Advanced Partial Differential Equations

In these lectures we provide an introduction to nonlinear partial differential equations and the basic methods useful for finding the solutions of these equations. Firstly, we will start with the classical solutions of the heat, potential and wave equations in terms of Green's functions, and properties of these solutions. Secondly, we will deal with nonlinear models, and variational principles. We will continue with quasi-linear and nonlinear equations using characteristics. The concept of weak solutions and asymptotic behavior is discussed related to hyperbolic equations.

We will switch gear and introduce modern techniques for finding solutions by use of singularity analysis.

We will conclude with the theory of solitions and inverse scattering transforms. We will include Backlund transforms and the Lax formulation. These techniques will be applied to interesting nonlinear dispersive models.

Table of Contents:

1. Introduction to PDEs

Heat: Green's functions, finite and infinite domain

Potential: source and dipole solution, Greens' functions

Wave equation: fundamental solution, Green's function, dispersive waves

1. Nonlinear models

Variational principle

1. First order equations

Quasi-linear equations

Method of characteristics

1. Weak solutions to hyperbolic equations

Discontinuous Solutions

Shock formation

Asymptotic behavior

1. Hyperbolic systems

Riemann's method

Hodograph method

1. Modern perspectives

Exact solutions of nonlinear PDEs

Singularity analysis

Paineleve analysis

1. Nonlinear waves

Solitary waves

The scattering problem

Dispersive models

References:

[1] Lokenath Debnath, Nonlinear Partial Differential equations (Birkhauser, Berlin 1997)

[2] P.G. Drazin and R.S. Johnson, Solitons: an introduction (Cambridge University Press, Cambridge 1996)

[3] J. Billingham and A. C. King, Wave motion (Cambridge University Press, Cambridge 2000)

[4] R. Conte, Exact solutions of nonlinear partial differential equations by singularity analysis (arxiv.org)