Indian Institute of Technology Jodhpur

Minor-II PHL1010: Electromagnetism & Optics

Time:1 hr | 18 October 2023 | Total Marks: 25

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1. A point charge q is residing at the center of a sphere of linear dielectric material (susceptibility χ_e and radius R). Find the electric field, the polarization and the surface and volume bound charge densities. [4]

2. A thin uniform ring of radius R, carrying positive charge Q and mass M rotates anticlockwise about z axis with angular frequency ω. (a) Find the ratio between magnetic dipole moment and its angular momentum. (b) Plot the magnetic field for this configuration. [2+1]

3. A charge Q is uniformly distributed over a solid cylinder of radius R and length L. If the cylinder is rotated at a constant angular velocity ω about its axis, find the volume current density at any point inside the cylinder. [2]

4. Using the primitive model of a Hydrogen atom calculate its atomic polaraizablity. The Bohr radius is 0.53 $\mathring{A}$ and Coulomb constant is $9 \times 10^9 Nm^2C^{-2}$. [2]

5. What is refractive index? Consider propagation of E.M. waves in linear and homogeneous medium. Setup the Maxwell equations. Obtain the refractive index. [3]

$\frac{\frac{W}{m^2} \times \frac{1}{}}{\frac{m}{1}}$

6. The intensity of sunlight hitting the earth is about $1300 W/m^2$. If sunlight strikes a perfect absorber, what pressure does it exert? How about a perfect reflector? [2]

$\frac{W m^{-2}}{m}$

7. Find the skin depth for a typical metal ($\sigma \approx 10^7 (\Omega m)^{-1}$) in the visible range ($\omega \approx 10^{15} s^{-1}$), assuming $\epsilon \approx \epsilon_0$ and $\mu \approx \mu_0$. [3]

8. (a). Suppose you imbedded some free charge in a piece of glass. About how long would it take for the charge to flow to the surface? [2]

(b). Suppose you were designing a microwave experiment to operate at a frequency of $10^{10} Hz$. How thick would you make the silver coatings? [2]

(c). Find the wavelength and propagation speed in copper for radio waves at 1 MHz. Compare the corresponding values in air (or vacuum). [2]

Assume refractive index of glass is 1.5, $\epsilon_0 = 8.85 \times 10^{-12}\ C^2/(N.m^2)$, $\mu_0 = 4\pi \times 10^{-7}\ H/m$, restivity of glass, silver and copper is 10^{12}, 1.59×10^{-8} and $1.68 \times 10^{-8}\ \Omega m$, respectively.

$\lambda = \frac{2\pi}{k}$

$\boxed{v = \frac{\omega}{k}}$

$J = \sigma v$

$c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}$

$\boxed{v = \frac{1}{\sqrt{\mu_r \cdot \varepsilon_r}}}$

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Time: 1 Hr Marks: 15

1. (a) Provide one evidence (outside laboratory) which demonstrates the unification of electricity, magnetism and optics.
(b) *"The speed of light in free space is independent of the frame of reference."* This is one of the postulates of the special theory of relativity. Can you find a connection of this statement with any concept related to electromagnetic theory? **[1+1]**

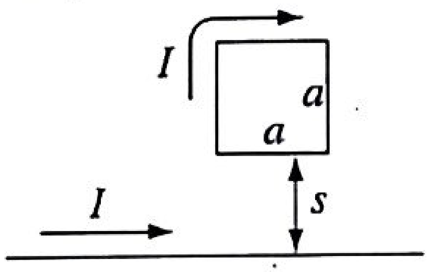
2. Consider Maxwell's equations in the following form:

$$\text{(i)}\ \nabla \cdot \mathbf{E} = \frac{1}{\epsilon_0}\rho, \qquad \text{(iii)}\ \nabla \times \mathbf{E} + \frac{\partial \mathbf{B}}{\partial t} = 0,$$

$$\text{(ii)}\ \nabla \cdot \mathbf{B} = 0, \qquad \text{(iv)}\ \nabla \times \mathbf{B} - \mu_0\epsilon_0\frac{\partial \mathbf{E}}{\partial t} = \mu_0 \mathbf{J}$$

Here ρ and J are the sources of the EM field and they are supposed to be specified and you are suppose to calculate E and B. So there are six unknown in the given problem and eight equations if we go by components ((i) and (ii) are scalar equations whereas (iii) and (iv) are vector equations). *So, there are eight equations and only 6 unknowns.* In general, you don't have a solution: such a system is said to be overdetermined or inconsistent Yet we know that there are electric and magnetic fields all around us and these equations work. So what would you say is happening here? **[2]**

3. (a) Find the force on a square loop placed as shown below, near an infinite straight wire. Both the loop and the wire carry a steady current I.

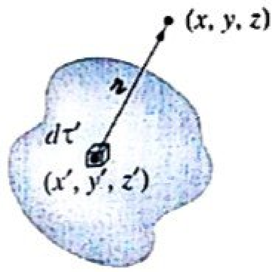


[2]

(b) A long solenoid, of radius a, is driven by an alternating current, so that the field inside is sinusoidal: $\mathbf{B}(t) = B_0 \cos(\omega t)\ \hat{z}$. A circular loop of wire, of radius $a/2$ and resistance R, is placed inside the solenoid, and coaxial with it. Find the current induced in the loop, as a function of time. [3]

4. Find the curl and Laplacian of the following *magnetic* vector potential:

$$\mathbf{A}(\mathbf{r}) = \frac{\mu_0}{4\pi}\int \frac{\mathbf{J}(\mathbf{r}')}{\mathscr{r}}\, d\tau'.$$



[4]

5. Maxwell's equations in terms of auxiliary functions (or potential functions) A and ϕ can be written as:

$$\frac{\partial}{\partial t}(\nabla \cdot A) + \nabla^2\phi = -\frac{\rho}{\varepsilon_0}$$

$$\nabla(\nabla \cdot A + \frac{1}{c^2}\frac{\partial \phi}{\partial t}) + \frac{1}{c^2}\frac{\partial^2 A}{\partial t^2} - \nabla^2 A = \mu_0 J$$

The first equation looks like Poission's equations but has A sitting in it whereas the second equation would look like a wave equation (with source term on the RHS) if the first term was not there. Therefore it seems that it will be a very difficult task to solve these equations unless we get rid of the first term in either of these equations, i.e., it would be glorious if we could put

$$\nabla \cdot A = 0$$

Or

$$\nabla \cdot A + \frac{1}{c^2}\frac{\partial \phi}{\partial t} = 0$$

Show that we can always set $(\nabla \cdot A)$ equal to zero. [2]