

1. Threshold frequency for electrons to be emitted from Na metal is 665×10^{12} Hz. Calculate the kinetic energy of electrons (in J) emitted from Na metal when radiation of wavelength 200 nm falls on it. (3 points)
2. Show that the wavefunctions $\psi_1(x)$ and $\psi_2(x)$ of particle in a 1D box problem are orthogonal. Particle in a 1D box wavefunctions are given by $\psi_n(x) = (\sqrt{2/l}) \sin(n\pi x/l)$. (3 points)
3. Consider an electron moving in a 1D box bound by infinite potential energy walls. When the electron jumps from $n = 5$ level to $n = 2$ level, it emits a photon of frequency $6 \times 10^{14} \text{ s}^{-1}$. Find the length of the box. (3 points)
4. Vibrational motion of a diatomic molecule such as HCl can be modeled using quantum harmonic oscillator. Suppose HCl molecule shows a vibrational frequency of 54.322×10^{13} Hz when it jumps from $v = 1$ to $v = 0$ level, calculate its force constant. Also calculate the zero point energy of HCl molecule. (mass of H atom, $m_H = 1.674 \times 10^{-27}$ kg, mass of Cl atom, $m_{Cl} = 5.887 \times 10^{-26}$ kg) (4 points)
5. Show that the particle in a ring wavefunctions ($\psi_{m_l}(\phi) = \frac{e^{im_l\phi}}{\sqrt{2\pi}}$) are normalized. (2 points)
6. Identify whether the following ions are Hydrogenic or not. (a) Li^+ (b) Li^{2+} (c) Be^{3+} (d) C^{5+} (2 points)

Useful Information

Planck's constant, $h = 6.626 \times 10^{-34} \text{ J.s}$

Speed of light, $c = 3 \times 10^8 \text{ m.s}^{-1}$

Mass of electron, $m_e = 9.109 \times 10^{-31} \text{ kg}$

Mass of proton, $m_p = 1.673 \times 10^{-27} \text{ kg}$

Rydberg constant, $R_H = 109677.6 \text{ cm}^{-1}$

1 eV = $1.602 \times 10^{-19} \text{ J}$

①

$$h\nu = h\nu_0 + KE \quad (1 \text{ mark for formula})$$

3 points

$$KE = \frac{hc}{\lambda} - h\nu_0$$

$$= \frac{6.626 \times 10^{-34} \text{ Js} \times 3 \times 10^8 \text{ ms}^{-1}}{200 \times 10^{-9} \text{ m}} - 6.626 \times 10^{-34} \text{ Js} \times 665 \times 10^{12} \text{ s}^{-1}$$

(1 mark for correct substitution)

$$KE = 5.533 \times 10^{-19} \text{ J} \quad (1 \text{ mark for final answer, less } \frac{1}{2} \text{ mark if unit is not written})$$

3 points

②

$$\psi_1(x) = \sqrt{\frac{2}{l}} \sin\left(\frac{\pi x}{l}\right)$$

$$\psi_2(x) = \sqrt{\frac{2}{l}} \sin\left(\frac{2\pi x}{l}\right)$$

orthogonality

$$\int_0^l \psi_1^*(x) \psi_2(x) dx = 0 \quad (1 \text{ mark})$$

$$\frac{2}{l} \int_0^l \sin\left(\frac{\pi x}{l}\right) \sin\left(\frac{2\pi x}{l}\right) dx = \frac{1}{l} \int_0^l \left[\cos\left(\frac{\pi x}{l}\right) - \cos\left(\frac{3\pi x}{l}\right) \right] dx \quad (1 \text{ mark})$$

$[2 \sin A \sin B = \cos(A-B) - \cos(A+B)]$

$$= \frac{1}{l} \left[\left(\frac{l}{\pi} \right) \sin\left(\frac{\pi x}{l}\right) - \left(\frac{l}{3\pi} \right) \sin\left(\frac{3\pi x}{l}\right) \right]_0^l$$

$$= 0 \quad (1 \text{ mark})$$

③

$$E_n = \frac{n^2 h^2}{8ml^2}$$

3 points

$$\Delta E = h\nu = E_5 - E_2 = \frac{25h^2}{8ml^2} - \frac{4h^2}{8ml^2} = \frac{21h^2}{8ml^2} \quad (1 \text{ mark})$$

$$\Rightarrow \lambda^2 = \frac{21h}{8m\nu}$$

$$= \frac{21 \times 6.626 \times 10^{-34} \text{ Js}}{8 \times 9.109 \times 10^{-31} \text{ kg} \times 6 \times 10^{14} \text{ s}^{-1}}$$

$$\lambda^2 = 3.18 \times 10^{-18} \text{ m}^2$$

$$\lambda = 1.78 \text{ nm} \quad (2 \text{ marks, less 1 mark for wrong unit})$$

(4)

$$\omega = \sqrt{\frac{k}{\mu}}$$

$$\mu = \frac{m_H m_{Li}}{m_H + m_{Li}}$$

$$= \frac{1.674 \times 10^{-27} \times 58.87 \times 10^{-27}}{60.544 \times 10^{-27}}$$

$$\mu = 1.628 \times 10^{-27} \text{ kg (1 mark)}$$

$$k = \mu \omega^2 = 1.628 \times 10^{-27} \text{ kg} \times 54.322^2 \times 10^{26} \text{ s}^{-2}$$

$$k = 480 \text{ kg s}^{-2} \text{ (or } \text{Nm}^{-1} \text{ (2 marks, less 1 mark for wrong unit))}$$

$$\text{ZPE} = \frac{1}{2} \hbar \omega = \frac{\hbar \omega}{4\pi}$$

$$= \frac{6.626 \times 10^{-34} \text{ Js} \times 54.322 \times 10^{13} \text{ s}^{-1}}{4 \times 3.14}$$

$$\text{ZPE} = 28.7 \times 10^{-21} \text{ J (1 mark, less } \frac{1}{2} \text{ mark for wrong unit)}$$

(5)

Normalization $\int_{-\infty}^{+\infty} \psi^*(x) \psi(x) dx = 1$

$$\int_0^{2\pi} \frac{1}{\sqrt{2\pi}} e^{-im\phi} \frac{1}{\sqrt{2\pi}} e^{im\phi} d\phi = \frac{1}{2\pi} \int_0^{2\pi} d\phi$$

(1 mark)

$$= \frac{1}{2\pi} [\phi]_0^{2\pi}$$

$$= \frac{1}{2\pi} [2\pi - 0]$$

$$= \underline{\underline{1}} \text{ (1 mark)}$$

2 points

(6)

a) NO ($\frac{1}{2}$)b) YES ($\frac{1}{2}$)c) YES ($\frac{1}{2}$)d) YES ($\frac{1}{2}$)

2 Points

11.1
a
7

TAU E
 $\nu_H B_1 t_p = \theta$ (0.5 mark)

$\Rightarrow \nu_H B_1 t_p = \frac{\pi}{2}$ (0.5 mark)

$\frac{\nu_H B_1}{2\pi} t_p = \frac{1}{4}$

$\frac{\nu_H B_1}{2\pi} = \frac{1}{4 t_p} = 10 \text{ kHz}$

$\Rightarrow t_p = \frac{1}{4 \times 10^4 \text{ Hz}} = 25 \times 10^{-6} \text{ s} = \underline{25 \mu\text{s}}$ (0.5 mark)

$KE = \frac{1}{2} m_e v^2$

1.5 points

- final ans = 1 mark

- no unit cost 1/2 mark

b

TAU E

$s = \frac{1}{2}$

$\left. \begin{aligned} \vec{\mu}_s &= \frac{-e\hbar}{2m_e} \vec{s} \\ \mu_{sz} &= \frac{-e\hbar}{2m_e} s_z \end{aligned} \right\}$ (0.5 mark)

1.5 points

$\left. \begin{aligned} |\mu_{sz}| &= \frac{-e\hbar}{2m_e} s_z = \frac{-e\hbar}{2m_e} \frac{1}{2} \\ |\vec{\mu}_s| &= \frac{-e\hbar}{2m_e} \sqrt{s(s+1)} = \frac{-e\hbar}{2m_e} \frac{\sqrt{3}}{2} \end{aligned} \right\}$ (0.5 mark)

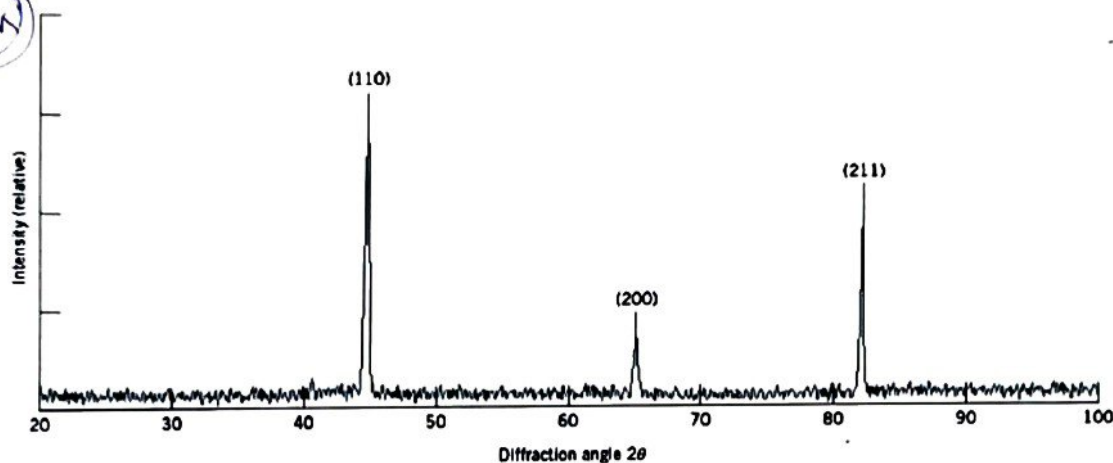
$\Rightarrow \frac{|\mu_{sz}|}{|\vec{\mu}_s|} = \frac{1}{\sqrt{3}}$ (0.5 mark)

All questions are compulsory.

Q.1. Describe the process of adsorption from thermodynamic point of view. Describe all the parameters that influence the process of adsorption [2+3=5 marks]

Q.2. Explain surface tension. Describe capillary rise method for the determination of surface tension. A capillary tube of internal diameter 0.21 mm is dipped into a liquid whose density is 0.79 g cm^{-3} . The liquid rises in this capillary to a height of 6.30 cm. Calculate the surface tension of the liquid. ($g = 980 \text{ cm sec}^{-2}$). [1+2+2=5 Marks]

Q.3. Write the information we get from XRD pattern. How can you calculate crystallite size from peak width of a XRD pattern. Below is the typical diffraction pattern for polycrystalline α -iron (BCC)



Calculate the d-spacing for (110) and (200) planes (given 2θ values for (110) and (200) are 45, 65 respectively; $\lambda = 1.5406 \text{ \AA}$). [2+2+2 = 6 Marks]

Q.4. Why is the edge centered cubic cell not considered a Bravais lattice? Why are there only 14 Bravais lattices. [2+2 = 4 Marks]

Q.5. Explain how Bloch theorem and Kronig-Penney Model help to understand the formation of electronic band structure in metals and semiconductors. [3+3=6 marks]

Q.6. Write short notes on surface plasmon and quantum confinement. [2+2=4 Marks]

$$h = 2\pi$$

$$\text{sgn}(\vec{r}) = (2\pi r)(r \cos \theta)$$