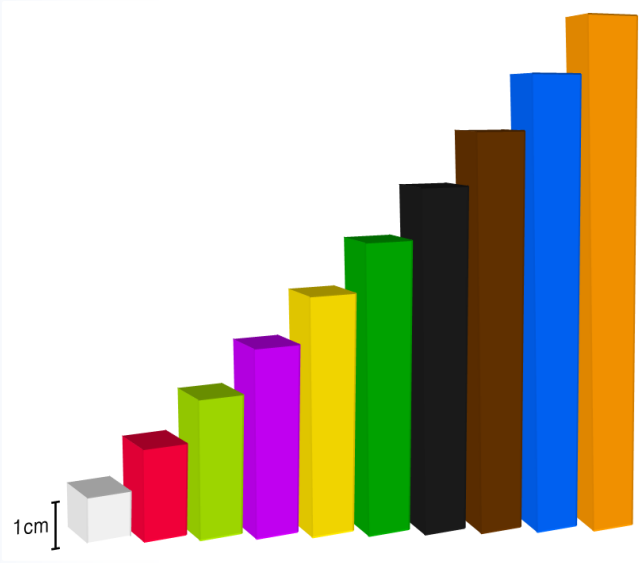
Arithmetic Progression – 2

Sequences & Series

2



jee LIVE daily 3.0

Sameer Chincholikar

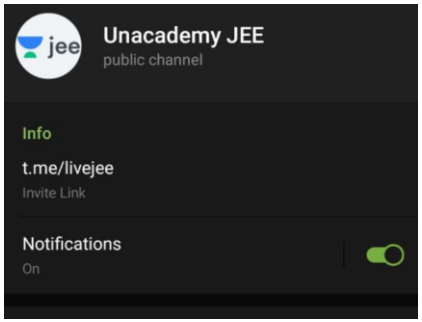
B.Tech, M.Tech - IIT-Roorkee

- 10+ years Teaching experience
- Taught 1 Million+ Students
- 100+ Aspiring Teachers Mentored

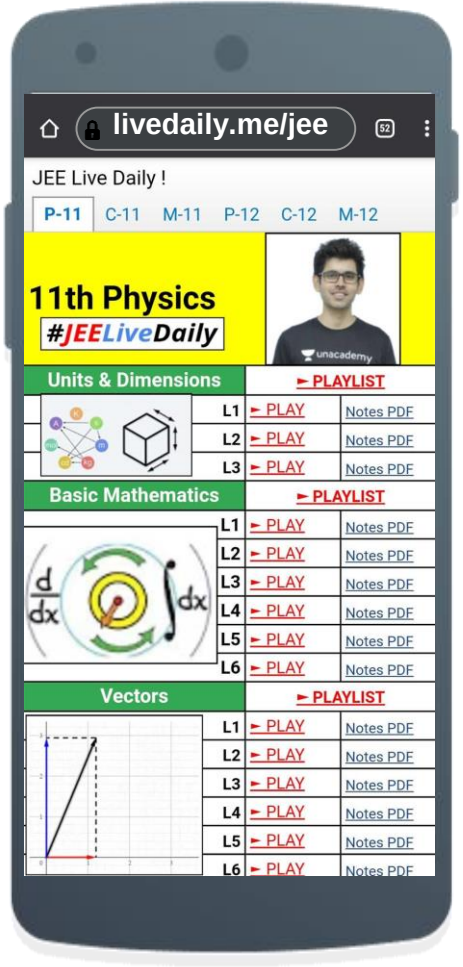
sameer_iitr

#JEE Live Daily

Telegram Channel

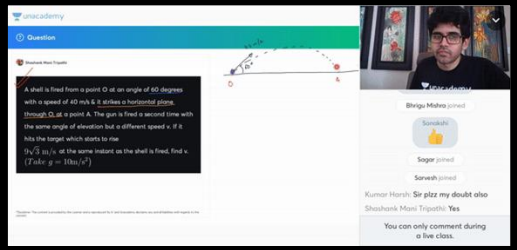


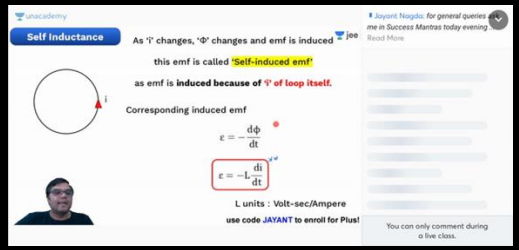
livedaily.me/jee

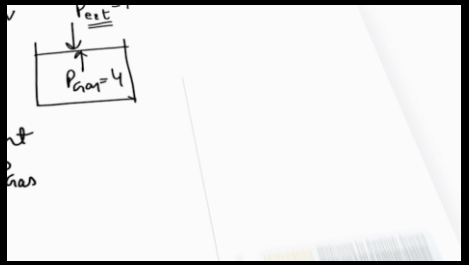


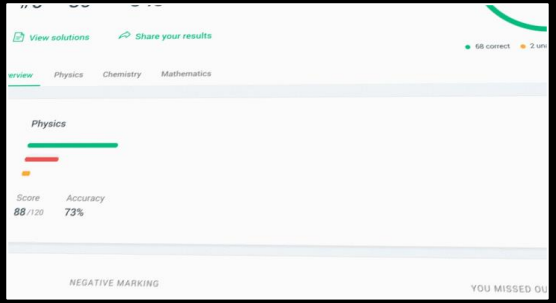
Unacademy Subscription

jee









+ LIVE Class Environment

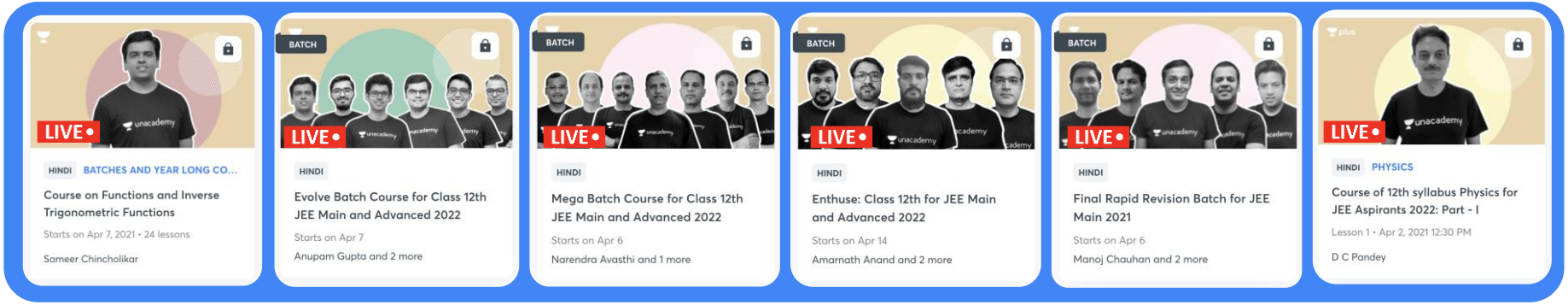
- + LIVE Polls & Leaderboard
- + LIVE Doubt Solving
- + LIVE Interaction

+ Performance Analysis

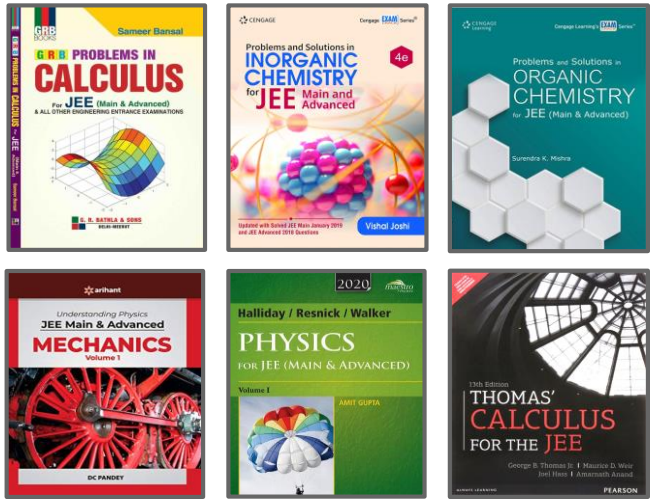
- + Weekly Test Series
- + DPPs & Quizzes

India's **BEST** Educators

Unacademy Subscription



If you want to be the **BEST**
"Learn" from the **BEST**



jee

Top Results

Bratin Mondal

100 %ile

Amaiya Singhal
99.97

Adnan
99.95

Ashwin Prasanth
99.94

Tanmay Jain
99.86

Kunal Lalwani
99.81

Utsav Dhanuka
99.75

Aravindan K Sundaram
99.69

Manas Pandey
99.69

Mihir Agarwal
99.63

Akshat Tiwari
99.60

Sarthak Kalankar
99.59

Vaishnovi Arun
99.58

Devashish Tripathi
99.52

Maroof
99.50

Tarun Gupta
99.50

Siddharth Kaushik
99.48

Mihir Kothari
99.39

Sahil
99.38

Vaibhav Dhanuka
99.34

Pratham Kadam
99.29

Shivam Gupta
99.46

Shrish
99.28

Yash Bhaskar
99.10

Subhash Patel
99.02

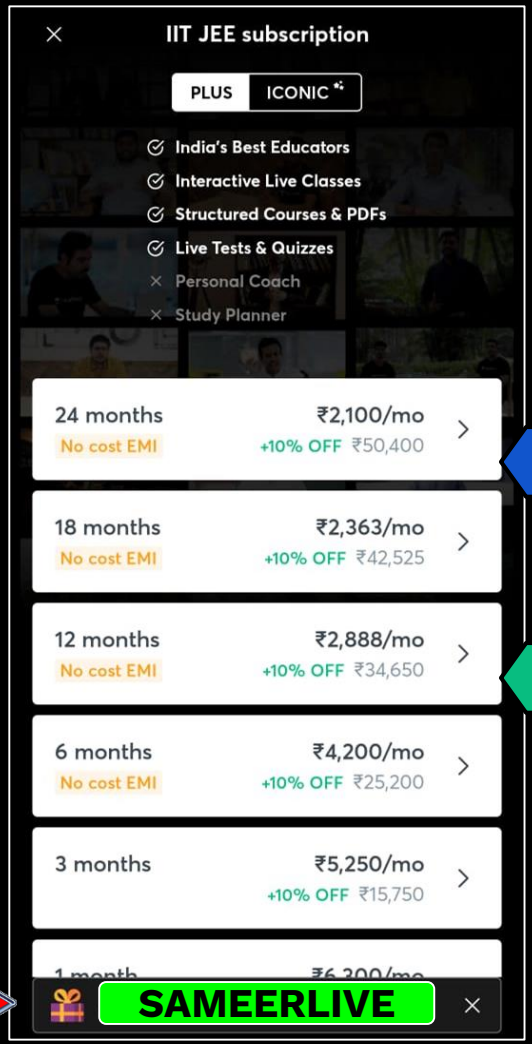
Ayush Kale
98.85

Ayush Gupta
98.67

Megh Gupta
98.59

Naman Goyal
98.48

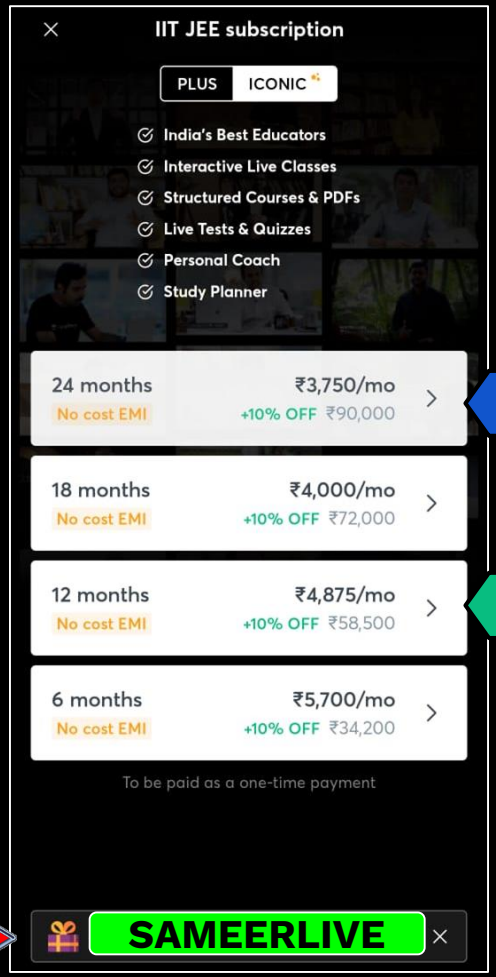
MIHIR PRAJAPATI
98.16



jee

11th / 9, 10

12th / Drop



11th / 9, 10

12th / Drop

jee

LET'S BEGIN!!

Homework Question

The sum of integers from 1 to 100 that are divisible by 2 or 5 is

A. 2150 B. 3050 C. 2500 D. None of these

$A: 2, 4, 6, \cdots 100$; $n(A) = 50$ | $S_A = \frac{50}{2}(2+100)$

$B: 5, 10, 15, \cdots, 100$; $n(B) = 20$ | $S_B = \frac{20}{2}(5+100)$

$C: 10, 20, 30, \cdots 100$; $n(C) = 10$ | $S_C = \frac{10}{2}(10+100)$

$$S_A + S_B - S_C$$

$$= 50 \times 51 + 10 \times 105 - (550)$$

$$= \boxed{3050}$$

Important Results

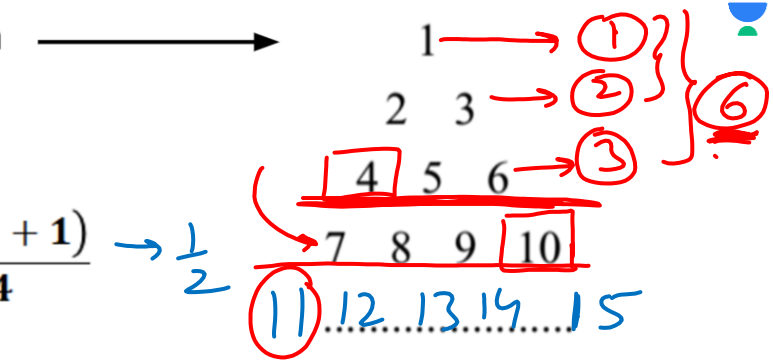
Important Result !

Sum of first n natural numbers

$$1+2+3+\cdots+n=\frac{n}{2}\left(2(1)+(n-1)1\right)$$

$$=\boxed{\frac{n(n+1)}{2}}$$

Eg: $\frac{10(10+1)}{2}$ Eg. $\frac{9(9+1)}{2}$

If the natural numbers are written as shown 

Then, the sum of the terms if the n^{th} row is

A. $\frac{n(n^2-1)}{2}$ → 0

B. $\frac{n(n^2+1)}{4}$ → $\frac{1}{2}$

C. $\frac{n(n^2+1)}{2}$ → 1

D. None of these

M-1 : Shortcuts:

n = 1

n = 2

M-2 : (No of elements in n^{th} row) = n

Common Diff = 1

first element = ?

$$\left(\begin{array}{c}\text{No. of elements}\\ \text{used till}\\ (n-1)^{th}\text{ row}\end{array}\right) = 1+2+3+---+(n-1)$$

$$= \frac{(n-1)((n-1)+1)}{2}$$

$$= \left(\frac{n^2-n}{2}\right)$$

$$\Rightarrow \left(\begin{array}{c}1^{st}\text{ element of}\\ n^{th}\text{ row}\end{array}\right) = \left(\frac{n^2-n}{2}+1\right) = \left(\frac{n^2-n+2}{2}\right)$$

$$\text{Sum} = \frac{n}{2}\left(2\left(\frac{n^2-n+2}{2}\right)+(n-1)(1)\right)$$

$$= \frac{n}{2}\left(n^2-n+2+n-1\right)$$

$$= \boxed{\frac{n}{2}\left(n^2+1\right)}$$

Important Result !

Sum of first n odd natural numbers

$1 + 3 + 5 + 7 + \text{- - - - } n\text{-terms}$

$a = 1$

$d = 2$

$n = n$

$\rightarrow \text{Sum} = \frac{n}{2}\left(2(1) + (n-1)(2)\right)$

$= \frac{n}{2}\left(2 + 2n - 2\right)$

$= \boxed{n^2}$

The coefficient of x^{49} in the product $(x - 1)(x - 3) .... (x - 99)$ is

A. -99^2 **B.** 1 **C.** -2500 **D.** None of these

$(x-1)(x-3)(x-5)\cdots\cdots(x-99)$

No. of brackets: 1, 3, 5, ---- 99

T_n

$T_n = 1 + (n-1)2 = 99$

$n = 50$

Eg: $(x-1)(x-2)$

$2 + (-x-2x) + x^2$

Eg: $(x-1)(x-2)(x-3)$

$x^3 + (-3-2-1)x^2 + x(6+3+2)$

Sol^:

$$x^{49}\left(-1-3-5-\cdots-99\right)$$

$$x^{49}\left(-\left(\underbrace{1+3+5+\cdots+99}_{50\text{-terms}}\right)\right)$$

$$x^{49}\left(-(50)^2\right)$$

Important Result !

$$T_n = S_n - S_{n-1}$$

$$S_n = T_1 + T_2 + T_3 + \cdots + T_{n-2} + T_{n-1} + T_n$$

$$S_{n-1} = T_1 + T_2 + T_3 + \cdots + T_{n-2} + T_{n-1}$$

$$S_n - S_{n-1} = T_n$$

If the sum of n terms of an AP is given by $S_n = 3n + 2n^2$, then the common difference of the AP is

A. 3 B. 2 C. 6 D. 4

M-1: Shortcut:

$S_2 = T_1 + T_2 = 3(2) + 2(2)^2 = 14$

$S_1 = T_1 = 3(1) + 2(1)^2 = 5$

$\ominus$: $T_2 = 9 \Rightarrow d = T_2 - T_1 = 9 - 5 = 4$

Method-2:

$$S_n = 3n + 2n^2$$

$$S_{n-1} = 3(n-1) + 2(n-1)^2$$

$\ominus$: $T_n = 3(1) + 2(n^2 - (n-1)^2)$

$$T_n = 3 + 2(2n-1)$$

$$\boxed{T_n = 4n+1}$$

$$T_n = 4n + 1$$

$$T_{n-1} = 4(n-1) + 1$$

$\ominus$: $T_n - T_{n-1} = 4$

$$\boxed{d = 4}$$

If S_n denotes the sum of n terms of an A.P., then

$S_{n+3} - 3S_{n+2} + 3S_{n+1} - S_n =$

A. $2S_n$ **B.** S_{n+1} **C.** $3S_n$ **D.** 0

$$S_{n+3} - 3S_{n+2} + 3S_{n+1} - S_n$$

$$= (S_{n+3} - S_{n+2}) - 2S_{n+2} + 2S_{n+1}$$

$$+ (S_{n+1} - S_n)$$

$$S_n - S_{n-1} = T_n$$

$$S_{n+1} - S_n = T_{n+1}$$

$$= T_{n+3} - 2(T_{n+2}) + T_{n+1}$$

$$= (T_{n+3} - T_{n+2}) - (T_{n+2} - T_{n+1})$$

$$(d) - (d)$$

$$= 0$$

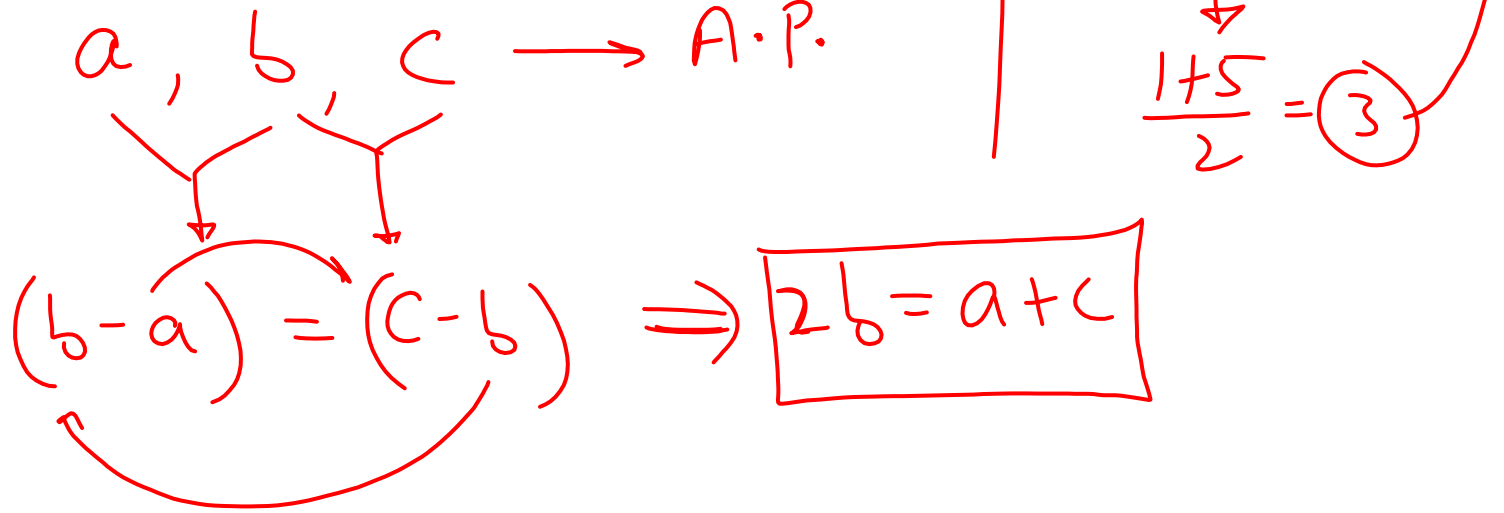
Properties of A.P.

Properties of A.P.

1. The common difference can be zero, positive or negative.

Properties of A.P.

2 If a, b, c are in A.P. $\Rightarrow 2b = a + c$



If $\log_3 2, \log_3(2^x - 5)$ and $\log_3\left(2^x - \frac{7}{2}\right)$ are in arithmetic progression. Determine the value of x

$\because$ in AP :

$$2 \cdot \log_3(2^x - 5) = \log_3 2 + \log_3\left(2^x - \frac{7}{2}\right)$$

$$\Rightarrow \log_3(2^x - 5)^2 = \log_3(2(2^x) - 7)$$

$$\Rightarrow (2^x - 5)^2 = 2(2^x) - 7$$

Let: $2^x = t$

$$(t-5)^2 = 2t - 7$$

$$t^2 - 10t + 25 = 2t - 7$$

$$t^2 - 12t + 32 = 0$$

$$(t-4)(t-8) = 0$$

$t = 4$

$\Rightarrow 2^x = 4$

$2^x = 2^2$

$x = 2$

Reject

$t = 8$

$2^x = 8$

$2^x = 2^3$

$x = 3$

Accept

Properties of A.P.

3 Considering Numbers in A.P.

3 terms: $(a-d), a, (a+d)$

4 terms: $(a-3d), (a-d), (a+d), (a+3d)$

5 terms: $(a-2d), (a-d), a, (a+d), (a+2d)$

The sum of three numbers in A.P. is 27 and the sum of their squares is 293, find the numbers.

$(a-d)+a+(a+d)=27$

$\Rightarrow 3a=27$

$\boxed{a=9}$

& also:

$(a-d)^2+a^2+(a+d)^2=293$

$3a^2+2d^2=293$

$3(9)^2+2d^2=293$

$2d^2=293-243$

$2d^2=50$

$d^2=25$

$\boxed{d=\pm 5}$

$(a-d), (a), (a+d)$

$(9-5), 9, (9+5) \Rightarrow \boxed{4, 9, 14}$

$(9+5), 9, (9-5) \Rightarrow \boxed{14, 9, 4}$

The real numbers x_1, x_2, x_3 satisfying the equation $x^3 - x^2 + \beta x + \gamma = 0$ are in A.P. The intervals in which β and γ lie are

A. $(-\infty, \frac{1}{3}], [-\frac{1}{27}, \infty)$

B. $(-8, \frac{1}{3}), [-\frac{1}{27}, \infty)$

C. $(-\infty, \frac{1}{3}], (-\frac{1}{27}, \infty)$

D. None of these

(i) $x_1 + x_2 + x_3 = 1$

(ii) $x_1x_2 + x_2x_3 + x_3x_1 = \beta$

(iii) $x_1x_2x_3 = -\gamma$

$x_1 \to a - d$

$x_2 \to a$

$x_3 \to a + d$

(i) $3a = 1 \Rightarrow \boxed{a = \frac{1}{3}}$

(ii) $(a^2 - ad) + (a^2 + ad) + (a^2 - d^2) = \beta$

(iii) $a(a^2 - d^2) = -\gamma$

Using Eqn (2):

$$\beta = 3a^2 - d^2$$

$$\beta = 3\left(\frac{1}{3}\right)^2 - d^2$$

$$\beta = \frac{1}{3} - d^2$$

$$\boxed{\beta = \left(-\infty, \frac{1}{3}\right]}$$

Eqn (3):

$$\gamma = a(d^2 - a^2)$$

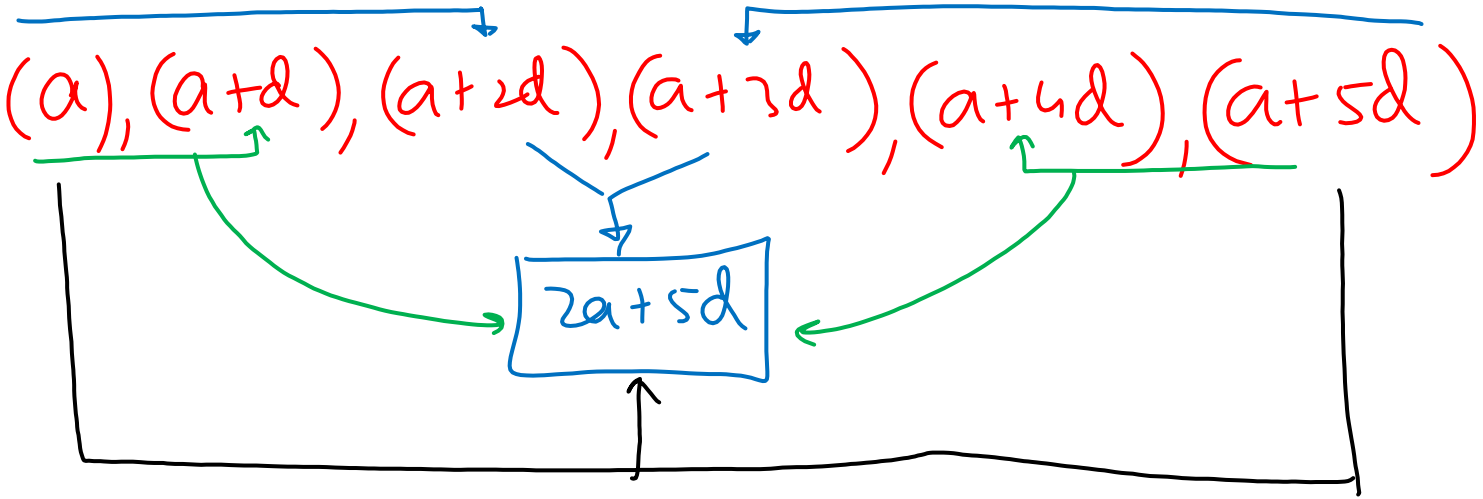
$$\gamma = \frac{1}{3}\left(d^2 - \frac{1}{9}\right)$$

$$\boxed{\gamma \in \left[-\frac{1}{27}, \infty\right)}$$

Properties of A.P.

4

The sum of the terms of an A.P. equidistant from the beginning & end is constant and equal to the sum of first & last terms.



Properties of A.P.

5 Every term of an AP is average of terms equidistant from it.

$$(a), (a+d), (a+2d), \boxed{(a+3d)}, (a+4d), (a+5d)$$

$$(T_k - nd) \longleftarrow T_k \longrightarrow (T_k + nd)$$

If $\sum_{j=1}^{21} a_j$ = 693, where a_1, a_2, ..., a_{21}, are in A.P., then $\sum_{i=0}^{10} a_{2i+1}$ is

A. 361 **B.** 396 **C.** 363 **D.** Data insufficient

HW-1

Properties of A.P.

6 If each term of an A.P. is increased, decreased, multiplied or divided by the same non zero number, then the resulting sequence is also an A.P..

Eg: 2, 4, 6, 8, 10 ⟶ A.P.

4, 8, 12, 16, 20 ⟶ A.P.

1, 2, 3, 4, 5 ⟶ A.P.

3, 5, 7, 9, 11 ⟶ A.P.

If $\frac{b+c-a}{a}, \frac{c+a-b}{b}, \frac{a+b-c}{c}$ are in AP, Then show that

$\frac{1}{a}, \frac{1}{b}, \frac{1}{c}$ are also in AP.

$\left(\frac{b+c-a}{a}\right), \left(\frac{c+a-b}{b}\right), \left(\frac{a+b-c}{c}\right) \longrightarrow$ A.P.

+2 +2 +2

$\left(\frac{b+c-a}{a}+2\right)$; $\left(\frac{c+a-b}{b}+2\right)$; $\left(\frac{a+b-c}{c}+2\right)$

$\longrightarrow$ A P.

$\left(\frac{a+b+c}{a}\right)$, $\left(\frac{a+b+c}{b}\right)$; $\left(\frac{a+b+c}{c}\right)$

$\longrightarrow$ A.P.

$\div (a+b+c)$

$\frac{1}{a}, \frac{1}{b}, \frac{1}{c} \longrightarrow$ A P.

#JEELiveDaily Schedule

jee

11th

Namo Sir | Physics

6:00 - 7:30 PM

Ashwani Sir | Chemistry

7:30 - 9:00 PM

Sameer Sir | Maths

9:00 - 10:30 PM

12th

Jayant Sir | Physics

1:30 - 3:00 PM

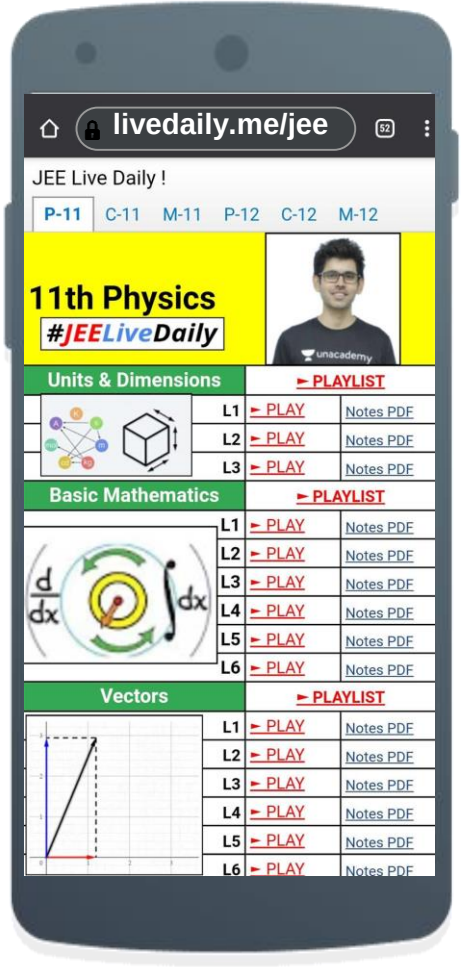
Anupam Sir | Chemistry

3:00 - 4:30 PM

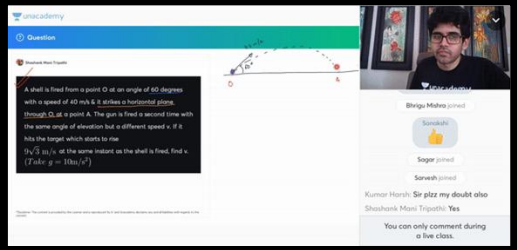
Nishant Sir | Maths

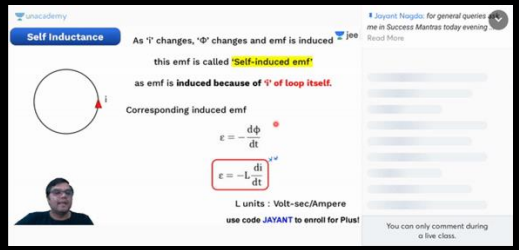
4:30 - 6:00 PM

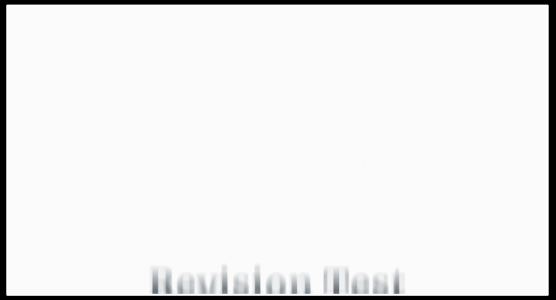
livedaily.me/jee

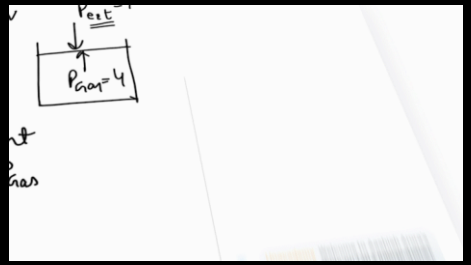


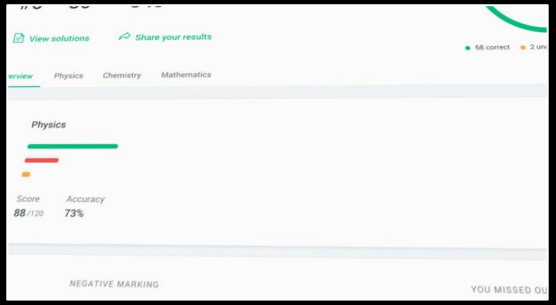
Unacademy Subscription











+ LIVE Class Environment

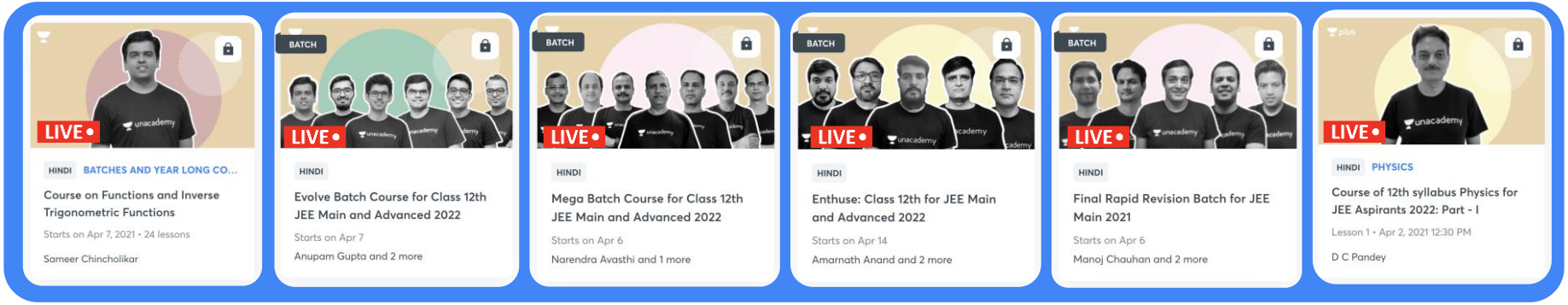
- + LIVE Polls & Leaderboard
- + LIVE Doubt Solving
- + LIVE Interaction

+ Performance Analysis

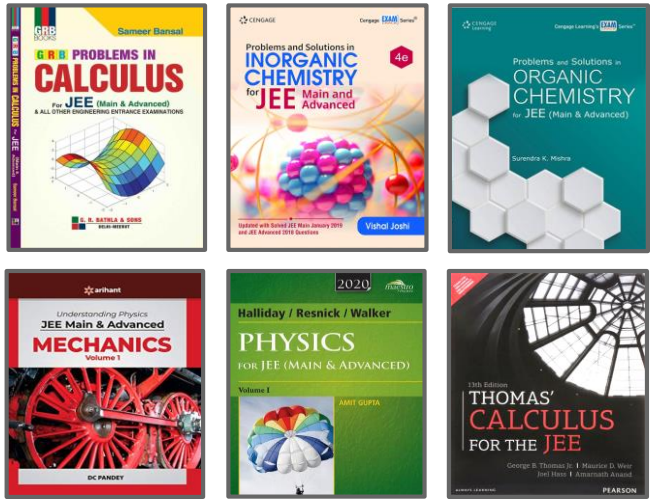
- + Weekly Test Series
- + DPPs & Quizzes

India's **BEST** Educators

Unacademy Subscription



If you want to be the **BEST**
"Learn" from the **BEST**



jee

Top Results

Bratin Mondal

100 %ile

Amaiya Singhal
99.97

Adnan
99.95

Ashwin Prasanth
99.94

Tanmay Jain
99.86

Kunal Lalwani
99.81

Utsav Dhanuka
99.75

Aravindan K Sundaram
99.69

Manas Pandey
99.69

Mihir Agarwal
99.63

Akshat Tiwari
99.60

Sarthak Kalankar
99.59

Vaishnovi Arun
99.58

Devashish Tripathi
99.52

Maroof
99.50

Tarun Gupta
99.50

Siddharth Kaushik
99.48

Mihir Kothari
99.39

Sahil
99.38

Vaibhav Dhanuka
99.34

Pratham Kadam
99.29

Shivam Gupta
99.46

Shrish
99.28

Yash Bhaskar
99.10

Subhash Patel
99.02

Ayush Kale
98.85

Ayush Gupta
98.67

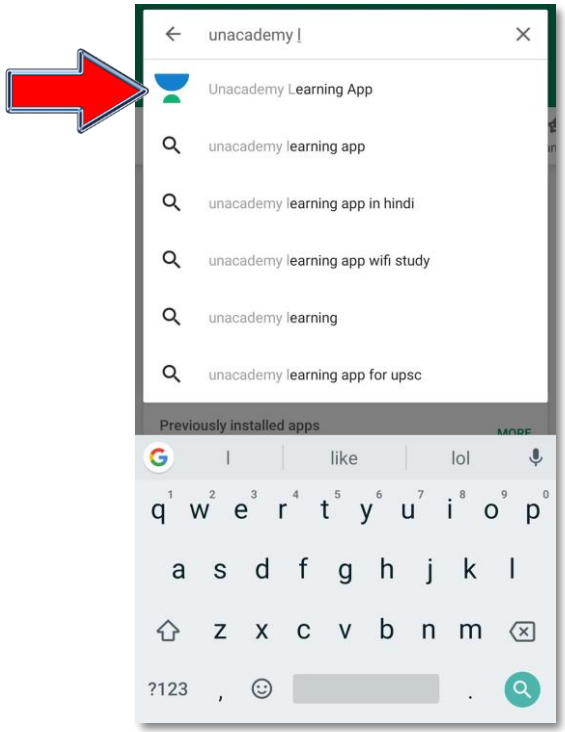
Megh Gupta
98.59

Naman Goyal
98.48

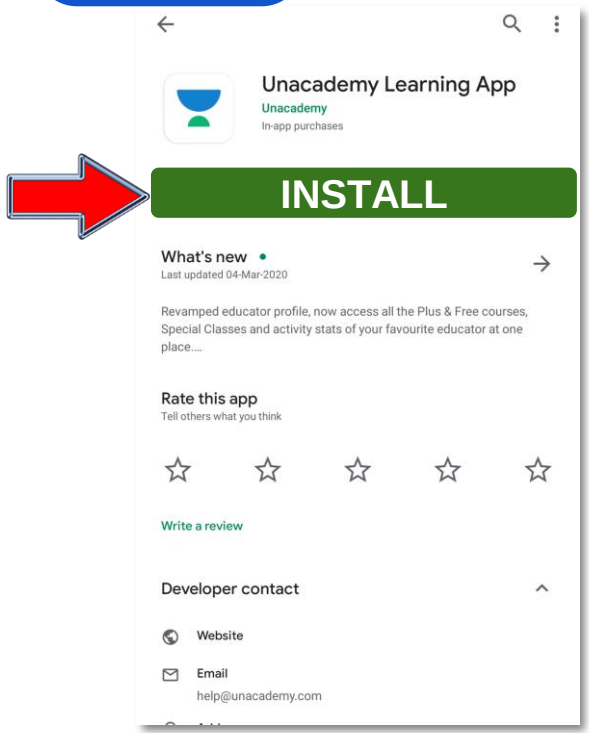
MIHIR PRAJAPATI
98.16

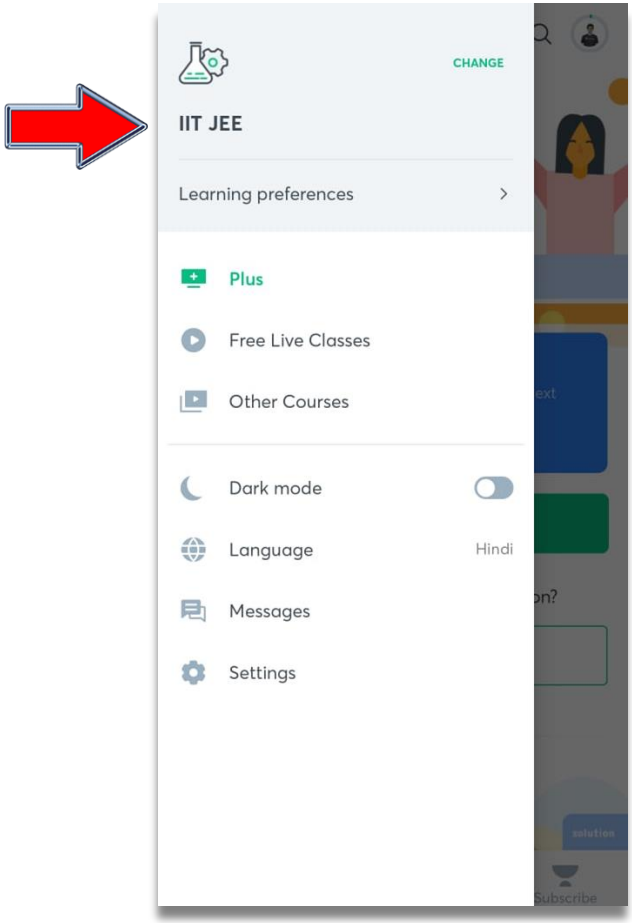
jee

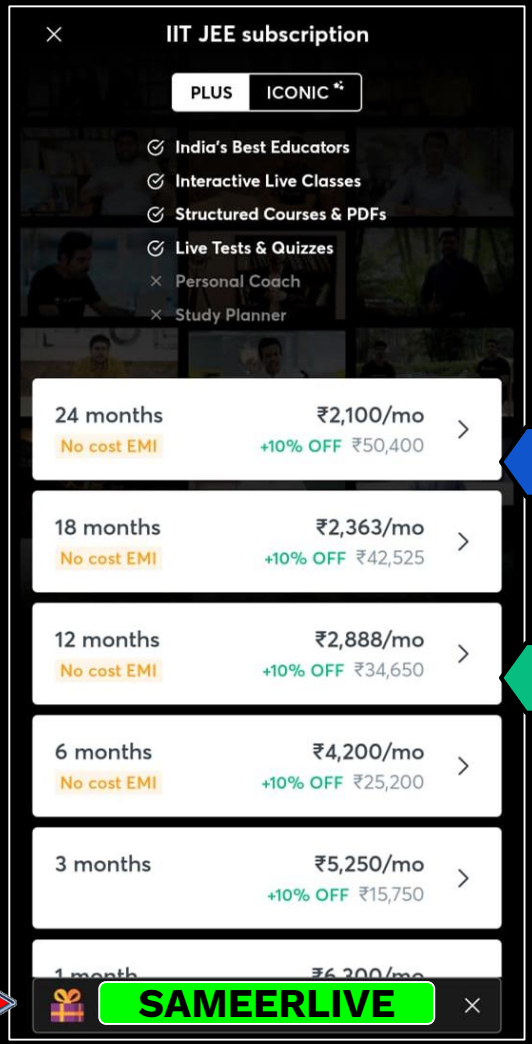
Step 1



Step 2



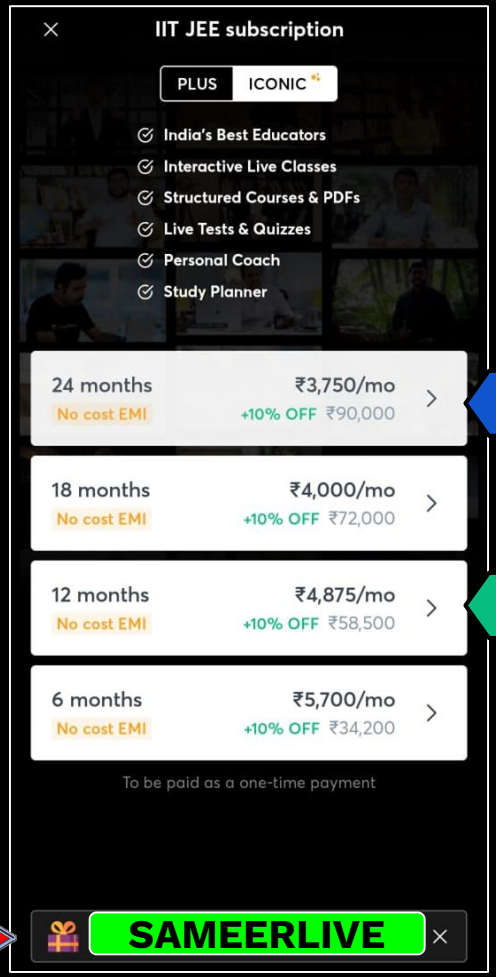

CHANGE
IIT JEE
Learning preferences
Plus
Free Live Classes
Other Courses
Dark mode
Language
Hindi
Messages
Settings



jee

11th / 9, 10

12th / Drop



11th / 9, 10

12th / Drop

jee

unacademy

jee LIVE daily

Test Series 2022

Test Series 2023

9th & 23rd June | 9 AM to 12 PM

jee

EMERGE 3.0 BATCH

JEE Main & Advanced 2023

Started on 12th May

Upcoming Batches in June

unacademy

All Stars Batch : JEE Main 2021 — Started on 9th June 2021

Emerge Batch (Class 11th) : JEE Main & Advanced 2023 — Starts on 16th June 2021

Early Excel Batch for Droppers : JEE Main & Advanced 2022 — Starts on 16th June 2021

Evolve Batch (Class 12th) : JEE Main & Advanced 2022 — Starts on 16th June 2021

UNACADEMY COMBAT
COMPETE. CRACK. CONQUER.

INDIA'S BIGGEST WEEKLY SCHOLARSHIP TEST

For IIT-JEE Aspirants

SCAN NOW TO ENROLL

Enroll for **Free**

Win **Scholarship** from a pool of

₹ 4 Crore

Terms and conditions apply*

Take it live from **android**

IIT-JEE COMBAT

Every Sunday at 11 AM

To unlock, use code

SAMEERLIVE

Rank 1 - 3

1 year IIT-JEE Plus Subscription

Rank 4 - 10

75% Scholarship

Rank 11 - 50

50% Scholarship

Rank 51 - 150

25% Scholarship

Thank You

@sameer_iitr

#JEE Live Daily

+ SUBSCRIBE

jee

LIKE & SHARE

PDF Download Now !