Minor-2

Engineering Mechanics MEL1010

16th October 2023

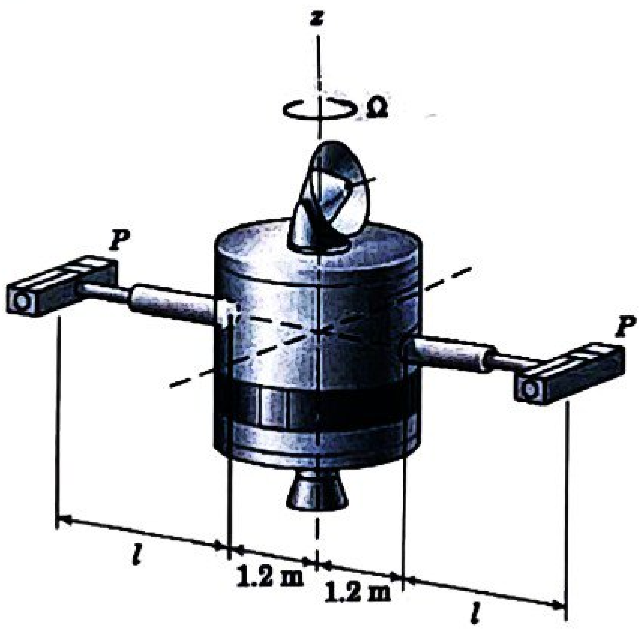
Total Marks: 5+5+8+8+2=28

Total Questions: 5

Total Time: 1 hour (11:30 a.m. - 12:30 p.m.)

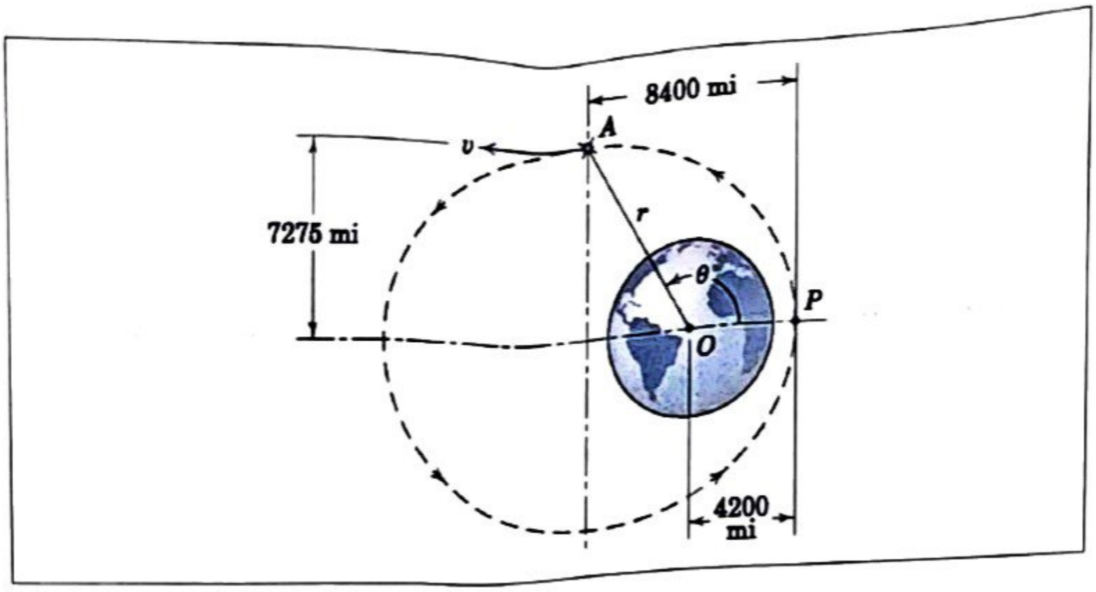
Choice in Question 1 as well as in Question 2

1. An internal mechanism is used to maintain a constant angular rate $\Omega = 0.05\ \text{rad/s}$ about the z-axis of the spacecraft as the telescopic booms are extended at a constant rate. The length l is varied from essentially zero to 3 m. The maximum acceleration to which the sensitive experiment modules P may be subjected is $0.011\ \text{m/s}^2$. Determine the maximum allowable boom extension rate $\dot{l}$.



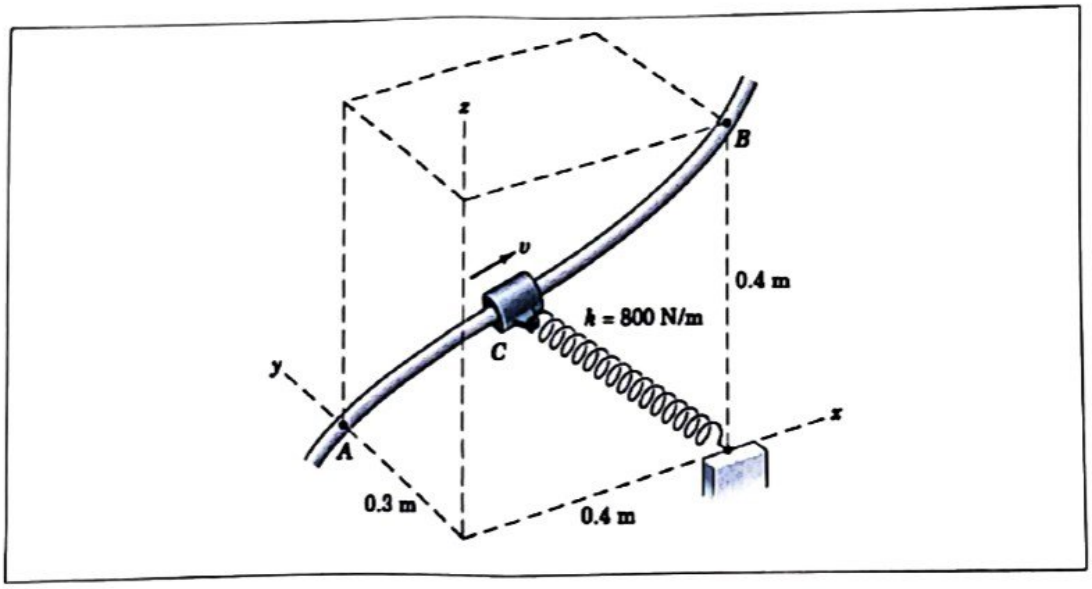
OR

1. An earth satellite travelling in the elliptical orbit shown has a velocity $v = 12{,}149$ mi/hr as it passes the end of the semiminor axis at A. The acceleration of the satellite at A is due to gravitational attraction and is $32.23[3959/8400]^2 = 7.159$ ft/sec^2 directed from A to O. For position A calculate the values of $\dot{r}, \ddot{r}, \dot{\theta}, and\ \ddot{\theta}$.



Marks: 5

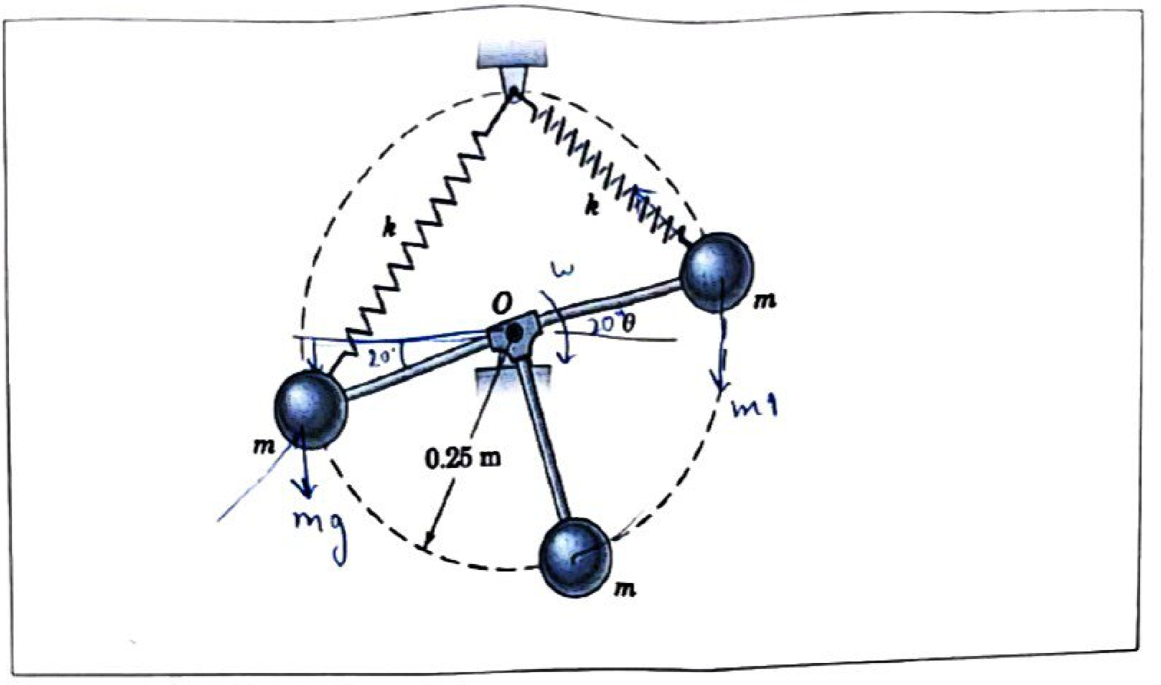
2. The 1.5-kg slider C moves along the fixed rod under the action of the spring whose unstretched length is 0.3 m. If the velocity of the slider is 2 m/s at point A and 3 m/s at point B, calculate the work U_f done by friction between these two points. Also, determine the average friction force acting on the slider between A and B if the length of the path is 0.70 m. The x-y plane is horizontal.



OR

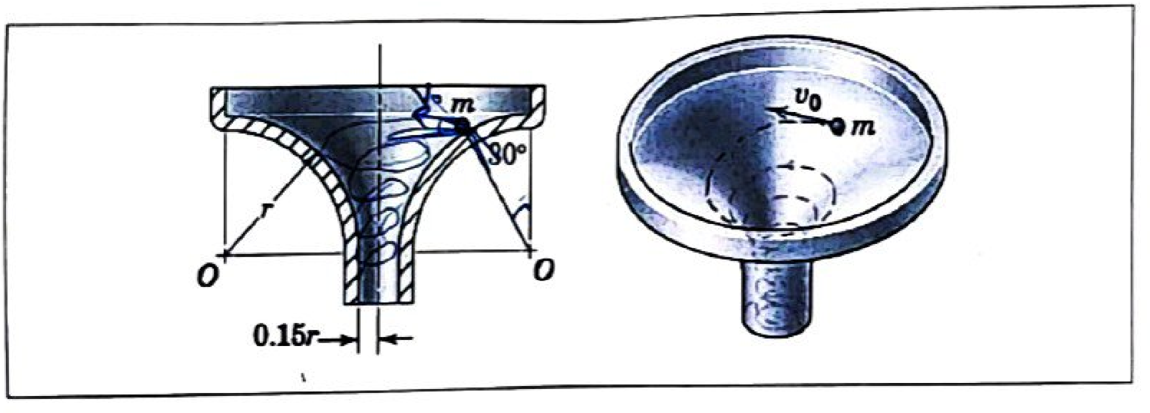
2. The two springs, each of stiffness k= 1.2 kN/m, are of equal length and undeformed when $\theta = 0$. If the mechanism is released from rest in the position $\theta = 20°$, determine its angular

velocity $\dot{\theta}$ when $\theta = 0$. The mass m of each sphere is 3 kg. Treat the spheres as particles and neglect the masses of the light rods and springs.



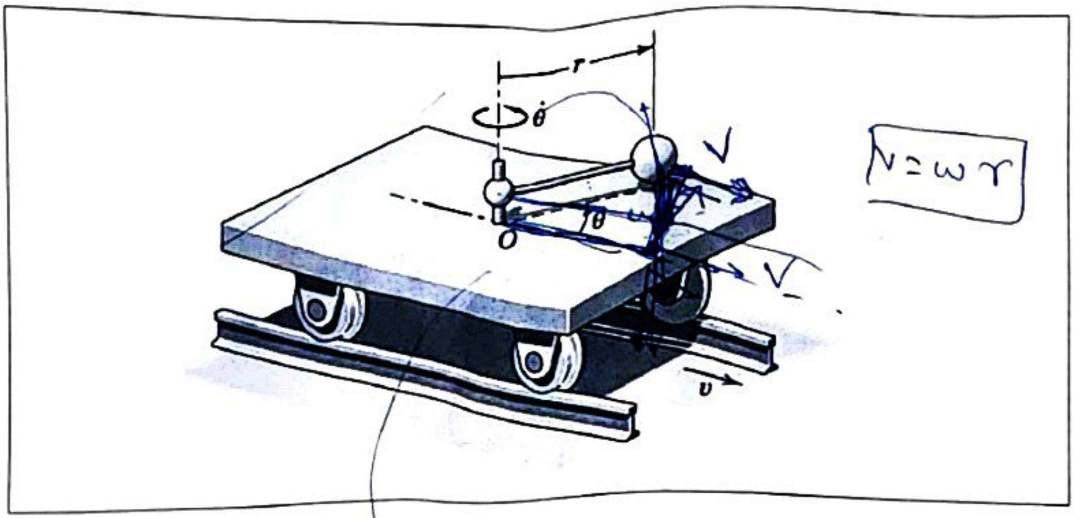
Marks: 5

3. A particle is launched with a horizontal velocity $v_0 = 0.55$ m/s from the 30° position shown and then slides without friction along the funnel-like surface. Determine the angle θ which its velocity vector makes with the horizontal as the particle passes level O-O. The value of r is 0.9 m.



Marks: 8

4. The small car, which has a mass of 20 kg, rolls freely on the horizontal track and carries the 5-kg sphere mounted on the light rotating rod with $r = 0.4$ m. A geared motor drive maintains a constant angular speed $\dot{\theta} = 4$ rad/s of the rod. If the car has a velocity $v = 0.6$ m/s when $\theta = 0$, calculate v when $\theta = 60°$. Neglect the mass of wheels and any friction.

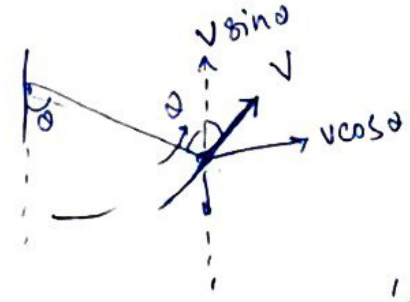


Marks: 8

5. What is coefficient of restitution?

Marks: 2

ω = 4 rad/s



$v - v' \sin\theta$

$v = \omega r$

$\boxed{v - \omega r \sin\theta}$

$v - \frac{\sqrt{3}}{2} \times 0.6 \times 4$ $\frac{1}{2}$

Minor-2 Solutions

1) a) **At point P**

5 marks

$$\dot{\theta} = 0.05 \text{ rad/s}$$

$$\ddot{\theta} = 0$$

Radial distance from the origin is given by,

$$r = l + 1.2$$

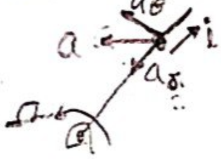
Differentiating w.r.t to "t"

$$\dot{r} = \dot{l}$$

Differentiating again, we will have,

$$\ddot{r} = \ddot{l}$$

As we know, Boom is extended with a constant rate, hence accl^n is zero.



$$\ddot{r} = 0$$

Now, Acceleration of point P is given by :-

$$a = \sqrt{(\underbrace{\ddot{r} - r\dot{\theta}^2}_{a_r})^2 + (\underbrace{r\ddot{\theta} + 2\dot{r}\dot{\theta}}_{a_\theta})^2} \quad \text{—}$$

where, $\dot{r} = \dot{l}$, $\dot{\theta} = 0.05$ rad/s, $\ddot{\theta} = 0$, $\ddot{r} = 0$

(2)

After Putting the value,

$$a = \sqrt{(0 - r \times 0.05^2)^2 + ((l+1.2) \times 0 + 2 \times \dot{l} \times 0.05)^2}$$

$$a^2 = (r \times 0.05^2)^2 + 0.01\,\dot{l}^2$$

(1)

$\boxed{l_{eff} = l + 1.2}$

Now, for maximum acceleration,

$$a_{max}^2 = \left[r_{max} \times 0.05^2\right] + 0.01\,\dot{l}_{max}^2$$

where, $r_{max} = l_{max} + 1.2$

$a_{max} = 0.11\ m/s^2$

$l_{max} = 3m$

After putting the values,

$$0.011^2 = \left[(3+1.2) \times 0.05^2\right]^2 + 0.01\dot{l}^2$$

$$\dot{l}_{max} = 0.0328\ m/s = 32.8\ mm/s$$

Hence the maximum boom extension rate is,

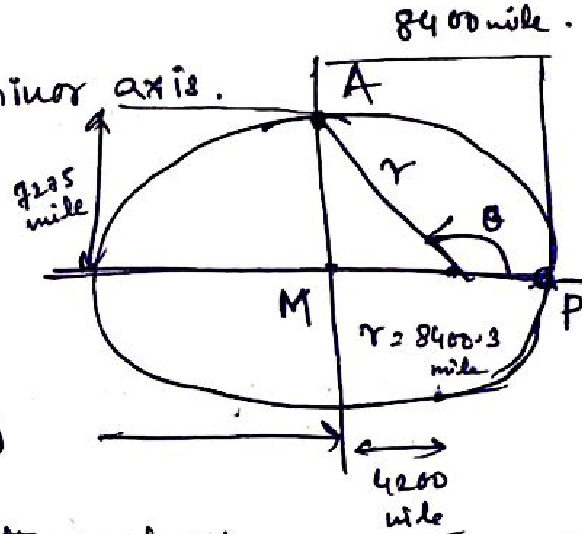
$$\boxed{\dot{l}_{max} = 32.8\ mm/s}$$

2

1(b) <u>5 Marks</u>

1 mi = 5280 mile

Let a & b be the semi-major & semi minor axis.



Given

$a = 8400$ mile $= 8400 \times 5280$ ft

$b = 7275$ mil $= 7275 \times 5280$ ft

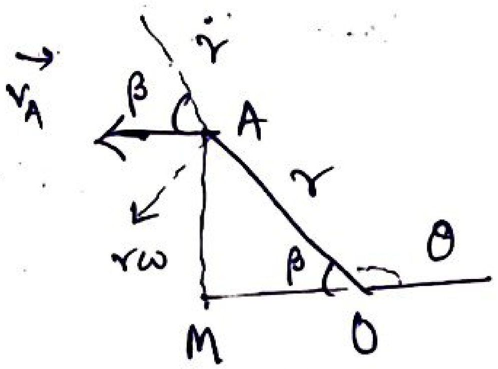
$R_e = 3959$ mile. (Radius of Earth)

$g_e = \cancel{32.5}\ 32.23$ ft/s² (Accelⁿ of Earth surface)

$$g_A = \frac{g_e r_e^2}{r^2} = 32.23 \left(\frac{3959}{r^2}\right)^2 = 32.23 \left(\frac{3959}{8400.3}\right)^2$$

$$= 7.1588 \text{ ft/s}^2$$

$$\boxed{g_A = 7.159 \text{ ft/s}^2}$$



$\vec{R}_A = r\hat{e}_r$

$v_A = \dot{r}\hat{e}_r + r\omega\hat{\theta}$

$r\dot{\theta}\hat{e}_\theta$.

$\dot{r} = v\cos\beta$

$$= 12149 \times \frac{4200}{\sqrt{4200^2 + 7275^2}} = 6.0743 \times 10^3 \text{ mile/hr}$$

$$= 6.0743 \times \frac{5280}{3600} \times 10^3$$

$$= 8909 \text{ ft/sec}$$

(a) $\boxed{\dot{r} = 8909 \text{ ft/sec}}$ — 1

$$\dot{\theta} = \frac{v}{r}\sin\theta = \frac{12149}{8400.3} \times \frac{7275}{8400.3}$$

$$= 1.252 \text{ rad/hr}$$

$$= 3.4778 \times 10^{-4} \text{ rad/s}$$

$\boxed{\dot{\theta} = 3.4778 \times 10^{-4} \text{ rad/s}}$ — 1

$$\vec{a} = \frac{d}{dt}(\dot{r}\hat{e}_r) + \frac{d}{dt}(r\dot{\theta}\hat{e}_\theta)$$

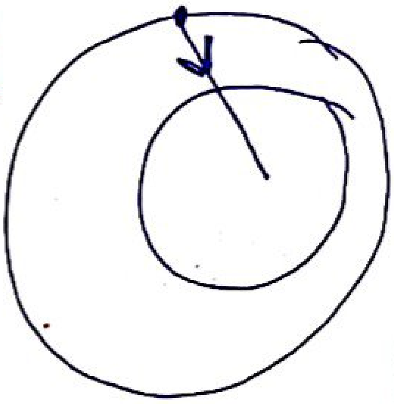
$$= (\ddot{r} - r\dot{\theta}^2)\hat{e}_r + (2\dot{r}\dot{\theta} + r\ddot{\theta})\hat{e}_\theta$$

$$a_r = \ddot{r} - r\dot{\theta}^2$$

$a_r = g_A = \ddot{r} - r\dot{\theta}^2$

$-7.159 = \ddot{r} - (8400.3 \times 5280) \times (3.4778 \times 10^{-4})^2$

$\boxed{\ddot{r} = \cancel{-7.1054\ \text{ft}}/\text{sec}^2 \quad -1.7944\ \text{ft/s}^2}$ 1.5



$a_\theta = 0$ ✓

$= 2\dot{r}\dot{\theta} + r\ddot{\theta}$

$\Rightarrow \ddot{\theta} = -\dfrac{2\dot{r}\dot{\theta}}{r} = \dfrac{-2 \times 8909 \times 3.4778 \times 10^{-5}}{8400.3 \times 5280}$

$\boxed{\ddot{\theta} = -13971 \times 10^{-8}\ \text{rad/s}^2}$ 1.5

~~1790~~

$= -1.3971 \times 10^{-7}$ rad

24 a) ~~5 marks~~

$m = 1.5\,kg$

$v_1 = 2\,m/s$ (Velocity of slider at point A)

$v_2 = 3\,m/s$ (Velocity of slider at point B)

$h = 0.4\,m$ (Height through which P.E changes)

$L = 0.3\,m$ (Unstrecthed length of the spring)

$K = 800\,N/m$ (spring constant)..

Work done by friction can be calculated as:-

$$U_f = mgh + E_{spring} + \frac{1}{2}m(v_2^2 - v_1^2)$$

P.E in the slider due to vertical movement of 0.4m.

$$= mgh = (1.5) \times 9.81 \times 0.4 = 5.886 \text{ J}$$ (1)

$$E_{spring} = \frac{1}{2}kx^2 \text{ (Gen. eqn)}$$

$$E_{spring} \text{ for system} = \frac{1}{2}k\left[(L_B - L)^2 - (L_A - L)^2\right]$$

Stretched length of spring at point A.

$$L_A = \sqrt{0.3^2 + 0.4^2} = 0.5\text{m}.$$

$$L_B = 0.4\text{m}.$$

$$E_{spring} = \frac{1}{2} \times 800 \times \left[(0.4 - 0.3)^2 - (0.5 - 0.3)^2\right] = -12 \text{ J}.$$ (1)

K.E of slider as it moves from point A to point B.

$$= \frac{1}{2}m(v_2^2 - v_1^2) = \frac{1}{2} \times 1.5 \times (3^2 - 2^2) = 3.75 \text{ J}$$

So, $U_f = (5.886 - 12 + 3.75) \text{ J} = -2.364 \text{ J}$ (2)

Work done, $W = F \cdot d$

It can be written as, $U_f = F \times (0.7) \rightarrow F$ (friction force)

So. $F = \frac{U_f}{0.7} = \frac{2.364}{0.7} = 3.38\,N$

1

$$\boxed{\begin{aligned} U_f &= -2.364\,J \\ F &= 3.38\,N \end{aligned}}$$

avg force by friction.

$c = \sqrt{a^2+b^2-2ab\cos\theta}$

$c = ?$

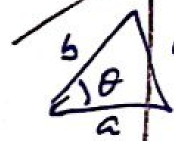
5 marks

2(b) 5marks

Undeformed length, $l_0 = \sqrt{(0.25)^2 + (0.25)^2} = 0.354\ m.$ 0.5

From law of cosines at $\theta = 20°$, we have,

$$l_1 = \sqrt{(0.25)^2 + (0.25)^2 - 2 \times (0.25) \times (0.25)\cos(90-\theta)}$$

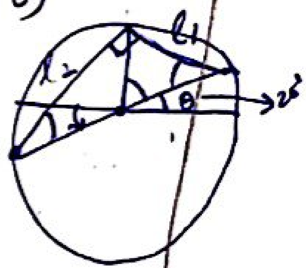


$$= 0.287\ m$$

$c = \sqrt{a^2+b^2-2ab\cos\theta}$

$$l_2 = \sqrt{(0.25)^2 + (0.25)^2 - 2\times(0.25)\times(0.25)\cos(90+\theta)}$$ 0.5

$$= 0.410\ m.$$



Deformations are :-

$$x_1 = l_0 - l_1 = \cancel{3.5}\ 0.354 - 0.287 = 0.067\ m$$

$$x_2 = l_2 - l_0 = 0.410 - 0.354 = 0.056\ m.$$

$$U'_{1-2} = \Delta T + \Delta V_g + \Delta V_e = 0$$

$$T_1 = 0$$

$$T_2 = 3 \cdot \frac{1}{2} m v^2 = \frac{3}{2} \cdot 3\left(\dot\theta(0.25)\right)^2 = 0.281\left(\dot\theta\right)^2$$ 1

If we make $\theta = 0°$ as datum.

The gravitational P.E's are given by.

$V_{g,1} = -mg(0.25)\sin 20^\circ - mg(0.25)\cos 20^\circ + mg(0.25)\sin 20^\circ$

$= -6.914\ J$

$V_{g,2} = -mg(0.25) = -7.358\ J$ (1)

Initial and final elastic spring P.E's are given by:-

$V_{e,1} = \frac{1}{2}kx_1^2 + \frac{1}{2}kx_2^2 = \frac{1}{2}(1.2)(10^3)\left((0.067)^2 + (0.056)^2\right)$

$= 4.575\ J$ (1)

Inserting it into the equation,

$0.281(\dot{\theta})^2 - 0 - 7.358 - (-6.914) + 0 - 4.575 = 0$

$(\dot{\theta})^2 = \frac{5.019}{0.281}$ ~~= 17.86 rad/s~~

$\boxed{\dot{\theta} = 4.226\ rad/s}$ (1)

Q3 8 marks

3) Let us consider the particle makes an angle θ with horizontal dir^n at level O-O'.

From conservation of angular momentum between two points

$$m v_0 r_0 = m v_1 r_1 \cos\theta$$

$$\Rightarrow \cos\theta = \frac{v_0 r_0}{v_1 r_1}$$

2

From the figure in question, we have

$r_0 = (0.15r + r) - r\sin 30°$

$\Rightarrow \quad r_0 = 0.65r$

and $\quad r_1 = 0.15r$

1

~~Late~~ To determine, θ, we have to find v_1

— Considering zero level of gravitational P.E along O-O',

Initial

$E_i = \frac{1}{2}mv_0^2 + mgh = \frac{1}{2}mv_0^2 + mgr\cos 30°$

Final

$E_f = \frac{1}{2}mv_1^2$

From conservation of Energy, we have, $E_f = E_i$

$\frac{1}{2}mv_1^2 = \frac{1}{2}mv_0^2 + mgr\cos 30°$

$\Rightarrow \quad v_1^2 = v_0^2 + 2gr\cos 30°$

$\Rightarrow \quad v_1^2 = 0.55^2 + 2 \cdot 9.81 \cdot 0.9 \cdot \cos 30°$.

3

$\boxed{v_1 = 3.95\,m/s}$

Now, from previous eqn we have,

$\cos\theta = \frac{v_0 r_0}{v_1 r_1} = \frac{0.55 \times 0.65r}{0.15r \times 3.95}$

$\boxed{\theta = \cos^{-1}(0.6034) = 52.9°}$ Ans

2

44

$\boxed{\theta = 52.7}$

Q4 8 Marks

4b $m = 20 kg$ $\qquad \dot{\theta} = 4 rad/s$

$m_s = 5 kg$ $\qquad$ when $\theta = 0$, $v = 0.6 m/s$

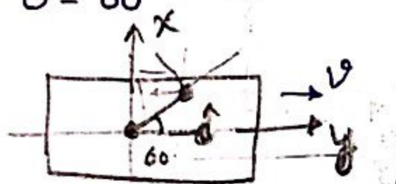
$r = 0.4 m$ (Distance from centre O)

~~In~~ To calculate v when $\theta = 60°$, we use conservation of linear momentum G_o about centre O. along v.

$\therefore \; G_{O_1} = G_{O_2}$ — 2

$G_{O_1} \Rightarrow$ when $\theta = 0°$, G_{O_2} when $\theta = 60°$

$G = \sum m_i v_i$



Velocity of the sphere in y direction, ~~v_s~~ when $\theta = 0$,

$v_{s_1} = r \cdot \dot{\theta} \cdot \sin\theta = 0.4 \times 4 \times \sin 0° = 0 m/s$.

Velocity of the sphere in y direction when $\theta = 60°$,

$v_{s_2} = r \cdot \dot{\theta} \sin\theta = 0.4 \cdot 4 \sin 60 = 1.386 m/s$. — 2

When $\theta = 0°$, ~~Total~~ $G_{O_1} = (m + m_s) v - m_s \cdot v_{s_2}$

$= (20 + 5) \cdot 0.6 - 5 \times \cancel{1.386}\ 0$

$= 15 kg\ m/s$

$G_{O_2} = (m + m_s) v - m_s v_{s_2} = (20 + 5) v - 5 \times 1.386$ — 3

$G_{O_2} = 25 \cdot v - 6.93$

By applying conservation of linear momentum.

$$G_{O_1} = G_{O_2}$$

$$15 = 25v - 6.93$$

$$v = \frac{15 + 6.93}{25}$$

$$\boxed{v = 0.877\, m/s}$$

1

P

Indian Institute of Technology Jodhpur

MEL1010: Engineering Mechanics

Minor 2

Duration: 60 mins **Maximum marks: 30**

Instruction:

1. The question paper consists of 5 questions and all the questions are equally weighted. Make sure all are printed.
2. Use of Scientific calculators are allowed.
3. Take suitable assumption whenever necessary.
4. Question paper is printed in bilingual format (English and Hindi); however, in case of any discrepancy, matter written in English is considered as final.

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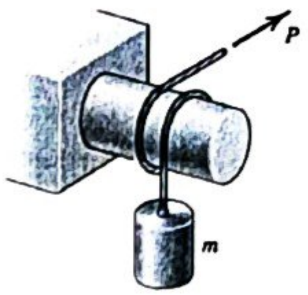
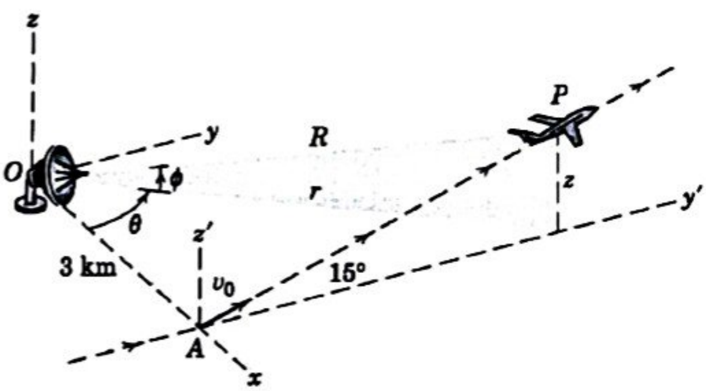
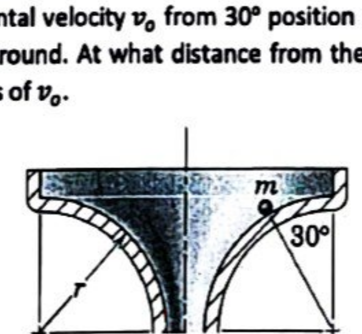
1. A force $P = mg/6$ is required to lower the cylinder at a constant slow speed with the cord making 1.25 turns around the fixed shaft. Calculate the coefficient of friction μ between the cord and the shaft.
सिलेंडर को स्थिर शाफ्ट के चारों ओर 1.25 चक्कर लगाती रस्सी से निरंतर धीमी गति से नीचे लाने के लिए एक बल P = mg/6 की आवश्यकता होती है। रस्सी और शाफ्ट के बीच घर्षण के गुणांक μ की गणना करें।

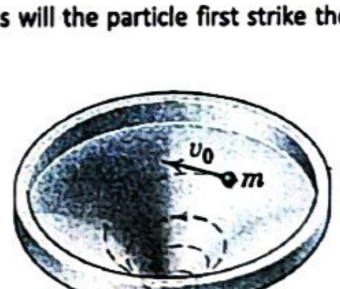


2. An aircraft P takes off at A with a velocity v_o of 250 km/h and climbs in the vertical y'-z' plane at the constant 15° angle with an acceleration along its flight path of 0.8 m/s². Flight progress is monitored by radar at point O. (a) Resolve the velocity of P into cylindrical-coordinate components 60 seconds after takeoff and find $\dot{r}, \dot{\theta}$ *and* $\dot{z}$ for that instant. (b) Resolve the velocity of the aircraft P into spherical-coordinate components 60 seconds after takeoff and find $\dot{R}, \dot{\theta}$ *and* $\dot{\phi}$ for that instant.
एक विमान P, 250 किमी/घंटा के वेग v_o के साथ A पर से उड़ान भरता है और अपने उड़ान पथ पर 0.8 m/s² के त्वरण के साथ निरंतर 15° कोण पर ऊर्ध्वाधर y'-z' समतल में चढ़ता है। उड़ान की प्रगति की निगरानी बिंदु O पर मौजूद रडार द्वारा की जाती है। (a) उड़ान भरने के 60 सेकंड बाद P के वेग को बेलनाकार- निर्देशांक घटकों में हल करें और उस क्षण के लिए $\dot{r}, \dot{\theta}$ *and* $\dot{z}$ ज्ञात करें। (b) उड़ान भरने के 60 सेकंड बाद विमान P के वेग को गोलाकार– निर्देशांक घटकों में हल करें और उस क्षण के लिए $\dot{R}, \dot{\theta}$ *and* $\dot{\phi}$ ज्ञात करें।



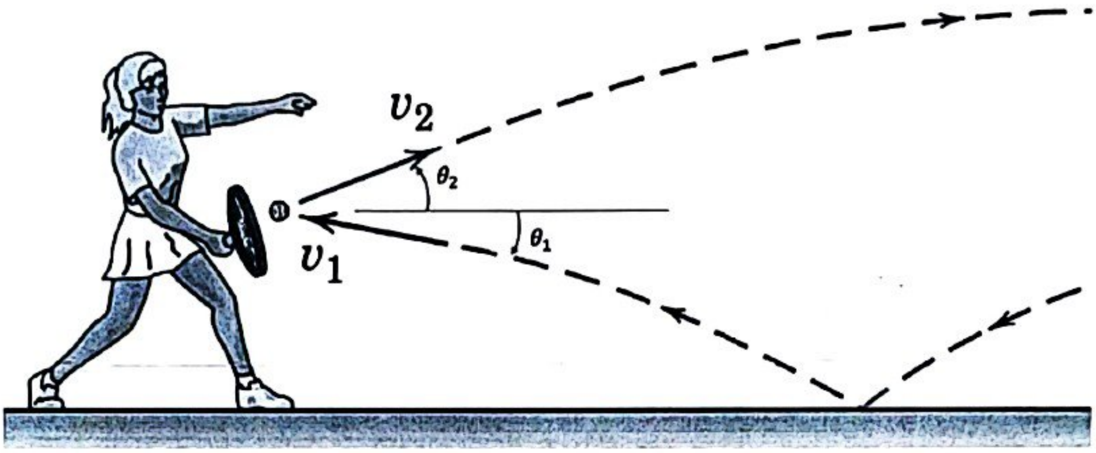
3. A particle is launched with a horizontal velocity v_o from 30° position as shown. At O it exits the funnel. If r=1m and O is located 1 m above ground. At what distance from the axis will the particle first strike the ground. Write your answers in terms of v_o.
दर्शाए गये चित्र के अनुसार, एक कण को 30° स्थिति से क्षैतिज वेग v_o के साथ प्रक्षेपित किया जाता है। O पर यह फ़नल से बाहर निकलता है। यदि r=1m और O जमीन से 1 मीटर ऊपर स्थित है। अक्ष से कितनी दूरी पर कण सबसे पहले जमीन से टकराएगा। अपने उत्तर को v_o के रूप में लिखें।





4. An image processing-based force/torque calculating system is required to be designed for lawn tennis game to calculate the impact force of racket and the direction of force during the impact based on given condition. A tennis player strikes the tennis ball with her racket while the ball is still rising. The ball speed before impact with the racket is v_1 and after the impact its speed is v_2, with directions $\theta_1 and\ \theta_2$ with horizontal as shown in the figure. If the ball of mass m is in contact with the racket for Δt time, determine the magnitude of the average force **R** exerted by the racket on the ball. Find the angle β made by **R** with the horizontal. Assume the weight of the ball is negligible in the context of reaction force **R**. Also, draw an impulse-momentum diagram.

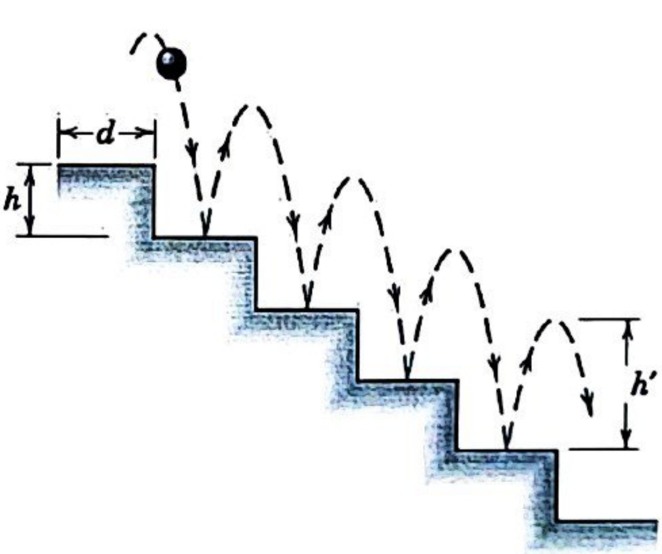
लॉन टेनिस गेम के लिए एक इमेज प्रोसेसिंग-आधारित बल/टॉर्क गणना प्रणाली को डिज़ाइन करने की आवश्यकता है ताकि दी गयी स्थिति के आधार पर प्रभाव के दौरान रैकेट के प्रभाव बल और प्रभाव बल की दिशा की गणना की जा सके। एक टेनिस खिलाड़ी अपने रैकेट से टेनिस गेंद पर प्रहार करता है जबकि गेंद अभी भी ऊपर उठ रही है। दर्शाए गये चित्र के अनुसार रैकेट से टकराने से पहले गेंद की गति क्षैतिज के साथ θ_1 दिशा में v_1 है और प्रभाव के बाद इसकी गति क्षैतिज के साथ θ_2 दिशा में v_2 है। यदि m द्रव्यमान की गेंद Δt समय के लिए रैकेट के संपर्क में है, तो गेंद पर रैकेट द्वारा लगाए गए औसत बल R का परिमाण निर्धारित करें और R द्वारा क्षैतिज के साथ बनाया गया कोण β भी ज्ञात कीजिए। प्रतिक्रिया बल R के तुलना में गेंद का वजन नगण्य है। आवेग-गति का रेखा-चित्र भी बनाइए।



5. The coefficient of restitution $e = 0.866$ which will allow the ball to bounce down the step as shown. The tread and riser dimensions, $d = 40\ cm$, and $h = 15\ cm$ respectively, are same for every step and the ball bounces for the same height h' above each step. What horizontal velocity v_x is required so that the ball lands in the center of each tread?

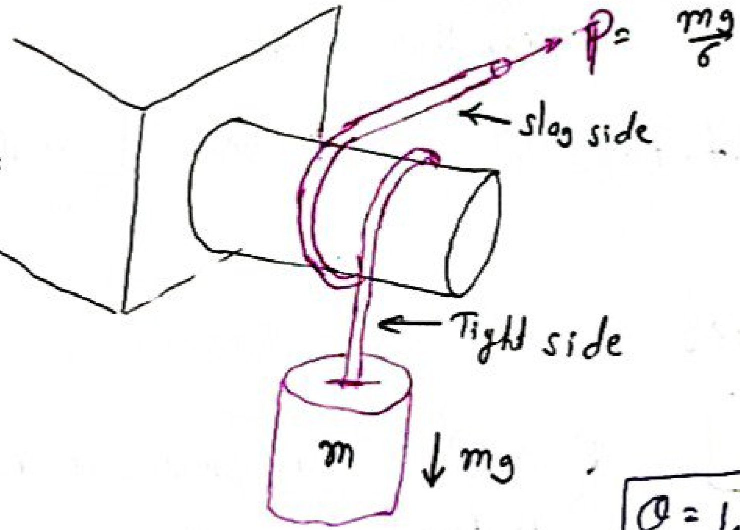
Also determine the height (h') for which ball bounces after the impact.

दर्शाए गये अनुसार, गेंद पुनर्स्थापन गुणांक $e = 0.866$ के साथ सीढ़ियों पर उछलती है। प्रत्येक सीढ़ी की लम्बाई और ऊंचाई के आयाम, क्रमशः $= 40\ cm$, और $h = 15\ cm$ है, और गेंद प्रत्येक सीढ़ी के ऊपर समान ऊंचाई h' तक उछलती है। किस क्षैतिज वेग v_x की आवश्यकता है ताकि गेंद प्रत्येक सीढ़ी के केंद्र में गिरे? यह भी निर्धारित करें कि गेंद संघात के बाद कितनी ऊंचाई (h') तक उछलती है।



Q.1

$$\frac{mg}{mg/6} = e^{\mu\theta} \quad - \text{Marks } 2$$



$$\frac{1}{1/6} = e^{\mu\theta}$$

$$\mu\theta = \ln 6$$

ANS $\boxed{\mu = \frac{\ln 6}{2.5\pi} = 0.228}$ ← 3 Marks

From the values of m & P it is quite clear that

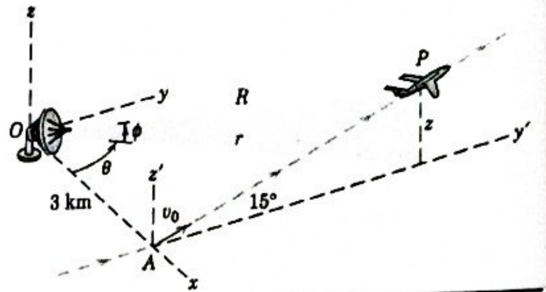
$mg > P$

[also mass is moving down this will also give some result]

$\boxed{\theta = 1.25 \text{ turns} = 2.5\pi}$

← 1 Marks

Q2:



An aircraft P takes off at A with a velocity v_0 of 250 km/h and climbs in the vertical y'-z' plane at the constant 15° angle with an acceleration along its flight path of 0.8 m/s². Flight progress is monitored by radar at point O. (*a*) Resolve the velocity of P into cylindrical-coordinate components 60 seconds after takeoff and find $\dot{r}$, $\dot{\theta}$, and $\dot{z}$ for that instant. (*b*) Resolve the velocity of the aircraft P into spherical-coordinate components 60 seconds after takeoff and find $\dot{R}$, $\dot{\theta}$, and $\dot{\phi}$ for that instant.

Solution. (a) The accompanying figure shows the velocity and acceleration vectors in the y'-z' plane. The takeoff speed is

$$v_0 = \frac{250}{3.6} = 69.4 \text{ m/s}$$

and the speed after 60 seconds is

$$v = v_0 + at = 69.4 + 0.8(60) = 117.4 \text{ m/s}$$

The distance s traveled after takeoff is

$$s = s_0 + v_0 t + \frac{1}{2}at^2 = 0 + 69.4(60) + \frac{1}{2}(0.8)(60)^2 = 5610 \text{ m}$$

The y-coordinate and associated angle θ are

$$y = 5610 \cos 15^\circ = 5420 \text{ m}$$

$$\theta = \tan^{-1}\frac{5420}{3000} = 61.0^\circ$$

From the figure (*b*) of x-y projections, we have

$$r = \sqrt{3000^2 + 5420^2} = 6190 \text{ m}$$

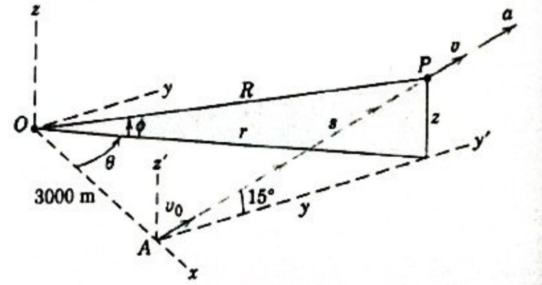
$$v_{xy} = v \cos 15^\circ = 117.4 \cos 15^\circ = 113.4 \text{ m/s}$$

$$v_r = \dot{r} = v_{xy} \sin\theta = 113.4 \sin 61.0^\circ = 99.2 \text{ m/s} \qquad \textit{Ans.}\ 1$$

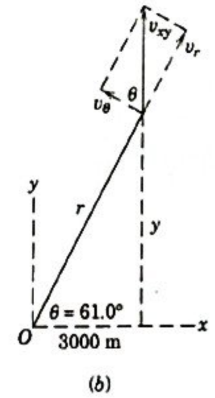
$$v_\theta = r\dot{\theta} = v_{xy}\cos\theta = 113.4 \cos 61.0^\circ = 55.0 \text{ m/s}$$

So $$\dot{\theta} = \frac{55.0}{6190} = 8.88(10^{-3}) \text{ rad/s} \qquad \textit{Ans.}\ 1$$

Finally $$\dot{z} = v_z = v \sin 15^\circ = 117.4 \sin 15^\circ = 30.4 \text{ m/s} \qquad \textit{Ans.}\ 1$$



(*a*)



(*b*)

(b) Refer to the accompanying figure (c), which shows the x-y plane and various velocity components projected into the vertical plane containing r and R. Note that

$$z = y \tan 15^\circ = 5420 \tan 15^\circ = 1451 \text{ m}$$

$$\phi = \tan^{-1}\frac{z}{r} = \tan^{-1}\frac{1451}{6190} = 13.19^\circ$$

$$R = \sqrt{r^2 + z^2} = \sqrt{6190^2 + 1451^2} = 6360 \text{ m}$$

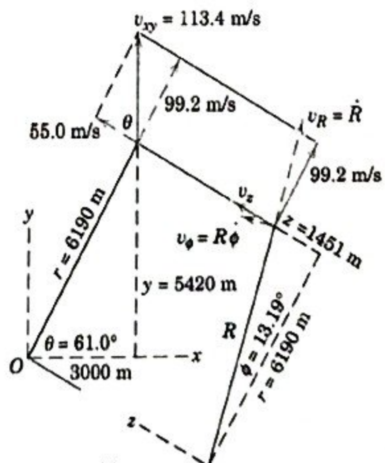
From the figure,

$$v_R = \dot{R} = 99.2 \cos 13.19^\circ + 30.4 \sin 13.19^\circ = 103.6 \text{ m/s} \qquad \textit{Ans.}\ 1$$

$$\dot{\theta} = 8.88(10^{-3}) \text{ rad/s, as in part } (a) \qquad \textit{Ans.}\ 1$$

$$v_\phi = R\dot{\phi} = 30.4 \cos 13.19^\circ - 99.2 \sin 13.19^\circ = 6.95 \text{ m/s}$$

$$\dot{\phi} = \frac{6.95}{6360} = 1.093(10^{-3}) \text{ rad/s} \qquad \textit{Ans.}\ 1$$

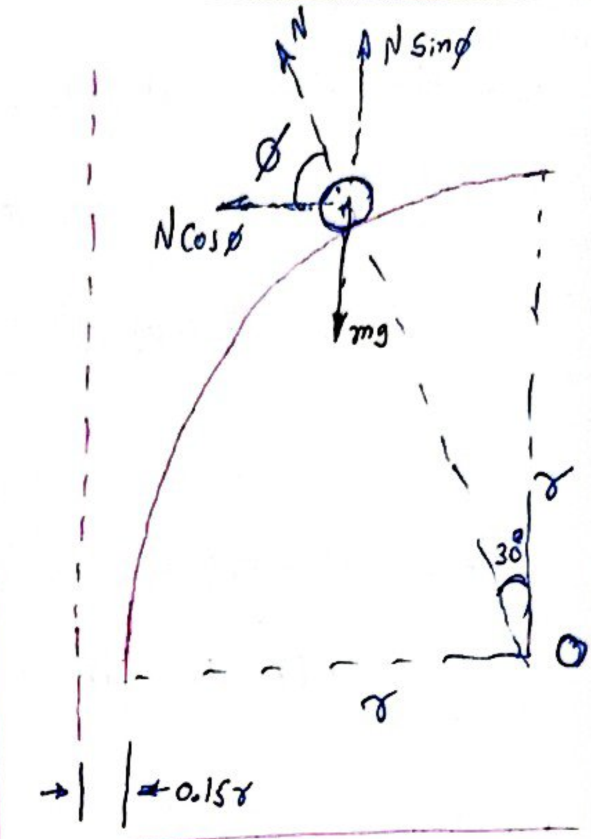


(c)

Every Ans = 1 marks
0.5 marks for value
& 0.5 marks for unit

Q-3

2 forces are acting
- → Gravity force
- → Normal reaction



$r = 1\ m$

$g = 9.81\ m/s^2$

Considering intinal position of partical as datum → Zero Potential Energy at Initial

$$P.E._0 + K.E._0 = P.E._1 + K.E._1 \quad \text{(1/2) Marks}$$

$$\frac{1}{2} m v_0^2 = \frac{1}{2} m v_1^2 - mgh$$

$(h = r\cos 30°)$ (1/2) Marks

$$v_1 = \sqrt{v_0^2 + 2 g r \cos 30°}$$

$$\boxed{v_1 = \sqrt{v_0^2 + 17}} \quad \text{(1) Marks}$$

* Since neither force exerts a moment about the axis of symmetry of the funnel like object Angular momentum is conserved about this axis

$$m v_0 r_0 = m v_1 r_1 \cos\theta \quad \text{(1/2) Marks}$$

$r_0 = r + 0.15r - r\sin 30°$

$\boxed{r_0 = 0.65\ m}$ (1/2) Marks

$\boxed{r_1 = 0.15r = 0.15\ m}$

$$\boxed{\cos\theta = \frac{v_0 r_0}{v_1 r_1} = \frac{0.65\, v_0}{0.15 \times \sqrt{v_0^2+17}}}$$

$$\Rightarrow \theta = \cos^{-1}\left[\frac{4.33\, v_0}{\sqrt{v_0^2+17}}\right] \quad \text{(1) Marks}$$

$$\cos\theta = \frac{4.33\, v_0}{\sqrt{v_0^2+17}}$$

$$\sin\theta = \sqrt{\frac{17 - 17.7489\, v_0^2}{17}}$$

From the above, it is clear that

in order to have $\sin^2\theta \geq 0$

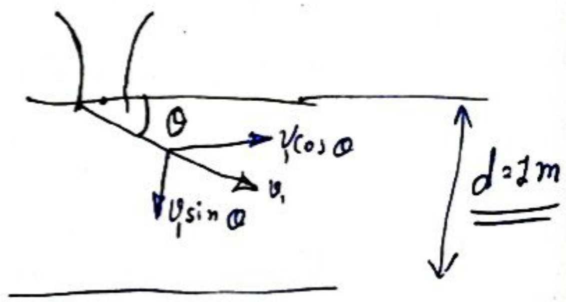
$$17 \geq 17.7489\, v_0^2 \Rightarrow \boxed{\begin{array}{l} v_0^2 \leq 0.9578 \\ v_0 \leq 0.98 \end{array}}$$

Hence $v_0^2 = 0.9578$ is very small as compared to 17 Hence

$$\boxed{v_1 = \sqrt{v_0^2 + 17} \approx \sqrt{17} = 4.123 \ m/s}$$

& $\cos\theta = 1.05\, v_0$

$\sin\theta = \sqrt{1 - 1.1025\, v_0^2}$



★ Distance at which the Partical strikes the floor first is

$$d = v_1 \cos\theta\, t$$

$$= 4.123 \times 1.05\, v_0 \times 0.420 \left[\sqrt{2.154 - 1.1025 v_0^2} - \sqrt{1 - 1.1025 v_0^2}\right]$$

$$\boxed{d \approx 1.82 \left[\sqrt{2.154 - 1.1025 v_0^2} - \sqrt{1 - 1.1025 v_0^2}\right]}$$

① Marks

$$v_1 \sin\theta\, t + \frac{1}{2} g t^2 = 1$$

$$\underbrace{4.123\sqrt{1 - 1.1025 v_0^2}}_{b}\, t + \underbrace{\frac{1}{2} 9.81}_{a} t^2 - \underbrace{1}_{c} = 0$$

$$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\cancel{t = \frac{0.420}{4.123}\left[\sqrt{2.154 - 1.1025 v_0^2} - \sqrt{1 - 1.1025}\right.}$$

① Marks

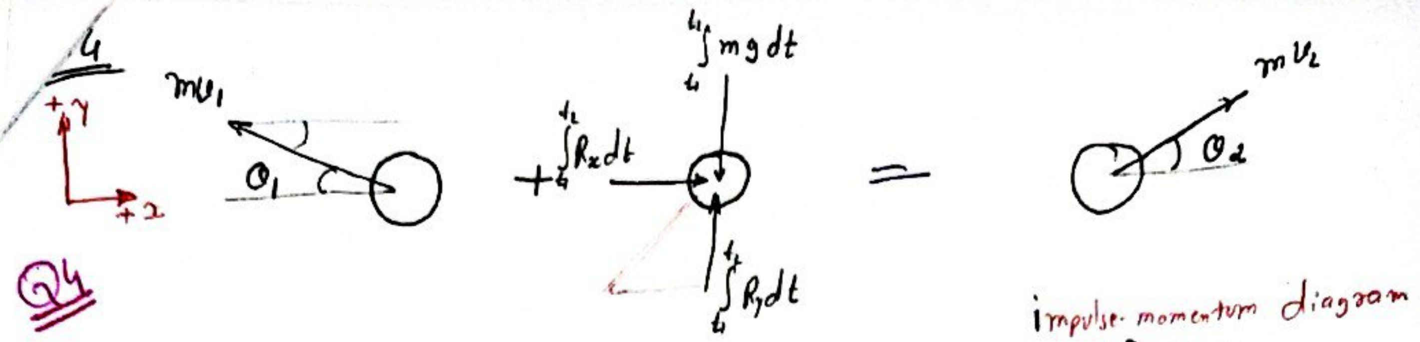
$$\boxed{t = 0.420 \left[\sqrt{2.154 - 1.1025 v_0^2} - \sqrt{1 - 1.1025 v_0^2}\right]}$$

Note → this is only Possible Ans as t must be ≥ 0 the other Ans

$$t = 0.420 \left[-\sqrt{2.154 - 1.1025 v_0^2} - \sqrt{1 - 1.1025 v_0^2}\right]$$

give $t \leq 0$ Hence Not a feasible solution

4



Q4

impulse-momentum diagram
2 Marks

In Vector form

$$\vec{v}_1 = -v_1\cos\theta_1\,\hat{i} + v_1\sin\theta_1\,\hat{j}$$
$$\vec{v}_2 = v_2\cos\theta_2\,\hat{i} + v_2\sin\theta_2\,\hat{j}$$

momentum Conserve

x direction

$$m(v_1)_x + R_x\,\Delta t = m(v_2)_x$$

$$R_x = \frac{m[(v_2)_x - (v_1)_x]}{\Delta t}$$

$$\boxed{R_x = \frac{m}{\Delta t}[v_2\cos\theta_2 + v_1\sin\theta_1]}$$ 1 Marks

y direction

$$m(v_1)_y + (R_y - mg)\,\Delta t = m(v_2)_y$$

(Neglecting weight of Ball in Contrast to R_y) as given in statement

$$\boxed{R_y = \frac{m}{\Delta t}[v_2\sin\theta_2 - v_1\sin\theta_1]}$$ 1 Marks

$$\boxed{\text{Reaction } R = \sqrt{R_x^2 + R_y^2} = \frac{m}{\Delta t}\sqrt{v_1^2 + v_2^2 + 2v_1v_2\cos(\theta_1+\theta_2)}}$$ Ans 1 Marks

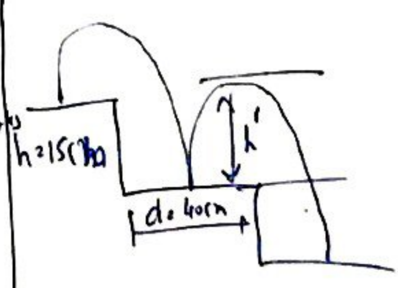
$$\boxed{\text{angle } \beta = \tan^{-1}\left(\frac{R_y}{R_x}\right) = \tan^{-1}\left(\frac{v_2\sin\theta_2 - v_1\sin\theta_1}{v_2\cos\theta_2 + v_1\cos\theta_1}\right)}$$ Ans 1 Marks

Q-5 The height from which ball fall = $h + h'$ ½ Marks

velocity Before impact = $v_1 = \sqrt{2g(h+h')}$ ½ Marks

The height at which ball ~~raise~~ raise after impact = h' ½ Marks

Velocity after impact, $v_2 = \sqrt{2gh'}$ ½ Marks



$$\text{Coefficient of restitution} = \frac{\text{relative velocity after collision}}{\text{relative velocity before collision}}$$

$$0.866 = \sqrt{\frac{h'}{h+h'}} \Rightarrow \boxed{h' = 45\ \text{cm}}$$ Ans 1 Marks for value +0.5 marks for Unit

Time of flight = $T = T_{up} + T_{Down}$

$$= \sqrt{\frac{2h'}{g}} + \sqrt{\frac{2(h+h')}{g}}$$

$$v_x = \frac{d}{T} = \frac{\sqrt{\frac{g}{2}}\,d}{\sqrt{h+h'} + \sqrt{h'}} = \frac{2.2147 \times 0.4}{\sqrt{0.45} + \sqrt{0.6}}$$

Ans $\boxed{v_x = 0.613\ m/s}$ 1 Marks for value + 0.5 Marks for Unit