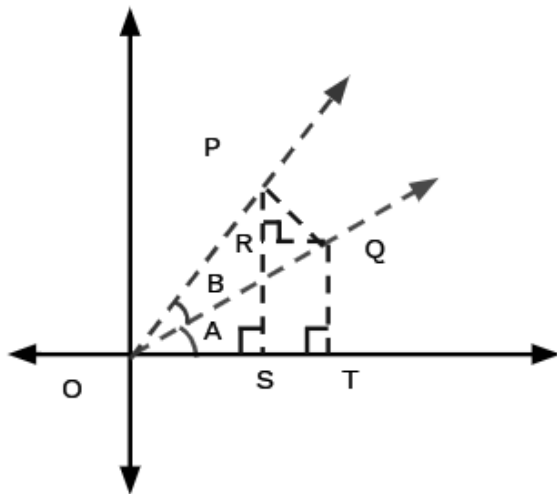


# Range of T-Expressions

## Trigonometry

15



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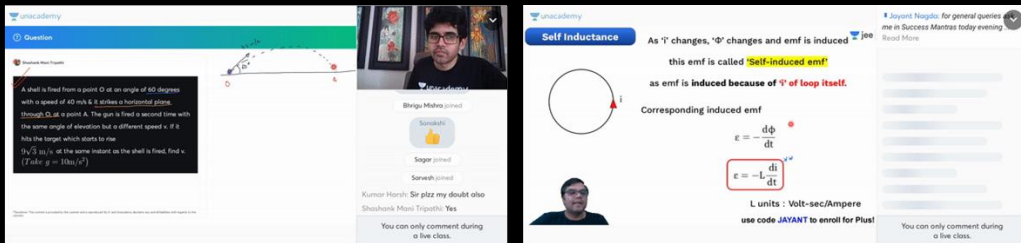
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A shell is fired from a point O at an angle of 60 degrees with a speed of 40 m/s. It strikes a horizontal plane through O at a point A. The gun is fired a second time with the same angle of elevation but a different speed  $v$ . If it hits the target which starts to rise  $(\sqrt{3}/2) \text{ m/s}^2$  at the same instant as the shell is fired, find  $v$ . (Take  $g = 10 \text{ m/s}^2$ )

Shreyas Mishra joined

Sagar joined

Saravali joined

Kumar Harsh: Sir plz my doubt also

Shashank Masi Tripathi: Yes

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Self Inductance

As  $\vec{I}$  changes,  $\vec{\Phi}$  changes and emf is induced

this emf is called **Self-induced emf**

as emf is induced because of  $\vec{I}$  of loop itself.

Corresponding induced emf

$$\mathcal{E} = -\frac{d\Phi}{dt}$$

$$\mathcal{E} = -L \frac{dI}{dt}$$

L units: Volt-sec/Ampere

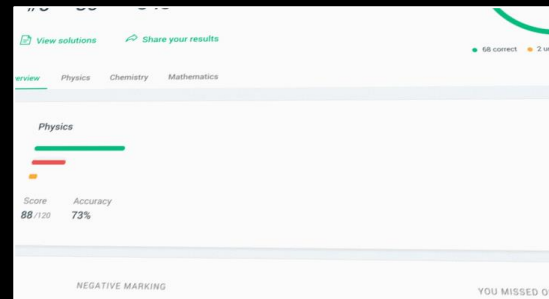
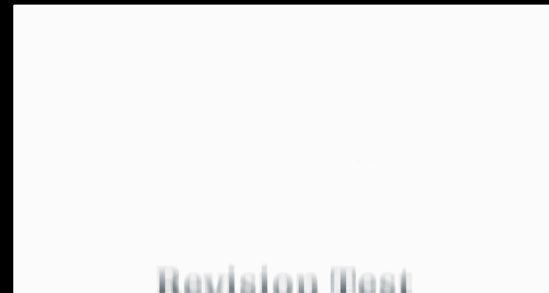
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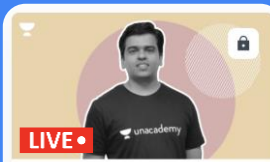


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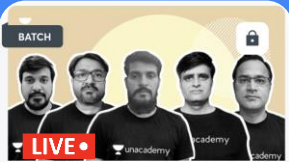
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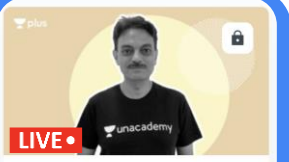
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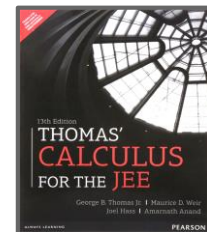
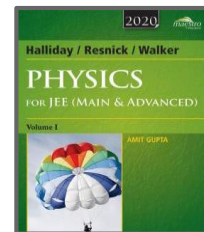
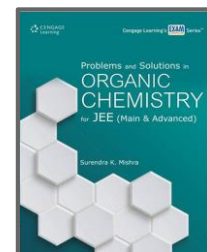
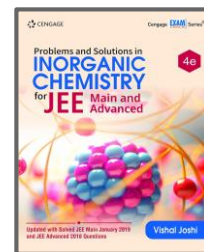
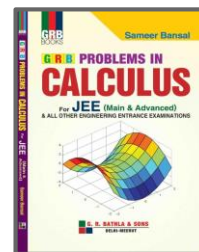
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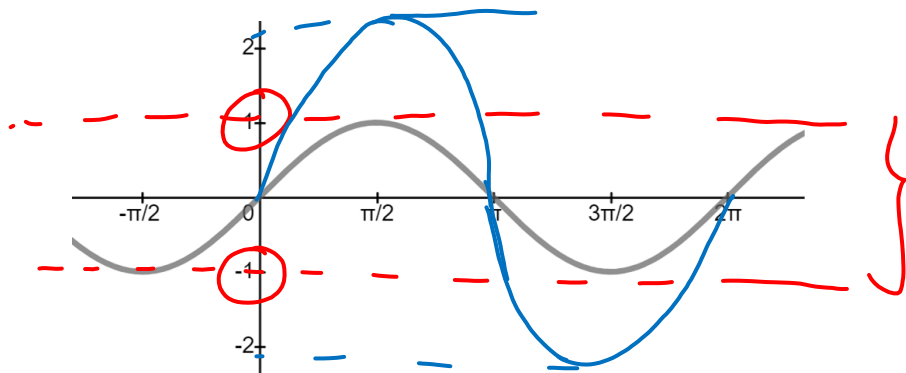
# Simple T-Expressions





Find the range of:

1.  $2\sin x$
2.  $3\sin(x + \pi/6)$
3.  $2\sin(5x) - 3$
4.  $\sin^2 x$



①  $2\sin x$

$\downarrow$

$[-1, 1]$

$\uparrow$

$[-2, 2]$

② ③  $\sin(x + \frac{\pi}{6})$

$\downarrow$

$[-1, 1]$

$\uparrow$

$[-3, 3]$

③  $2 \sin(5x) - 3$

$\downarrow$   
 $[-1, 1]$   
 $\downarrow$   
 $[-2, 2]$   
 $\downarrow$   
 $[-5, -1]$

④  $\sin^2 x$

$\begin{cases} \cos 2\theta = 1 - 2\sin^2 \theta \\ \sin^2 \theta = \left( \frac{1 - \cos 2\theta}{2} \right) \end{cases}$   
 $\left( \frac{1 - \cos 2x}{2} \right) \rightarrow [0, 1]$   
 $\downarrow$   
 $\left[ -\frac{1}{2}, \frac{1}{2} \right]$



Minimum value of  $\sin^6 x + \cos^6 x$  is equal to:

A.  $\frac{1}{2}$

B.  $\frac{3}{4}$

☒ C.  $\frac{1}{4}$

D. 1

$$\begin{aligned} \sin^6 x + \cos^6 x &= (\sin^2 x + \cos^2 x)^3 - 3(\sin^2 x)(\cos^2 x)(\sin^2 x + \cos^2 x) \\ &= 1 - 3\sin^2 x \cos^2 x \end{aligned}$$

Handwritten derivation showing the identity  $(a+b)^3 = a^3 + b^3 + 3ab(a+b)$  applied to  $\sin^2 x$  and  $\cos^2 x$ .

$$1 - \frac{3}{4} \left( 4 \sin^2 x \cos^2 x \right)$$

$$= 1 - \frac{3}{4} (2 \sin x \cos x)^2$$

$$= \boxed{1 - \frac{3}{4} \sin^2 2x}$$

$\downarrow$   
 $[0, 1]$

$$\left[ \frac{1}{4}, 1 \right]$$



# T-Expressions of Quadratic form





Find the range of  $\sin^2 x - \sin x + 2$

A.  $[0, 6]$

B.  $[2, 4]$

C.  $[7/4, 4]$

D.  $[5/2, 4]$

$$\sin^2 x - \sin x + 2$$

Let:  $\sin x = t$

$$t^2 - t + 2$$

$$\left( t - \frac{1}{2} \right)^2 - \frac{1}{4} + 2$$

$$= \left( t - \frac{1}{2} \right)^2 + \frac{7}{4}$$

$$\left[ \left( \sin x - \frac{1}{2} \right)^2 + \frac{7}{4} \right]$$

$$\underline{\text{min}} : 0 + \frac{7}{4} = \left( \frac{7}{4} \right)$$

$$\underline{\text{max}} : \left( -1 - \frac{1}{2} \right)^2 + \frac{7}{4} = \frac{9}{4} + \frac{7}{4} = \frac{16}{4} = (4)$$

# T-Expressions of type - $a \sin x \pm b \cos x$





## Range of $a \sin x \pm b \cos x$

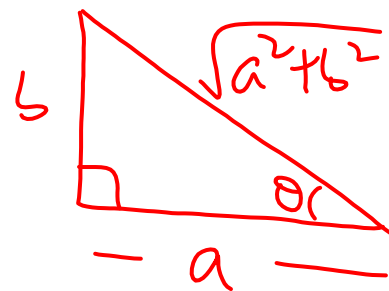
$$a \sin x \pm b \cos x$$

$$\sqrt{a^2 + b^2} \left( \frac{a}{\sqrt{a^2 + b^2}} \sin x + \frac{b}{\sqrt{a^2 + b^2}} \cos x \right)$$

$\downarrow$   $\cos \theta$                        $\downarrow$   $\sin \theta$

Let

$$\cos \theta = \frac{a}{\sqrt{a^2 + b^2}}$$



$$\begin{aligned} & \sqrt{a^2+b^2} \left( \sin x \cos \theta + \sin \theta \cos x \right) \\ & \quad \downarrow \\ & \left( \sqrt{a^2+b^2} \right) \sin(x + \theta) \\ & \quad \downarrow \\ & \quad \rightarrow [-1, 1] \\ & \left[ -\sqrt{a^2+b^2}, \sqrt{a^2+b^2} \right] \end{aligned}$$





The maximum value of :  $(12 \sin^2 x - 5 \cos^2 x + 1)$  is

A. 13

✓ B. 14

C. 8

D. 18

$$12 \sin^2 x - 5 \cos^2 x + 1$$

Diagram illustrating the expression  $12 \sin^2 x - 5 \cos^2 x + 1$ . The coefficients 12 and 5 are identified as 'a' and 'b' respectively, with arrows pointing from them to the labels 'a' and 'b'. A bracket underneath the expression indicates the range of the quadratic form, which is  $[-13, 13] + 1$ .





Find the **minimum value** of :  $4 \sin 2\theta - 6 \sin^2 \theta$

A. -5

B. -2

☒ C. -8

D. 0

$$4 \sin \underline{2\theta} - 6 \sin^2 \theta$$

$$4 \sin 2\theta - 3(1 - \cos 2\theta)$$

$$4 \sin 2\theta + 3 \cos 2\theta - 3$$

$$[-5, 5] - 3$$





The **minimum value** of:  $5 \sin \theta + 3 \sin \left( \theta - \frac{\pi}{3} \right) + 10$

**A.** 3

**B.** 5

**C.** -13

**D.** -8

$$5 \sin \theta + 3 \left( (\sin \theta) \left( \frac{1}{2} \right) - (\cos \theta) \left( \frac{\sqrt{3}}{2} \right) \right) + 10$$

$$\left( \frac{13}{2} \sin \theta - \frac{3\sqrt{3}}{2} \cos \theta \right) + 10$$

↓  
a

↓  
b

$$\left\{ \begin{aligned} a^2 + b^2 &= \frac{169}{4} + \frac{27}{4} \\ &= \frac{196}{4} \\ &= 49 \end{aligned} \right.$$

$$\therefore [-7, 7] + 10$$







Find the **maximum value** of the expression:

$$\frac{1}{(\sin^2 \theta + 3 \sin \theta \cos \theta + 5 \cos^2 \theta)}$$

$$\sin^2 \theta + 3 \sin \theta \cos \theta + 5 \cos^2 \theta$$

$$= 1 + \frac{3}{2} (2 \sin \theta \cos \theta) + 2 (2 \cos^2 \theta)$$

$$= 1 + \frac{3}{2} (\sin 2\theta) + 2 (1 + \cos 2\theta)$$

$$= 3 + \left( \frac{3}{2} \sin 2\theta + 2 \cos 2\theta \right)$$

$$= 3 + \left[ -\sqrt{\frac{9}{4} + 4}, \sqrt{\frac{9}{4} + 4} \right]$$

$$\begin{aligned} & 3 + \left[ -\frac{5}{2}, \frac{5}{2} \right] \\ & \hline & = \left[ \frac{1}{2}, \frac{11}{2} \right] \end{aligned}$$

max value of  
given expression

$$= \frac{1}{\left(\frac{1}{2}\right)}$$

$$= 2$$





The maximum value of the **sum of the squares of the roots** of the equation

$$x^2 + \underbrace{(\cos\theta - 1)x} + \frac{1}{2}\cos^2\theta = 0 \text{ is.}$$

A.  $1/2$

B. 2

☒ C. 3

D. 8

$$\begin{cases} \alpha + \beta = -\frac{(\cos\theta - 1)}{1} \\ \phantom{\alpha + \beta = -\frac{(\cos\theta - 1)}{1}} = (1 - \cos\theta) \end{cases}$$

$$\begin{cases} \alpha\beta = \left(\frac{1}{2}\cos^2\theta\right) \end{cases}$$

$$\alpha^2 + \beta^2$$

$$= (\alpha + \beta)^2 - 2(\alpha\beta)$$

$$= (1 - \cos\theta)^2 - 2\left(\frac{1}{2}\cos^2\theta\right)$$

$$= 1 + \cancel{\cos^2 \theta} - 2\cos \theta - \cancel{\cos^2 \theta}$$

$$= 1 - 2\cos \theta$$

$$\begin{array}{c} \downarrow \\ [-1, 1] \end{array}$$

$$\begin{array}{c} \downarrow \\ [-2, 2] \\ \downarrow \\ [-1, 3] \end{array}$$







If  $a + b = 3 - \cos 4\theta$  &  $a - b = 4 \sin 2\theta$ , then  $ab$  is always less than or equal to

A.  $\frac{1}{2}$

B.  $\frac{3}{4}$

C.  $\frac{2}{3}$

✓ D. 1

$$\begin{cases} (a+b)^2 = (3 - \cos 4\theta)^2 \\ (a-b)^2 = (4 \sin 2\theta)^2 \end{cases}$$

$$4ab = (9 + \cos^2 4\theta - 6\cos 4\theta) - (16 \sin^2 2\theta)$$

$$\begin{aligned} 4ab &= (9 + \underline{\cos^2 4\theta} - 6\cos 4\theta) \\ &\quad - 8(1 - \cos 4\theta) \end{aligned}$$

$$4ab = (1 + \cos^2 4\theta + 2\cos 4\theta)$$

$$4ab = (1 + \cos 4\theta)^2$$

$$\boxed{ab = \frac{1}{4}(1 + \cos 4\theta)^2} \Rightarrow (ab)_{\min} \neq 1$$





$$\text{If } u = \sqrt{a^2 \cos^2 \theta + b^2 \sin^2 \theta} + \sqrt{a^2 \sin^2 \theta + b^2 \cos^2 \theta}$$

then the **difference between** the maximum & minimum values of  $u^2$  is given by

- A.**  $(a-b)^2$       **B.**  $2\sqrt{a^2 + b^2}$       **C.**  $(a+b)^2$       **D.**  $2(a^2 + b^2)$

$$\begin{aligned} u^2 &= (\underbrace{a^2 \cos^2 \theta}_{\text{blue underline}} + \underbrace{b^2 \sin^2 \theta}_{\text{blue wavy}}) + (\underbrace{a^2 \sin^2 \theta}_{\text{blue underline}} + \underbrace{b^2 \cos^2 \theta}_{\text{blue wavy}}) \\ &\quad + 2\sqrt{(\underbrace{a^2 \cos^2 \theta + b^2 \sin^2 \theta}_{\text{blue underline}})(\underbrace{a^2 \sin^2 \theta + b^2 \cos^2 \theta}_{\text{blue underline}})} \end{aligned}$$

$$u^2 = (a^2 + b^2) + 2 \sqrt{a^4 \sin^2 \theta \cos^2 \theta + a^2 b^2 \cos^4 \theta + a^2 b^2 \sin^4 \theta + b^4 \sin^2 \theta \cos^2 \theta}$$

$$= (a^2 + b^2) + 2 \sqrt{(a^4 + b^4) \sin^2 \theta \cos^2 \theta + a^2 b^2 (\sin^4 \theta + \cos^4 \theta)}$$

$$= (a^2 + b^2) + 2 \sqrt{(a^4 + b^4) \sin^2 \theta \cos^2 \theta + a^2 b^2 (1 - 2 \sin^2 \theta \cos^2 \theta)}$$

$$u^2 = (a^2 + b^2) + 2\sqrt{a^2b^2 + \sin^2\theta b^2\cos^2\theta(a^4 + b^4 - 2a^2b^2)}$$

$$u^2 = (a^2 + b^2) + 2\sqrt{a^2b^2 + \frac{1}{4}(\sin^2 2\theta)(a^2 - b^2)^2}$$

$$u^2)_{\max} = (a^2 + b^2) + \cancel{2}\sqrt{4a^2b^2 + a^4 + b^4 - 2a^2b^2}$$

$$= (a^2 + b^2) + \cancel{2}\sqrt{(a^2 + b^2)^2}$$

$$= \boxed{2(a^2 + b^2)}$$

$$\boxed{u^2_{\min} = (a+b)^2}$$

Ans:

$$(2a^2 + 2b^2) - (a^2 + b^2 + 2ab)$$

$$= a^2 + b^2 - 2ab$$

$$= (a - b)^2$$







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6:00 - 7:30 PM



Ashwani Sir | Chemistry

7:30 - 9:00 PM



Sameer Sir | Maths

9:00 - 10:30 PM

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1:30 - 3:00 PM



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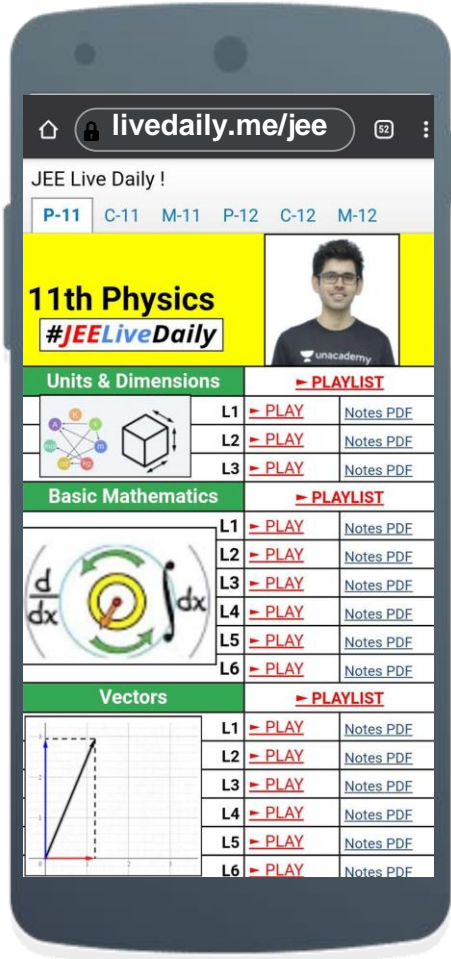
3:00 - 4:30 PM



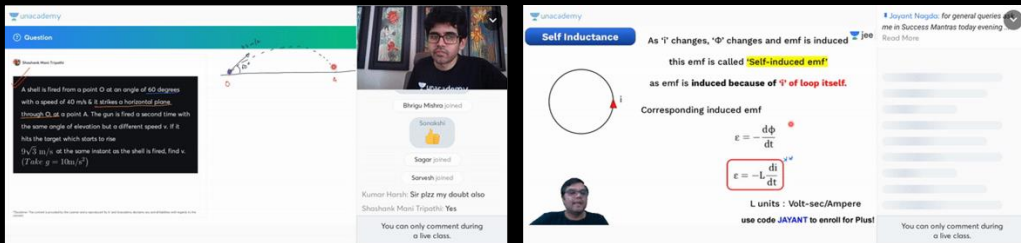
Nishant Sir | Maths

4:30 - 6:00 PM

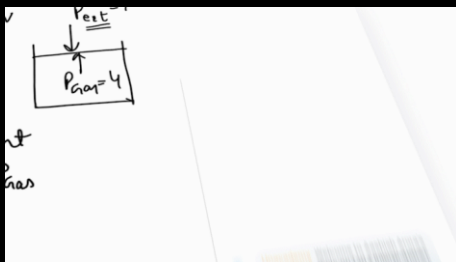
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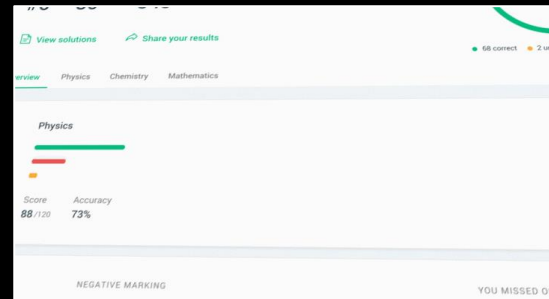
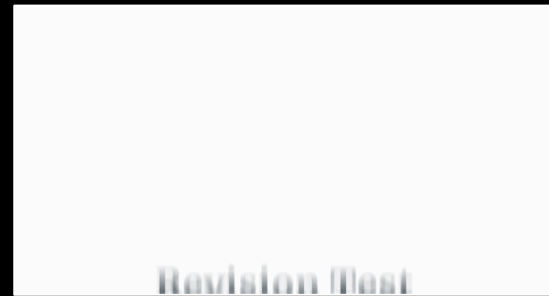


The image shows two screenshots of the Unacademy platform. The left screenshot displays a physics question: "A shell is fired from a point O at an angle of 60 degrees with a speed of 40 m/s. It strikes a horizontal plane through O at a point A. The gun is fired a second time with the same angle of elevation but a different speed  $v$ . If it hits the target which starts to rise  $(\sqrt{3}/2) \text{ m/s}^2$  at the same instant as the shell is fired, find  $v$ . (Take  $g = 10 \text{ m/s}^2$ )". The right screenshot shows a lecture on "Self Inductance" with the text: "As  $\vec{I}$  changes,  $\vec{\Phi}$  changes and emf is induced. This emf is called **Self-induced emf** as emf is induced because of  $\vec{I}$  of loop itself." It also includes the formula for induced emf:  $\mathcal{E} = -\frac{d\Phi}{dt}$  and  $\mathcal{E} = -L \frac{di}{dt}$ , and mentions "L units: Volt-sec/Ampere".



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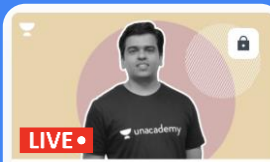


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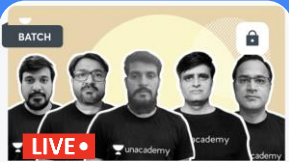
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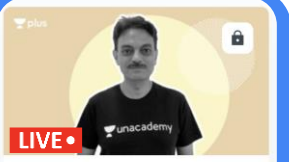
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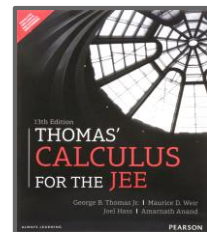
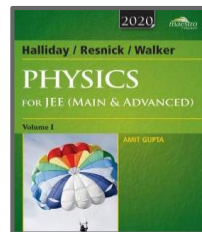
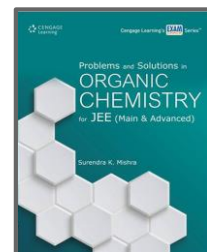
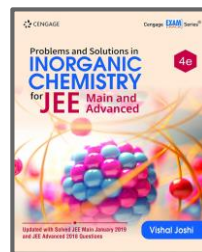
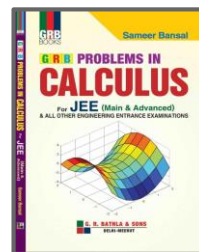
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Aravindan K  
Sundaram  
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Manas Pandey  
99.69



Mihir Agarwal  
99.63



Akshat Tiwari  
99.60



Sarthak  
Kalankar  
99.59



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99.50



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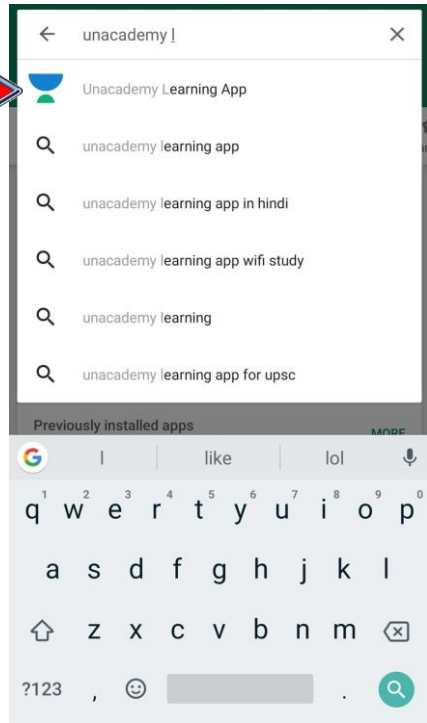


Naman Goyal  
98.48

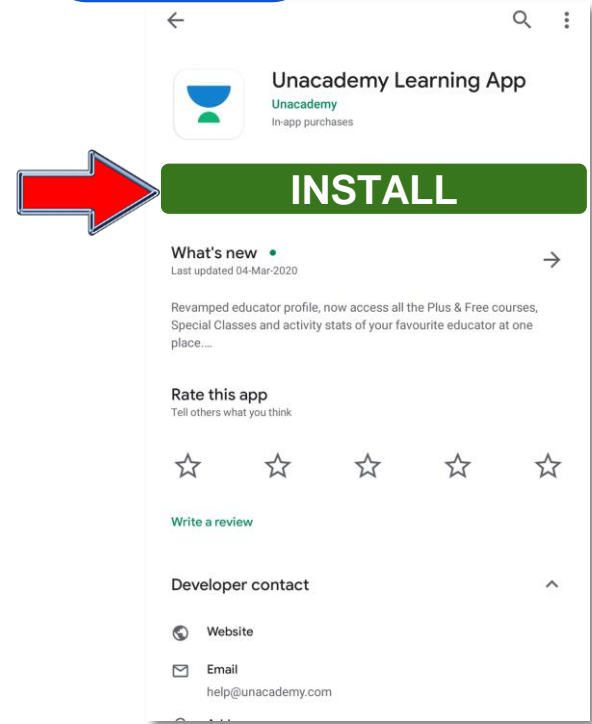


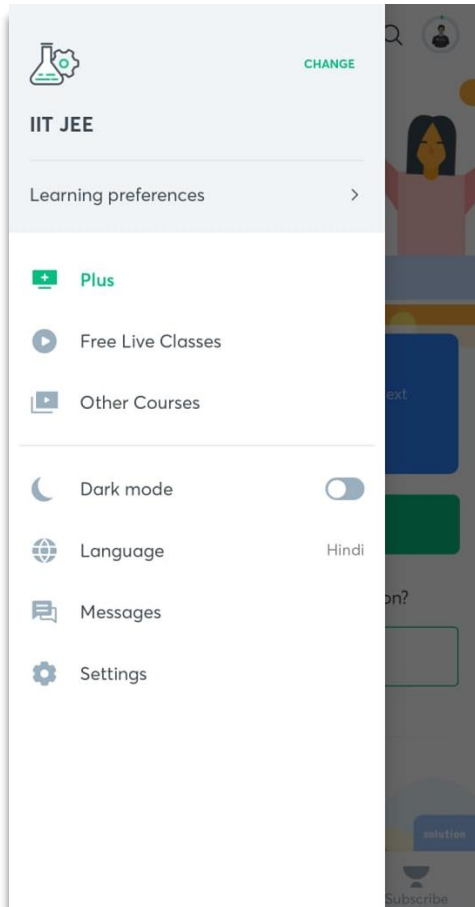
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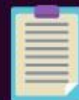
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