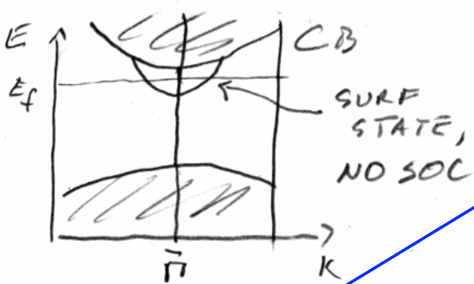
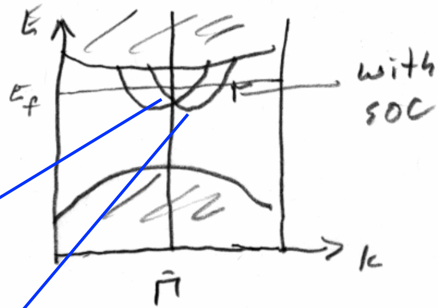
Surface Dirac cone

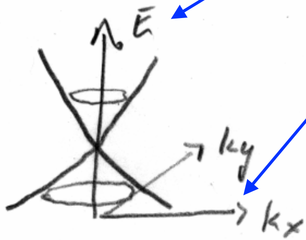
First, normal insulator (or at least, $\nu_0 = 0$)





2 Bands means we can write the ham in 2x2 matrix form

Blow up:



$$H_{\underline{k}} = E_0 + \overbrace{\boxed{\lambda_R (k_x \sigma_y - k_y \sigma_x)}}^{\text{Rashba Term}} \overbrace{\boxed{+ \frac{\hbar^2}{2m^*} k^2}}^{\text{Quadratic term}}$$

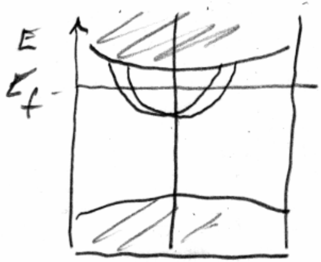
$$E_{\underline{k}} = E_0 + \frac{\hbar^2 k^2}{2m^*} \pm \lambda_R \sqrt{k_x^2 + k_y^2}$$

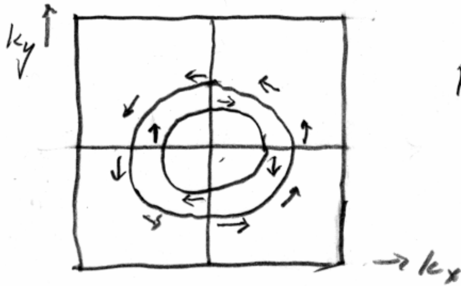
Terminology:

2D: This is "Dirac cone" (2 bands) c.f. graphene

3D: Dirac cone requires 4 bands

3D, two bands: "Weyl cone"



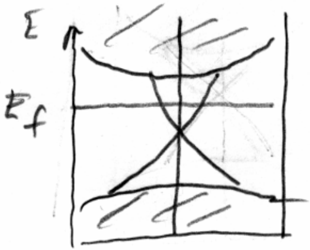


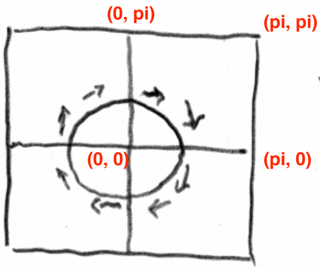
SPIN TEXTURE

$$H^R = \lambda_R (k_x \sigma_y - k_y \sigma_x)$$

So Dirac cones are allowed, but make <u>two</u> Fermi loops. Count crossings: $\nu_1 = \nu_1' = \nu_2 = \nu_2' = 0 \quad \nu_0 = 0$

<u>Strong TI</u>





$\nu_1 = \nu_2 = 1$

$\nu_1' = \nu_2' = 0$

$\nu_0 = 1$

$\boxed{\text{STI}}$

Single crossing up to half BZ. => strong TI

Map onto Bloch sphere:

$$H_{2\times2}(\underline{k}) = f_0(\underline{k})\,\mathbb{1}_{2\times2} + f_1(\underline{k})\,\sigma_1 + f_2(\underline{k})\,\sigma_2 + f_3(\underline{k})\,\sigma_3$$

$$E(\underline{k}) = f_0(\underline{k}) \pm \sqrt{f_1^2+f_2^2+f_3^2} = f_0(k) \pm f(\underline{k})$$

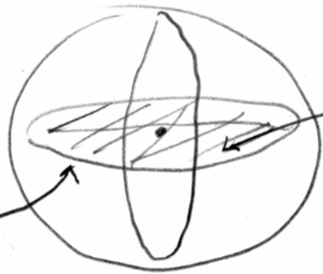
Eigenfunctions are indep. of f_0, so focus on

$$\underset{\sim}{f}(\underline{k}) = (f_x, f_y, f_z), \quad f(k) = |\underline{f}(\underline{k})|$$

Our case:

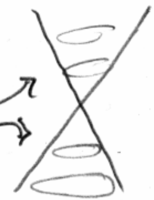
$$H = k_x \sigma_y - k_y \sigma_x \quad \text{or} \quad \underline{f} = (-k_y, k_x, 0)$$

Bloch sphere:



on this plane.

$\Omega_{xy} = 0$.



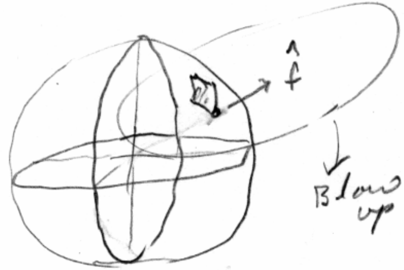
But $\phi = \pi$

Stokes?

[Could also be $\underset{\sim}{f} = (k_x, k_y, 0)$ or $(k_y, k_x, 0)\ldots$]

Spinor as function of two parameters

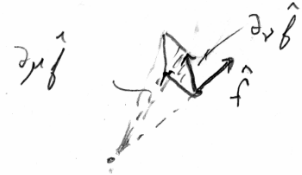
$$H = f_1(\underline{\lambda})\,\sigma_1 + f_2(\underline{\lambda})\,\sigma_2 + f_3(\underline{\lambda})\,\sigma_3$$



$\underline{\lambda} = (\mu, \nu)$ $\quad$ $(= (k_x, k_y)$ in k-space$)$

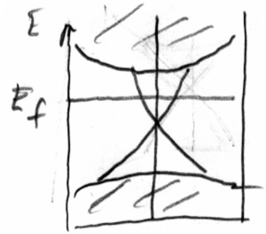
$d\omega$ = element of solid angle in $\hat{f}$ orientation space.

$$\frac{\partial^2 \omega}{\partial \mu\, \partial \nu} = \frac{(\partial_\mu \underline{f}) \times (\partial_\nu \underline{f}) \cdot \underline{f}}{f^3}$$



$$\frac{\partial \phi_{Berry}}{\partial \omega} = -\frac{1}{2} \quad \Rightarrow \quad \Omega_{\mu\nu} = -\frac{1}{2}\, \frac{(\partial_\mu \underline{f}) \times (\partial_\nu \underline{f}) \cdot \underline{f}}{f^3}$$

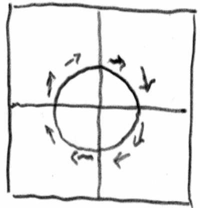
Or $\boxed{\Omega_{\mu\nu} = -\frac{1}{2} (\partial_\mu \hat{f}) \times (\partial_\nu \hat{f}) \cdot \hat{f}}$



Question: If Berry phase $= \pi$, does it mean that

$$\sigma_{AHE}^{surf} = \frac{e^2}{h}\frac{\phi}{2\pi} = \frac{e^2}{2h} \quad \text{(half quantum)?}$$

No: TR not broken, $\Rightarrow \sigma_{AHE}^{surf} = 0$



Hint: There is "bulk contribution" of π

$$\text{Total} = \pi + \pi = 2\pi = 0 \quad (??)$$

Break TR at surface (but not in the bulk):

$$H_{\underline{k}} = \underbrace{\lambda_R (k_x \sigma_2 - k_y \sigma_1)}_{\text{conserves TR}} + \underbrace{\Delta \sigma_3}_{\text{Breaks TR};\ \underline{B} \parallel \hat{z}}$$

Note, at $k_x = k_y = 0$, $\Theta \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$, $\Theta \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} -1 \\ 0 \end{pmatrix}$

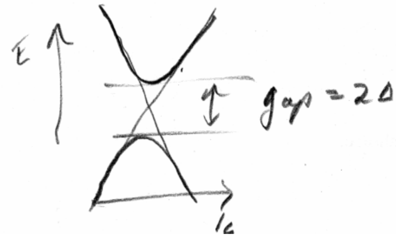
They "act like spins." "PSEUDOSPIN"

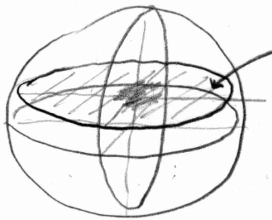
TR: $\underline{k} \to -\underline{k}$, $\underline{\sigma} \to -\underline{\sigma}$, $H_R \to +H_R$, $H_z \to -H_z$

$\hookrightarrow k_x \sigma_2 - k_y \sigma_1$ $\hookrightarrow \sigma_3$

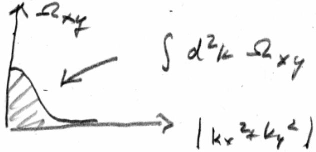
$$H_{\underline{k}} = \lambda_R (k_x \sigma_2 - k_y \sigma_1) + \Delta \sigma_3$$

$$E_{\underline{k}} = \pm \sqrt{\lambda_R^2 k^2 + \Delta^2}$$ "Gapped Dirac cone"



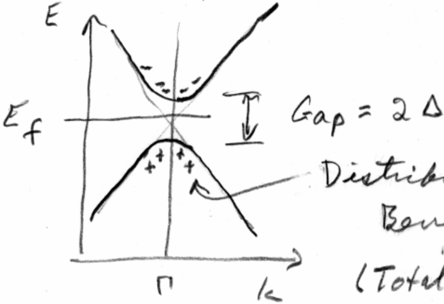


f_3 = const plane



$$\int d^2k \, \Omega_{xy} = \pm \pi$$

$$\boxed{\Omega_{\mu\nu} = -\frac{1}{2} (\partial_\mu \hat{f}) \times (\partial_\nu \hat{f}) \cdot \hat{f}}$$

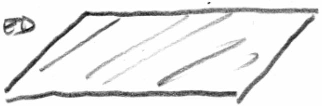


Distribution of Berry curvature (Total flux = ± π)

"Gapped surface reveals bulk contribution of ± π"

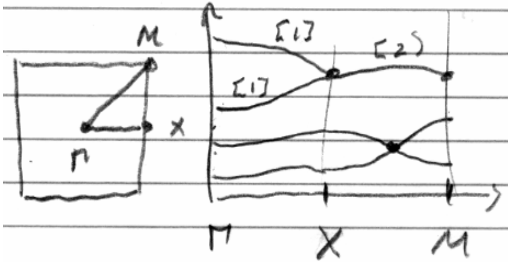
BERRY PHASE ϕ

	INSULATING SURF	METALLIC SURF
ISOLATED 2D	$\sigma^{2D}_{AHC} = 0 \mod \frac{e^2}{h}$	$\sigma^{2D}_{AHC} = \frac{\phi}{2\pi}\frac{e^2}{h} \mod \frac{e^2}{h}$
SURFACE OF NORMAL INS.	$\sigma^{SURF}_{AHC} = 0 \mod \frac{e^2}{h}$	$\sigma^{SURF}_{AHC} = \frac{\phi}{2\pi}\frac{e^2}{h} \mod \frac{e^2}{h}$
SURFACE OF STRONG TI	NOTE $\sigma^{SURF}_{AHC} = \pi \mod \frac{e^2}{h}$	NOTE $\sigma^{SURF}_{AHC} = \frac{(\phi+\pi)}{2\pi}\frac{e^2}{h} \mod \frac{e^2}{h}$



TYPO
it has to be e^2/2h mod e^2/h

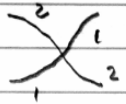
Band structure in 2D:



Band touchings?

- TR: Kramers degen at TRIM
- Crystalline symmetries

But are there generic "accidental" crossings?



Approx $H_{2\times 2} = f_0 \mathbb{1} + \underline{f} \cdot \underline{\sigma}$ $\qquad E = f_0 \pm |\underline{f}|$

So for degen. we need:
$$\begin{cases} f_x(k_x, k_y) = 0 \\ f_y(k_x, k_y) = 0 \\ f_z(k_x, k_y) = 0 \end{cases}$$

3 eq. for 2 unknowns ⇒ does <u>not</u> generically have a solution.

Codimension

V = full k-space (here $\dim[V] = 2$)

W = dim. of node in k-space (here $\dim[W] = 0$)

Codim = $\dim[V] - \dim[W]$ (here $= 2$)

Codim must $= \dim[\{\psi\}]$ (here $= 3$)

to get such a node generically.

- Can we have generic line nodes in 3D?
- Can we have generic point nodes in 3D?