Assignment-I (PHL 1010)

Total Marks: 30

(1) Consider the function (Gaussian function)

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left[-\frac{(x-a)^2}{2\sigma^2}\right] \quad \sigma > 0$$

(a) Show that $\int_{-\infty}^{\infty} f(x)\,dx = 1$

(b). Plot $f(x)$ for $a=0$, $a=2$ & $\sigma = 1.0, 5.0$ & 10.0

(c) Show that $\lim_{\sigma\to 0} \frac{1}{\sigma\sqrt{2\pi}} \exp\left[-\frac{(x-a)^2}{2\sigma^2}\right] = \delta(x-a)$

Represents Dirac delta function. [5 Marks]

(2) The potential in some region is given by

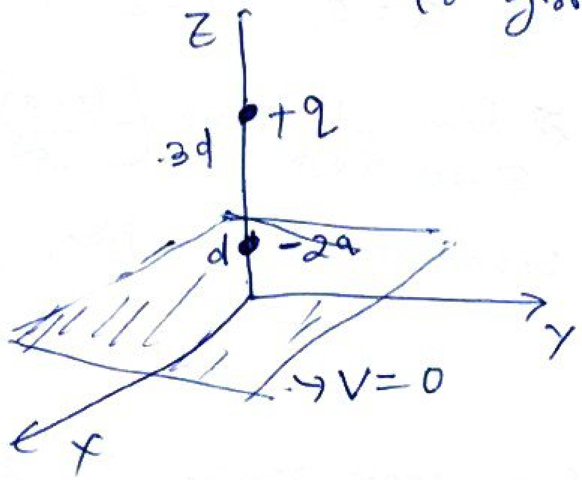
$$V = \alpha r^2 z \sin\phi$$

Find (i) Electric field and (ii) Charge density in cylindrical coordinate system. [5 Marks]

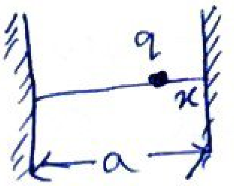
(3) An inverted hemispherical bowl of radius 'R' carries a uniform surface charge density σ. Find the potential difference between the "north pole" and the centre. [5 Marks]

(4) Prove Earnshaw's Theorem: A charged particle can not be held in a stable equilibrium by electrostatic forces alone. [5 Marks]

(5) Find the force on charge $+q$ for the configuration shown below. The XY plane is a conductor ~~[illegible]~~ connected to ground. [5 Marks]



(6) (a) Find the force q on the Charge for below configuration, where the Planes are grounded ($V=0$).



(b) Find the force if

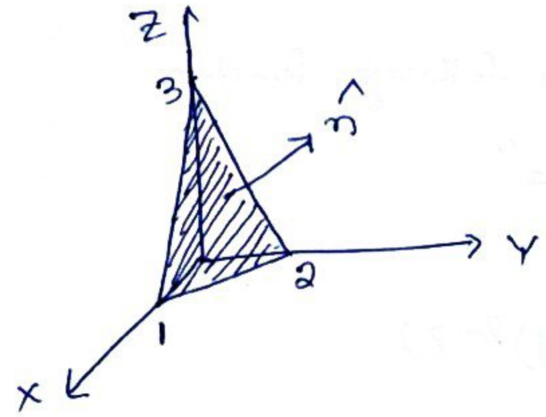
(i) $a \to \infty$ (ii) $x = \frac{a}{2}$

[5 Marks]

Tutorial Problems

(1) Find the angle between the body diagonal of a unit cube ($a = 1$)

(2) Find the components of the unit vector $\hat{n}$ perpendicular to the plane shown in Figure.



(3) The transformation of coordinate frame is represented by the following relation

$$\begin{pmatrix} A_y' \\ A_z' \end{pmatrix} = \begin{pmatrix} \cos\phi & \sin\phi \\ -\sin\phi & \cos\phi \end{pmatrix} \begin{pmatrix} A_y \\ A_z \end{pmatrix}$$

Prove that the Dot Product of two vectors $\vec{A}$ and $\vec{B}$ remain invariant under such rotation.

(4) Find the transformation matrix $\ddot{R}$ that describes a rotation by 120° about an axis from the origin through the point (1,1,1). The rotation is clockwise as you look down the axis toward the origin.

(5) (a) How do the components of a vector transform under an inversion of coordinates? $[x' = -x,\ y' = -y,\ z' = -z]$

(b) How do the components of a cross product transform under inversion?

(c) How does the scalar triple product transform under this inversion?

(6) Find Gradient of the following function

(a) $f(x,y,z) = x^2 + y^3 + z^4$

(b) $f(x,y,z) = x^2 y^3 z^4$

(c) $f(x,y,z) = e^x \sin(y) \ln(z)$

(7) The separation vector between two points (x', y', z') and (x,y,z) is represented by $\vec{\boldsymbol{r}}$ (magnitude is $\boldsymbol{r}$).

(a) ~~Find~~ out $\vec{\nabla}(\boldsymbol{r}^2)$, (b) $\vec{\nabla}\left(\frac{1}{\boldsymbol{r}}\right)$

(8) Find the divergence of following vectors

(a) $\vec{V}_a = x^2 \hat{x} + 3xz^2 \hat{y} - 2xz \hat{z}$

(b) $\vec{V}_b = xy \hat{x} + 2yz \hat{y} + 3zx \hat{z}$

(c) $\vec{V}_c = y^2 \hat{x} + (2xy + z^2) \hat{y} + 2yz \hat{z}$

(9) Sketch the vector function $\vec{v} = \frac{\hat{r}}{r^2}$

Compute the divergence.

Practice Problem 2

(1) Construct a vector function that has zero divergence and zero curl (Function should not be a constant function).

(2) Prove the following.

(i) $\vec{\nabla}(fg) = f\,\vec{\nabla}g + g\,\vec{\nabla}f$

(ii) $\vec{\nabla}\cdot(\vec{A}\times\vec{B}) = \vec{B}\cdot(\vec{\nabla}\times\vec{A}) - \vec{A}\cdot(\vec{\nabla}\times\vec{B})$

(iii) $\vec{\nabla}\times(f\vec{A}) = f(\vec{\nabla}\times\vec{A}) - \vec{A}\times(\vec{\nabla}f)$

(3) (a) Compute $(\hat{r}\cdot\vec{\nabla})\,\hat{r}$, where $\hat{r}$ is unit vector

(b) $\vec{V}_a = x^2\hat{x} + 3xz^2\hat{y} - 2xz\hat{z}$, $\vec{V}_b = xy\hat{x} + 2yz\hat{y} + 3zx\hat{z}$

find out $(\vec{V}_a\cdot\vec{\nabla})\vec{V}_b$

(4) Compute $\vec{\nabla}\cdot\left(\frac{\vec{A}}{g}\right)$ and $\vec{\nabla}\times\left(\frac{\vec{A}}{g}\right)$

(5) Calculate Laplacian of the following functions.

$T_a = x^2 + 2xy + 3z + 4$

$T_c = e^{-5x}\sin 4y\cos 3z$

$\vec{V} = x^2\hat{x} + 3xz^2\hat{y} - 2xz\hat{z}$

(6) (a) Find ~~Prove~~ the divergence of curl of vector $\vec{V}_a$ & $\vec{V}_b$ [As defined in 3(b)]

(b) Calculate curl of gradient $f(x,y,z) = e^x \sin y \ln(z)$

(7) Calculate the line integral of the function $\vec{v} = x^2\hat{x} + 2yz\,\hat{y} + y^2\hat{z}$ from the origin to $(1,1,1)$ through the path

(a) $(0,0,0) \to (1,0,0) \to (1,1,0) \to (1,1,1)$

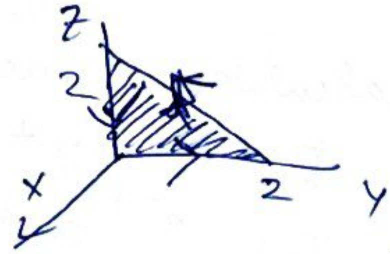
(b) $(0,0,0) \to (0,0,1) \to (0,1,1) \to (1,1,1)$

(8) Calculate the surface integral of $\vec{v} = 2xz\,\hat{x} + (x+2)\hat{y} + y(z^2-3)\hat{z}$ over the sides of the cubical box (side 2).

(9) Calculate volume integral of $T = z^2$ over tetrahedron with corners at $(0,0,0)$, $(1,0,0)$, $(0,1,0)$ & $(0,0,1)$

(10) Check the divergence theorem using the function $\vec{v} = y^2\hat{x} + (2xy + z^2)\hat{y} + (2yz)\hat{z}$ and a unit cube at the origin.

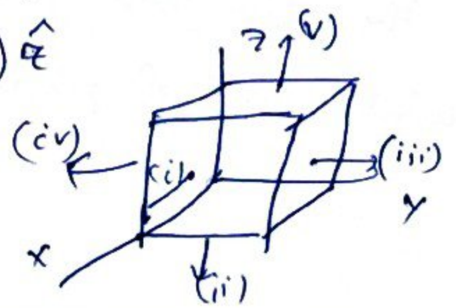
(11) Check Stokes' theorem for the function $\vec{v} = (xy)\hat{x} + (2yz)\hat{y} + (3zx)\hat{z}$ using the triangular shaded area shown below.



(12) Calculate $\int(\vec{\nabla}\times\vec{v})\cdot d\vec{a}$ and show that it depends only on boundary line, not on particular surface used.

$\vec{v} = (2xz + 3y^2)\,\hat{y} + (4yz^2)\,\hat{z}$

The back of the cube is open

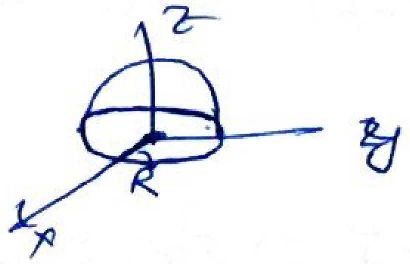


Practice Problem (3)

(1) Compute the divergence of the function

$$\vec{V} = (r\cos\theta)\hat{r} + (r\sin\theta)\hat{\theta} + (r\sin\theta\cos\phi)\hat{\phi}$$

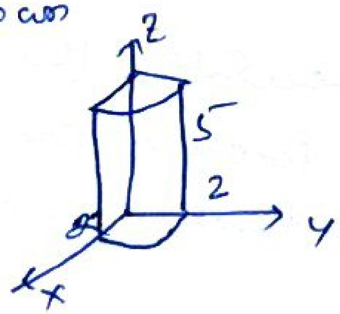
Check the divergence theorem for this function using the volume enclosed by hemispherical bowl inverted on xy plane centred at origin with radius R.



(2) (a) $\vec{V} = s(2+\sin^2\phi)\hat{s} + s\sin\phi\cos\phi\,\hat{\phi} + 3z\hat{z}$

Find the divergence and curl of this function.

(b) Test the divergence theorem using the quarter cylinder ($r=2$, $h=5$) as shown



(3) Evaluate the following integrals

(a) $\int_0^5 \cos x\,\delta(x-17)\,dx$, (b) $\int_0^3 x^3\delta(x+1)\,dx$

(c) $\int_0^2 (x^3+3x+2)\,\delta(1-x)\,dx$ (d) $\int_{-\infty}^{a} \delta(x-b)\,dx$

(4) $\theta(x)$ is a step function $\theta(x) = \begin{cases} 1, & x>0 \\ 0, & x\le 0 \end{cases}$

Show that $\frac{d\theta}{dx} = \delta(x)$

(5) ~~What is the volume charge density of an electric dipole, consisting of a point charge $-q$ at the origin and $+q$ at $\vec{a}$?~~

(6) Evaluate (a) $\int_{\text{space}} (r^2 + \vec{r}\cdot\vec{a} + a^2)\, \delta^3(\vec{r} - \vec{a})\, d\tau$

$\vec{a}$ = fixed vector = $a\hat{a}$

(b) $\int_{V} (\vec{r} - \vec{b})^2\, \delta^3(\vec{r})\, d\tau$, where V is a cube of side 2 centred at origin and $\vec{b} = 4\hat{y} + 3\hat{z}$

(c) $\int_{V} \vec{r}\cdot(\vec{d} - \vec{r})\, \delta^3(\vec{e} - \vec{r})\, d\tau$, $\vec{d} = (1,2,3)$, $\vec{e} = (3,2,1)$ and V is sphere of radius 1.5 centred at (2, 2, 2)

(7) (a) $\vec{F}_1 = x^2\hat{z}$, $\vec{F}_2 = x\hat{x} + y\hat{y} + z\hat{z}$.

Calculate the divergence and curl of the function. Which function can be written as the gradient of a scalar? Which function can be written as curl of a vector?

(b) Show $\vec{F}_3 = yz\hat{x} + zx\hat{y} + xy\hat{z}$ can be written both as the gradient of a scalar and as the curl of a vector. Find scalar and vector potential for this function.

Practice problem (4)

(1) Find the electric field at a distance z above one end of a straight line of Length L, that carries a uniform line charge density λ.

(2) Find the electric field a distance z from the centre of a spherical surface of radius R that carries a uniform charge density σ. Treat the case $z < R$ (inside) and $z > R$ (outside).

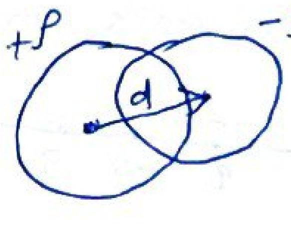
(3) Find the electric field inside a uniformly charged solid sphere (charge density ρ).

(4) A thick spherical ~~col~~ shell carries charge density

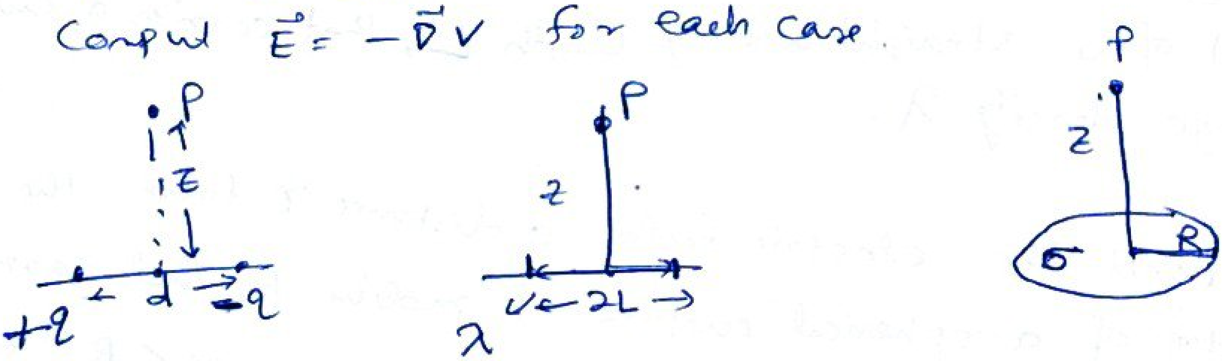
$$\rho = \frac{k}{r^2} \quad (a \le r \le b)$$

Find the electric field in the three regions (i) $r < a$ (ii) $a < r < b$ (iii) $r > b$. plot $|\vec{E}|$ as a function of r, for the case $b = 2a$

(5) Two sphere of readius R each and carrying volume charge densities $+\rho$ & $-\rho$, respectively, Placed so that they partially overlap. The vector Joint the ~~centre~~ two centres is $\vec{d}$ as show in figure. Find the value of $\vec{E}$ in the overlap region.



(6) Find the Potential at distance z above the centre of charge distribution in the configuration below. Compute $\vec{E} = -\vec{\nabla} V$ for each case.

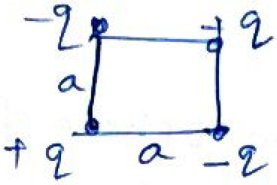


(7) Find the potential on the axis of a uniformly charged solid cylinder a distance z from the centre. The length of cylinder is L, radius is R & charge density is ρ. Calculate the $\vec{E}$ field.

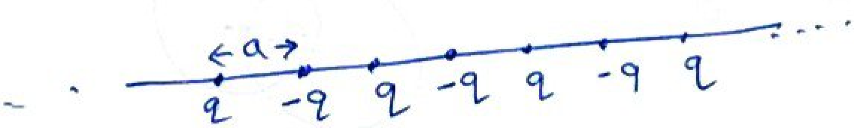
(8) Calculate ~~the~~ ~~discont~~ $\vec{\nabla} V_{above} - \vec{\nabla} V_{below}$ for a uniformly charged spherical shell of radius R.

$\vec{\nabla} V_{above}$ is $(r > R)$ $\quad \vec{\nabla} V_{below}$ is $r < R$

(9) How much work does it take to assemble a configuration of four charges on the corner of a square.

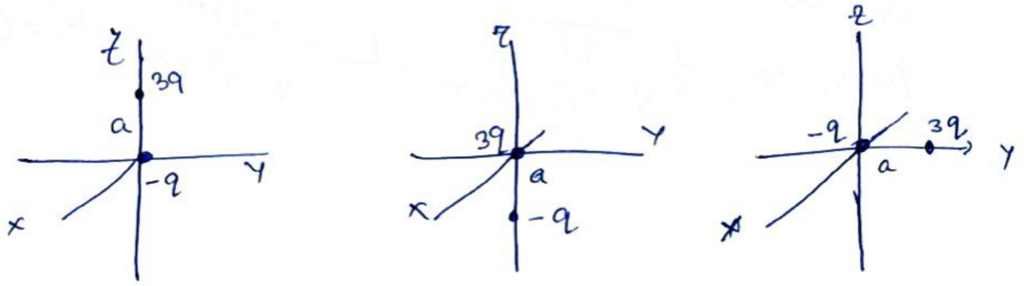


(10) Consider an infinite chain of point charges $\pm q$ fixed along X-axis each at a distance a from nearest neighbour. Find the work per particle to assemble this system.

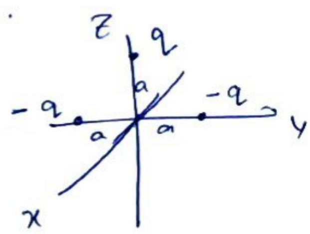


Practice Problem 5

(1) Two point charges $3q$ & $-q$ are separated by a distance a. For each of the configuration show below, find (i) the monopole moment, (ii) the dipole moment and (iii) the approximate potential (in spherical polar coordinates) at large r.



(2) Three point charges are located as show in Fig. each a distance 'a' from the origin. Find the approximate electric field and potential at point far away from origin in spherical polar coordinate. Use monopole & dipole terms.



(3) A solid sphere of radius R. is centred at origin. The northern hemisphere carries a uniform charge density ρ_0 and the southern hemisphere carries a charge density $-\rho_0$. Find the Electric field $\vec{E}(r,\theta)$ for $r >> R$. Ans: $\vec{E} = \frac{\pi \rho_0 R^4}{8\pi\epsilon_0 r^3}\left(2\cos\theta\,\hat{r} + \sin\theta\,\hat{\theta}\right)$

(4) A stationary electric dipole $\vec{p} = p\hat{z}$ is situated at the origin. A positive point charge 'q', mass 'm', executes circular motion (radius s) at constant speed in the field of the dipole. Find the speed, angular momentum & total energy of the charge in the orbit.

Ans: $v = \frac{1}{s}\sqrt{\frac{qp}{3\sqrt{3}\,\pi\epsilon_0 m}}$, $L = \sqrt{\frac{qpm}{3\sqrt{3}\,\pi\epsilon_0}}$, $W = 0$

Practicle problem 6

① In hydrogen atom, the electron cloud charge density is given by

$$\rho(\vec{r}) = \frac{q}{\pi a^3} e^{-2r/a}$$

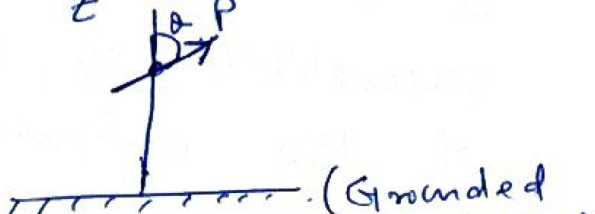
where 'a' is Bohr radius & q is charge of electron. Find the atomic polarizability of the atom.

② A point charge q is situated a large distance r from a neutral atom of polarizability α. Find the force of attraction between them.

$$\text{Ans. } F = 2\alpha\left(\frac{q}{4\pi\epsilon_0}\right)^2 \frac{1}{r^5}$$

attractive

③ Find the tourque on $\vec{p}$ for below configuration. If the dipole is free to rotate, in what orientation will it come to rest?



④ Show that the interaction energy of two dipoles separated by a displacement $\vec{r}$ is

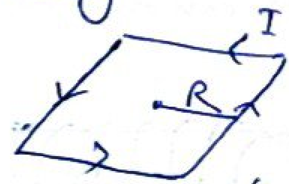
$$U = \frac{1}{4\pi\epsilon_0} \frac{1}{r^3}\left[\vec{p_1}\cdot\vec{p_2} - 3(\vec{p_1}\cdot\hat{r})(\vec{p_2}\cdot\hat{r})\right]$$

Practicle Problem 7

(1) The magnetic field in some regions has the form $\vec{B} = kz\hat{x}$, K is a constant. Find the force on the square loop of (side a), lying in the yz plane and centred at the origin. It carries current I, flowing counter clockwise, when you look down the x-axis.

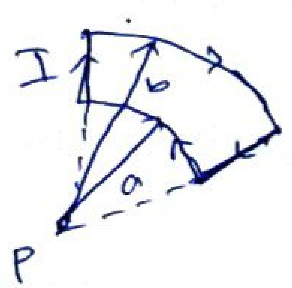
(2) A uniform charged solid sphere of radius R and total charge Q is centred at the origin & spinning at a constant angular velocity ω about the z-axis. Find the current density $\vec{J}$ at any point (r, θ, ϕ) within the sphere.

(3)(a) Find the magnetic field at the centre of a square loop, which carries a steady current I.

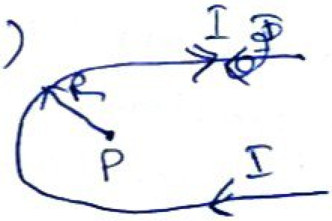


(b) Find the magnetic field at point P for each of the configuration below.

(i)

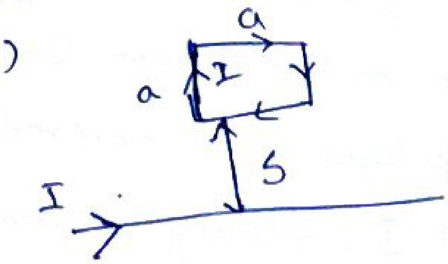


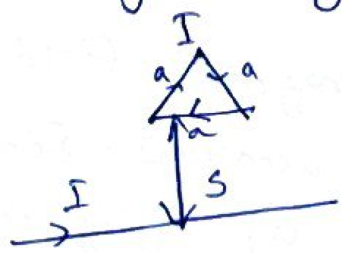
(ii)



(4) Find the force on a current loop for configuration shown below. Both the loop carry steady current I.

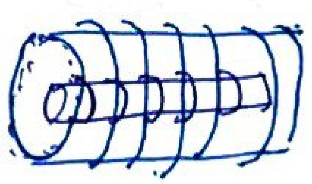
(i)





(5) Calculate the magnetic field at the centre of a uniformly charged spherical shell, of radius R and total charge Q, spinning at constant angular velocity ω.

(6) Two long coaxial solenoids each carry current I in opposite direction as shown in Fig below. The inner solenoid (radius a) has n_1 turns per unit length and the outer solenoid (radius b) has n_2. Find $\vec{B}$ in (i) inside the inner solenoid (ii) between them (iii) Outside.



(7) A large parallel plate capacitor with uniform charge 'σ' on the upper plate & $-\sigma$ on lower plate is moving with constant speed v.

(i) Find the magnetic field between the plates & also above & below the plates

(ii) ~~At~~ what speed v would the magnetic force ~~bta~~ balance the electrical force?

(8) The vector potential is $\vec{A} = k\hat{\varphi}$, k is constant. What is the current density in cylindrical coordinates?

(9) Prove that $\vec{A}(\vec{r}) = -\frac{1}{2}(\vec{r} \times \vec{B})$ represents a uniform magnetic field $\vec{B}$.

(10) Show that the magnetic field due to a dipole can be written as $\vec{B}_{dip}(\vec{r}) = \frac{\mu_0}{4\pi}\frac{1}{r^3}\left[3(\vec{m}\cdot\hat{r})\hat{r} - \vec{m}\right]$

(11) A circular loop of wire, with radius R, lies on XY plane (centred at origin) and carries current I along counterclockwise direction when viewed from the positive z-axis.

(i) What is the magnetic dipole moment?

(ii) What is the magnetic field at points far away from the origin?

(12) Calculate the magnetic force of attraction between the northern and southern hemispheres of a spinning charged spherical shell? Ans: $\frac{\pi}{4}\mu_0\sigma^2\omega^2R^4$

shells have radius R, carry surface charge density σ & 'ω' is the angular velocity.

Quiz 1: EM

Maximum Marks: 20

1. A long solenoid, of radius a, is driven by an alternating current, so that the field inside is sinusoidal: $B(t) = B_0 \cos(\omega t)\hat{z}$. A circular loop of wire, of radius $a/2$ and resistance R, is placed inside the solenoid, and coaxial with it. Find the current induced in the loop, as a function of time. [2]

2. A square loop of wire, with sides of length a, lies in the first quadrant of the xy plane, with one corner at the origin. In this region there is a nonuniform time-dependent magnetic field $B(y,t) = ky^3t^2\hat{z}$ (where k is a constant). Find the emf induced in the loop. [2]

3. At $t = 0$, a battery is connected to a series arrangement of a resistor and an inductor. At what multiple of the inductive time constant will the energy stored in the inductor's magnetic field be 0.5 its steady-state value? [5]

4. A uniform magnetic field $\vec{B}$ is perpendicular to the plane of a circular loop of diameter 10 cm formed from wire of diameter 2.5 mm and resistivity $1.69 \times 10^{-8}\,\Omega m$. At what rate must the magnitude of $\vec{B}$ change to induce a 10 A current in the loop? [4]

5. At a certain place, Earth's magnetic field has magnitude $B = 0.59$ Gauss and is inclined downward at an angle of 70.0° to the horizontal. A flat horizontal circular coil of wire with a radius of 10.0 cm has 1000 turns and a total resistance of 85.0 Ω. It is connected in series to a meter with 140 Ω resistance. The coil is flipped through a half-revolution about a diameter, so that it is again horizontal. How much charge flows through the meter during the flip? [5]

6. A circular loop of wire 50 mm in radius carries a current of 100 A. Find the (a) magnetic field strength and (b) energy density at the center of the loop. [2]

Quiz 2: EM

Maximum Marks: 20

1. A square loop of wire, of side a, lies midway between two long wires, $3a$ apart, and in the same plane. (Actually, the long wires are sides of a large rectangular loop, but the short ends are so far away that they can be neglected.) A clockwise current I in the square loop is gradually increasing: $dI/dt = k$ (a constant). Find the emf induced in the big loop. [3]

2. When you look at the North Star (Polaris), you intercept light from a star at a distance of 431 light years and emitting energy at a rate of 2.2×10^3 times that of our Sun ($P_{sun} = 3.90 \times 10^{26} W$). Neglecting any atmospheric absorption, find the *rms* values of the electric and magnetic fields when the starlight reaches you. Note that $E_{rms} = E_{average}/\sqrt{2}$. [4]

3. The maximum electric field $10m$ from an isotropic point source of light is $2.0V/m$. What are (a) the maximum value of the magnetic field and (b) the average intensity of the light there? (c) What is the power of the source? [3]

4. When current flows down a wire, work is done, which shows up as Joule heating of the wire. Compute the energy per unit time delivered to the wire using the Poynting vector. [3]

5. Calculate the (time averaged) energy density of an electromagnetic plane wave in a conducting medium. Does the electric or the magnetic contribution dominate. Justify. [3]

6. Consider a rectangular wave guide with dimensions $2.28\ cm \times 1.01\ cm$. What TE modes will propagate in this wave guide, if the driving frequency is $1.70 \times 10^{10} Hz$? Suppose you wanted to excite only one TE mode; what range of frequencies could you use? [4]

Indian Institute of Technology Jodhpur

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Electromagnetism and Optics [PHL1010]

INSTRUCTOR: B. M. KRISHNA MARISERLA

Assignment_Optics

Answer all questions.

1. Two glass plates are placed on top of one another and on one side a cardboard is introduced to form a thin wedge of air. A beam of wavelength 600 nm is incident normally, and that are 100 interference fringes per centimeter, calculate the wedge angle. [3M]
2. An equiconvex lens is placed on another equiconvex lens. The radii of curvature of the two surfaces of the upper lens are 50 cm and those of the lower lens are 100 cm. The light of wavelength 600 nm reflected from the upper and lower surface of the air film (formed between the two lenses) interfere to produce Newtons rings. Calculate the radii of the dark rings. [3M]
3. In the Michelson interferometer experiment, if one of the mirrors is moved by a distance 0.08 mm, 250 fringes cross the field of view. Calculate the wavelength. [3M]
4. A soap film floating in the air has an index of refraction 1.34. Under illumination, if a region of the film strongly reflects a wavelength of 804 nm, what is the minimum thickness of the film? [3M]
5. Light of wavelength 500 nm enters a human eye. Although pupil diameter varies from person to person, let's estimate a daytime diameter of 2 mm. a) Estimate the limiting angle of resolution for this eye, assuming its resolution is limited only by diffraction. b) Determine the minimum separation distance d between two-point sources that the eye can distinguish if the point sources are a distance of 25 cm from the observer [4M]
6. A helium–neon laser emits light that has a wavelength of 632.8 nm. The circular aperture through which the beam emerges has a diameter of 0.500 cm. Estimate the diameter of the beam 10.0 km from the laser. [3M]
7. A diffraction grating has 4 200 rulings/cm. On a screen 2.00 m from the grating, it is found that for a particular order m, the maxima corresponding to two closely spaced wavelengths of sodium (589.0 nm and 589.6 nm) are separated by 1.54 mm. Determine the value of m. [3M]
8. Plane-polarized light is incident on a single polarizing disk with the direction of $\vec{E}_0$ parallel to the direction of the transmission axis. Through what angle should the disk be rotated so that the intensity in the transmitted beam is reduced by a factor of (a) 3 and (b) 10? [3M]

<u>Submission should be on or before 26th Nov 2023, 11:59 pm.</u>