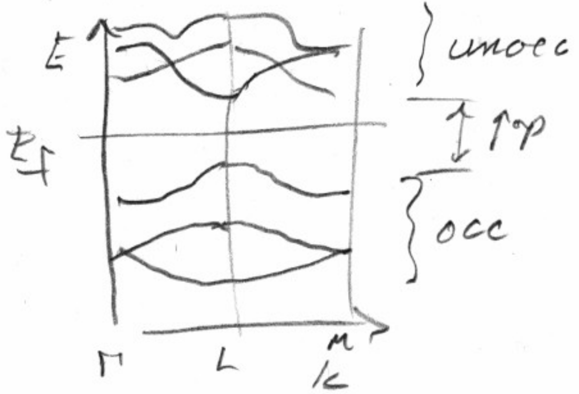
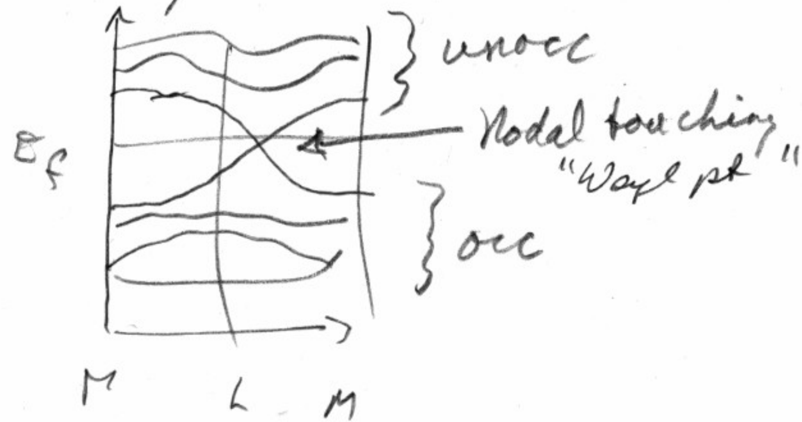
Weyl semimetals

Normal insulator



Weyl semimetal

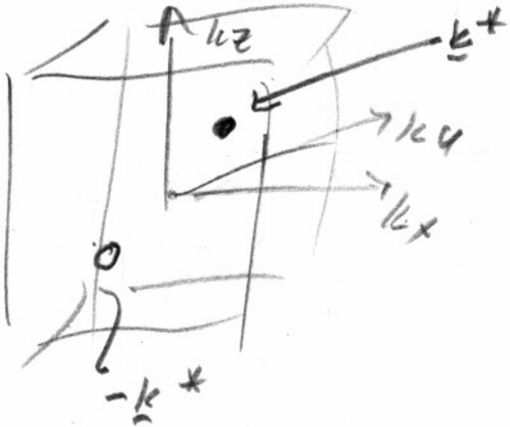


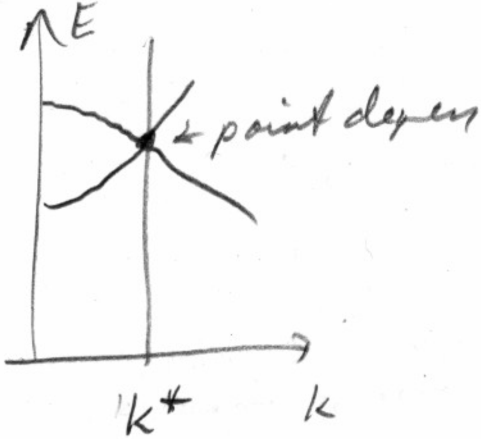
Note: There might be lots of Weyl points deep in valence or conduction bands.

We are mainly interested in ones between valence + conduction bands.

Origin? : (a) Symmetry (b) Generic

Generic Weyl nodes:





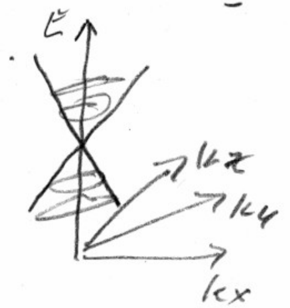
At arbitrary $\underline{k}$, neither I nor TR is a good symmetry

[Special case: crystal has I and TR;
then I*TR maps $\underline{k} \to \underline{k}$, get Kramers degen
at all $\underline{k}$. For now, assume this is not the case.]

drop

$H_{2\times 2}(\underline{k}) = f_0(\underline{k}) + \underline{f}(\underline{k})\cdot\underline{\sigma}$ Really $\underline{k}' = \underline{k} - \underline{k}^*$

Example: $\underline{f}(\underline{k}) = v_0(k_x, k_y, k_z)$ $H = v_0\left(k_x\sigma_x + k_y\sigma_y + k_z\sigma_z\right)$



Hard to draw on 2D paper!

$E = \pm v_0 k$ $v_0 =$ "Fermi veloc."

Chirality:

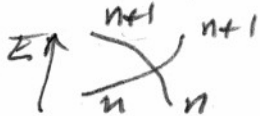
Let $A_{ij} = \dfrac{\partial f_i}{\partial k_j}$ $A = \begin{pmatrix} \partial f_1/\partial k_x & \partial f_1/\partial k_y & \partial f_1/\partial k_z \\ \partial f_2/\partial k_x & \cdots & \cdots \\ \partial f_3/\partial k_x & \cdots & \cdots \end{pmatrix}$

(above: $A = v_0 \mathbb{1}_{3\times 3}$)

$$\boxed{\chi = \operatorname{sgn} \det A}$$

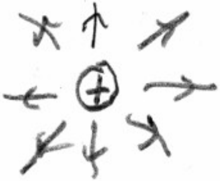
E.g., $\underline{f} = v_0(k_x, -k_y, k_z)$ has $\chi = -1$

If $\chi = +1$:



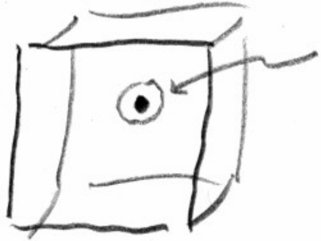
$$\left.\begin{array}{l}\text{States in band } n+1 \text{ see } -2\pi \text{ Berry flux} \\ \quad\text{"} \quad\ \text{"} \quad\ \text{"} \quad n \text{ see } +2\pi \text{ Berry flux}\end{array}\right\}$$

"Magnetic monopoles in k-space"



band n

band $n+1$



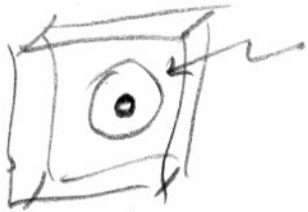
$C_n = +1$

$C_{n+1} = -1$

If $\chi = -1$:

band n

band $n+1$

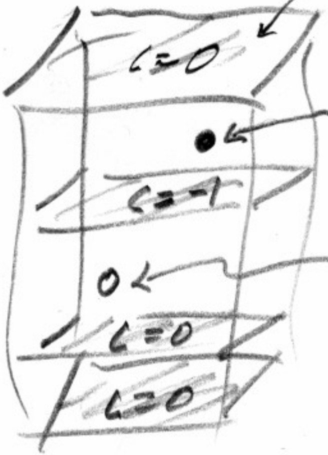


$C_n = -1$

$C_{n+1} = +1$

Nielsen-Ninomiya theorem

c = Chern # of band n (or sum over $1 \dots n$)



$\chi = +1$

$\chi = -1$

Slice Chern number must return

$$\boxed{\sum_j \chi_j = 0}$$

to all Weyl points between n and $n+1$ in 3D BZ

see p. 260 in text for careful argument.