2$$

$$= \sum_{i=1}^{10} (16i^2 + 8i + 1)$$

$$= 16 \sum_{i=1}^{10} i^2 + 8 \sum_{i=1}^{10} i + \sum_{i=1}^{10} 1$$

$$= 16 \left(\frac{10 \times 11 \times 21}{6} \right) + 8 \left(\frac{10 \times 11}{2} \right) + 10$$

using

Σ^n ② & Σ^n ③

in Σ^n ①:

23145

3



Evaluate:

$$\sum_{1 \leq i < j} i \cdot \left(\frac{1}{2}\right)^j$$

$$1 \left(\frac{1}{2}\right)^2 + 1 \left(\frac{1}{2}\right)^3 + 1 \cdot \left(\frac{1}{2}\right)^4 + \dots \infty$$

$$2 \cdot \left(\frac{1}{2}\right)^3 + 2 \left(\frac{1}{2}\right)^4 + \dots \infty$$

$$+ 3 \cdot \left(\frac{1}{2}\right)^4 + \dots \infty$$

using sum of ∞ G.P. in each row.

$$S = \frac{\left(\frac{1}{2}\right)^2}{\left(1 - \frac{1}{2}\right)} + 2 \cdot \left(\frac{\left(\frac{1}{2}\right)^3}{\left(1 - \frac{1}{2}\right)} \right) + 3 \frac{\left(\left(\frac{1}{2}\right)^4\right)}{\left(1 - \frac{1}{2}\right)}$$

$$S = \frac{1}{2} + 2\left(\frac{1}{2}\right)^2 + 3\left(\frac{1}{2}\right)^3 + \dots - \infty$$

$$S = \frac{1}{2} + \frac{2}{2^2} + \frac{3}{2^3} + \dots \infty$$

$$\frac{S}{2} = \frac{1}{2^2} + \frac{2}{2^3} + \dots \infty$$

$$\frac{S}{2} = \frac{1}{2} + \frac{1}{2^2} + \frac{1}{2^3} + \dots \infty$$

$$S = 2 \left(\frac{1/2}{1 - 1/2} \right) = \boxed{2}$$



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7:30 - 9:00 PM



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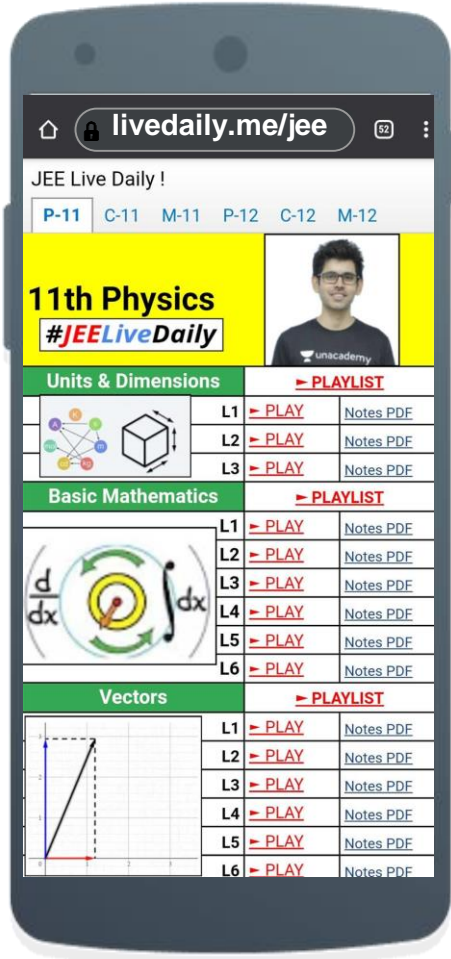
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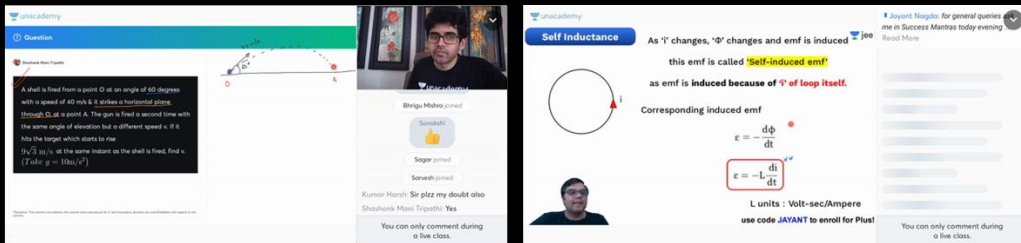
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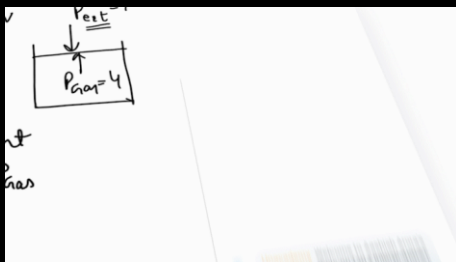
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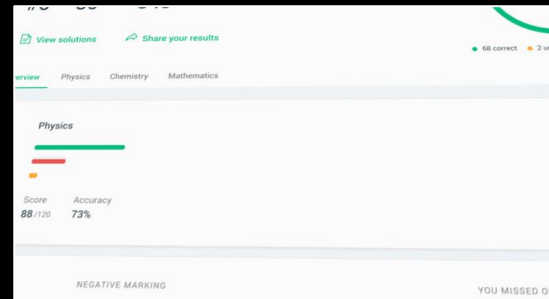
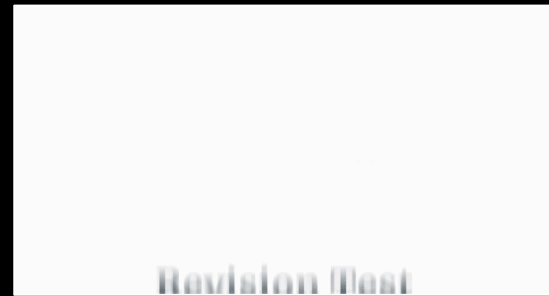


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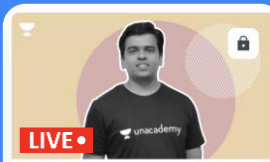


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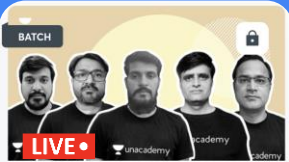
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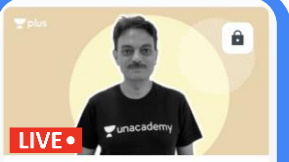
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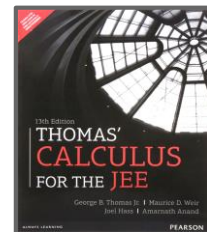
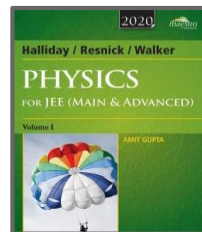
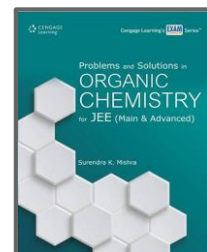
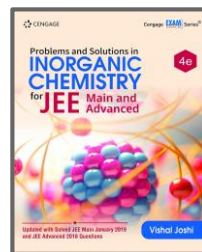
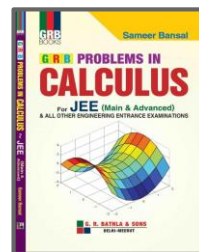
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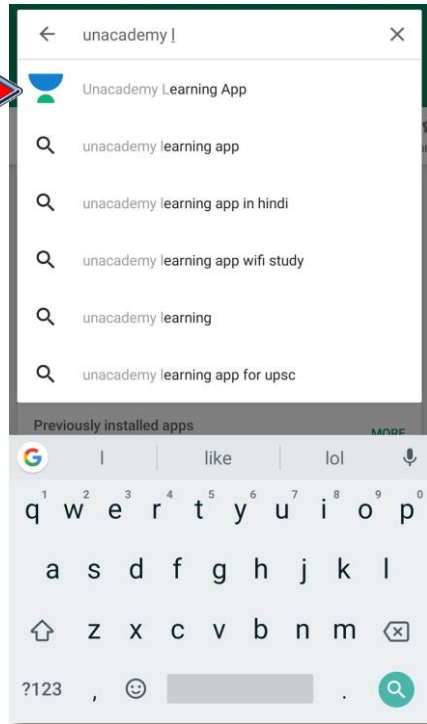


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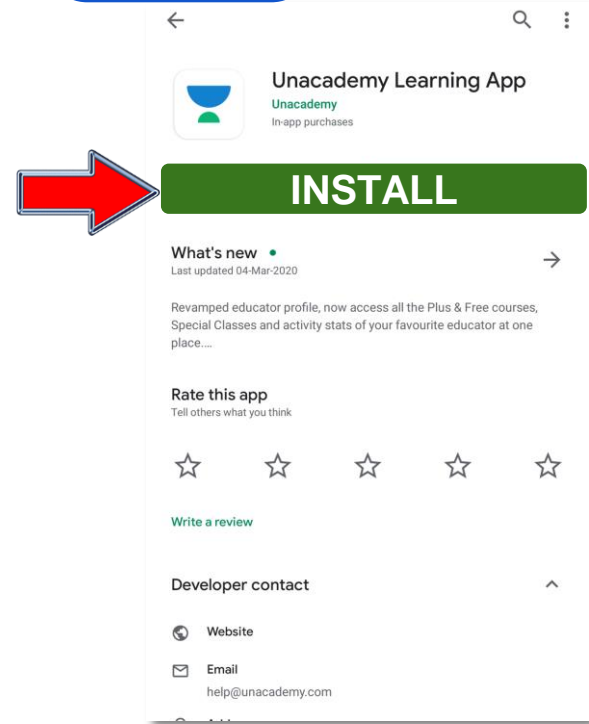


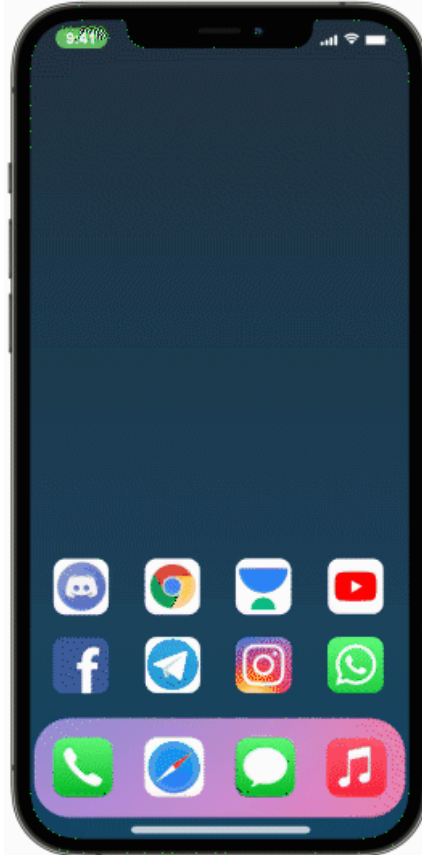
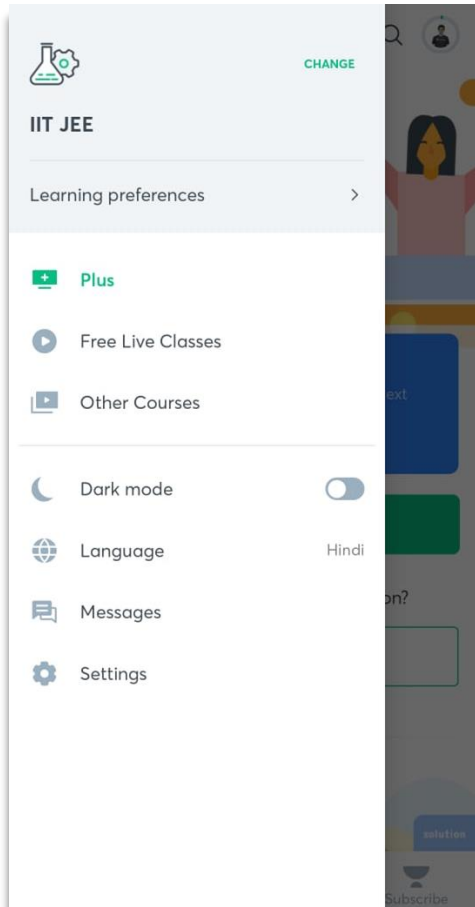
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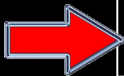
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