IOE 551

Analysis of English Premier League football teams and DEA analysis of Manchester United FC

End Term Project

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Introduction

Data Envelopment Analysis (DEA) is a non parametric method of economics and operations research for the estimation of efficiency frontiers. It is used to empirically measure productive efficiency of Decision Making Units (DMUs). In addition for its use in economic analysis, DEA has also been used for benchmarking in operations management, where a set of measures (inputs and outputs) are chosen and analyzed to benchmark the performance of service operations. The name of the methodology is derived from the enveloping property of the dataset's efficient DMUs constituting the efficiency frontier against which all other DMUs are compared. DEA essentially uses linear programming functions to determine optimality polygon of efficient DMUs in the dataset under study.

Importance of DEA for this problem

Premier League, also known as the English Premier League (EPL) is the top level soccer competition of the English Football League system. There are over 20 teams in the league competing for the title each year. There has been an emergence of the Top 4 teams in the league over the years, which include Liverpool, Manchester City, Chelsea, and Manchester United.

For a very long time, football teams and players are assessed by the performance of the whole team and rarely based on individual contributions. It is a fact that the 11 players in the team combinedly contribute to the success of the team. When the manager of a team assesses the performance of a player, there are a variety of parameters that come into play. These parameters which can be used to rate a player differs for players playing in different positions in the game namely Forward, Midfield and Defense position. It has been observed from our study that players in each of these positions have around 3-4 input parameters which determines their level of performance in 3-4 parameters(output). Data Envelopment Analysis is a perfect tool to analyze this type of data with multiple input and output parameters to find the most efficient teams and players.

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This project combines data obtained from a variety of sources (described later) of 4 major English Premier League teams- Chelsea, Manchester United, Manchester City, and Liverpool. We initially performed DEA on the performance statistics of these teams over the period 2016 and 2021 to identify the weakest performing team. For the weakest performing team we perform another set of DEA analysis for the players in each position to identify the players who have to be transferred out of the team. Then we perform another set of DEA analysis to scout potential players who can be brought to this team to improve the performance of the team.

Literature Review

Data Envelopment Analysis has been used widely by various businesses to benchmark their competitors. Thomas R. Sexton et al[1] wrote a paper titled Improving Pupil Transportation in North Carolina which used DEA to conduct a large scale study to devise funding strategy for school districts in the state of North Carolina. They identified a set of measures for conducting the study and came up with inferences for changing bus routes, bus quantities, schedules etc. The result of this study led to savings of over $25 million in capital costs and $27 million in operations costs between the years 1990 and 1993.

Luis Aizemberg et al[2] studied the use of DEA for efficiency evaluation of NBA basketball teams in the seasons from 2006 to 2010. They used the expenditure on the team and average audience attendance as the inputs while the number of wins and average points per game were chosen as the outputs. Different teams were chosen as DMUs and the BCC model was employed to find an efficient frontier of teams. Window analysis was used to find whether a team improved over the duration of this analysis.

Data used for analysis

The data used in this project mainly came from the following two sources

- https://www.fifaindex.com/ : This website contains input data to be used in the DEA for the teams and players considered in the analysis. The input data considered are team and player attributes in Forward, Midfield and Defense allocated by FIFA every year based on the performance of teams and players.
- https://fbref.com/en/ : This website contains output parameters for our analysis which are the performance statistics of each player of a particular team for different calendar year seasons. The output parameters include goals scored, assists, tackles won, passes made are captured from the matches by agencies and recorded in this website for player ratings which we use to perform our analyses.

Analysis performed

We performed 4 analyses in this project.

1) **Identifying the lowest performing team**
   The top four teams playing in the English Premier League are Liverpool, Manchester City, Chelsea and Manchester United. We initially perform an analysis to find out the lowest performing team among the top four teams.

2) **Transfer out low performing players**
   For the analysis, we divided the players into forward, midfield and defenders. This was done because we could compare a player only against his own type. For each type of player, we identified certain attributes that would characterize him. This was assumed to

be intrinsic to each of the players. This could be thought of as being computed from the past few years's performance. The outputs were the performance metrics in the years 2020 and 2021. For forward players metrics like the number of goals scored, assists, dribles etc. were used, whereas for defenders we used metrics like the number of blocks.

A window analysis was separately done for the 3 categories of players, and the scores obtained were used to compare relative performances. As a result of the analysis the lowest performing players were weeded out.

3) **Add better performing players to the team**
   Since we have weeded out bad performing players, we would need to replace them with better performing players. This can be done by buying players into the team who are actively available in the player market. About 5 to 10 players from each category were identified. Care was taken to avoid buying players from rival teams. This included not buying players from teams under the current analysis.
   An analysis similar to the window analysis performed for weeding out players was performed. We carried out the analysis this time with all the new players in the market and the players from the teams apart from the lowest performing team.

4) **Benchmarking the current players with the newly added players using categorical variables DEA**
   From the players newly added to the team in each position, we now perform a DEA with the existing players not removed from the team and benchmark the existing players with the efficient new players and suggest to the manager which players need more attention in their training and overall development to make them more effective and competitive.

Analysis 1 - Identifying lowest performing team

As mentioned previously the first step of our project consists of performing a Window DEA analysis of the previously mentioned 4 teams for the years 2016, 2017, 2018, 2019, 2020, and 2021.

We used the following set of measures for this:

1. **Inputs:** Attack, Midfield, Defense
   The set of input parameters are the total attributes of the respective teams as generated by FIFA, the governing body of football across the world. The attributes of the team in Attacking, Midfielding and Defense is considered here.

2. **Outputs:** Goals Scored divided by Goals Conceded, Points
   The output parameters for the teams in this analysis are those parameters which can be considered as the team achievements. We used goals scored / goals conceded which tells us about the attacking as well as the defensive prowess of the teams. As a good attacking side means more goals and a good defensive side means less goals conceded,

this ratio is a very good output parameter. The other output parameter is the number of points scored in a particular season. As the number of points depends on the number of wins and losses, this output parameter is perfect to determine if a particular team is performing well or not.

We calculated points using the following formula:

Points = 1*No. Of Wins + 0.5*No. Of Draws

Using the Output Oriented VRS Window Analysis DEA model we found the following results (Average through Windows) for each of the six windows:

1. **Window 1**

	2016	2017	2018	2019	2020	2021
Liverpool	0.999997	0.999996	1.000000	1.000000	0.884476	1.000000
Manchester City	0.907589	1.000000	1.000000	1.000000	1.000000	1.000000
Chelsea	1.000000	0.868214	0.701493	0.999990	0.999999	0.999984
Manchester United	1.000000	0.759687	0.999975	0.846618	0.999998	0.999997

2. **Window 2**

	2016-2017	2017-2018	2018-2019	2019-2020	2020-2021
Liverpool	0.954566	0.999995	1.000000	0.865672	0.942238
Manchester City	0.943547	1.000000	0.910448	0.976747	1.000000
Chelsea	0.922110	0.761432	0.850739	0.999989	0.879869
Manchester United	0.871211	0.736624	0.843467	0.879117	0.999997

3. **Window 3**

	2016-2017-2018	2017-2018-2019	2018-2019-2020	2019-2020-2021
Liverpool	0.947316	0.998758	0.910448	0.910448
Manchester City	0.941230	0.936275	0.900498	0.955216
Chelsea	0.840955	0.840939	0.900471	0.916656
Manchester United	0.819674	0.718243	0.844933	0.919410

4. **Window 4**

	2016-2017-2018-2019	2017-2018-2019-2020	2018-2019-2020-2021
Liverpool	0.959558	0.930550	0.904746
Manchester City	0.908128	0.919118	0.894018
Chelsea	0.813036	0.857841	0.850712
Manchester United	0.785126	0.788680	0.883697

5. **Window 5**

	2016-2017-2018-2019-2020	2017-2018-2019-2020-2021
Liverpool	0.912831	0.921968
Manchester City	0.900032	0.910210
Chelsea	0.813278	0.820754
Manchester United	0.777268	0.817521

6. **Window 6**

	2016-2017-2018-2019-2020-2021
Liverpool	0.908633
Manchester City	0.895790
Chelsea	0.776830
Manchester United	0.771243

Results from DEA analysis of teams

Usually, every football club in the English Premier League has a cycle of ups and downs every 3 to 4 years. This is mainly due to the fact that the managers of these clubs are replaced after this duration. Every manager coming to these top clubs tries to make an efficient team, establishes a good team for 3-4 years and after that starts to see a dip in the performance due to player aging and worn-out tactics. That is when managers are replaced in these teams. In the above performed analysis windows 3,4 and 5 provides sensible outputs as it can capture how the teams performed under a manager. Hence when we analyze the outputs of these teams in windows 3,4 and 5 we get to see that Manchester United is the team which is performing the least among all other teams with an average efficiency of 77.1% among the six years considered for analysis.

Conclusion from DEA analysis on teams

From the analysis we identified Manchester United as the least performing team among the top 4 teams in the English Premier League at the end of 2021. Interestingly as we follow football we got to know that the most recent coach of Manchester United ‘Ole Gunnar Solskjaer’, got sacked at the end of 2021 and the team is looking out for a new coach to manage the team. In April 2022, Manchester United decided to appoint dutch coach ‘Erik ten Haag’ as the new coach for the 2022-2023 season. He is also entrusted with a transfer budget of $200m to buy potential players who can be used to transform the team to the best team in the Premier League. Eyeing this news, our team decided to help Mr. ten Haag to offload the inefficient players and suggest new players who can fit the team to make it stronger.

Now we go on to the second part of our analysis which is to identify players who do not perform well in Manchester United.

Analysis 2 - Transfer out low performing players

The inputs and outputs for different categories of players for the purpose of performing DEA is given below:

For Forward

Inputs: Attack Average, Minutes Played, Shooting Average, Passing Average
Outputs: Goals Scores, Shots on Target, Dribles, Assists

For Mid Field

Inputs: Shooting Average, Minutes, Movement Average, Passing Average
Outputs: Goals Scores, Dribbles, Assists, Passes Completed/ Passes Attempted

For Defense

Inputs: Passing Average, Minutes Played, Defense Average, Movement Average
Outputs: Tackles Won/Tackles Attempted, Number of Blocks,

As we have now arrived at the team which performs the least, we now perform a DEA analysis comparing Manchester United players with players of the most efficient teams in the Premier League - Liverpool and Manchester City. The players in Manchester City and Liverpool are the best players in the league in their particular positions and have chances to form the effective DEA frontier with which Manchester United players can be compared to. As the objective of the whole analysis is for the team to compete for the premier league title, this set of DMUs looks perfect for the analysis. The players in these teams are divided into Forwards, Midfielders and Defenders and separate DEA analysis is performed for players in these positions to identify weak players playing in each of these positions for Manchester United.

DMUs for this analysis

Defense Players (DMU)	Midfield Players (DMU)	Forward Players (DMU)
Virgil Van Dyjk(LV)	Fabinho(LV)	Mohammad Salah(LV)
Aymeric Laporte(MC)	Thiago(LV)	Sadio Mane(LV)
John Stones(MC)	Jordan Henderson(LV)	Luis Diaz(LV)
Joel Matip(LV)	Kevin De Bruyne(MC)	Roberto Firmino(LV)
Trent Alexander(LV)	Rodri(MC)	Bernardo Silva(MC)
Kyle Walker(MC)	Fernandinho(MC)	Marcus Rashford(MU)
Ruben Dias(MC)	Bruno Fernandes(MU)	Edinson Cavani(MU)
Joao Cancelo(MC)	Paul Pogba(MU)	Jadon Sancho(MU)
Harry Maguire(MU)	Donny van de Beek(MU)	Cristiano Ronaldo(MU)
Victor Lindelof(MU)	Jesse Lingard(MU)	Mason Greenwood(MU)
Phil Jones(MU)	Fred(MU)	Daniel James(MU)
Alex Telles(MU)	Nemanja Matic(MU)	Anthony Martial(MU)
Luke Shaw(MU)	Scott Mctominay(MU)	Andreas Pereira(MU)
Raphael Varane(MU)	Juan Mata(MU)	
Diogo Dalot(MU)		
Aaron Wan Bissakka(MU)		

The players marked in green are the efficient players from Liverpool and Manchester City and the players marked in orange are the players from Manchester United who need to be analyzed, identified and replaced from the team.

Results of DEA window analysis for Forward players

Average through Window	
	2020-2021
Mohammad Salah(LV)	1
Sadio Mane(LV)	0.927844976
Luis Diaz(LV)	0.883722012
Roberto Firmino(LV)	0.758940994
Bernardo Silva(MC)	0.774996719
Marcus Rashford(MU)	0.701166122
Edinson Cavani(MU)	0.999990158
Jadon Sancho(MU)	0.984538145
Cristiano Ronaldo(MU)	1
Mason Greenwood(MU)	0.911325811
Daniel James(MU)	0.999988789
Anthony Martial(MU)	0.698752971
Andreas Pereira(MU)	0.761625513

Results of DEA window analysis for Mid field players

Average through Window	
	2020-2021
Fabinho(LV)	0.978158
Thiago(LV)	0.984258
Jordan Henderson(LV)	0.938955
Kevin De Bruyne(MC)	1
Rodri(MC)	1
Fernandinho(MC)	1
Bruno Fernandes(MU)	0.948212
Paul Pogba(MU)	1
Donny van de Beek(MU)	1
Jesse Lingard(MU)	0.44084
Fred(MU)	0.999613
Nemanja Matic(MU)	0.744119
Scott Mctominay(MU)	1
Juan Mata(MU)	0.571026

Results of DEA window analysis for Defense players

Average through Window	
	2020-2021
Virgil Van Dyjk(LV)	0.983547824
Aymeric Laporte(MC)	1
John Stones(MC)	0.889777618
Joel Matip(LV)	0.940238501
Trent Alexander(LV)	0.818586390
Kyle Walker(MC)	0.874425604
Ruben Dias(MC)	0.957410029
Joao Cancelo(MC)	0.836977472
Harry Maguire(MU)	0.550884057
Victor Lindelof(MU)	0.888636305
Phil Jones(MU)	0.724193883
Alex Telles(MU)	0.900468357
Luke Shaw(MU)	0.915894365
Raphael Varane(MU)	0.930004024
Diogo Dalot(MU)	0.999997934
Aaron Wan Bissakka(MU)	0.949690497

From the above results we can clearly see that there are players who are not performing up to the mark. The analysis does justice to the ability of the players with regards to their input and their performance in the team as the output. The results of these DEA analyses show that certain players who have good attributes do not perform well in the team. These are the players who have to be removed from the team. We used a window analysis to average out performance across 2 recent years (used Window 2 results) to better analyze the players. The analysis was an output oriented variable returns to scale model. We get efficiency of players with regards to their ability and their level of output.

In the category of forward players we can see that- Marcus Rashford (70.11%) and Anthony Martial (69.87%) are the lowest performing players and hence can be chucked out. Both these players are actually talented with good input attributes but they have been on a form dip for 2 straight seasons and have to be sold. Daniel James interestingly has a 99.9% efficiency in this analysis. It was found that his playing time(input) was very less and he had good contributions in the games which he played which made him a very efficient player. In the mid-field category we could be getting rid of players like Jesse Lingard (44%), Juan Mata (57%), and Nemanja Matic (74%). Both Mata and Matic are veteran players and have low output for the games they play. Similarly Manchester United would be better off without their current captain Harry Maguire (55%) who has good input attributes but has been struggling with his form and Phil Jones (72%) who is a veteran player and has poor input and output attributes.

Players to be transferred out
Forward: Marcus Rashford ($77m), Anthony Martial ($30m)
Midfield: Jesse Lingard ($22m), Juan Mata ($3.3m), Nemanja Matic ($6.6m)
Defense: Harry Maguire ($44m), Phil Jones ($4.4m)

The amount put in brackets above is the current transfer value of these players. The transfer value depends on the talents of the players and also the age of the players. Younger players have more value compared to older close to retirement players. If we sell all these players and we assume we manage to sell all these players we get $187.3m which can be added to our existing transfer budget of $200m to buy new players. Here our total transfer budget becomes $387million.

Analysis 3 - Adding better players to the team
Inputs and Outputs used here for the analysis: same as that of Analysis 2

The next step in the analysis is to identify potential players to replace the transferred out players. For this analysis we looked at the potential transfer targets of Manchester United from web sources and shortlisted the players Manchester United are scouting for the next season. Manchester United now has a transfer budget of $387m to buy players.

The potential targets for Manchester United for each of the positions are as follows:

Forwards	
Player	Transfer Value
Harry Kane(Tottenham)	$110m
Darwin Nunez(Benfica)	$40m
Victor Osimhen (Napoli)	$60m
Dominic Calvert Lewin (Everton)	$46.2m

Midfielders	
Player	Transfer Value
Declan Rice(West Ham)	$75m
Jude Bellingham(Borussia Dortmund)	$82.5m
Kalvin Phillips(Leeds United)	$55m
Youri Tielemans (Leicester City)	$60.5m
Antony (Ajax)	$38.5m
Christopher Nkunku(RB Leipzig)	$71.5m
Aurélien Tchouameni(AS Monaco)	$44m

Defenders	
Player	Transfer Value
Pau Torres(Villareal)	$55m
Jurrien Timber (Ajax)	$33m
Kyle Walker Peters (Southampton)	$19.8m
Antonio Rudiger (Chelsea)	Free Agent
Ronald Araujo(Barcelona)	$40m

The brackets show the player's current team. We do not consider players from our rival teams and players from Liverpool and Manchester City as they are virtually impossible to buy. Similar to Analysis 2, we now perform an output oriented variable returns to scale time window based DEA with Liverpool and Manchester City (the efficient premier league players) and our potential targets to find out which players have good efficiencies. The cost of these players are also taken into consideration when finally selecting the players to buy in.

Results of DEA window analysis for Forward players

Average through Window	
	2020-2021
Mohammad Salah(LV)	1
Sadio Mane(LV)	0.77843
Luis Diaz(LV)	0.904897
Roberto Firmino(LV)	0.760617
Bernardo Silva(MC)	0.862667
Harry Kane(Tottenham)	0.921663
Darwin Nunez(Benfica)	1
Victor Osimhen (Napoli)	0.968515
Dominic Calvert Lewin (Everton)	0.92357

Results of DEA window analysis for Midfield players

Average through Window	
	2020-2021
Fabinho(LV)	0.991257
Thiago(LV)	0.984794
Jordan Henderson(LV)	0.934515
Kevin De Bruyne(MC)	1
Rodri(MC)	1
Fernandinho(MC)	0.990982
Declan Rice(West Ham)	1
Jude Bellingham(Borussia Dortmund)	0.963554
Kalvin Phillips(Leeds United)	0.945511
Youri Tielemans (Leicester City)	0.999999
Antony (Ajax)	1
Christopher Nkunku(RB Leipzig)	1
Aurélien Tchouameni(AS Monaco)	1

Results of DEA window analysis for Defense players

Average through Window	
	2020-2021
Virgil Van Dyjk(LV)	1
Aymeric Laporte(MC)	1
John Stones(MC)	0.890655
Joel Matip(LV)	0.946887
Trent Alexander(LV)	0.854401
Kyle Walker(MC)	0.868891
Ruben Dias(MC)	0.956332
Joao Cancelo(MC)	0.873896
Pau Torres(Villareal)	0.744562
Jurrien Timber (Ajax)	1
Kyle Walker Peters (Southampton)	0.902528
Antonio Rudiger (Chelsea)	0.716545
Ronald Araujo(Barcelona)	1

From the analysis we arrive at the conclusion that forwards Darwin Nunez (100%) and Victor Osimhen (96%) are perfect young players to be bought who lie in the efficient frontier and can improve the attacking performance of the team. Similarly Declan Rice (100%), Antony (100%) and Aurélien Tchouameni (100%) can be bought to improve the midfield. Jurrien Timber (100%) and Kyle Walker Peters (90%) are the players whom we have identified to fill the defense positions. All these players have close to 100% efficiency as they have similar characteristics to efficient Premier League players and would be the best to come to a team like Manchester United. We have avoided Ronald Araujo who has a score of 100% because his current team Barcelona would not release him as they see him as a talent whom they do not want to sell.

The new players include:

Players	Position	Transfer Value
Darwin Nunez(Benfica)	Forward	$40m
Victor Osimhen (Napoli)	Forward	$60m
Declan Rice(West Ham)	Midfield	$75m
Antony (Ajax)	Midfield	$38.5m
Aurélien Tchouameni(AS Monaco)	Midfield	$44m
Jurrien Timber (Ajax)	Defense	$33m
Kyle Walker Peters (Southampton)	Defense	$19.8m

Buying all these players would cost us $310m which is under our total transfer budget of $387m.

Analysis 4- Benchmarking the current players with the newly added players using categorical variables DEA

Inputs and Outputs used here for the analysis: same as that of Analysis 2

Within the forward category we have players who play the **Center Forward** (Category 1) position as well as the **Winger** (Category 2) position. In the midfield position there are players who play **Attacking mid**(Category 2) and others who play **Defensive mid**(Category 1). In defenders we have **Center Defenders**(Category 1)and **Wing Back Defenders**(Category 2) who play on the sides. We perform a categorical DEA based on the categories described above for our new team of players with efficient players from Liverpool and Manchester City and identify who players need improvement in their outputs. These players can be improved by identifying them and providing them selective training.

An output oriented VRS model is used for the categorical DEA analysis for year 2021 data

The results of the categorical DEA are given below:

Results of categorical DEA analysis for Forward players

Rank	DMU	Score	Category
1	Mohammad Salah(LV)	1	2
1	Sadio Mane(LV)	1	1
1	Luis Diaz(LV)	1	1
1	Jadon Sancho(MU)	1	1
1	Cristiano Ronaldo(MU)	1	2
1	Daniel James(MU)	1	1
1	Darwin Nunez(Benfica)	1	2
1	Victor Osimhen (Napoli)	1	1
9	Bernardo Silva(MC)	0.999976	1
10	Edinson Cavani(MU)	0.999858	2
11	Andreas Pereira(MU)	0.999582	1
12	Mason Greenwood(MU)	0.652124	1
13	Roberto Firmino(LV)	0.60514	2

From the above categorical analysis, we observe that some of the existing Manchester United players need to be given training so that they can perform at par with the other players both within the team and away from the team. Mason Greenwood could hence be coached by other category 2 forward players like Ronaldo.

Results of categorical DEA analysis for Midfield players

Rank	DMU	Score	Category
1	Fabinho(LV)	1	1
1	Jordan Henderson(LV)	1	1
1	Kevin De Bruyne(MC)	1	2
1	Rodri(MC)	1	1
1	Bruno Fernandes(MU)	1	2
1	Paul Pogba(MU)	1	2
1	Donny van de Beek(MU)	1	2
1	Fred(MU)	1	1
1	Scott Mctominay(MU)	1	1
1	Declan Rice(West Ham)	1	1
1	Antony (Ajax)	1	2
1	Aurélien Tchouameni(AS Monaco)	1	1
13	Fernandinho(MC)	0.989094	1
14	Thiago(LV)	0.978476	2

Since the players here are at par with everyone else on the team, we could carry out regular training sessions with everyone involved. There is no one who requires to be coached or mentored selectively.

Results of categorical DEA analysis for Defense players

Rank	DMU	Score	Category
1	Virgil Van Dyjk(LV)	1	1
1	Aymeric Laporte(MC)	1	1
1	John Stones(MC)	1	1
1	Joel Matip(LV)	1	1
1	Ruben Dias(MC)	1	1
1	Aaron Wan Bissakka(MU)	1	2
1	Jurrien Timber (Ajax)	1	1
1	Kyle Walker Peters (Southampton)	1	2
9	Raphael Varane(MU)	0.999992	1
10	Alex Telles(MU)	0.99998	2
11	Diogo Dalot(MU)	0.999964	2
12	Victor Lindelof(MU)	0.996104	1
13	Luke Shaw(MU)	0.903284	2
14	Kyle Walker(MC)	0.894552	2
15	Joao Cancelo(MC)	0.873861	2
16	Trent Alexander(LV)	0.807866	2

The new Manchester United team defenders appear highly efficient and would not require selective training for further improvements.

Improvements and Future Works

While we performed a time window and a categorical analysis separately, we were unable to implement both of them together within the same analysis. In the future, this problem could be solved with DEA solvers which can perform both Time Window and categorical variables in a single analysis.

Conclusions

Using DEA, we have been able to measure performance, for the purpose of reorganizing teams. We were able to identify relative player performance within their own category and added players so that the lower performing teams can perform just as well as the higher performing teams. We were also able to identify players who require training , so that they can perform up to their full potential.

The employment of DEA for the analysis, has also allowed us to deep dive into the expectations from each player in regards to their intrinsic abilities.

References

[1] Sexton, T. R., Sleeper, S., & Taggart, R. E. (1994). Improving pupil transportation in North Carolina. *Interfaces*, *24*(1), 87-103.

[2] Aizemberg, L., et al. (2014) Measuring the NBA Teams' Cross-Efficiency by DEA Game. American Journal of Operations Research, 4, 101-112