

Advanced Partial Differential Equations

In these lectures we provide an introduction to nonlinear partial differential equations and the basic methods useful for finding the solutions of these equations. Firstly, we will start with the classical solutions of the heat, potential and wave equations in terms of Green's functions, and properties of these solutions. Secondly, we will deal with nonlinear models, and variational principles. We will continue with quasi-linear and nonlinear equations using characteristics. The concept of weak solutions and asymptotic behavior is discussed related to hyperbolic equations.

We will switch gear and introduce modern techniques for finding solutions by use of singularity analysis. We will conclude with the theory of solitons and inverse scattering transforms. We will include Backlund transforms and the Lax formulation. These techniques will be applied to interesting nonlinear dispersive models.

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References:

- [1] Lokenath Debnath, Nonlinear Partial Differential equations (Birkhauser, Berlin 1997)
- [2] P.G. Drazin and R.S. Johnson, Solitons: an introduction (Cambridge University Press, Cambridge 1996)
- [3] J. Billingham and A. C. King, Wave motion (Cambridge University Press, Cambridge 2000)
- [4] R. Conte, Exact solutions of nonlinear partial differential equations by singularity analysis (arxiv.org)