Minor Exam-I

Indian Institute of Technology Jodhpur
Mathematics-II: MAL1020
(February 08, 2024)

Duration: 50 minutes **Maximum Marks: 15**

$(3 \times 5 = 15)$

Answer all questions:

1. (a) Check and justify whether U is a subspace of the vector space V over $\mathbb{R}$. U and V are as follows:
   (i) $U = \{(a,b) \in \mathbb{R}^2 : 2a + 3b = 0\}, V = \mathbb{R}^2$
   (ii) $U = \mathbb{Q}, V = \mathbb{R}$
   (iii) $U = \{a + 0i : a \in \mathbb{R}\}, V = \mathbb{C}$

   (b) Prove that, $P_2(\mathbb{R})$ is a subspace of $P_3(\mathbb{R})$ over the real field. [3 + 2]

2. (a) Check and justify whether the subset S is linearly independent or dependent in V. S and V are as follows:
   (i) $S = \{(1,3,2),(1,2,3),(2,4,6)\}, V = \mathbb{R}^3$
   (ii) $S = \{1, i\}, V = \mathbb{C}$
   (iii) $S = \{1, 1+t, 1+t^2\}, V = P_2(\mathbb{R})$

   (b) Find a basis and the dimension of the subspace W of $\mathbb{R}^3$, where
   $$W = \{(x,y,z) \in \mathbb{R}^3 : x + y + z = 0\}.$$ [3 + 2]

3. (a) The maps $f,\ g : \mathbb{R} \to \mathbb{R}$ defined by $f(x) = x^3,\ g(x) = \cos(x)$. Check and justify whether these are linear transformations or not.

   (b) Let $T_1,\ T_2 : V \to W$ be linear transformations. Let a and b be scalars. Define $T : V \to W$ by $T(x) = a\,T_1(x) + b\,T_2(x)$ for $x \in V$. Check and Justify whether T is a linear transformation or not.

   (c) The matrix of a linear mapping $T : \mathbb{R}^3 \to \mathbb{R}^2$ relative to the order bases $\{(0,1,1),(1,0,1),(1,1,0)\}$ of $\mathbb{R}^3$ and $\{(1,0),(1,1)\}$ of $\mathbb{R}^2$ is $\begin{pmatrix} 1 & 2 & 4 \\ 2 & 1 & 0 \end{pmatrix}$. Find the mapping T. Also, find the matrix of T relative to the ordered bases $\{(1,1,0),(1,0,1),(0,1,1)\}$ of $\mathbb{R}^3$ and $\{(1,1),(0,1)\}$ of $\mathbb{R}^2$. [1 + 1 + 3]

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