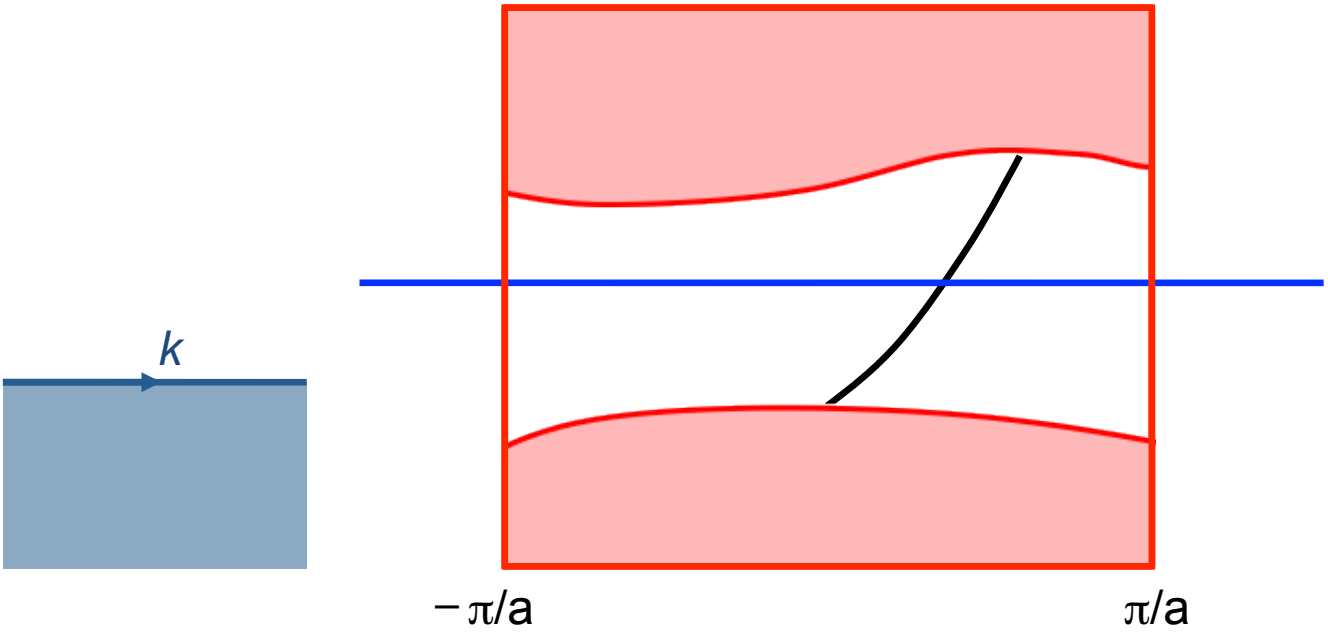
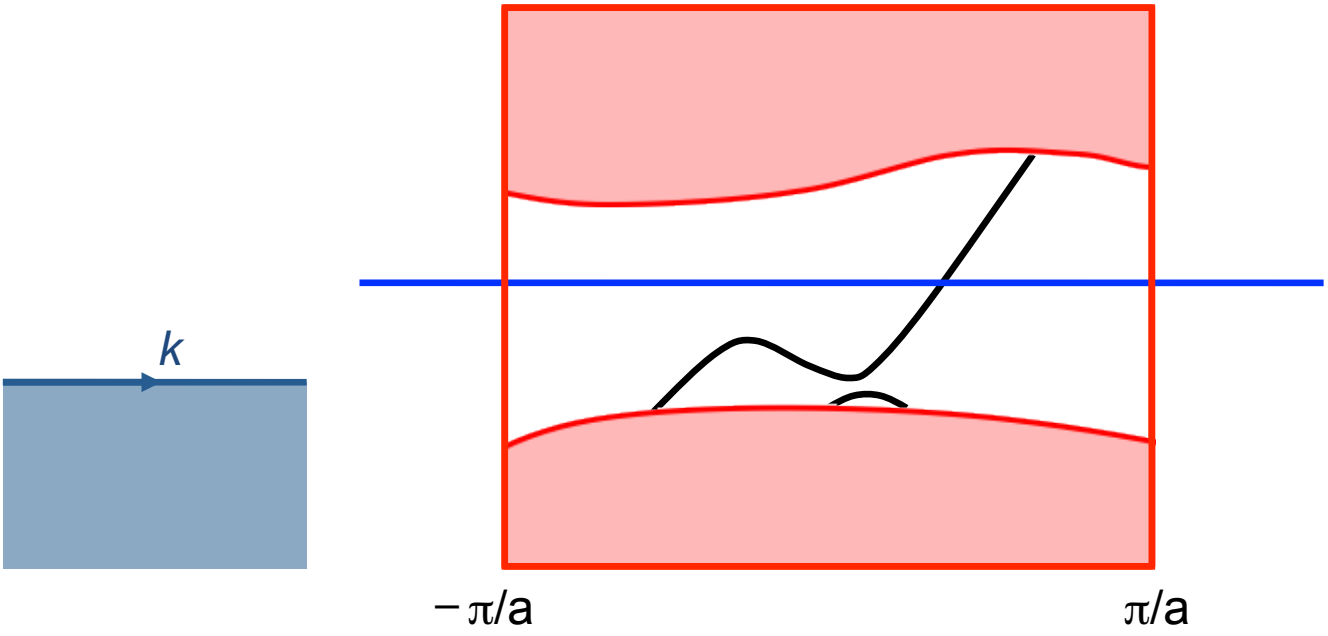
Edge states: 2D QAH insulator



$$Z = N_{up} - N_{down} = \text{Invariant}$$

Edge states: 2D QAH insulator



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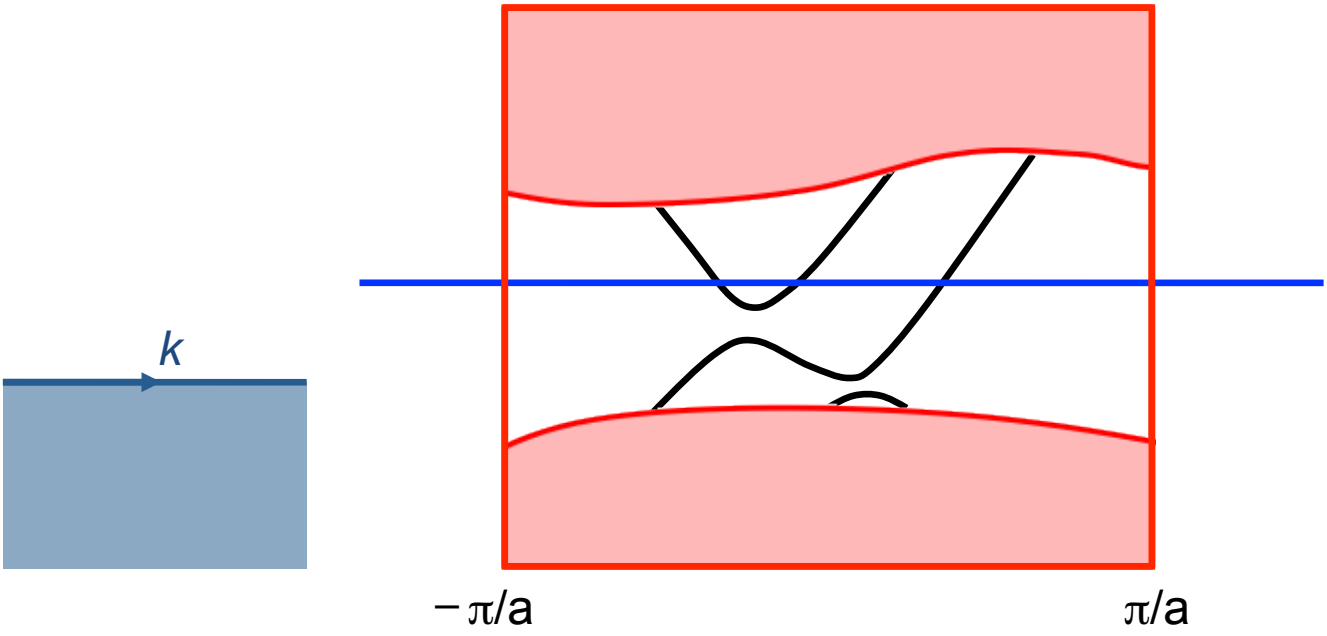

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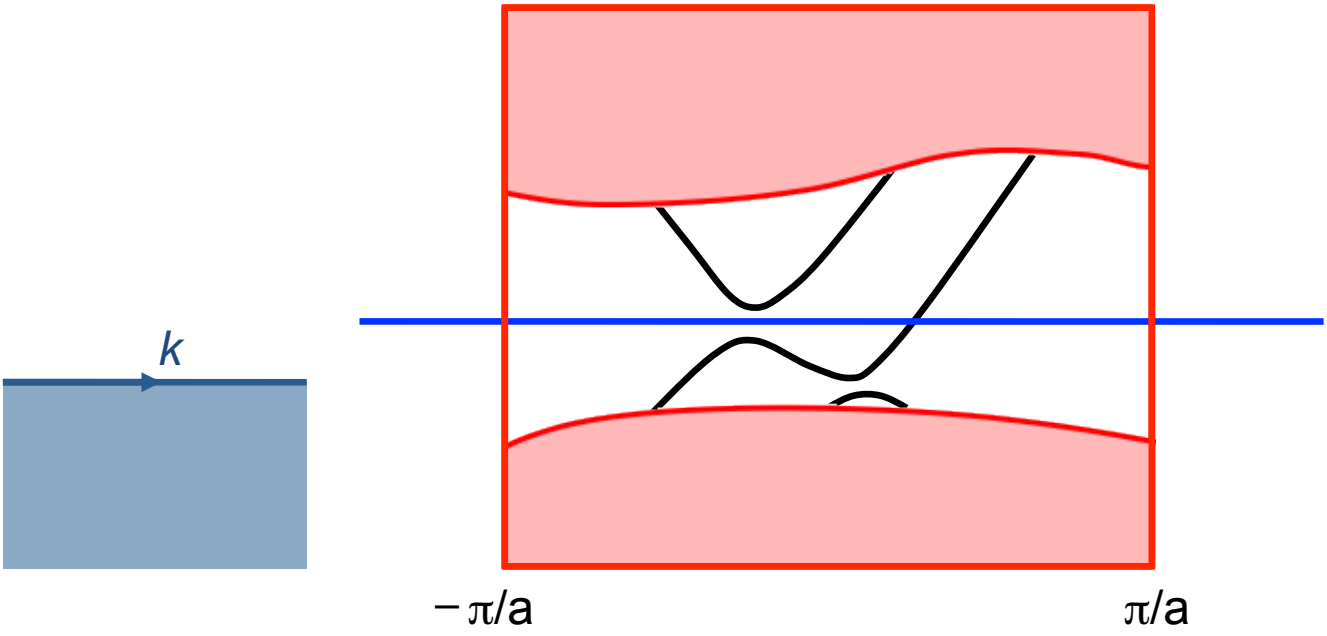
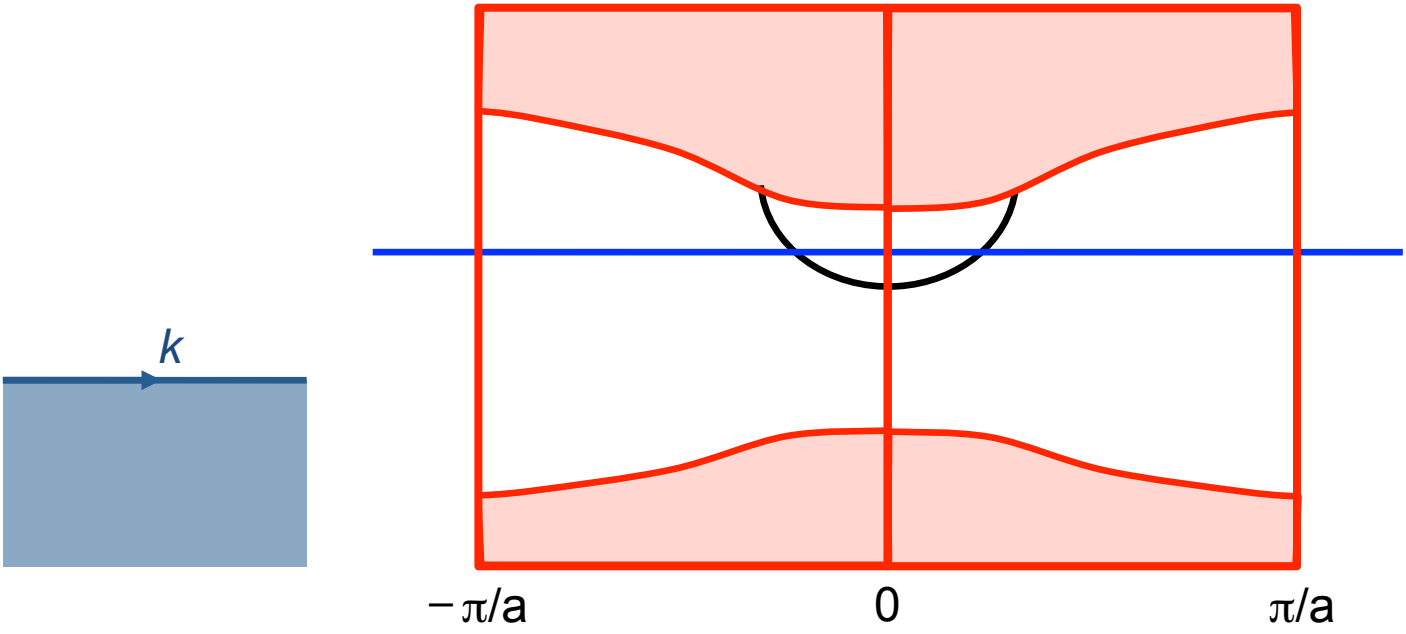


$$Z = N_{up} - N_{down} = \text{Invariant}$$

Edge states: 2D TR-invariant insulator



$$Z = N_{up} - N_{down} = 0$$

Time-reversal symmetry (TR)

H obeys TR if $H\Theta = \Theta H$, Θ = TR op.

Θ is anti-unitary: $\Theta(a|\psi\rangle) = a^*(\Theta|\psi\rangle)$

Bosons: $\Theta^2 = 1$ (integer spin systems)

Fermions: $\Theta^2 = -1$ ($\frac{1}{2}$-integer spins)

Ex: Spinor $\Theta\begin{pmatrix}1\\0\end{pmatrix} = \begin{pmatrix}0\\1\end{pmatrix}$ $\Theta\begin{pmatrix}0\\1\end{pmatrix} = \begin{pmatrix}-1\\0\end{pmatrix}$

Matrix rep: $\begin{pmatrix}0 & -1\\ 1 & 0\end{pmatrix} = -i\sigma_y$?

"$\Theta = -i\sigma_y K$" K = complex conjugation

Bloch state:

$\Theta|\psi_{n\underline{k}}\rangle = |\psi_{n,-\underline{k}}\rangle$ since $K e^{i\underline{k}\cdot\underline{r}} = e^{i(-\underline{k})\cdot\underline{r}}$

Kramers Theorem

Fermion case, $\Theta^2 = -1$

$H\Theta = \Theta H \Rightarrow$ if $H|\psi\rangle = E|\psi\rangle$

then $H(\Theta|\psi\rangle) = E(\Theta|\psi\rangle)$

Is $\Theta|\psi\rangle$ the same physical state as $|\psi\rangle$?

Assume yes: $\Theta|\psi\rangle = e^{i\varphi}|\psi\rangle$

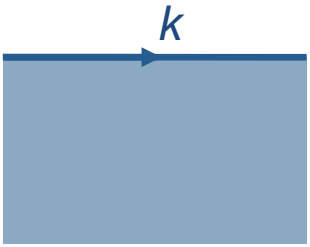
$$\Theta^2|\psi\rangle = \Theta(e^{i\varphi}|\psi\rangle)$$
$$= e^{-i\varphi}(\Theta|\psi\rangle)$$
$$= e^{-i\varphi} e^{i\varphi}|\psi\rangle = |\psi\rangle$$

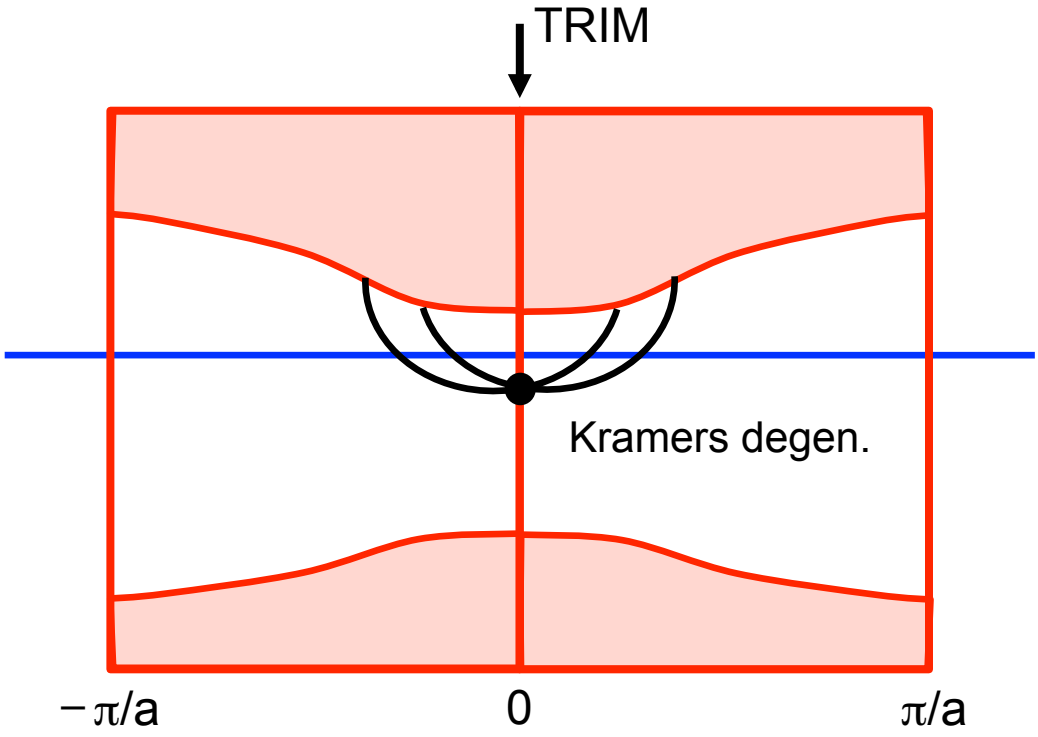
Inconsistent with $\Theta^2 = -1$!

$\Rightarrow$ $|\psi\rangle$ and $\Theta|\psi\rangle$ form degenerate "Kramers pair" or "Kramers doublet"

Edge states: 2D TR-invariant insulator

TRIM =
"Time reversal
invariant momenta"





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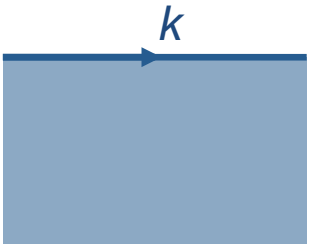

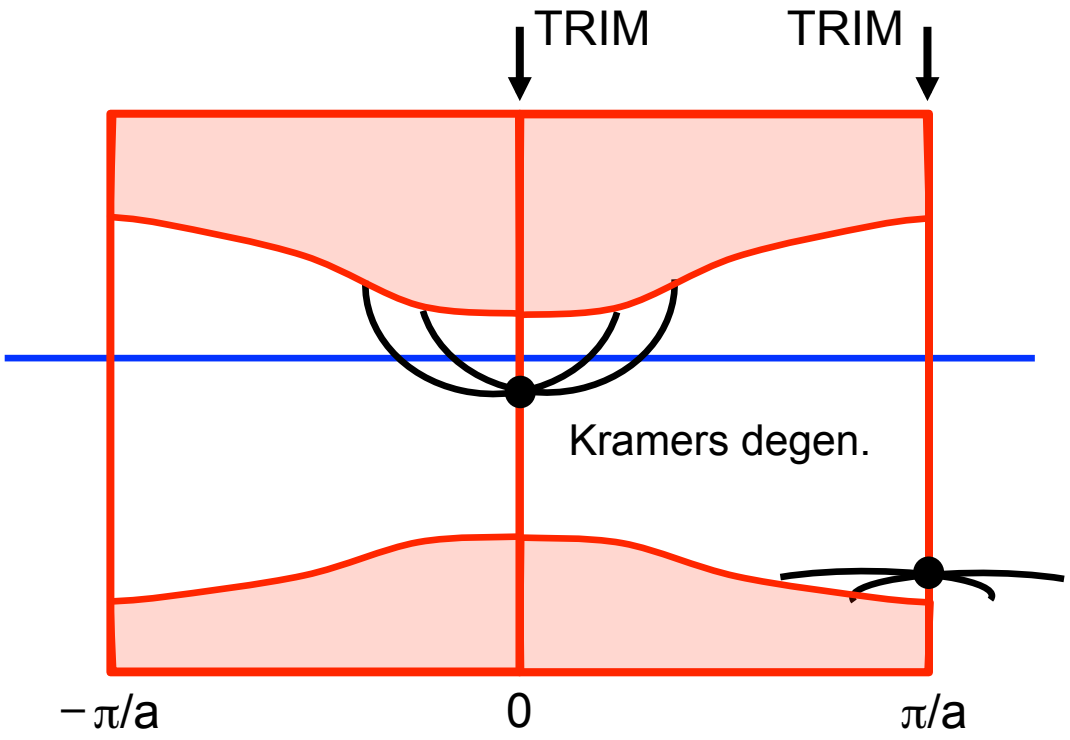

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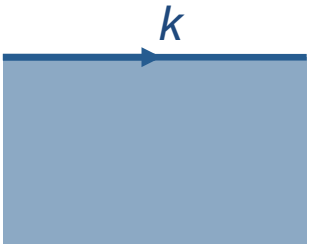

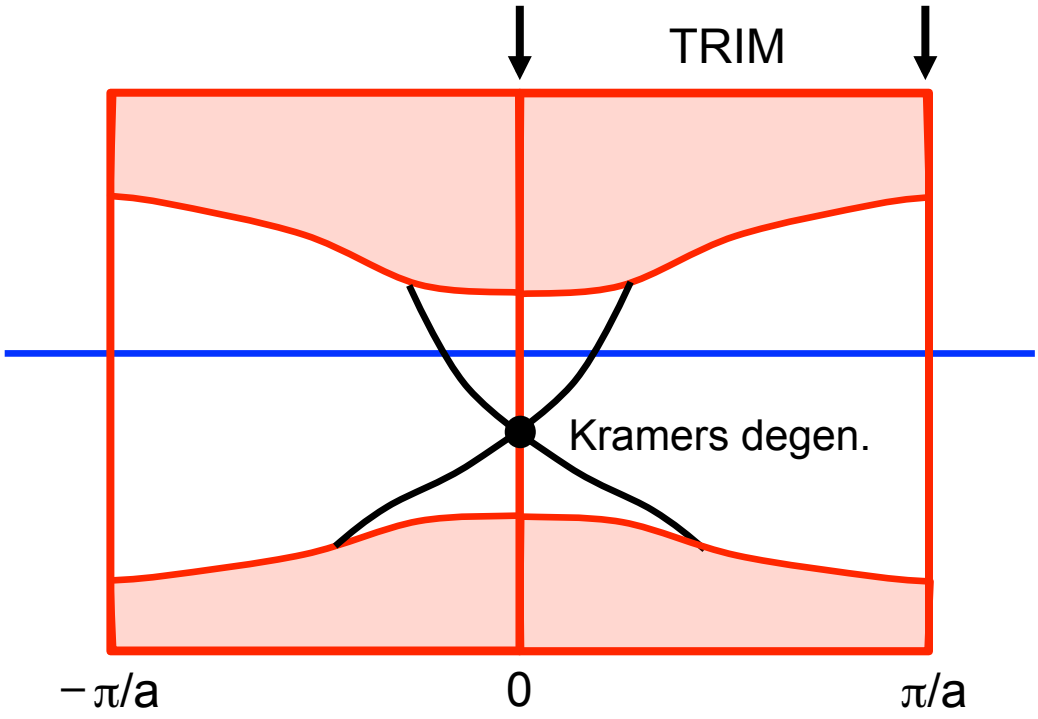

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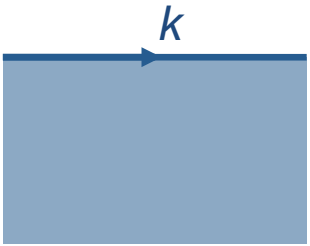

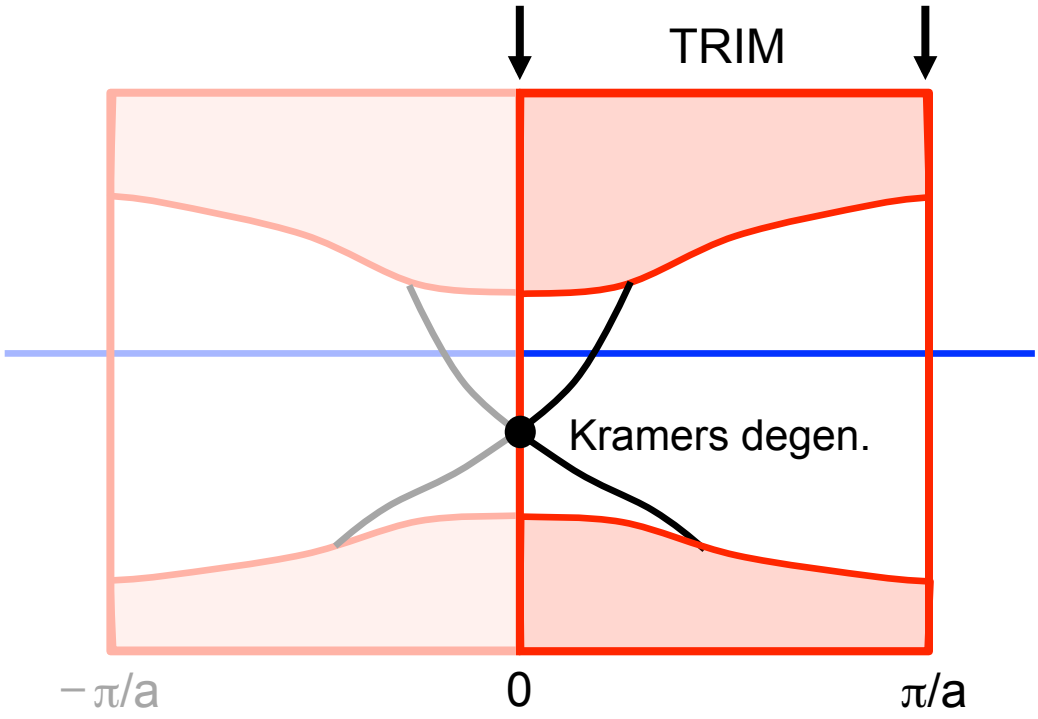

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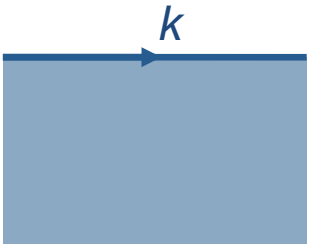

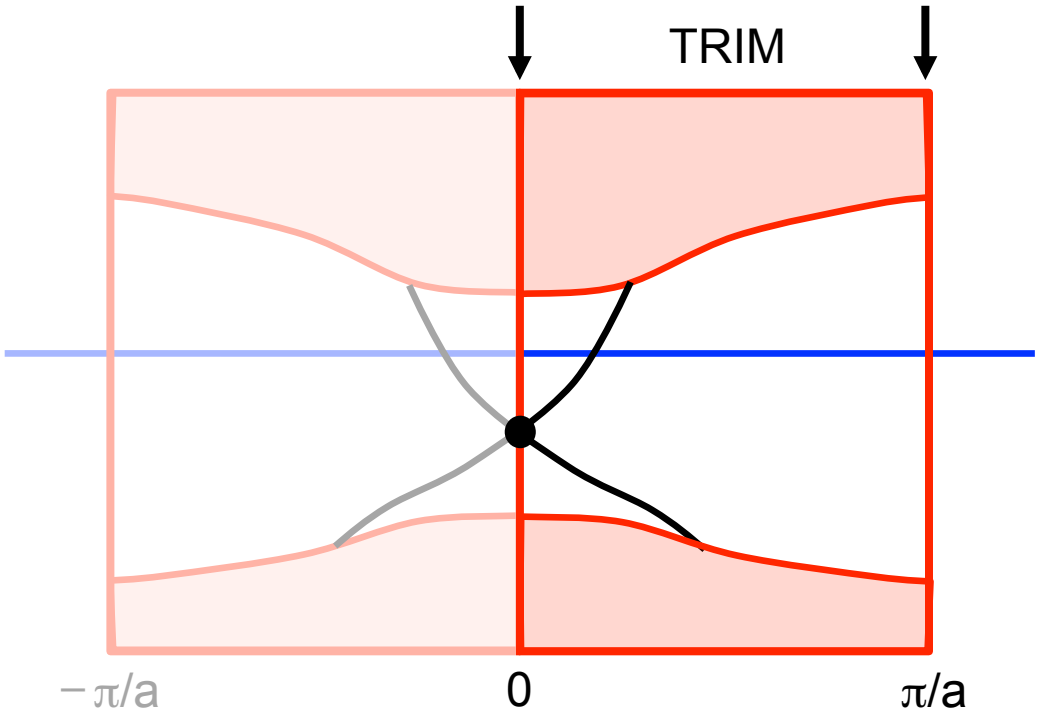

$Z_2 = N_{cross}$ (mod 2) = Invariant

Edge states: 2D TR-invariant insulator

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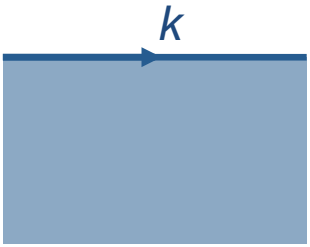

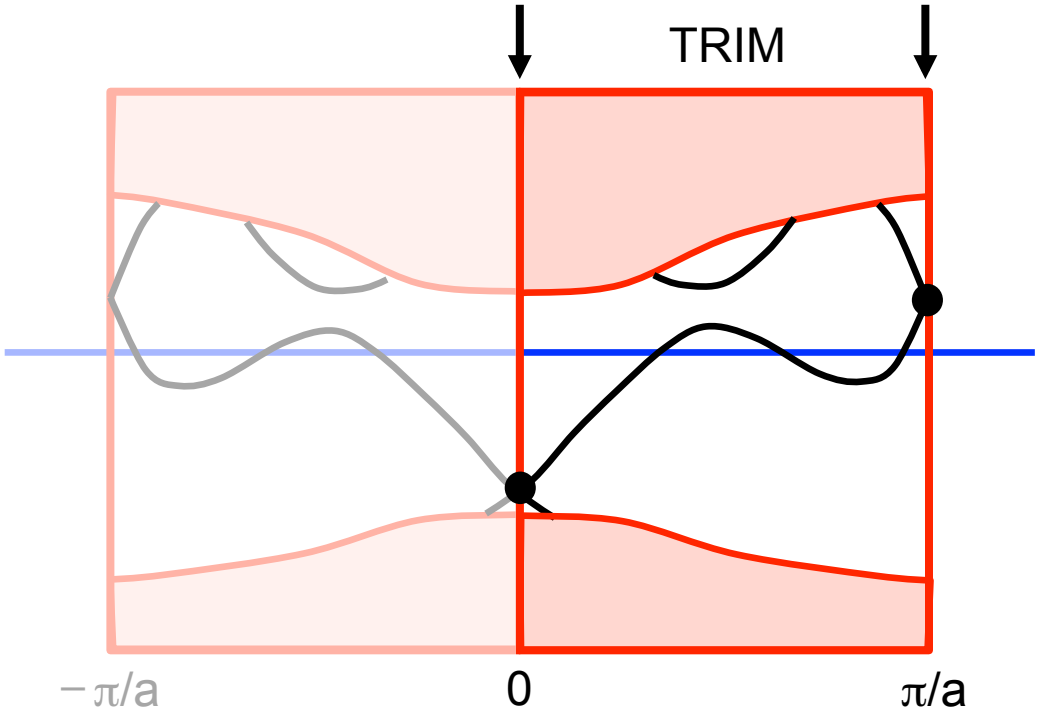

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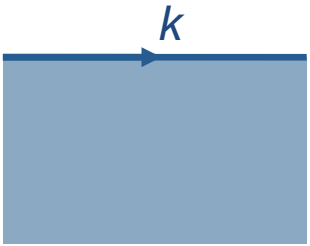

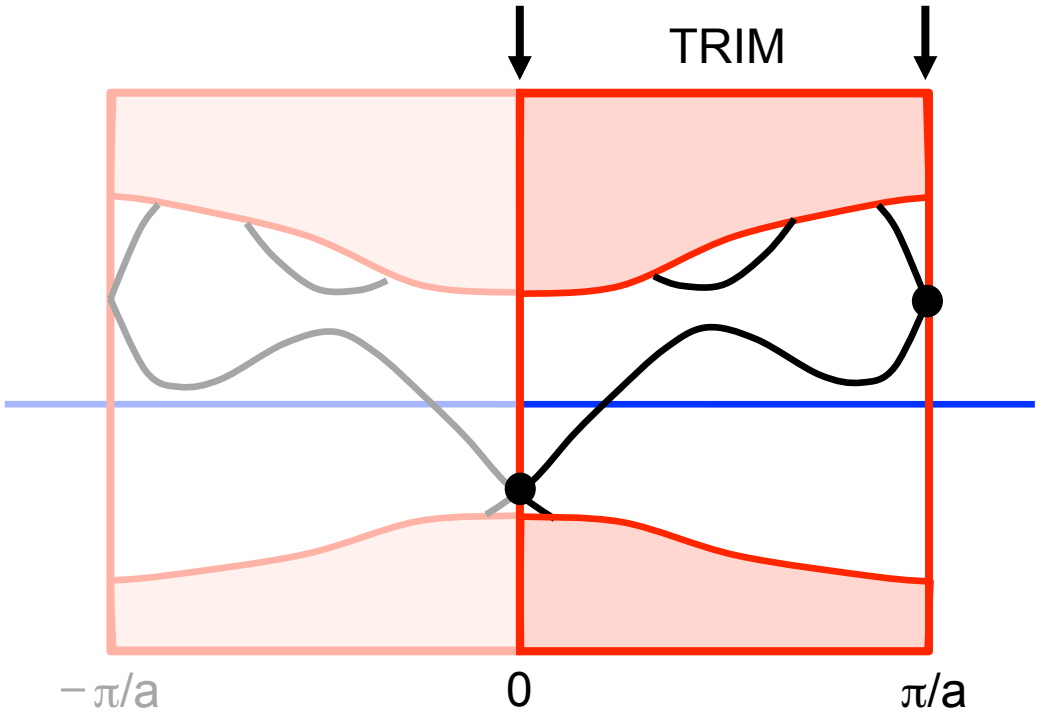

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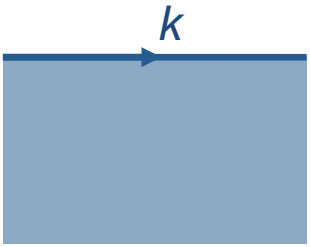

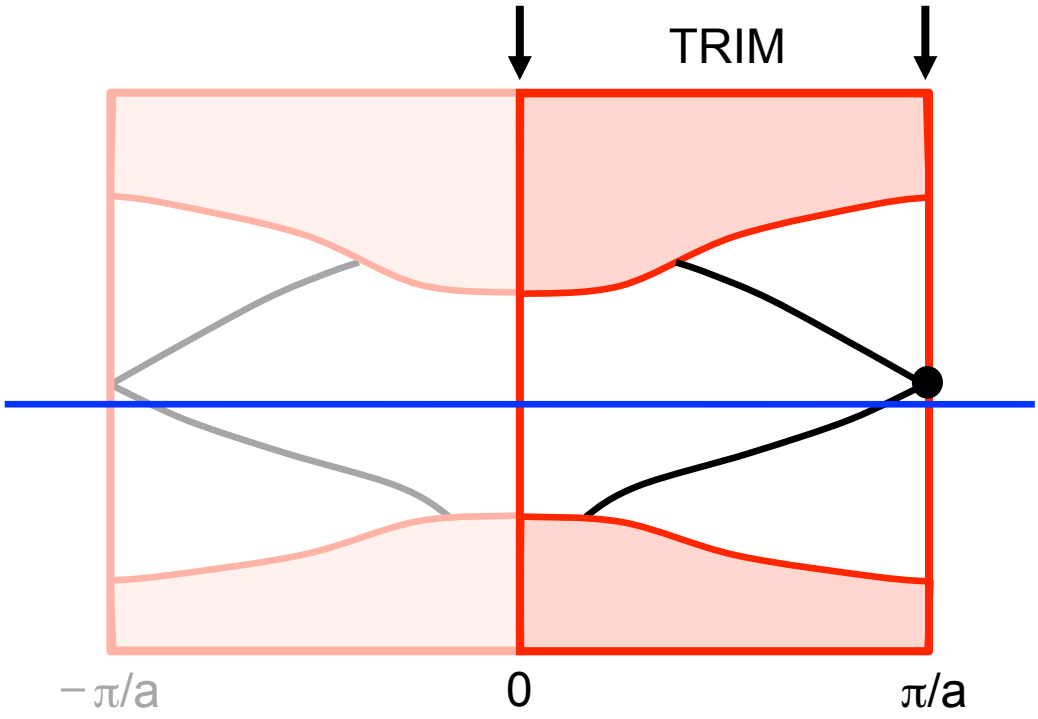

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