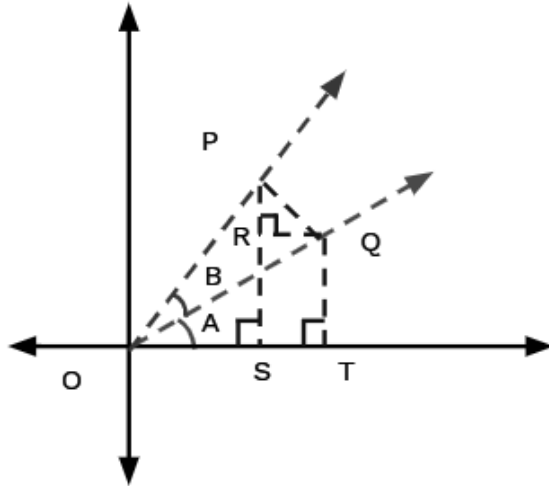


## Multiple & Submultiple Angle Formulas – 2

# Trigonometry

8



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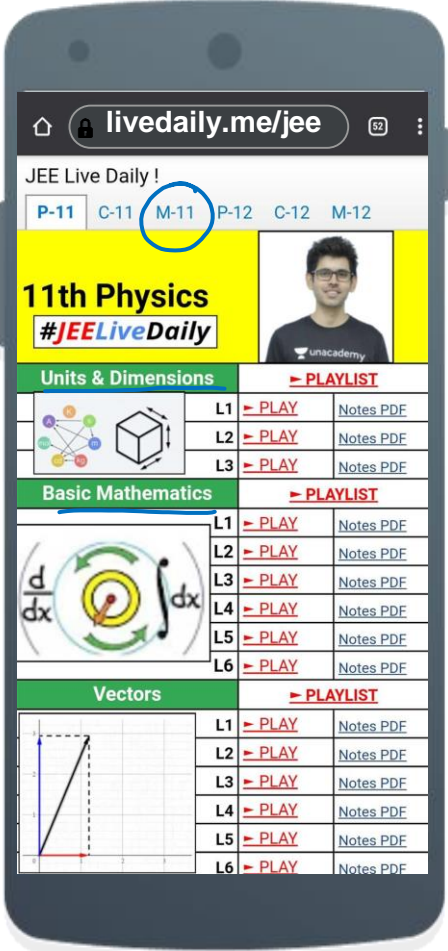
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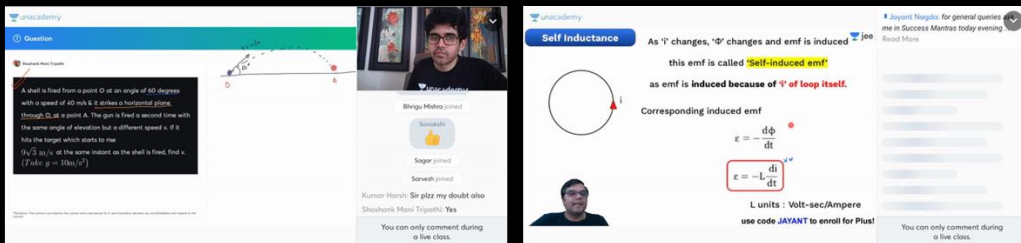
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A shell is fired from a point O at an angle of 60 degrees with a speed of 40 m/s & it strikes a horizontal plane through O at a point A. The gun is fired a second time with the same angle of elevation but a different speed  $v$ . If it hits the target which starts to rise  $(\sqrt{3}/2) \sin(\theta)$  at the same instant as the shell is fired, find  $v$ . (Take  $g = 10 \text{ m/s}^2$ )

Shruti Mishra joined

Sagar joined

Saravali joined

Kumar Harsh: Sir plz my doubt also

Shashank Masi Tripathi: Yes

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**Self Inductance**

As  $\Phi$  changes,  $\frac{d\Phi}{dt}$  changes and emf is induced

this emf is called **Self-induced emf**

as emf is induced because of  $\Phi$  of loop itself.

Corresponding induced emf

$$\mathcal{E} = -\frac{d\Phi}{dt}$$

$$\mathcal{E} = -L \frac{di}{dt}$$

Units: Volt-sec/Ampere

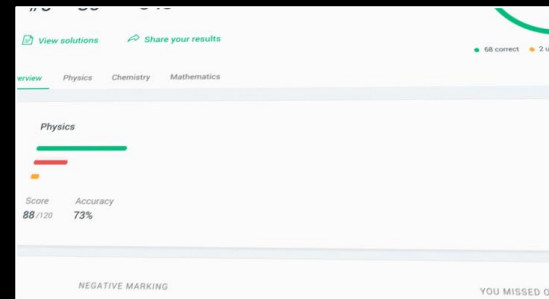
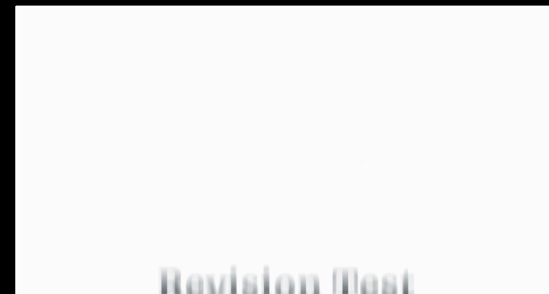
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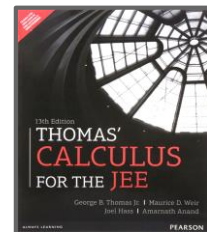
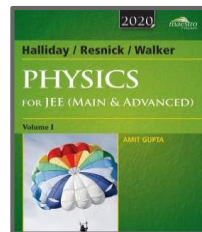
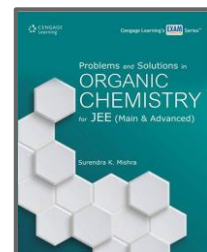
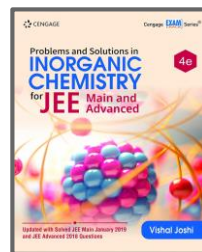
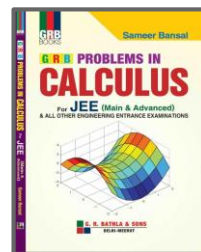
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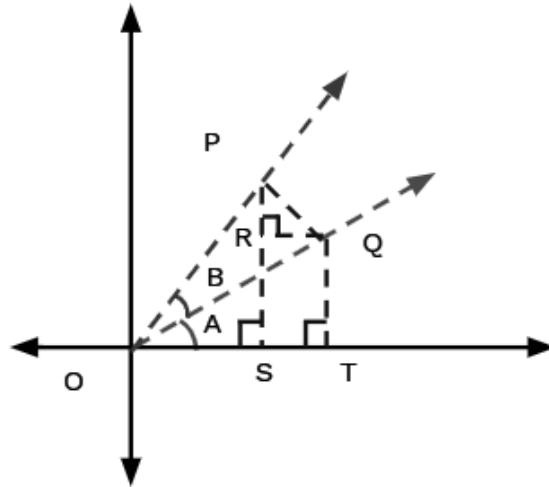




## Multiple & Submultiple Angle Formulas - 2

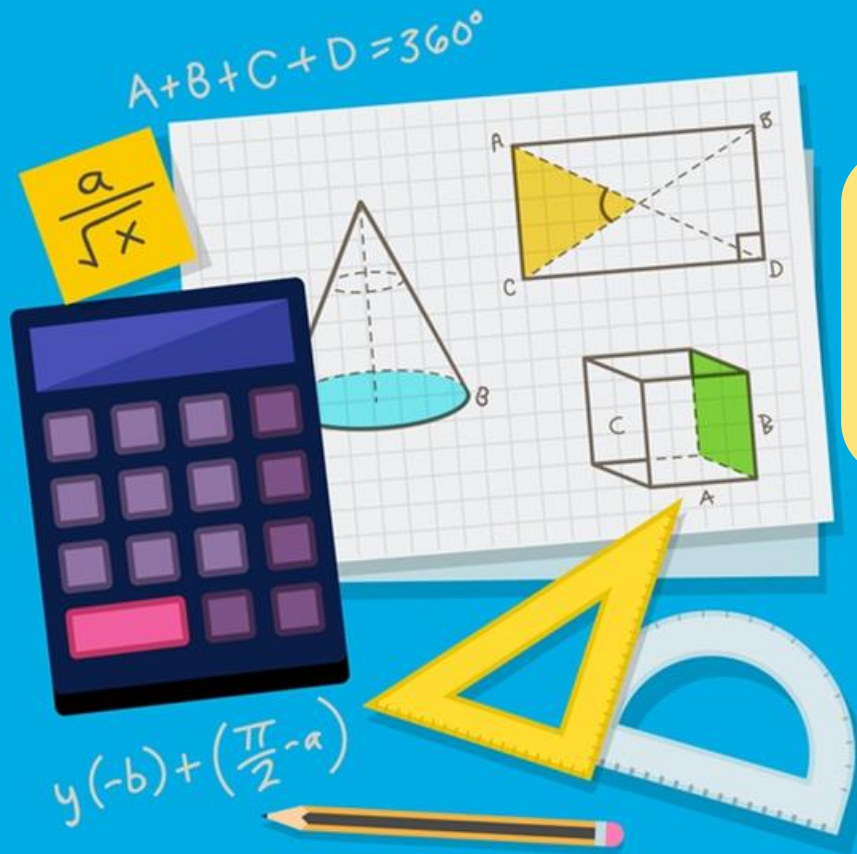
# Trigonometry

8



# LET'S BEGIN!!

# Homework Question





The value of  $\cos^3\left(\frac{\pi}{8}\right) \cdot \cos\left(\frac{3\pi}{8}\right) + \sin^3\left(\frac{\pi}{8}\right) \cdot \sin\left(\frac{3\pi}{8}\right)$  is

JEE Main 2020 (Jan)

A.  $\frac{1}{\sqrt{2}}$

B.  $\frac{1}{2\sqrt{2}}$

C.  $\frac{1}{2}$

D.  $\frac{1}{4}$

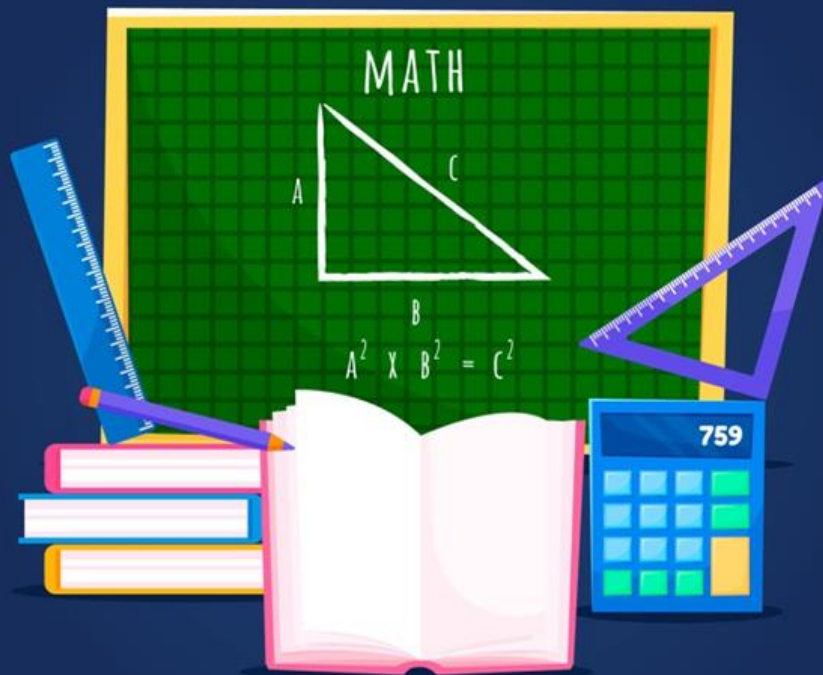
$$\frac{\pi}{8} + \frac{3\pi}{8} = \frac{4\pi}{8} = \frac{\pi}{2}$$

$$\begin{aligned} & \left( \cos^3 \frac{\pi}{8} \right) \left( \cos \left( \frac{\pi}{2} - \frac{\pi}{8} \right) \right) \\ & + \left( \sin^3 \left( \frac{\pi}{8} \right) \right) \left( \sin \left( \frac{\pi}{2} - \frac{\pi}{8} \right) \right) \\ & = \cos^3 \frac{\pi}{8} \sin \frac{\pi}{8} + \sin^3 \frac{\pi}{8} \cos \frac{\pi}{8} \end{aligned}$$

$$\sin \frac{\pi}{8} \cdot \cos \frac{\pi}{8} \left( \underbrace{\cos^2 \frac{\pi}{8} + \sin^2 \frac{\pi}{8}}_{=1} \right)$$

$$\frac{2 \sin \frac{\pi}{8} \cos \frac{\pi}{8}}{2} \rightarrow \frac{1}{2} \sin\left(\frac{\pi}{4}\right) \rightarrow \frac{1}{2\sqrt{2}}$$

# T-Ratios of Standard Angles







## T-Ratios of standard angles

1

$$\sin 22.5^\circ = \frac{\sqrt{2 - \sqrt{2}}}{2}$$

$$\theta = 22.5^\circ$$

$$2\theta = 45^\circ$$

$$\cos 2\theta = \cos 45^\circ$$

$$1 - 2\sin^2 \theta = \frac{1}{\sqrt{2}}$$

$$2\sin^2 \theta = 1 - \frac{1}{\sqrt{2}}$$

$$\sin^2 \theta = \frac{\sqrt{2} - 1}{2\sqrt{2}}$$



$$\underline{\underline{\sin \theta}} = \pm \sqrt{\frac{\sqrt{2}-1}{2\sqrt{2}}}$$

(ignore -ve value)  
 $\therefore \theta$  is in 1<sup>st</sup> Quad

$$\sin \theta = \frac{\sqrt{2-\sqrt{2}}}{2}$$



## T-Ratios of standard angles

2

$$\cos 22.5^\circ = \frac{\sqrt{2 + \sqrt{2}}}{2}$$

$$\theta = 22.5^\circ$$

$$2\theta = 45^\circ$$

$$\cos 2\theta = \frac{1}{\sqrt{2}}$$

$$2\cos^2 \theta - 1 = \frac{1}{\sqrt{2}}$$

$$\cos^2 \theta = \frac{1 + \sqrt{2}}{2\sqrt{2}}$$

$$\cos \theta = \frac{\sqrt{\sqrt{2} + 2}}{2}$$





## T-Ratios of standard angles

3

$$\tan 22.5^\circ = \sqrt{2} - 1$$

$$\begin{aligned}\tan 22.5^\circ &= \frac{\sin 22.5^\circ}{\cos 22.5^\circ} \\ &= \sqrt{\frac{2 - \sqrt{2}}{2 + \sqrt{2}}}\end{aligned}$$

$$\begin{aligned}&= \sqrt{\frac{\sqrt{2} - 1}{\sqrt{2} + 1}} \\ &= \sqrt{2} - 1\end{aligned}$$







## T-Ratios of standard angles

4

$$\sin 18^\circ = \frac{\sqrt{5} - 1}{4} = \cos 72^\circ$$

$$\theta = 18^\circ$$

$$5\theta = 90^\circ$$

$$2\theta + 3\theta = 90^\circ$$

$$2\theta = 90^\circ - 3\theta$$

$$\sin 2\theta = \sin (90^\circ - 3\theta)$$

$$\sin 2\theta = \cos 3\theta$$

$$2 \sin \theta \cos \theta = 4 \cos^3 \theta - 3 \cos \theta$$

$$\cos \theta \neq 0 \quad (\because \theta = 18^\circ)$$

$$2 \sin \theta = 4 \cos^2 \theta - 3$$



$$2 \sin \theta = 4(1 - \sin^2 \theta) - 3$$

$$2 \sin \theta = 4 - 4 \sin^2 \theta - 3$$

$$4 \sin^2 \theta + 2 \sin \theta - 1 = 0$$

$$\sin \theta = \frac{-2 \pm \sqrt{4 + 16}}{8}$$

$$\sin \theta$$

$$= \frac{-2 \pm 2\sqrt{5}}{8}$$

$$= \frac{-1 \pm \sqrt{5}}{4}$$

$$\Rightarrow \sin 18^\circ = \frac{\sqrt{5} - 1}{4}$$



## T-Ratios of standard angles

5

$$\cos 36^\circ = \frac{\sqrt{5} + 1}{4} = \sin 54^\circ$$

$$\begin{aligned}\cos 36^\circ &= \cos(2(18^\circ)) \\ &= 1 - 2\sin^2 18^\circ\end{aligned}$$

$$= 1 - 2 \left( \frac{\sqrt{5} - 1}{4} \right)^2$$

$$= 1 - 2 \frac{(\sqrt{5} - 1)^2}{16}$$

$$= \frac{8 - (5 + 1 - 2\sqrt{5})}{8}$$

$$= \frac{2 + 2\sqrt{5}}{8} \longrightarrow \boxed{\frac{\sqrt{5} + 1}{4}}$$





Find the value of  $\tan 9^\circ - \tan 27^\circ - \tan 63^\circ + \tan 81^\circ$

A. 2

B. 4

C. 8

D. 0

$$(\tan 9^\circ + \tan 81^\circ) - (\tan 27^\circ + \tan 63^\circ)$$

$$(\tan 9^\circ + \cot 9^\circ) - (\tan 27^\circ + \cot 27^\circ)$$

$$= \left( \frac{\sin 9^\circ}{\cos 9^\circ} + \frac{\cos 9^\circ}{\sin 9^\circ} \right) - \left( \frac{\sin 27^\circ}{\cos 27^\circ} + \frac{\cos 27^\circ}{\sin 27^\circ} \right)$$

$$= \frac{1 \times 2}{2 \sin 9^\circ \cos 9^\circ} - \frac{1 \times 2}{2 \sin 27^\circ \cos 27^\circ}$$

$$= \frac{2}{\sin 18^\circ} - \frac{2}{\sin 54^\circ}$$

$$= \frac{2}{\left(\frac{\sqrt{5}-1}{4}\right)} - \frac{2}{\left(\frac{\sqrt{5}+1}{4}\right)}$$

$$8 \left( \frac{1}{\sqrt{5}-1} - \frac{1}{\sqrt{5}+1} \right)$$

$$2 \cancel{8} \left( \frac{\cancel{\sqrt{5}+1} - \cancel{\sqrt{5}+1}}{4} \right)$$

$$\textcircled{4}$$





Value of  $\sin^4 \frac{\pi}{8} + \sin^4 \frac{3\pi}{8} + \sin^4 \frac{5\pi}{8} + \sin^4 \frac{7\pi}{8}$  is equal to:

A. 1

B. 2

C.  $\frac{3}{2}$

D.  $\frac{1}{2}$

$$\left\{ \begin{array}{l} \frac{\pi}{8} + \boxed{\frac{7\pi}{8}} = \pi \\ \frac{3\pi}{8} + \boxed{\frac{5\pi}{8}} = \pi \end{array} \right.$$

$$\sin^4 \frac{\pi}{8} + \sin^4 \frac{3\pi}{8} + \sin^4 \left( \pi - \frac{3\pi}{8} \right)$$

$$+ \sin^4 \left( \pi - \frac{\pi}{8} \right)$$

$$= \sin^4 \frac{\pi}{8} + \sin^4 \frac{3\pi}{8} + \sin^4 \frac{3\pi}{8} + \sin^4 \frac{\pi}{8}$$

$$= 2 \left( \sin^4 \frac{\pi}{8} + \sin^4 \frac{3\pi}{8} \right)$$

Now:

$$\frac{\pi}{8} + \boxed{\frac{3\pi}{8}} = \frac{4\pi}{8} = \frac{\pi}{2}$$

$$= 2 \left( \sin^4 \frac{\pi}{8} + \sin^4 \left( \frac{\pi}{2} - \frac{\pi}{8} \right) \right)$$

$$= 2 \left( \sin^4 \frac{\pi}{8} + \cos^4 \frac{\pi}{8} \right)$$

Let's  $\theta = \pi/8$

$$= 2 (\sin^4 \theta + \cos^4 \theta)$$

$$= 2 \left( (\sin^2 \theta + \cos^2 \theta)^2 - 2 \sin^2 \theta \cos^2 \theta \right)$$

$$= 2 \left( 1 - \frac{\sqrt{4 \sin^2 \theta \cos^2 \theta}}{2} \right)$$

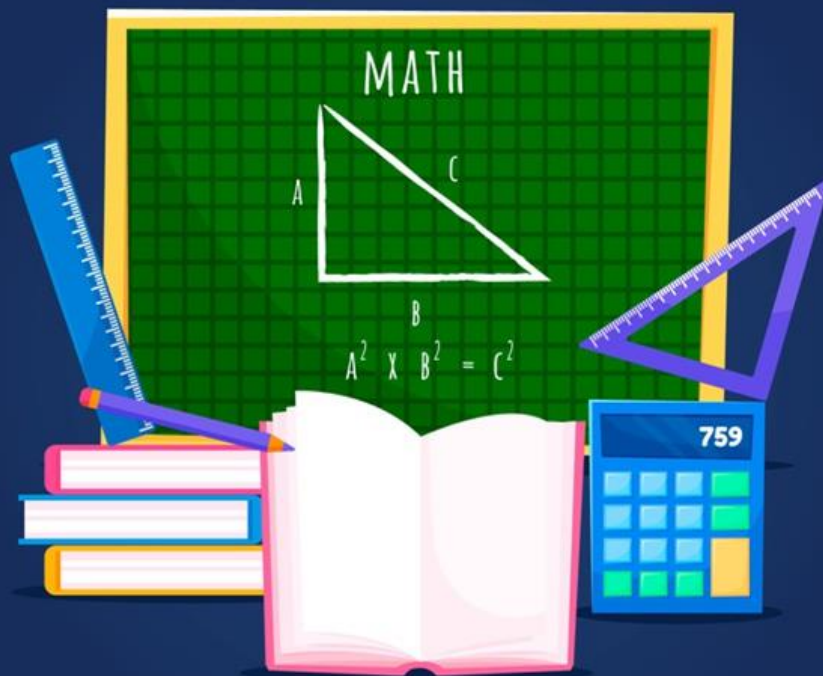
$$= 2 \left( 1 - \frac{1}{2} (\sin 2\theta)^2 \right)$$

$$= \frac{3}{2}$$

$$\frac{\pi}{4}$$



# Some Important Results





## Some Important Results

1

$$\sin \theta \sin(60^\circ - \theta) \sin(60^\circ + \theta) = \frac{1}{4} \sin 3\theta$$

LHS =

$$\sin \theta \left( \frac{\sqrt{3}}{2} \cos \theta - \frac{1}{2} \sin \theta \right) \left( \frac{\sqrt{3}}{2} \cos \theta + \frac{1}{2} \sin \theta \right)$$

$$\sin \theta \left( \frac{3}{4} \cos^2 \theta - \frac{\sin^2 \theta}{4} \right)$$

Ex:

$$\sin 20^\circ \sin 40^\circ \sin 80^\circ$$

↓

$$(60^\circ - 20^\circ)$$

↓

$$(60^\circ + 20^\circ)$$

$$= \frac{1}{4} \sin 60^\circ$$



$$\frac{1}{4} \sin \theta (3(1 - \sin^2 \theta) - \sin^2 \theta)$$

$$\frac{1}{4} \sin \theta (3 - \underbrace{3 \sin^2 \theta - \sin^2 \theta})$$

$$\frac{1}{4} (\underbrace{3 \sin \theta - 4 \sin^3 \theta})$$

$$\frac{1}{4} \sin 3\theta$$





## Some Important Results

2

$$\cos \theta \cos(60^\circ - \theta) \cos(60^\circ + \theta) = \frac{1}{4} \cos 3\theta$$





## Some Important Results

3

$$\tan \theta \tan(60^\circ - \theta) \tan(60^\circ + \theta) = \tan 3\theta$$

$$= \frac{\sin \theta \sin(60^\circ - \theta) \sin(60^\circ + \theta)}{\cos \theta \cos(60^\circ - \theta) \cos(60^\circ + \theta)}$$

$$= \frac{\cancel{\frac{1}{4}} \sin 3\theta}{\cancel{\frac{1}{4}} \cos 3\theta} = \boxed{\tan 3\theta}$$





## Some Important Results

4

$$\cot \theta \cot(60^\circ - \theta) \cot(60^\circ + \theta) = \cot 3\theta$$



Value of  $\frac{\cos 20^\circ + 8 \sin 10^\circ \sin 50^\circ \sin 70^\circ}{\sin^2 80^\circ}$  equal to

$$\cos 20^\circ + 8 \left( \frac{1}{4} \sin 3(10^\circ) \right)$$

$$\sin^2 80^\circ$$

$$\cos 20^\circ + 8 \left( \frac{1}{4} \times \frac{1}{2} \right)$$

$$\sin^2 80^\circ$$

$$\frac{\cos 20^\circ + 1}{\sin^2 80^\circ}$$

$$= \frac{2 \cancel{\cos^2 10^\circ}}{\cancel{\sin^2 80^\circ}}$$

$$= 2$$

$$\left\{ \begin{array}{l} \cos 2\theta = 2\cos^2\theta - 1 \\ 1 + \cos 2\theta = 2\cos^2\theta \\ \underline{1 + \cos 20^\circ} = 2\cos^2 10^\circ \end{array} \right.$$







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Namo Sir | Physics

6:00 - 7:30 PM



Ashwani Sir | Chemistry

7:30 - 9:00 PM



Sameer Sir | Maths

9:00 - 10:30 PM

12<sup>th</sup>



Jayant Sir | Physics

1:30 - 3:00 PM



Anupam Sir | Chemistry

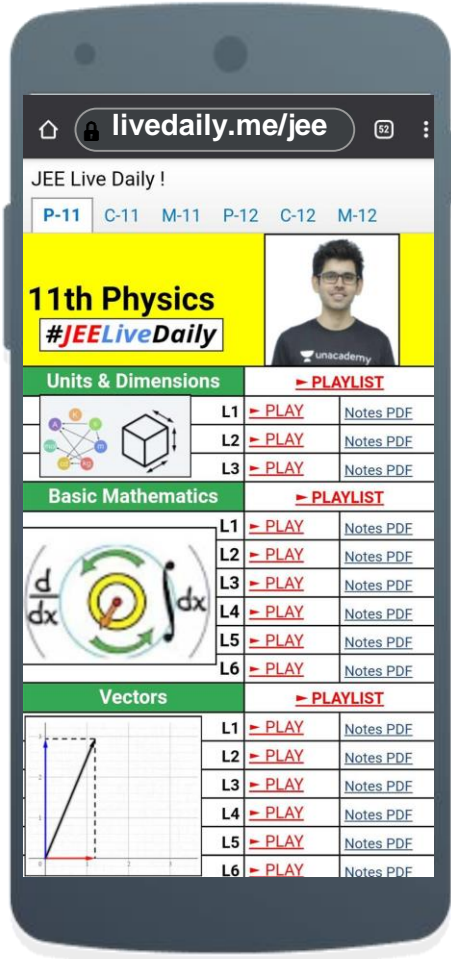
3:00 - 4:30 PM



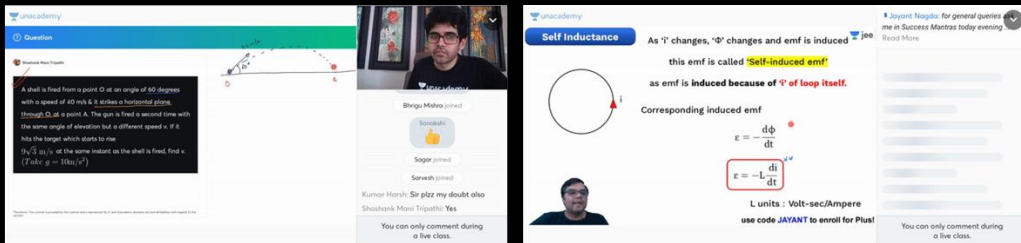
Nishant Sir | Maths

4:30 - 6:00 PM

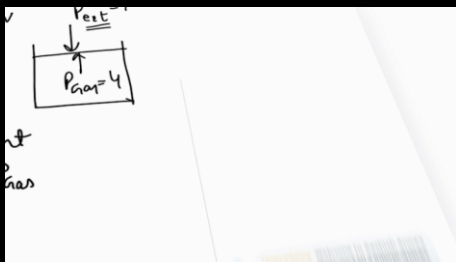
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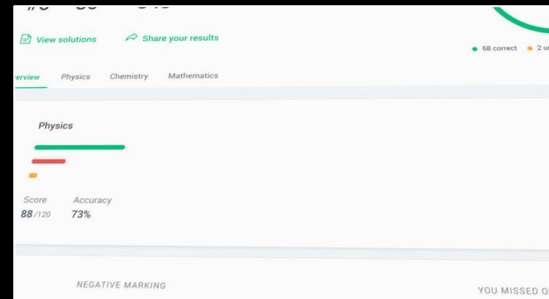
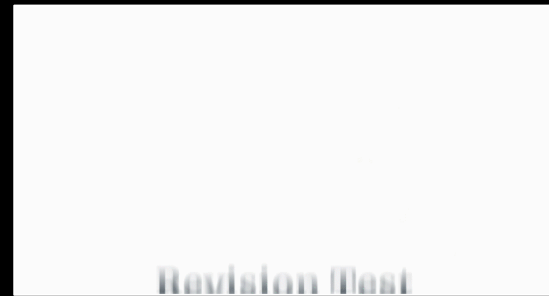


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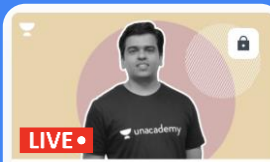


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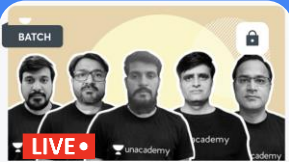
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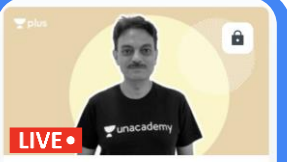
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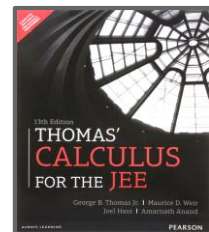
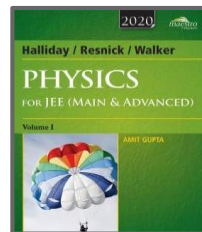
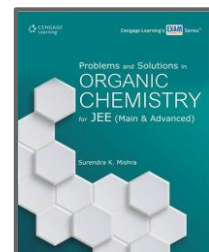
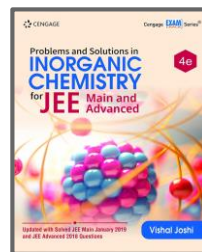
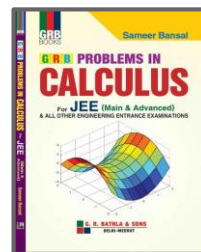
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99.69



Manas Pandey  
99.69



Mihir Agarwal  
99.63



Akshat Tiwari  
99.60



Sarthak  
Kalankar  
99.59



Vaishnovi Arun  
99.58



Devashish Tripathi  
99.52



Maroof  
99.50



Tarun Gupta  
99.50



Siddharth Kaushik  
99.48



Mihir Kothari  
99.39



Sahil  
99.38



Vaibhav Dhanuka  
99.34



Pratham Kadam  
99.29



Shivam Gupta  
99.46



Shrish  
99.28



Yash Bhaskar  
99.10



Subhash Patel  
99.02



Ayush Kale  
98.85



Ayush Gupta  
98.67



Megh Gupta  
98.59

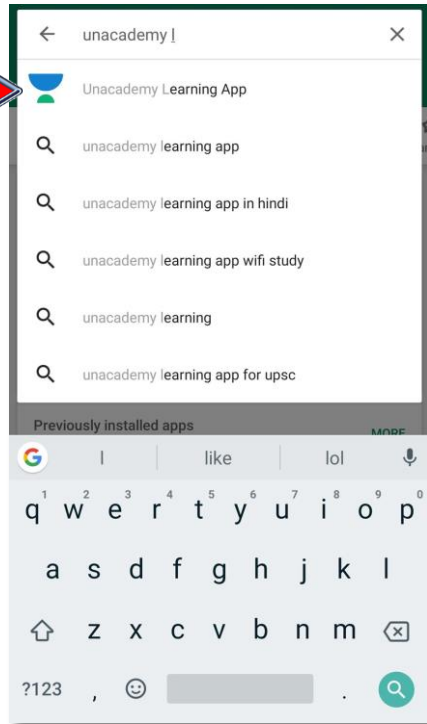


Naman Goyal  
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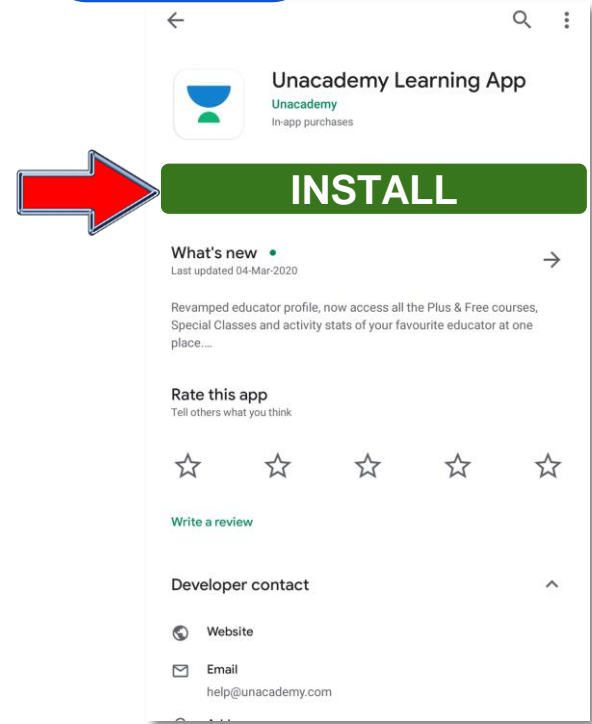


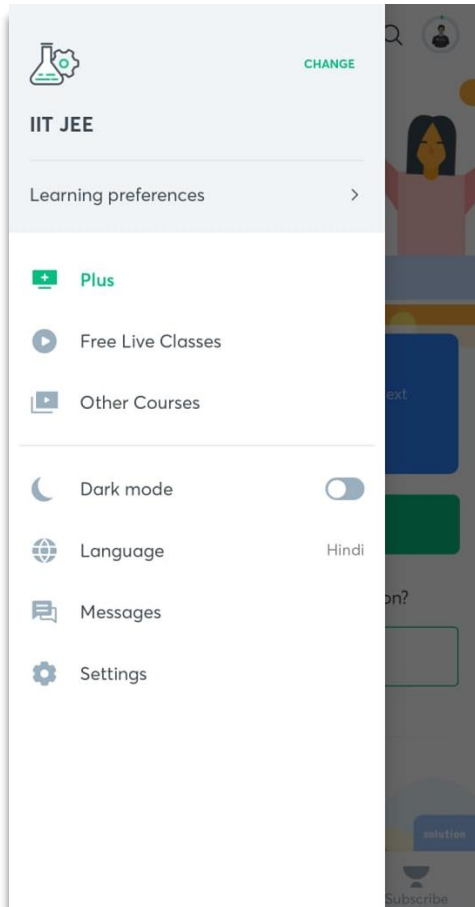
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