Telescoping Series (V_n Method)

Sequences & Series

9

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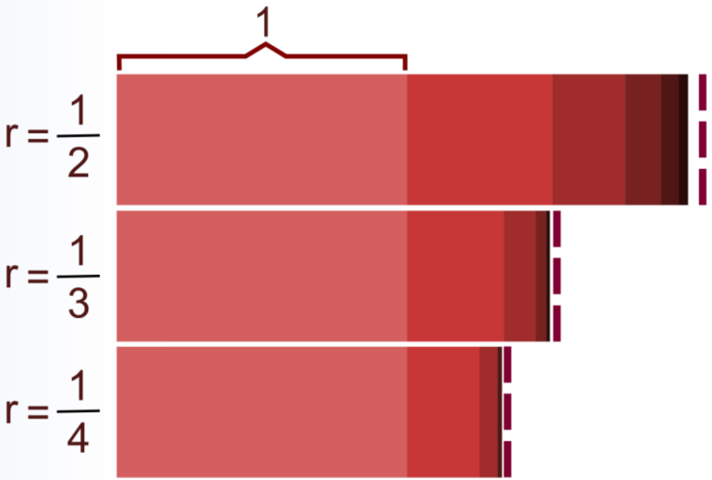

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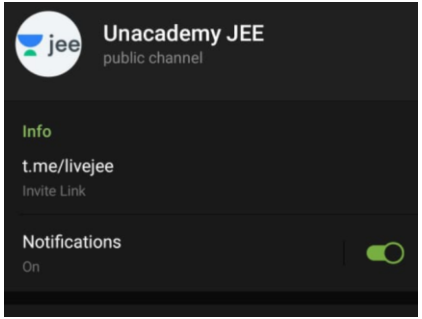

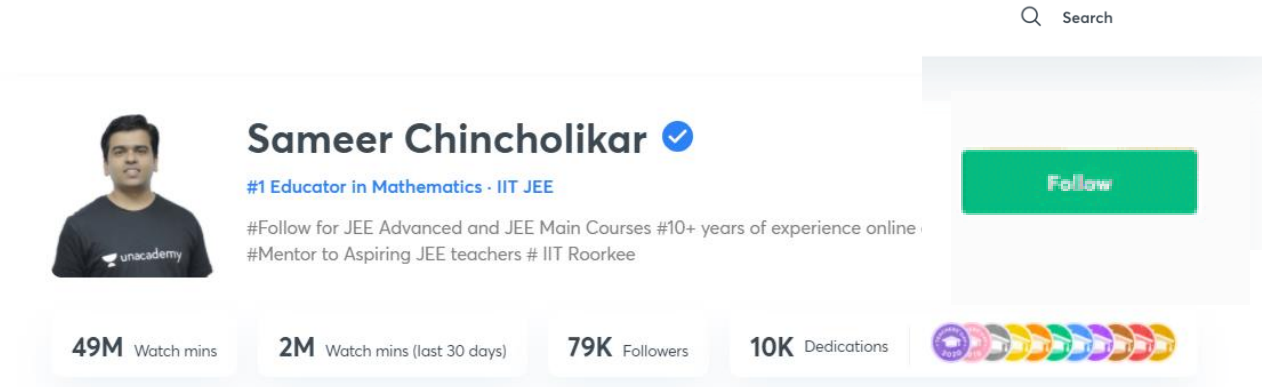

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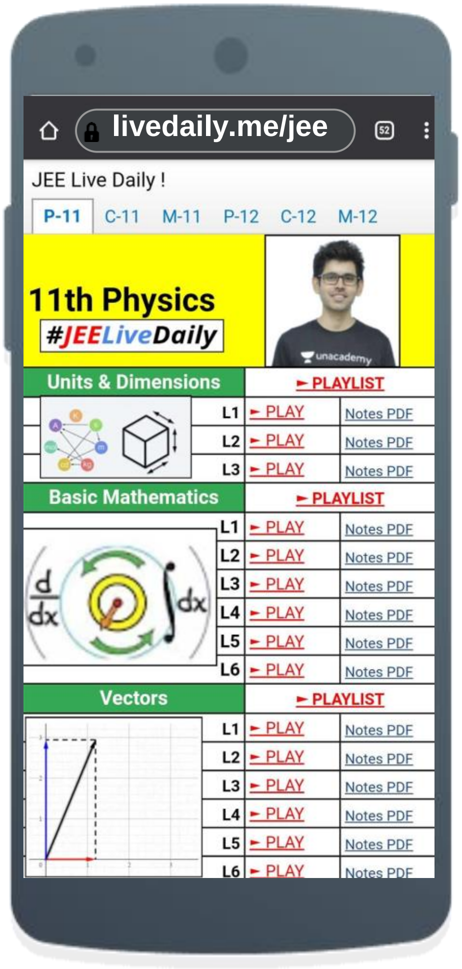

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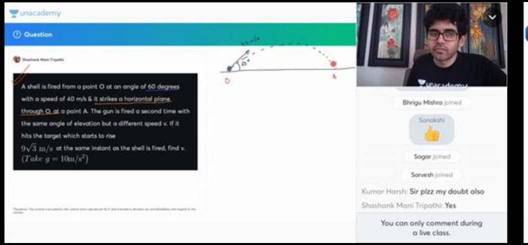

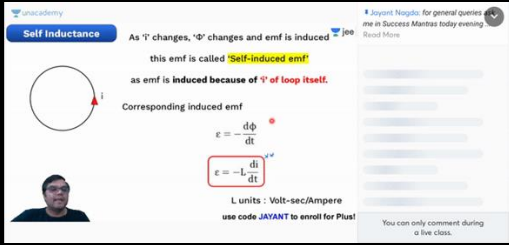

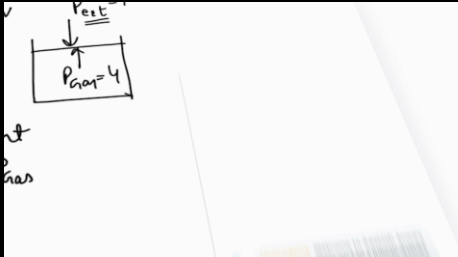

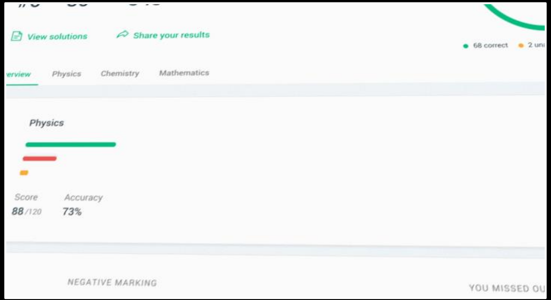

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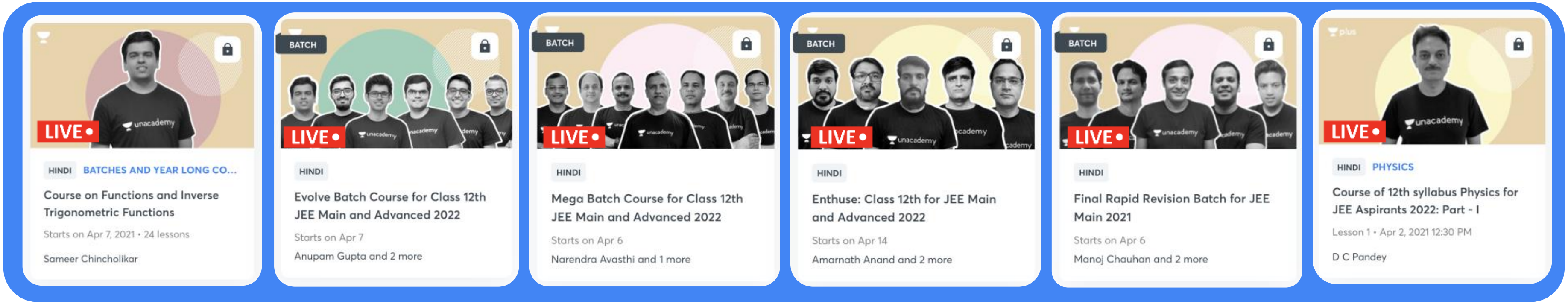

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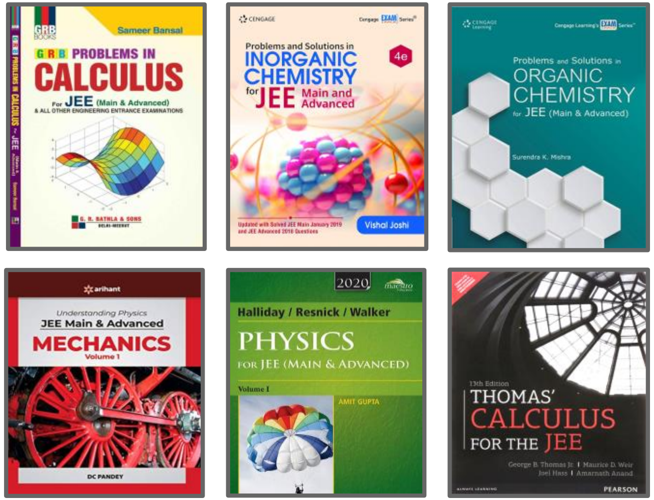


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Telescoping Series (V_n Method)

Sequences & Series

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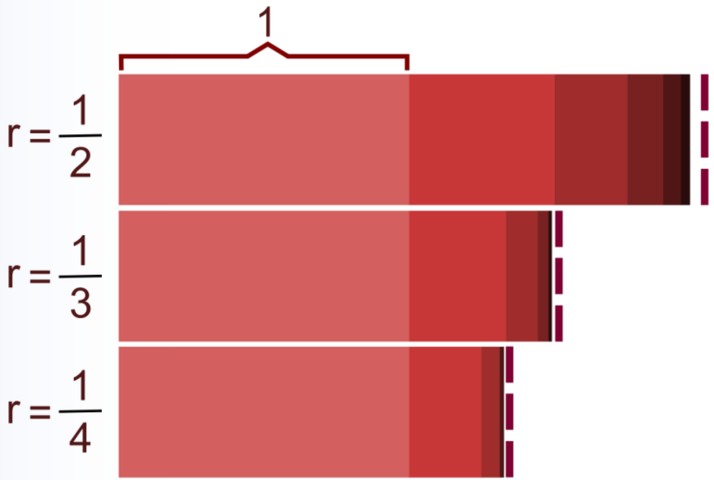

HomeWork Questions

Find the sum to **n-terms** : 2 + 12 + 36 + 80 + 150 + 252 + 392 ....

$2 + 12 + 36 + 80 + 150 + 252 + 392 + \ldots$

10 24 44 70 102 140

$\boxed{K=2}$: 14 20 26 32 38 $\rightarrow$ A.P.

$T_n = an^3 + bn^2 + cn + d$

$d=0$ $c=0$ $b=1$ $a=1$

$$T_1 = a + b + c + d = 2$$

$$T_2 = 8a + 4b + 2c + d = 12$$

$$T_3 = 27a + 9b + 3c + d = 36$$

$$T_4 = 64a + 16b + 4c + d = 80$$

$$\Rightarrow \boxed{T_n = n^3 + n^2}$$

$$S_n = \sum T_n$$

$$= \sum (n^3 + n^2)$$

$$= \sum n^3 + \sum n^2$$

$$= \frac{n^2(n+1)^2}{4} + \frac{n(n+1)(2n+1)}{6}$$

$$= \frac{n(n+1)(n+2)(3n+1)}{12}$$

Telescoping Series

Find the sum upto n terms: $\frac{1}{1\cdot 2}+\frac{1}{2\cdot 3}+\frac{1}{3\cdot 4}+\cdots$

$$\underbrace{\frac{1}{(1)(2)}}_{\left(\frac{1}{1}-\cancel{\frac{1}{2}}\right)}+\underbrace{\frac{1}{(2)(3)}}_{\left(\cancel{\frac{1}{2}}-\cancel{\frac{1}{3}}\right)}+\underbrace{\frac{1}{(3)(4)}}_{\left(\cancel{\frac{1}{3}}-\cancel{\frac{1}{4}}\right)}+----+\underbrace{\frac{1}{(n)(n+1)}}_{\left(\cancel{\frac{1}{n}}-\frac{1}{n+1}\right)}$$

$$=1-\frac{1}{n+1}=\boxed{\frac{n}{n+1}}$$

Telescoping Series

A telescoping series is a series whose partial sums eventually only have a fixed number of terms after **cancellation.**

Find the sum upto n terms: $\frac{1}{1\cdot 4}+\frac{1}{4\cdot 7}+\frac{1}{7\cdot 10}+\cdots - - - + \frac{1}{(\quad)(\quad)}$

$1, 4, 7, ----$ $t_n = 1+(n-1)3$

$= (3n-2)$

$$\frac{1}{3}\left[\frac{3}{1\cdot 4}+\frac{3}{4\cdot 7}+\frac{3}{7\cdot 10}+ - - - + \frac{3}{(3n-2)(3n+1)}\right]$$

$$= \frac{1}{3}\left[\left(\frac{1}{1}-\cancel{\frac{1}{4}}\right)+\left(\cancel{\frac{1}{4}}-\cancel{\frac{1}{7}}\right)+\left(\cancel{\frac{1}{7}}-\cancel{\frac{1}{10}}\right)+\cancel{-}\cancel{-}+\left(\cancel{\frac{1}{3n-2}}-\frac{1}{3n+1}\right)\right]$$

$$= \frac{1}{3}\left[1 - \frac{1}{3n+1}\right]$$

$$= \frac{1}{3}\left[\frac{3n+\cancel{1}-\cancel{1}}{3n+1}\right]$$

$$= \boxed{\frac{n}{3n+1}}$$

Sum the following series to n terms $\frac{1}{1 \cdot 3 \cdot 5}+\frac{1}{3 \cdot 5 \cdot 7}+\frac{1}{5 \cdot 7 \cdot 9}+\cdots$

?

Type-1 of Telescoping Series

Sum of n terms of a series, each term of which is composed of the product of r factors in A.P., the first factors of the several terms being in the same A.P.

Example: 1.2.3 + 2.3.4 + 3.4.5 + ...

M-1

$$T_n = n(n+1)(n+2)$$

$$= (\ )n^3 + (\ )n^2 + (\ )n + (\ )$$

Type-1 of Telescoping Series

In such cases we modify our general term by multiplying it with:

$$\boxed{\left[\frac{(Next\ factor) - (previous\ factor)}{(constant)}\right]}$$

Sum the following series to n terms **1.2.3 + 2.3.4 + 3.4.5 + ...**

$$T_n = n(n+1)(n+2)$$

$$= n(n+1)(n+2)\left[\frac{(n+3)-(n-1)}{4}\right]$$

$$T_n = \frac{n(n+1)(n+2)(n+3)}{4} - \frac{(n-1)(n)(n+1)(n+2)}{4}$$

$$S_n = T_1 + T_2 + T_3 + \cdots + T_n$$

$$T_1 = \cancel{\frac{1\cdot 2\cdot 3\cdot 4}{4}} - \frac{0\cdot 1\cdot 2\cdot 3}{4}$$

$$T_2 = \cancel{\frac{2\cdot 3\cdot 4\cdot 5}{4}} - \cancel{\frac{1\cdot 2\cdot 3\cdot 4}{4}}$$

$$T_3 = \cancel{\frac{3\cdot 4\cdot 5\cdot 6}{4}} - \cancel{\frac{2\cdot 3\cdot 4\cdot 5}{4}}$$

$$\vdots$$

$$T_n = \frac{n(n+1)(n+2)(n+3)}{4} - \cancel{\frac{(n-1)(n)(n+1)(n+2)}{4}}$$

add

$$S_n = \frac{n(n+1)(n+2)(n+3)}{4}$$

* Why V_n-Method:

$$T_n = n(n+1)(n+2)$$

$$T_n = \underbrace{\frac{n(n+1)(n+2)(n+3)}{4}}_{V_n} - \underbrace{\frac{(n-1)n(n+1)(n+2)}{4}}_{V_{n-1}}$$

NOTE:

In questions of Type-1 of telescoping series

$T_n = V_n - V_{n-1}$

$T_n = V_n - V_{n-1}$

$T_1 = \cancel{V_1} - V_0$

$T_2 = \cancel{V_2} - \cancel{V_1}$

$T_3 = \cancel{V_3} - \cancel{V_2}$

add

$T_n = V_n - \cancel{V_{n-1}}$

$$\boxed{S_n = V_n - V_0}$$

Sum the following series to n terms 1.4.7 + 4.7.10 + 7.10.13 + ...

$1, 4, 7, \ldots$ $t_n = 1 + (n-1)3$

$= 3n-2$

$$T_n = (3n-2)(3n+1)(3n+4)$$

$$T_n = (3n-2)(3n+1)(3n+4)\left[\frac{(3n+7) - (3n-5)}{12}\right]$$

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$$T_n = \frac{(3n-2)(3n+1)(3n+4)(3n+7)}{12}$$

$$- \frac{(3n-5)(3n-2)(3n+1)(3n+4)}{12}$$

$$T_n = V_n - V_{n-1}$$

$$3n-2 \longrightarrow 3(n-1)-2$$

$$S_n = V_n - V_0$$

$$= \frac{(3n-2)(3n+1)(3n+4)(3n+7)}{12}$$

$$- \left(\frac{-2 \cdot 1 \cdot 4 \cdot 7}{12}\right)$$

Type-2 of Telescoping Series

Sum of n terms of a series each term of which is composed of the reciprocal of the product of r factors in A.P., the first factors of the several terms being in the same A.P.

Example: $$\frac{1}{1.2.3}+\frac{1}{2.3.4}+\frac{1}{3.4.5}\cdots$$

Type-2 of Telescoping Series

In such cases we modify our general term by multiplying it with:

$$\boxed{\left[\frac{(\text{last factor}) - (\text{first factor})}{(\text{constant})}\right]}$$

Sum the following series to n terms $\frac{1}{1.2.3}+\frac{1}{2.3.4}+\frac{1}{3.4.5}....$

$$T_n = \frac{1}{n(n+1)(n+2)}\left[\frac{(n+2)-(n)}{2}\right]$$

$$T_n = \frac{1}{2n(n+1)} - \frac{1}{2(n+1)(n+2)}$$

$$T_n = \frac{1}{2}\left[\frac{1}{n(n+1)} - \frac{1}{(n+1)(n+2)}\right]$$

$$T_1 = \frac{1}{2}\left[\frac{1}{1\cdot 2} - \cancel{\frac{1}{2\cdot 3}}\right]$$

$$T_2 = \frac{1}{2}\left[\cancel{\frac{1}{2\cdot 3}} - \cancel{\frac{1}{3\cdot 4}}\right]$$

$$T_3 = \frac{1}{2}\left[\cancel{\frac{1}{3\cdot 4}} - \cancel{\frac{1}{4\cdot 5}}\right]$$

$$\vdots$$

$$T_n = \frac{1}{2}\left[\cancel{\frac{1}{n(n+1)}} - \frac{1}{(n+1)(n+2)}\right]$$

$$\rightarrow S_n = \frac{1}{2}\left[\frac{1}{2} - \frac{1}{(n+1)(n+2)}\right]$$

* Idea of V_n :

$$T_n = \frac{1}{2(n)(n+1)} - \frac{1}{2(n+1)(n+2)}$$

$$\boxed{T_n = V_n - V_{n+1}}$$

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NOTE:

In questions of Type-2 of telescoping series

$T_n = V_n - V_{n+1}$

$$\begin{cases} T_1 = V_1 - \cancel{V_2} \\ T_2 = \cancel{V_2} - \cancel{V_3} \\ T_3 = \cancel{V_3} - \cancel{V_4} \\ \vdots \\ T_n = \cancel{V_n} - V_{n+1} \end{cases}$$

$$\boxed{S_n = V_1 - V_{n+1}}$$

Sum the following series to n terms $\frac{1}{1\cdot3\cdot5}+\frac{1}{3\cdot5\cdot7}+\frac{1}{5\cdot7\cdot9}+\cdots$

$1, 3, 5, \ldots \quad t_n = 1+(n-1)2 = \boxed{2n-1}$

$$T_n = \frac{1}{(2n-1)(2n+1)(2n+3)}\left[\frac{(2n+3)-(2n-1)}{4}\right]$$

$$T_n = \frac{1}{4(2n-1)(2n+1)} - \frac{1}{4(2n+1)(2n+3)}$$

$2n-1 \longrightarrow 2(n+1)-1 \to \boxed{2n+1}$

$$T_n = V_n - V_{n+1}$$

$$S_n = V_1 - V_{n+1}$$

$$S_n = \frac{1}{4\cdot 1\cdot 3} - \frac{1}{4(2n+1)(2n+3)}$$

Find the sum upto n terms:

$$\frac{1}{1^3}+\frac{1+2}{1^3+2^3}+\frac{1+2+3}{1^3+2^3+3^3}+\ldots\ldots\ldots$$

$$T_n=\frac{1+2+3+---+n}{1^3+2^3+3^3+--+n^3}$$

$$=\frac{\sum n}{\sum n^3}$$

$$=\frac{\cancel{\left(\frac{n(n+1)}{2}\right)}}{\left(\frac{n(n+1)}{2}\right)^{\cancel{2}}}$$

$$=\boxed{\frac{2}{n(n+1)}}$$

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$$T_n = \frac{2}{n(n+1)}\left[\frac{(n+1)-(n)}{1}\right]$$

$$T_n = \frac{2}{n} - \frac{2}{n+1}$$

$$T_n = V_n - V_{n+1}$$

$$S_n = V_1 - V_{n+1}$$

$$= 2 - \frac{2}{n+1}$$

$$S_n = \frac{2n}{n+1}$$

The value of $\lim_{n\to\infty}\left[\frac{3}{1^2}+\frac{5}{1^2+2^2}+\frac{7}{1^2+2^2+3^3}+\frac{9}{1^2+2^2+3^2+4^2}+\text{.....to n terms}\right]$ is

A. 6 ✓

B. 7

C. 8

D. 9

$$T_n = \frac{(2n+1)}{\sum n^2}$$

$$T_n = \frac{\cancel{(2n+1)}}{\frac{n(n+1)\cancel{(2n+1)}}{6}}$$

$$T_n = \frac{6}{n(n+1)}\left[\frac{(n+1)-(n)}{1}\right]$$

$$\boxed{S_n = \frac{6n}{n+1}}$$

$$S_n = \frac{6n+6}{n+1} - \frac{6}{n+1}$$

$$\boxed{S_n = 6 - \frac{6}{n+1}}$$

$n \to \infty$. $S_n = 6$

Variations of Type-1 and Type-2

If the terms are not as per type-1 or type-2 then we first break our T_n into multiple parts where each part follows the conditions of Type-1 or Type-2

Sum the following series to n terms $\frac{4}{1\cdot2\cdot3}+\frac{5}{2\cdot3\cdot4}+\frac{6}{3\cdot4\cdot5}+\cdots$

$$T_n = \frac{(n+3)}{n(n+1)(n+2)}$$

$$T_n = \frac{\cancel{(n+2)}}{n(n+1)\cancel{(n+2)}} + \frac{1}{n(n+1)(n+2)}$$

$$= \frac{1}{n(n+1)} + \frac{1}{n(n+1)(n+2)}$$

$$T_n = \underbrace{(T_n)_1} + \underbrace{(T_n)_2}$$

$$S_n = (S_n)_1 + (S_n)_2$$

Sum the following series to n terms $\frac{1}{1\cdot2\cdot3}+\frac{3}{2\cdot3\cdot4}+\frac{5}{3\cdot4\cdot5}+\frac{7}{4\cdot5\cdot6}+\cdots$

$$T_n = \frac{(2n-1)}{n(n+1)(n+2)}$$

$$= \frac{2\cancel{(2n+4)}}{n(n+1)\cancel{(n+2)}} + \frac{-5}{n(n+1)(n+2)}$$

$$= \frac{2}{n(n+1)} - 5\left(\frac{1}{n(n+1)(n+2)}\right)$$

Sum the following series to n terms $1.4.7 + 2.5.8 + 3.6.9 + \ldots\ldots\ldots\ldots$

M-1

$$T_n = n(n+3)(n+6)$$

$$T_n = n^3 + 9n^2 + 18n$$

$$S_n = \sum(n^3 + 9n^2 + 18n)$$

M-2

$$T_n = n(n+3)(n+6)$$

$$= n(\overline{n+1}+2)(n+6)$$

$$= n(n+1)(n+6) + 2n(n+6)$$

$$= n(n+1)(\overline{n+2}+4) + 2n(\overline{n+1}+5)$$

$$= n(n+1)(n+2) + 4n(n+1) + 2n(n+1) + 10n$$

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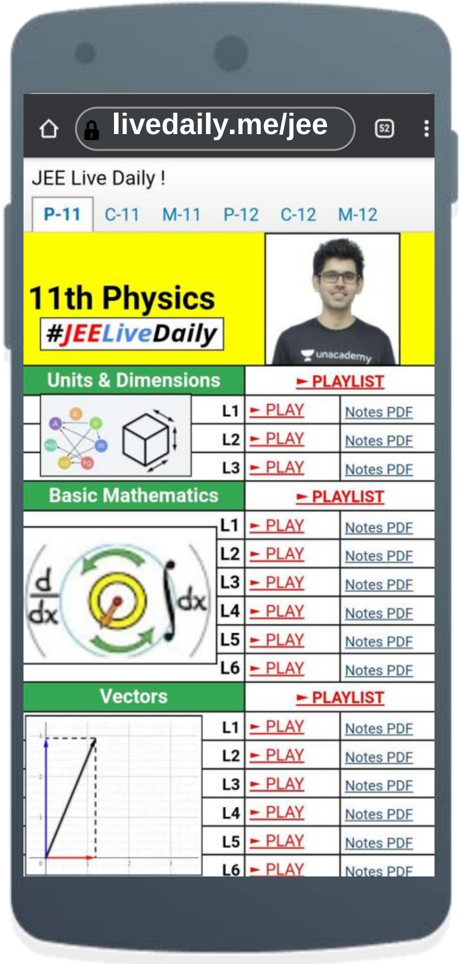

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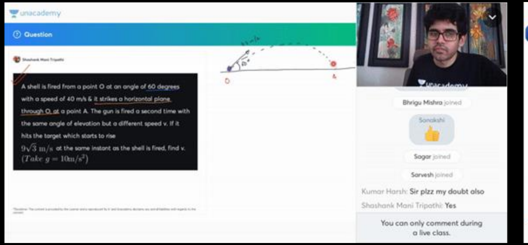

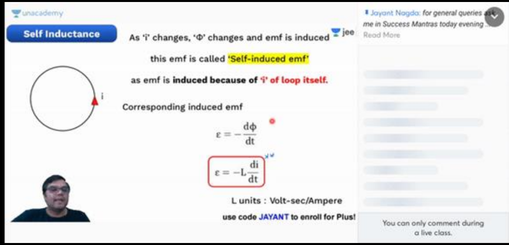

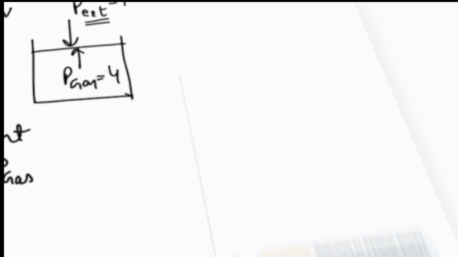

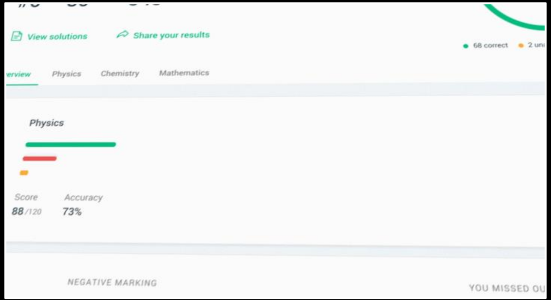

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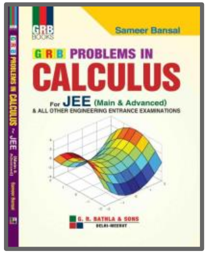

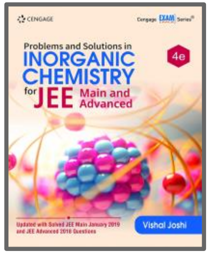

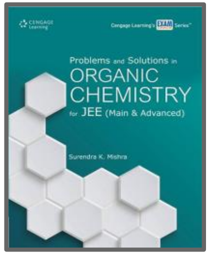

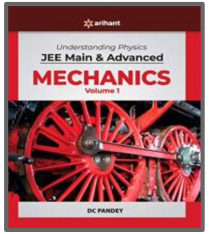

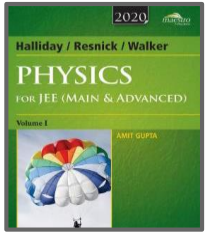

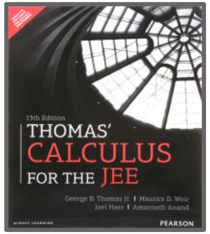


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Mihir Kothari 99.39
Sahil 99.38
Vaibhav Dhanuka 99.34
Pratham Kadam 99.29
Shivam Gupta 99.46
Shrish 99.28
Yash Bhaskar 99.10
Subhash Patel 99.02
Ayush Kale 98.85
Ayush Gupta 98.67
Megh Gupta 98.59
Naman Goyal 98.48
MIHIR PRAJAPATI 98.16

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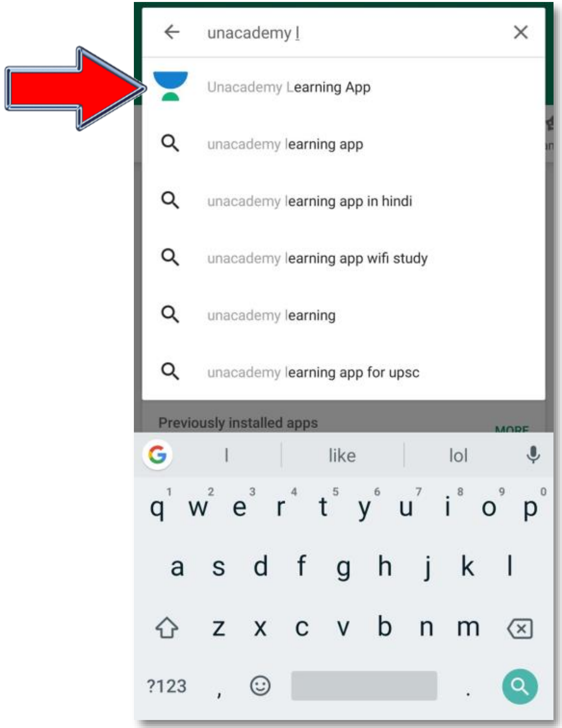
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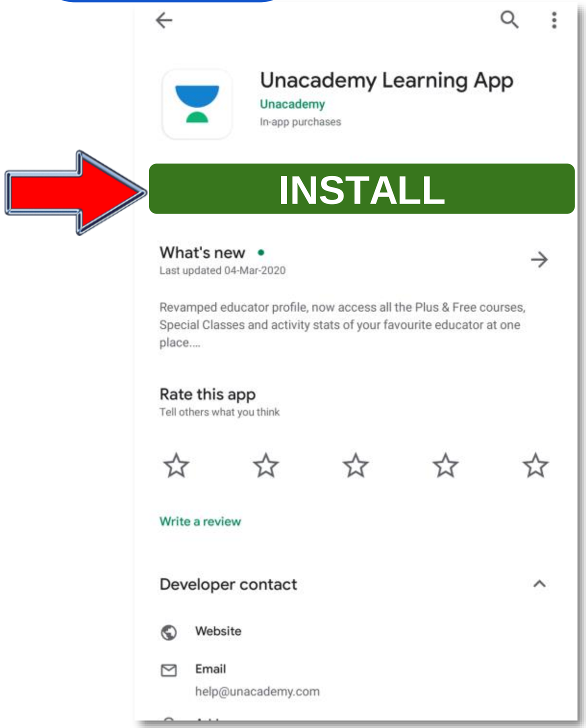
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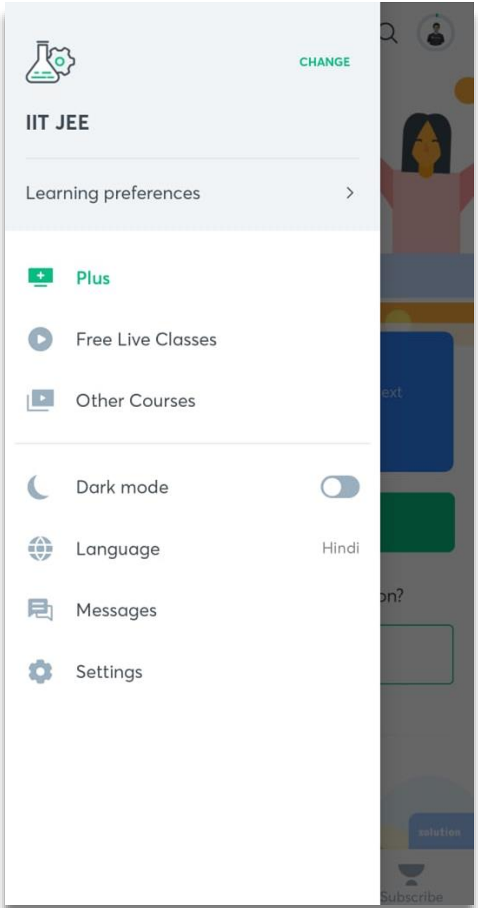
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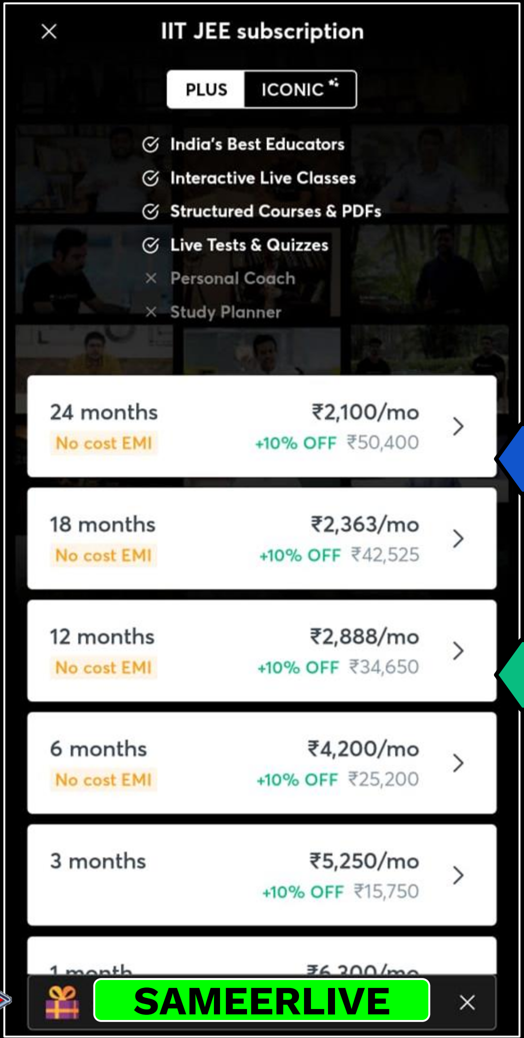

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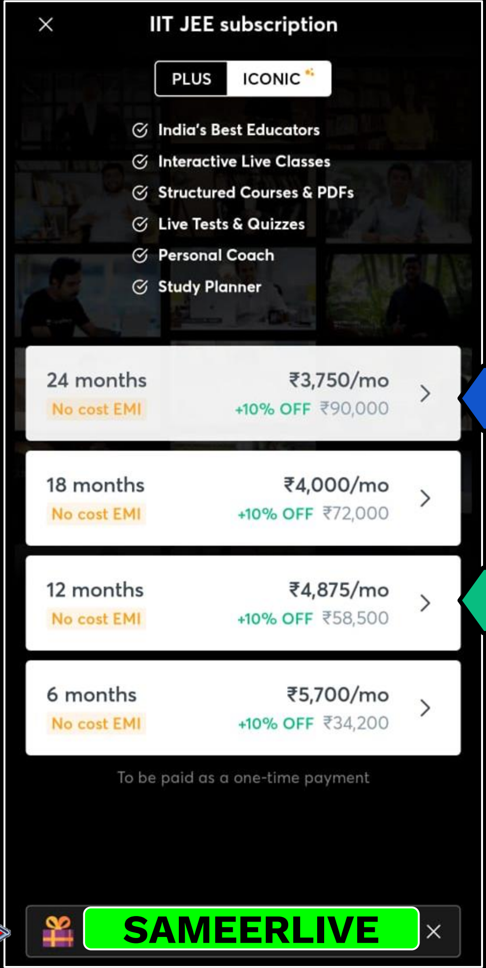

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