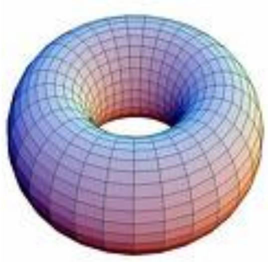
# Chern number for two "parameters"

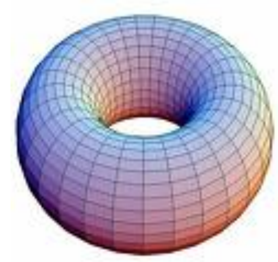
$$(\lambda_x, \lambda_y) \Rightarrow (k, \lambda) \Rightarrow (k_x, k_y)$$

Parametric Hamiltonian of molecule



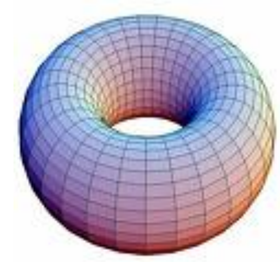
$C \neq 0$

Hamiltonian of chain with one control parameter



Quantum charge pump

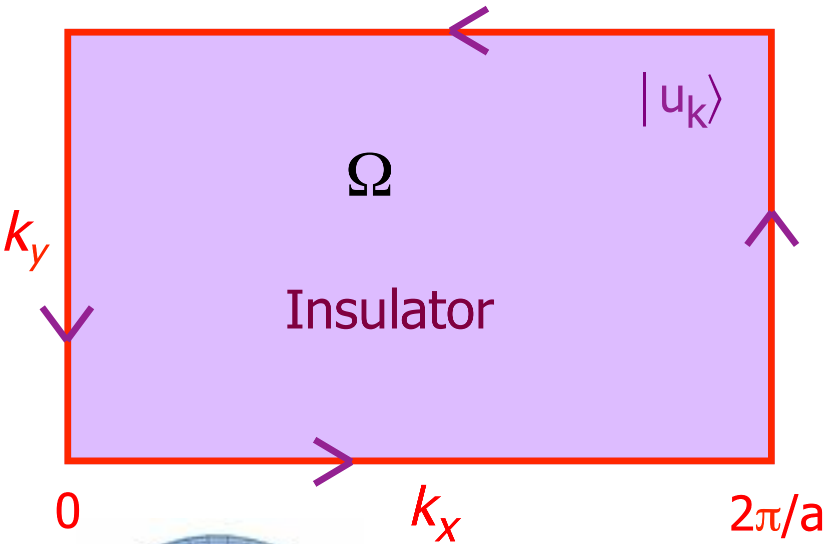
Hamiltonian of 2D insulator

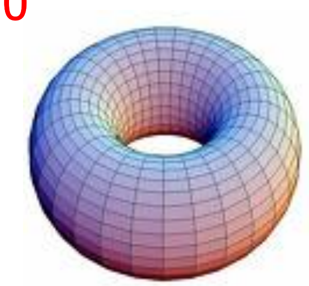


Quantum anomalous Hall insulator

RUTGERS

# Berry curvature in the Brillouin zone



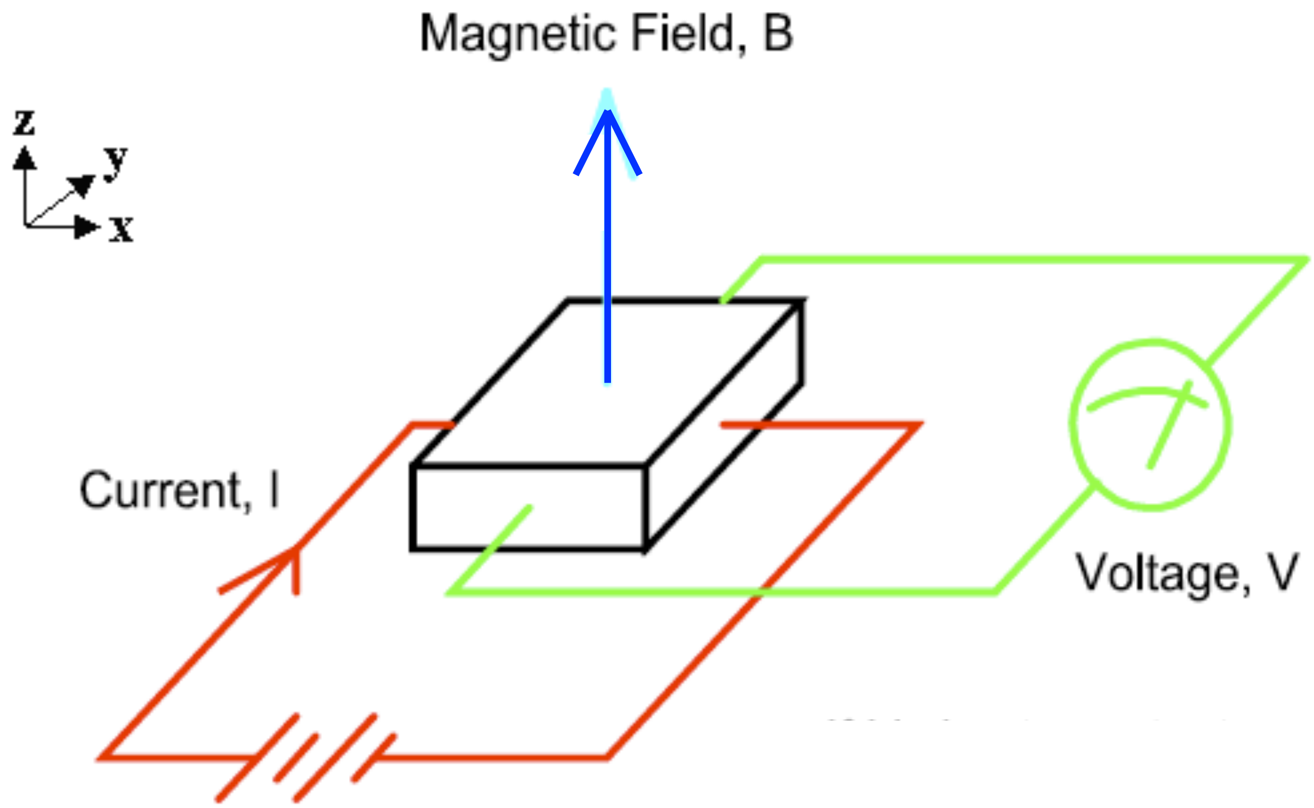
$$\boxed{\Omega_z(\mathbf{k}) = -2\,\mathrm{Im}\left\langle \frac{du}{dk_x} \middle| \frac{du}{dk_y} \right\rangle}$$

$$\int_{BZ} \Omega_z(\mathbf{k})\, d^2k = 2\pi C$$

Chern number

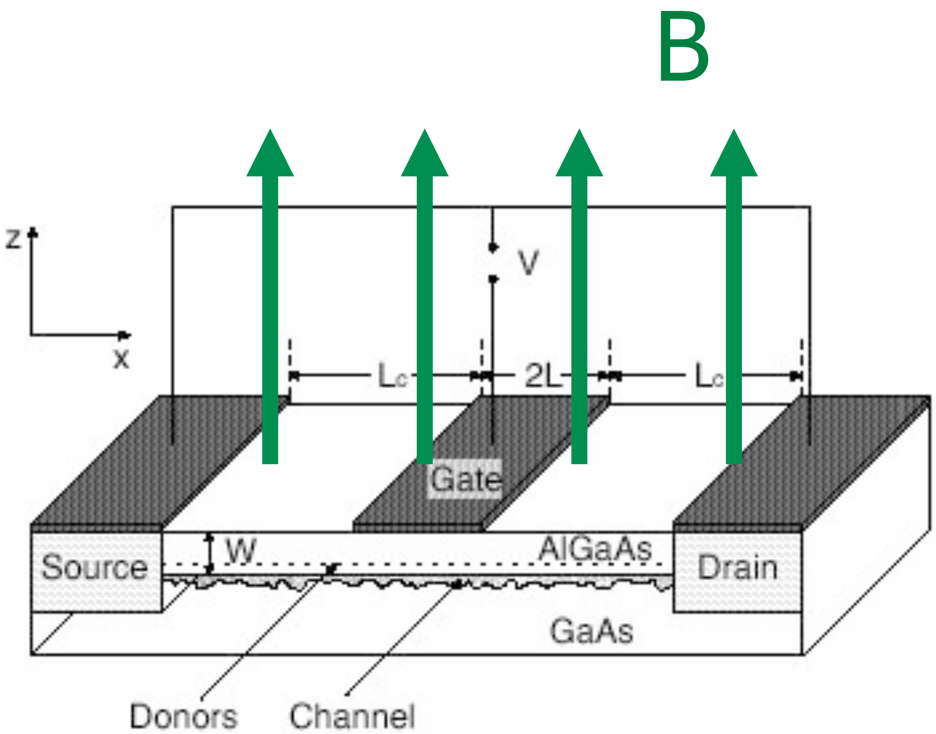
- $C$ is usually zero
  - Always if non-magnetic
  - Usually otherwise
- But what if it is not?

# Ordinary Hall conductivity

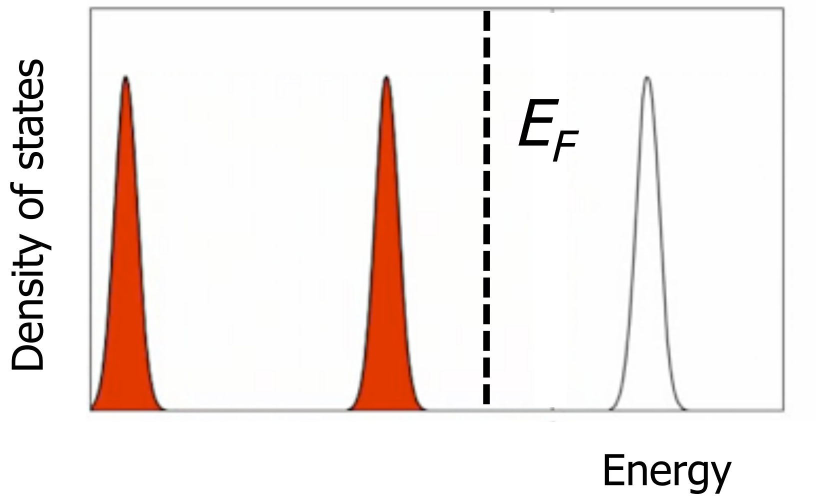


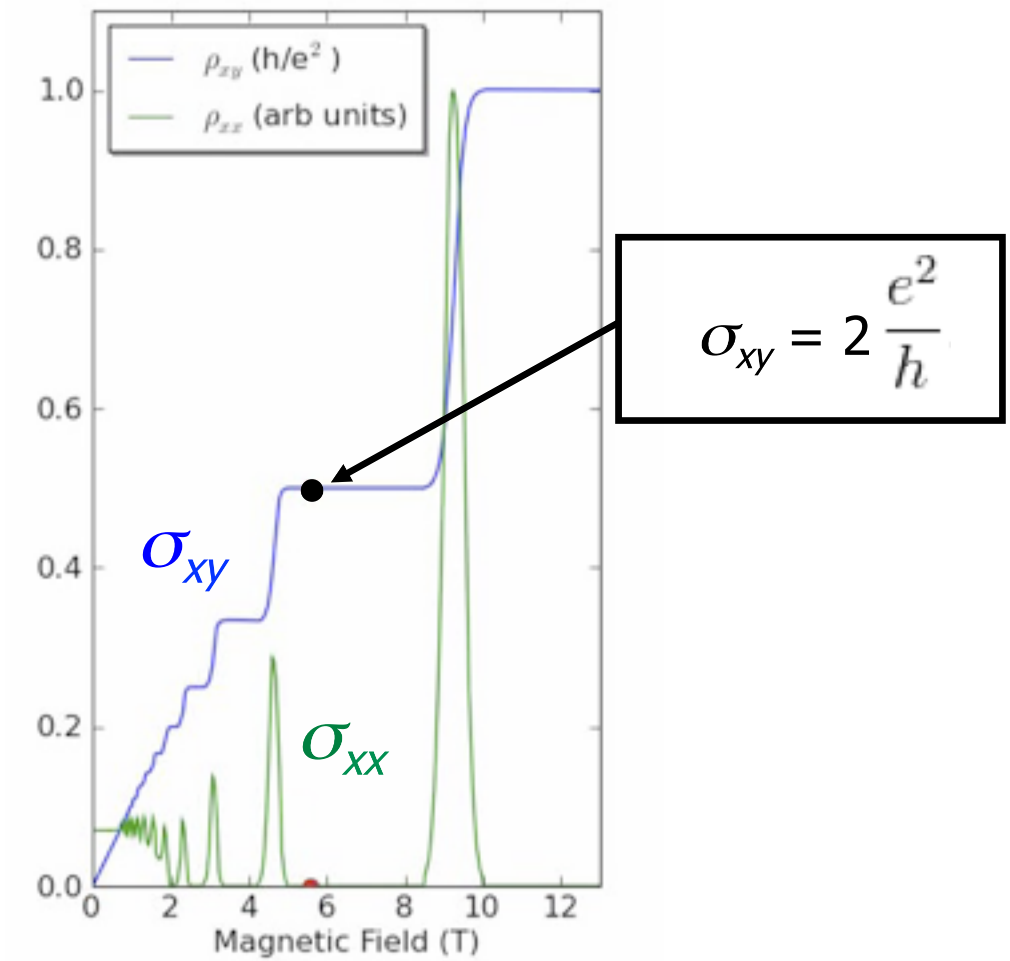

Measure $\sigma_{xy}$ in presence of *B*-field

# Quantum Hall effect



# Quantum Hall effect

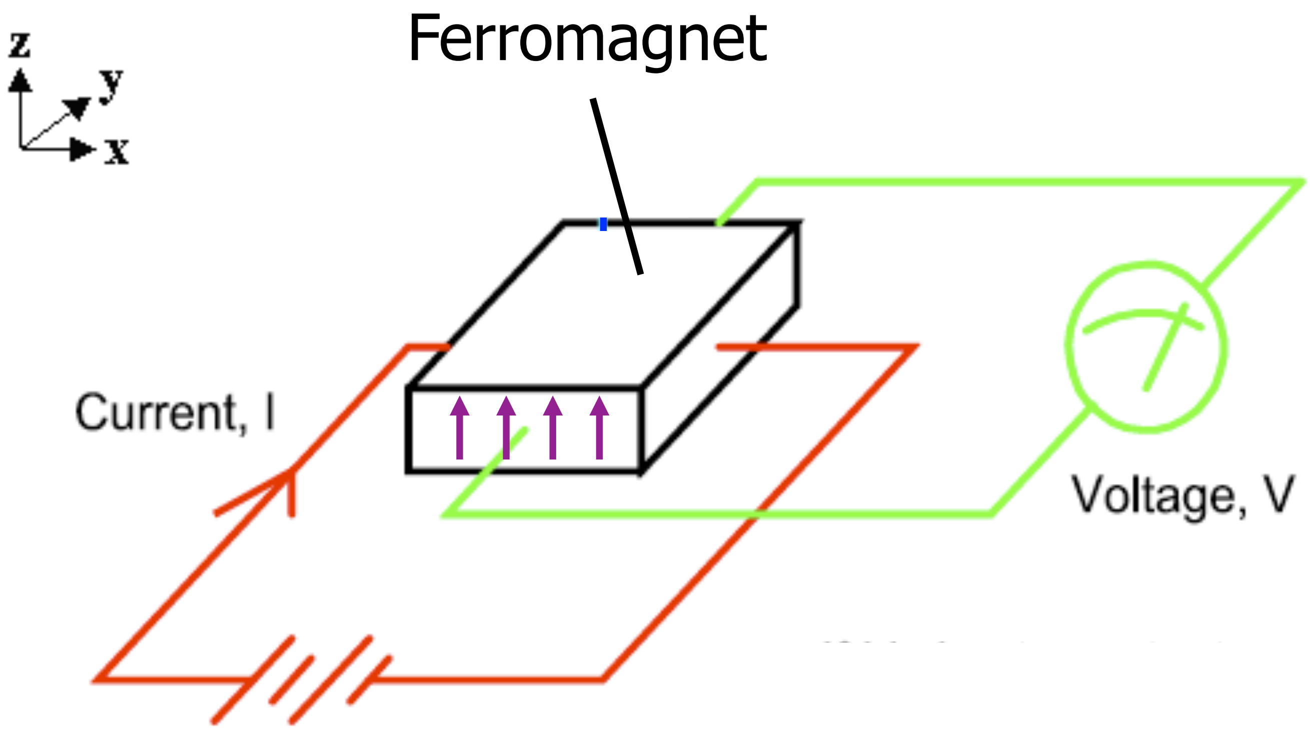


# Hall effects: The big picture

| | Induced by B-field | Ferromagnetic sample |
|---|---|---|
| Metal | Ordinary Hall (1879) | |
| Topological insulator | Quantum Hall (1980) | |

# Anomalous Hall conductivity (AHC)

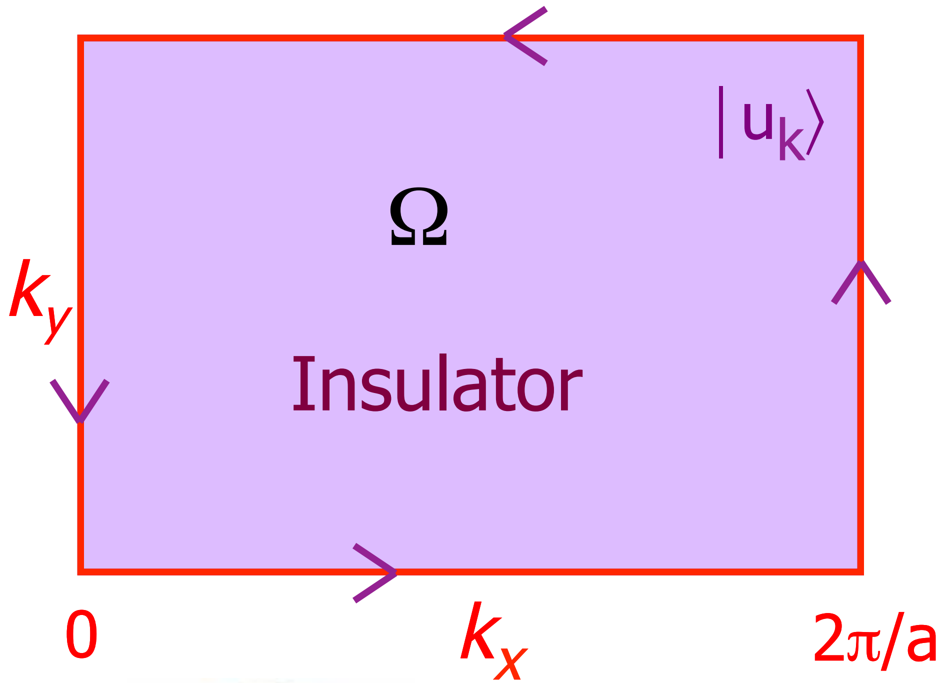



Measure $\sigma_{xy}$ in ***absence*** of *B*-field

# Hall effects: The big picture

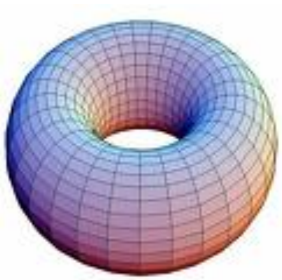
| | Induced by B-field | Ferromagnetic sample |
|---|---|---|
| Metal | Ordinary Hall (1879) | Anomalous Hall (1881) |
| Topological insulator | Quantum Hall (1980) | Quantum Anomalous Hall ? |

# QAH insulator




$$\boxed{\Omega_z(\mathbf{k}) = -2\mathrm{Im}\left\langle \frac{du}{dk_x} \middle| \frac{du}{dk_y} \right\rangle}$$

$$\int_{BZ} \Omega_z(\mathbf{k})\, d^2k = 2\pi C$$



Quantum Anomalous Hall: $\boxed{\sigma_{xy} = \frac{-e^2}{h} C}$

Chern number