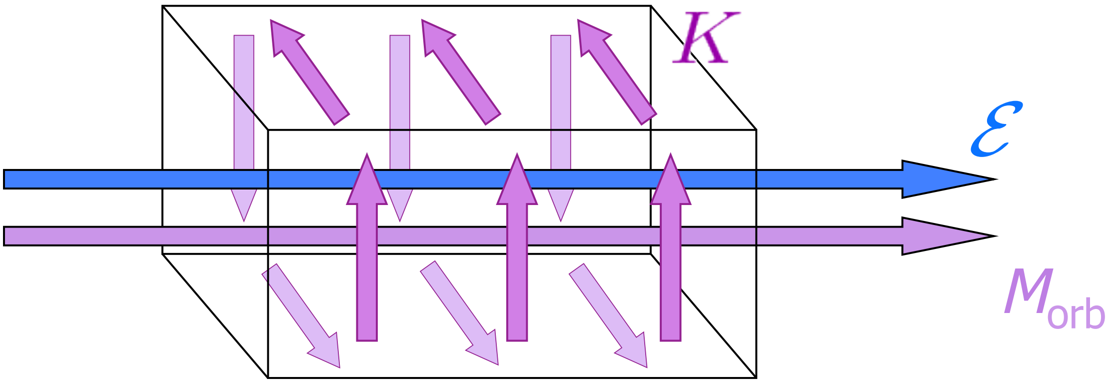
2D and 3D quantum anomalous Hall insulators

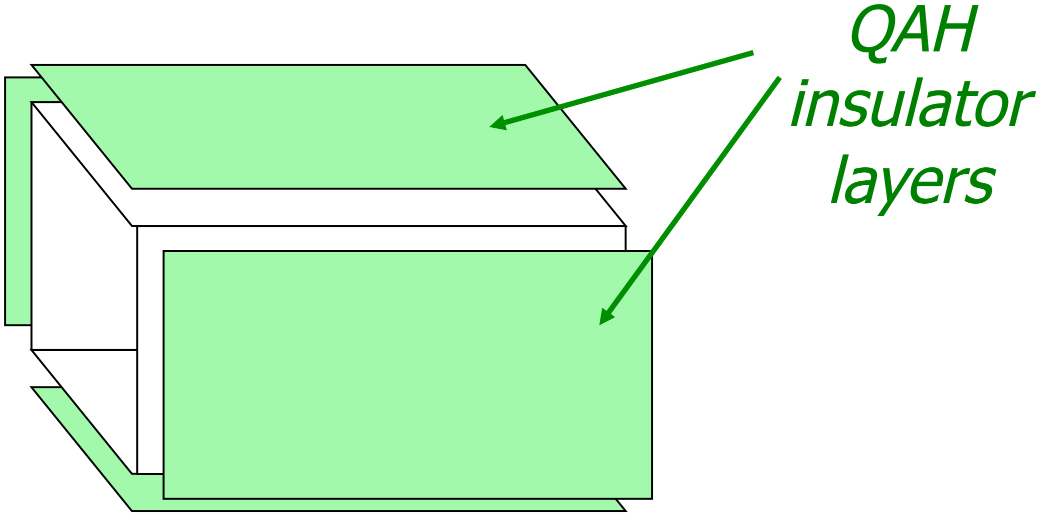
In the last lecture, I meant to make a comment about QAH insulators in 2D and 3D concerning applications as magnetoelectrics:

Orbital MEC ↔ Surface σ_{yx}

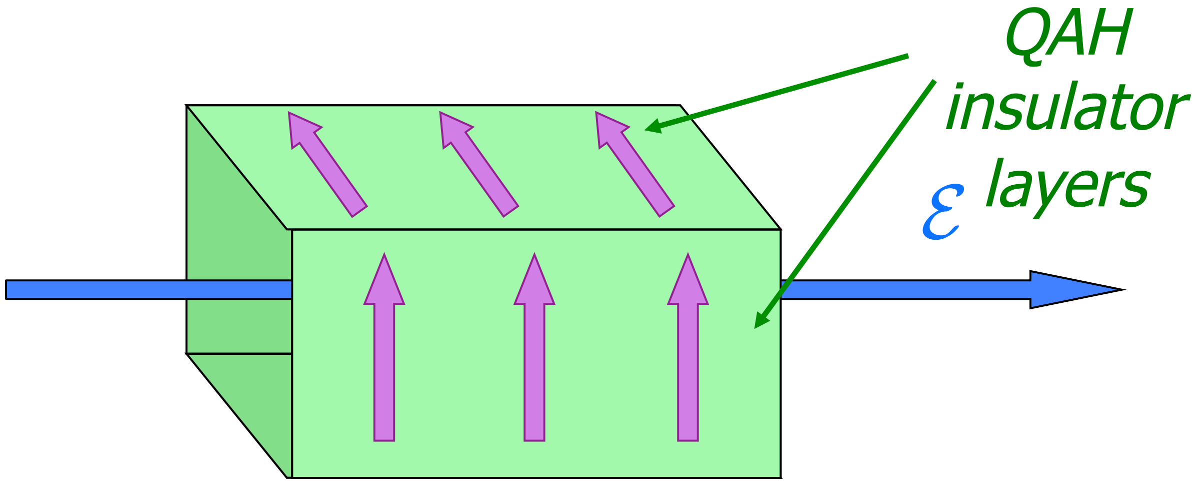


$$\alpha_{\mathrm{orb}} = \frac{dM_{\mathrm{orb}}}{d\mathcal{E}} = \frac{dK}{d\mathcal{E}} = \sigma_{yx}^{\mathrm{surf}}$$

How to build a magnetoelectric coupler



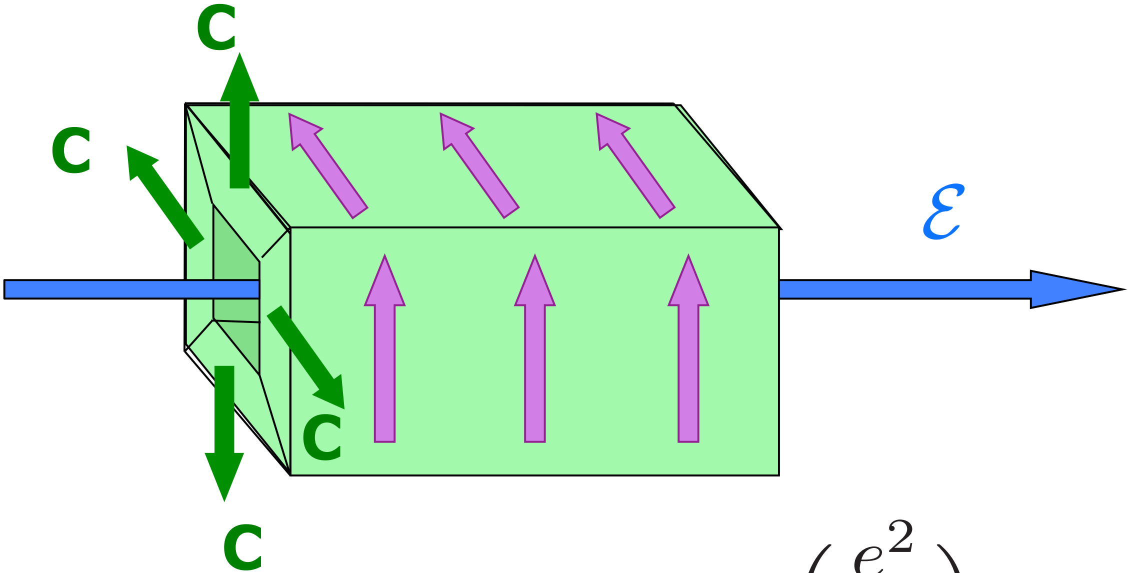
How to build a magnetoelectric coupler



$$\alpha_{\rm orb} = \frac{dK}{d\mathcal{E}} = \frac{e^2}{h} = \frac{1}{2\pi}\frac{1}{137}\ \text{g.u.}$$

For comparison, Cr_2O_3 has $\alpha \simeq 10^{-4}$ g.u.

How to build a magnetoelectric coupler

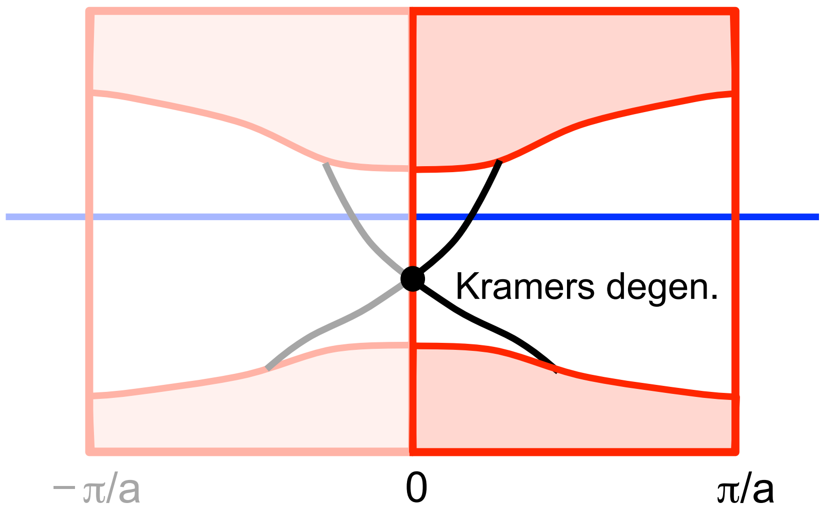


$$\alpha_{\rm orb} = N_{\rm layers}\left(\frac{e^2}{h}\right)$$

This can easily be 10^8 times that of Cr_2O_3 !

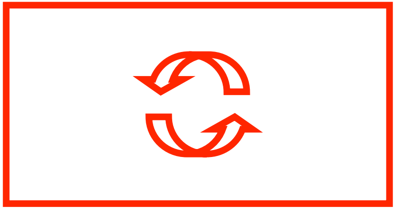
2D quantum spin Hall insulators

Started introduction last Wednesday:
Z_2 index is natural from point of view of edge states

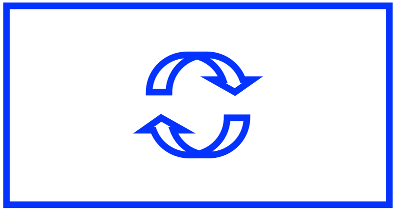


$Z_2 = N_{cross}$ (mod 2) = Invariant

Z_2 Topological Insulator

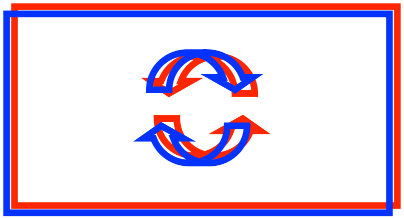


Spin up, $C = +1$



Spin down, $C = -1$

Z_2 Topological Insulator



Spin up, $C = +1$

Spin down, $C = -1$

Obeys T symmetry

Turn on spin-orbit:

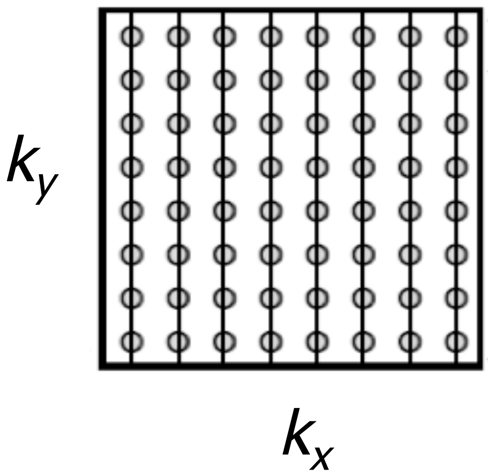
Z_2 Topological Insulator

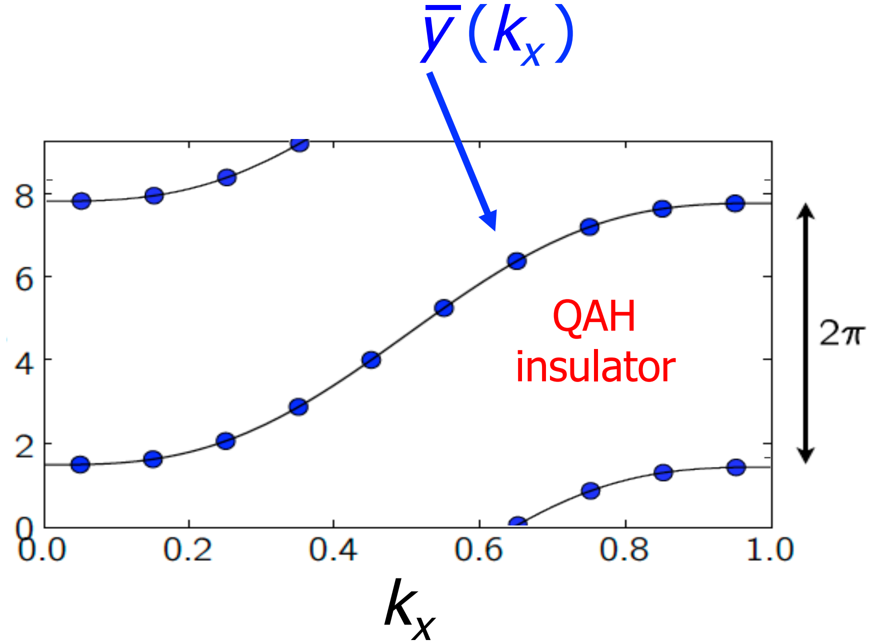
Z_2 Topological Insulator (QSH)

<u>Properties</u>:

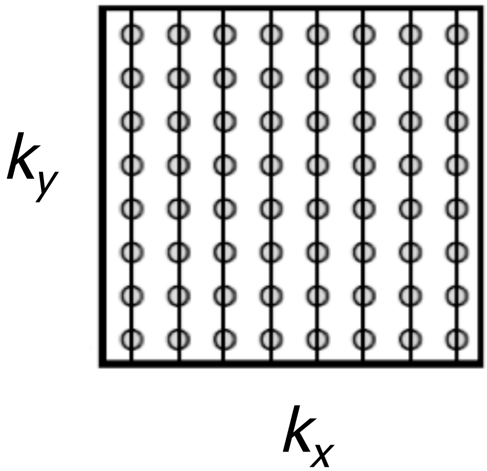
- Obeys T symmetry
- Total $C = 0$
- Z_2 invariant is odd
- (σ_{xy}^{spin} is not quantized)

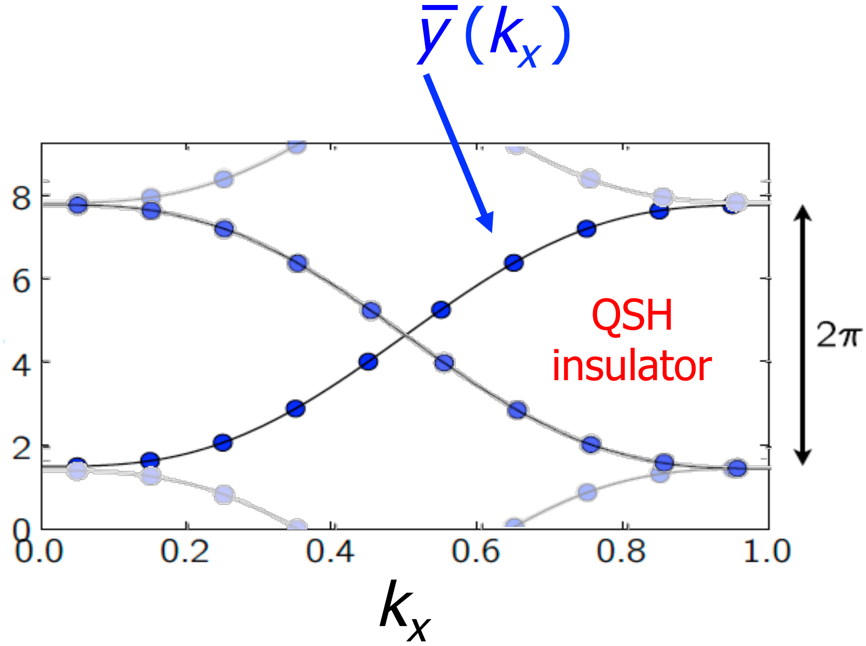
QAH: Hybrid WF centers $y(k_x)$





Z_2 QSH insulator: Hybrid WF centers $y\,(k_x)$

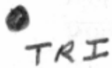




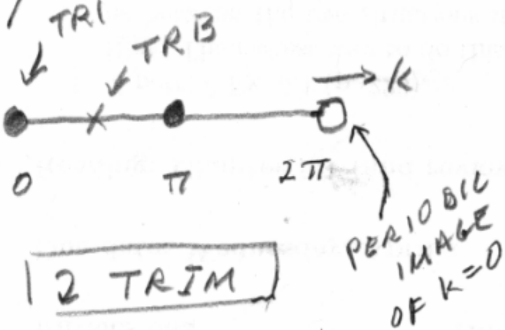
H_k IN TIME-REVERSAL-INVARIANT SYSTEMS

(TRI = TR INVARIANT   TRB = TR-BROKEN)

0D

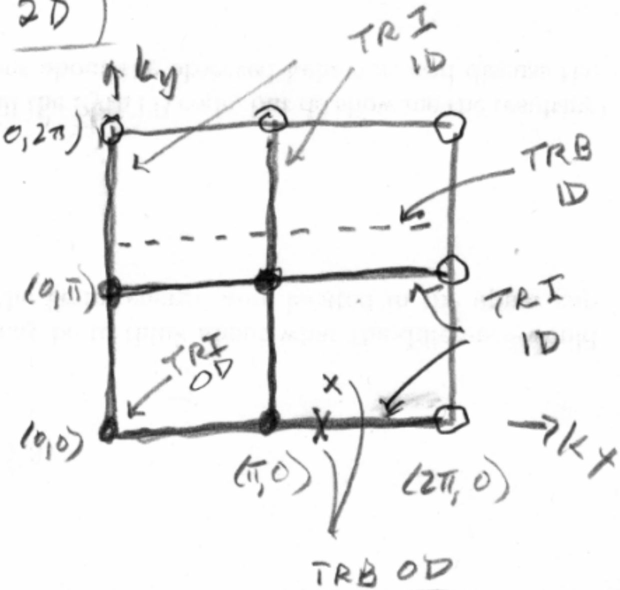


1D



$\boxed{\text{2 TRIM}}$

2D



$\boxed{\text{2 TRIM, 4 TRI LINES}}$

Kramers Theorem

Fermion case, $\Theta^2 = -1$

$H\Theta = \Theta H \Rightarrow$ If $H|\psi\rangle = E|\psi\rangle$

then $H(\Theta|\psi\rangle) = E(\Theta|\psi\rangle)$

Is $\Theta|\psi\rangle$ the same physical state as $|\psi\rangle$?

Assume yes: $\Theta|\psi\rangle = e^{i\varphi}|\psi\rangle$

$$\Theta^2|\psi\rangle = \Theta(e^{i\varphi}|\psi\rangle)$$
$$= e^{-i\varphi}(\Theta|\psi\rangle)$$
$$= e^{-i\varphi}e^{i\varphi}|\psi\rangle = |\psi\rangle$$

Inconsistent with $\Theta^2 = -1$!

$\Rightarrow$ $|\psi\rangle$ and $\Theta|\psi\rangle$ form degenerate

"Kramers pair" or "Kramers doublet"

Kramers Theorem

Also true for $H \rightarrow PxP$
So Wannier centers
are degenerate

Fermion case, $\Theta^2 = -1$

$H\Theta = \Theta H \Rightarrow$ If $H|\psi\rangle = E|\psi\rangle$

then $H(\Theta|\psi\rangle) = E(\Theta|\psi\rangle)$

Is $\Theta|\psi\rangle$ the same physical state as $|\psi\rangle$?

Assume yes: $\Theta|\psi\rangle = e^{i\varphi}|\psi\rangle$

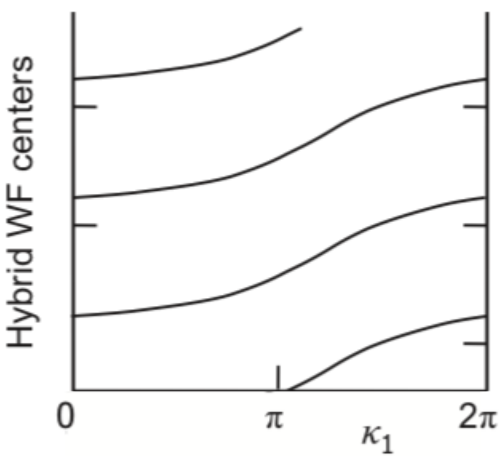
$$
\begin{aligned}
\Theta^2|\psi\rangle &= \Theta(e^{i\varphi}|\psi\rangle) \\
&= e^{-i\varphi}(\Theta|\psi\rangle) \\
&= e^{-i\varphi}e^{i\varphi}|\psi\rangle = |\psi\rangle
\end{aligned}
$$

Inconsistent with $\Theta^2 = -1$!

$\Rightarrow$ $|\psi\rangle$ and $\Theta|\psi\rangle$ form degenerate

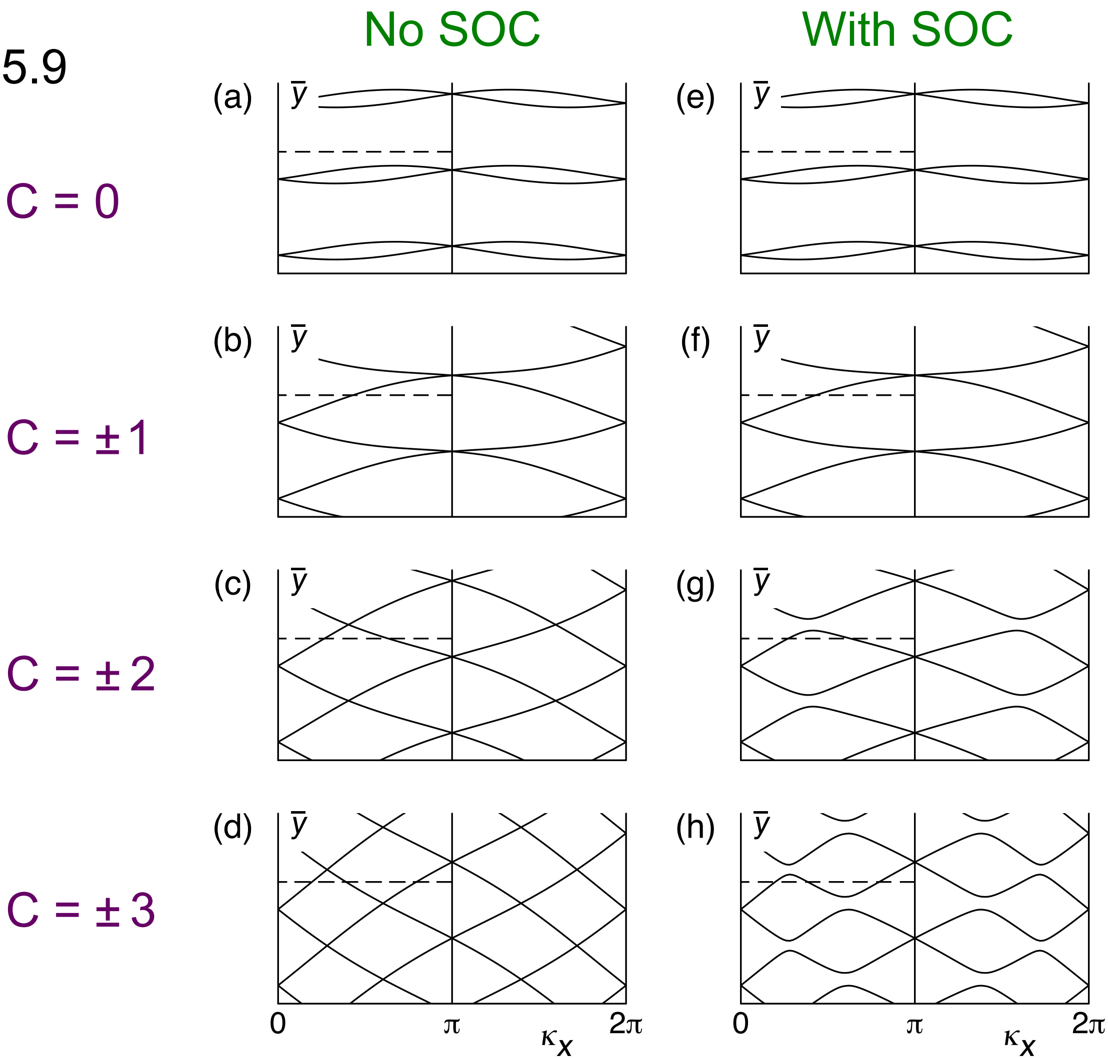
"Kramers pair" or "Kramers doublet"

Figure 5.9



QAH QSH

Fig. 5.9



<u>Methods for computing the Z2 invariant</u> (see pp. 234-6)

- If inversion symmetry is present

$$(-1)^{\nu} = \prod_{a=1}^{4} \prod_{m}^{N_{\text{occ}}/2} \xi_{am}$$

My preferences

- If inversion symmetry is absent
  - Flow of Wannier bands
  - Flow of edge states
  - Flow of entanglement spectrum
  - Berry curvature vs. Berry phase

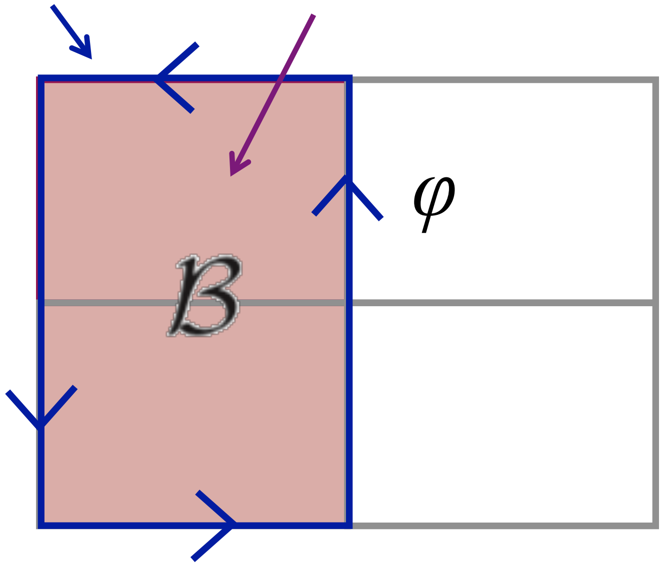
$$\nu = \frac{1}{2\pi}\left[\oint_{\partial\mathcal{B}} \mathbf{A}\cdot d\mathbf{l} - \int_{\mathcal{B}} \Omega\, d^2k\right] \bmod 2$$

  - Pfaffian

Z_2 index from Berry curvature vs. Berry phase

$$\nu = \frac{1}{2\pi}\left[\underbrace{\oint_{\partial\mathcal{B}} \mathbf{A}\cdot d\mathbf{l}} - \underbrace{\int_{\mathcal{B}} \Omega\, d^2k}\right] \bmod 2$$

With TR-imposed gauge restriction on boundary



Wannier obstruction

If the Z_2 index is odd:

- There is no smooth and periodic gauge over the 2D Brillouin zone that also respects TR symmetry
- As a result, it is impossible to construct WFs that respect TR symmetry, i.e., that come in Kramers pairs:

  $\Theta |w_{1a}\rangle = |w_{1b}\rangle$ and $\Theta |w_{1b}\rangle = - |w_{1a}\rangle$
- However, if this symmetry restriction is lifted, then it is possible.

Soluyanov & Vanderbilt, 2011