Minor Exam-II

Indian Institute of Technology Jodhpur
Mathematics-II: MAL1020
(March 21, 2024)

Duration: 60 minutes   **Maximum Marks: 15**

$(3 \times 5 = 15)$

Answer all questions with proper justifications:

1. (a) Find the solution of the following system of equations

$$2x_2 + 4x_3 = 2$$
$$2x_1 + 4x_2 + 2x_3 = 3$$
$$3x_1 + 3x_2 + x_3 = 1$$

(b) Let the transformation $T : \mathbb{R}^3 \to \mathbb{R}^3$ is defined as

$$T(a_1, a_2, a_3) = (a_1 + a_2 + a_3, a_1 - a_2 + a_3, a_1 + a_3)$$

Show that $(3, 3, 2)$ doesn't belong to the range of T.

$[3 + 2]$

2. (a) Find all the scalars $\lambda \in \mathbb{R}$ and non-zero polynomials $p(x) \in P_2(\mathbb{R})$ such that

$$(\lambda - 1)p(x) = (x + 1)p'(x),$$

where $p'(x)$ denotes the first derivative of $p(x)$.

(b) Let $T : P_2(\mathbb{R}) \to P_2(\mathbb{R})$ be defined as

$$T(f(x)) = f(x) + f'(x) + f''(x),$$

where $f'(x)$ and $f''(x)$ denote the first and second derivatives of $f(x)$. Find $T^{-1}(a_0 + a_1 x + a_2 x^2)$.

$[3 + 2]$

3. (a) Find the orthogonal matrix P such that $P^{-1}AP$ is a diagonal matrix where

$$A = \begin{pmatrix} 2 & 2 & -2 \\ 2 & 5 & -4 \\ -2 & -4 & 5 \end{pmatrix}$$

(b) Solve the following differential equation

$$(\cos y + y \cos x)dx + (\sin x - x \sin y)dy = 0$$

$[3 + 2]$

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