Major Examination

Electromagnetism and Optics: PHL1010

Time: 2 hours **Points: 50**

Answer all the questions.

1. Two parallel narrow horizontal slits in an opaque vertical screen are separated center-to-center by 2.644 mm. These are directly illuminated by yellow plane waves from a filtered discharge lamp. Horizontal fringes are formed on a vertical viewing screen 4.500 m from the aperture plane. The center of the fifth bright band is 5.000 mm above the center of the zeroth or central bright band. [3+2]

   (a) Determine the wavelength of the light in air.

   (b) If the entire space is filled with clear soybean oil ($n = 1.4729$), where would the fifth fringe now appear?

$r_m = \frac{(2m+1)\lambda}{2}$ Newton's Rings $R_{10} = ?$

2. A convex lens rests on an optical flat in a dust-free setup in the air. It is illuminated by green light from a mercury discharge at 546.07 nm. If the radius of curvature of the lens is 20.0 cm, how far from its center will we find the 10th bright fringe? $R = 2f$ [5]

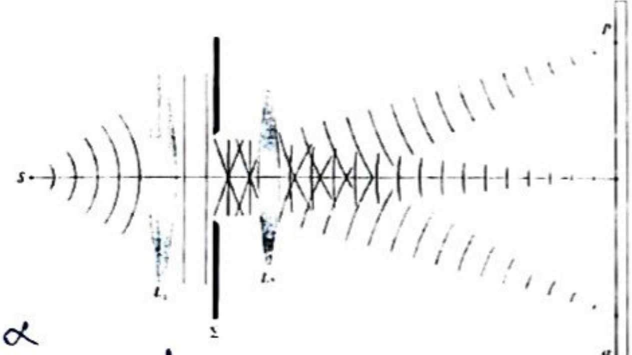
$a = b$

$a + b = D$

3. Fresnel biprism is used to obtain fringes from a point source that is placed 1.5 m from the screen, and the prism is midway between the source and the screen. Let the wavelength of the light be 500 nm and the index of refraction of the glass be $n = 1.5$. What is the prism angle if the separation of the fringes is 0.5 mm? [5]

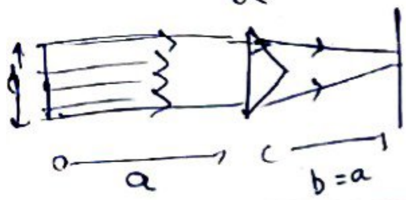
4. Consider the arrangement of the given figure, where a large lens L_2 is close to the long narrow (0.250 mm) slit in the aperture screen. The illumination is green magnesium light at 518.36 nm. Determine the width of the central maximum formed by L_2, which has a focal length of 65.0 cm, on the viewing screen. [5]

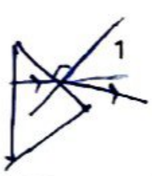
$W = \frac{0.66 f\lambda_0}{d}$



$I = I_0 \frac{\sin^2\beta}{\beta^2}$ $\beta = \frac{\pi b \sin\theta}{\lambda}$

α



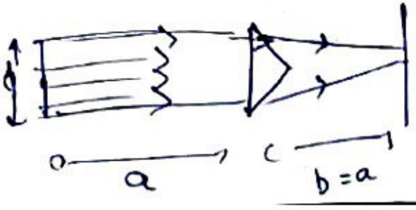


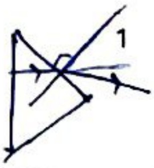
$\beta = \frac{\lambda D}{d}$ $A = 2a\alpha(n-1)$ $10^{-3} \times 1.5 = 0.75 \cdot \alpha$ $\alpha = 2 \times 10^{-3}$ rad

$\beta = \frac{\pi b \sin\theta}{\lambda}$

$= \frac{5\pi b \sin\theta}{\lambda}$ N-slit Fraunhoffer $I = I_0 \frac{\sin^2\beta}{\beta} \cdot \frac{\sin^2(N\gamma)}{\sin^2(\gamma)}$

b





$\beta = \frac{\lambda D}{d}$ $\quad \Delta = 2a\alpha$

$10^{-3} \times 1.5 = 0.75 \cdot \alpha$

$\alpha = 2\times 10^{-3}$ rad

$\beta = \frac{\pi b \sin\theta}{\lambda}$

$\gamma = \frac{5\pi b \sin\theta}{\lambda}$ N-slit Fraunhoffer $\quad I = I_0 \frac{\sin^2\beta}{\beta} \cdot \frac{\sin^2(N\gamma)}{\sin^2(\gamma)}$

5. Imagine 12 narrow, parallel, long slits, each b millimeters wide, each separated from the next slit by a center-to-center distance of $5b$. The apertures are illuminated normally by plane waves and produce a Fraunhofer diffraction pattern on a distant screen. Determine the relative irradiance of the first-order principal maximum compared to the zeroth-order principal maximum. [5]

6.483

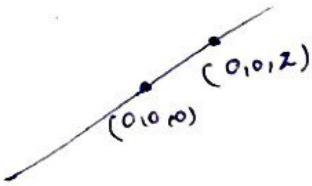
6. A circular hole in an opaque screen has a diameter of 4.98 mm. It is illuminated perpendicularly by light from a helium-neon laser of wavelength 543 nm and forms a Fraunhofer diffraction pattern on a distant screen. Determine the angular width, $2\Delta\theta$, of the Airy disk. How big would it be if the hole was made 10 times smaller? [3+2]

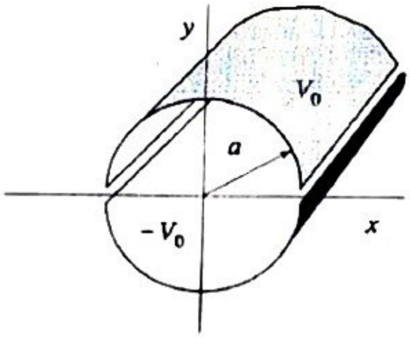
$\delta\theta = 1.22\frac{\lambda}{d}$

$2\Delta\theta = 1.22\frac{\lambda}{d}$

7. Consider an infinitely long conducting cylinder of radius "a" with its axis coinciding with the z-axis. One half of the cylinder (cut the long way) (y > 0) is kept at a constant potential V_0, while the other half (y < 0) is kept at a constant potential $-V_0$, (as shown in Figure below). Find out the potential for all points inside the cylinder and the electric field E along the z-axis. [5]

E =





$\left(\frac{2N+1}{2}\right)\pi$

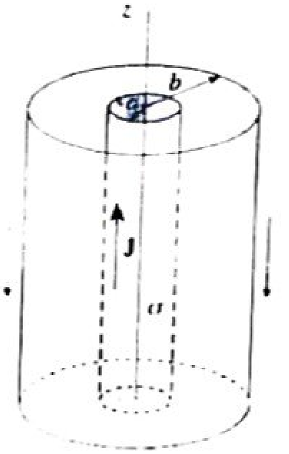
$\frac{5\pi}{2}$

8. A very long coaxial cable consists of an inner cylinder of radius "a" and isotropic conductivity σ, ad a concentric outer cylinder of radius b. The outer shell has infinite conductivity. The space between the cylinders is empty. A uniform, constant current density J, directed along the axial coordinate z, is maintained in the inner cylinder. Return current flows uniformly in the outer shell, as shown in figure below.

4

$12 \cdot (5\beta) = \frac{3\pi}{2}$

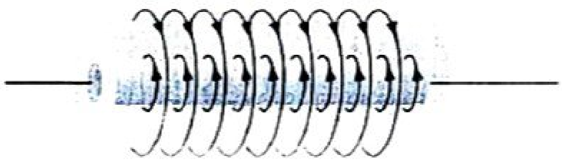
$\beta = \frac{\pi}{10}$



Find out the surface charge density on the inner cylinder as a function of the axial coordinate z, with the origin z = 0 chosen to be the plane halfway between the two ends of the cable. [5]

9. Two long coaxial solenoids each carry current I, but in opposite directions, as shown below:

$\boxed{B_{solenoid} = \mu_0 n I}$



The inner solenoid (radius a) has n_1 turns per unit length, and the outer one (radius b) has n_2. Find **B** in each of the three regions: [2+2+1]

(i) inside the inner solenoid,

(ii) between them, and

(iii) outside both.

10. Suppose a magnetic monopole q_m passes through a resistanceless loop of wire with self-inductance L. What current is induced in the loop? [5]

$\frac{\mu_0 q_m}{4L}$

q

$E = \frac{kq}{r^2}$

$\mathcal{E} = -L\frac{di}{dt}$

$\boxed{i = \frac{dq}{dt}}$

L

$\phi = LI$

R

$B = \frac{\mu_0}{4\pi} \cdot \frac{q_m}{r^2}$

$I = \frac{\Phi}{L} = \frac{B \cdot A}{L}$

$= \frac{\frac{\mu_0}{4\pi} \cdot \frac{q_m}{R^2} \cdot (\pi R^2)}{L}$

Major Examination

PHL1010: **Electromagnetism & Optics**

Time: 2 Hours | Date: 02.12.2023 | Maximum Marks: 50

1. Explain Fraunhofer diffraction by N-slits and find the intensity at an arbitrary point P. **[4]**

2. Consider a set of two slits each of width b = 0.05 cm and separated by a distance d = 0.1 cm, illuminated by a monochromatic light of wavelength 632.8 nm. If a convex lens of focal length 10 cm is placed beyond the double slit arrangement, calculate the positions of the minima inside the first diffraction minimum. **[3]**

3. A circular aperture of radius 0.4 mm illuminated normally by a plane wave of wavelength 500 nm. Find the position of the brightest and darkest point along the axis. **[3]**

4. Find the polarizing angle (Brewster's angle) for the air-glass interface, $n_1=1$ and $n_2=1.5$ and also for the air-water interface, $n_1=1$ and $n_2=1.33$. **[4]**

5. Discuss the state of polarization when the x and y components of the electric field are given by the following equations: **[5]**

a) $E_x = E_0 \sin(\omega t + kz)$, $E_y = E_0 \sin(\omega t + kz)$;

b) $E_x = E_0 \cos(\omega t + kz)$, $E_y = \frac{-1}{\sqrt{2}} E_0 \cos(\omega t + kz + \pi)$;

6. Draw the optical diagrams of polarization, phase contrast and confocal microscopy with proper labeling. **[6]**

7. (a). High-power lasers are used to compress a plasma (a gas of charged particles) by radiation pressure. A laser generating radiation pulses with peak power 1.5×10^3 MW is focused onto 1.0 mm^2 of high-electron-density plasma. Find the pressure exerted on the plasma if the plasma reflects all the light beams directly back along their paths. **[2]**

(b). Radiation from the Sun reaching Earth (just outside the atmosphere) has an intensity of 1.4 kW/m^2. (a) Assuming that Earth (and its atmosphere) behaves like a flat disk perpendicular to the Sun's rays and that all the incident energy is absorbed, calculate the force on Earth due to radiation pressure. (b) For comparison, calculate the force due to the Sun's gravitational attraction. Given radius of earth is 6.37×10^6 m, gravitational constant is 6.67×10^{-11} Nm^2/kg^2, mass of sun is 2×10^{30} kg, mass of earth is 5.98×10^{24} kg, distance between earth and sun is 1.5×10^{11} m. **[3]**

8. In the Figure 1, a 12 V ideal battery, a 20 Ω resistor, and an inductor are connected by a switch at time $t = 0$. At what rate is the battery transferring energy to the inductor's field at $t = 1.61\,\tau_L$? [5]

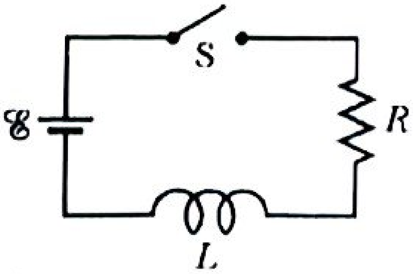


Fig 1

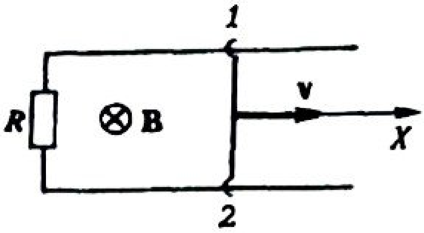
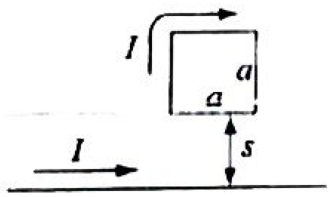


Fig 2

9. A jumper 1-2 of mass m slides without friction along two long conducting rails separated by a distance l, see Figure 2. The system, is in a uniform magnetic field perpendicular to the plane of the circuit. The left ends of the rails are shunted through resistor R. At the instant $t = 0$, the jumper received the initial velocity v_0 directed to the right. Find the velocity of the jumper as a function of time, ignoring the resistances of the jumper and of the rails as well as self-inductance of the circuit. [5]

10. (a) Find the force on a square loop placed as shown in figure below near an infinite straight wire. Both the wire and the loop carry steady current I. [3]



(b) Prove that the vector potential $\vec{A}(\vec{r}) = -\frac{1}{2}\left(\vec{r} \times \vec{B}\right)$ represents a uniform magnetic field $\vec{B}$.

[2]

11. (a) The electric potential of a charge configuration is given by $V(\vec{r}) = A\frac{e^{-\lambda r}}{r}$ where λ and A are constants. Calculate the (a) electric field and (b) Charge density of the configuration. [2]

(b) A sphere of linear dielectric material has embedded in it a uniform free charge density ρ. Find the potential at the centre of the sphere (relative to infinity) if its radius is R and the dielectric constant is ε_r. [3]

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