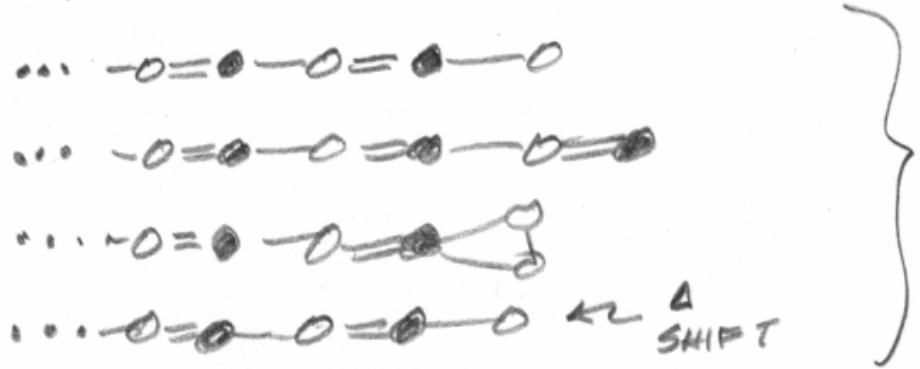
Surface charge theorem

Recall $P = \frac{e}{a}\left(\underbrace{\sum_j z_j \tau_j}_{\text{ions}} - \underbrace{\sum_n \bar{x}_n}_{\text{electrons}}\right) \quad (\text{mod } e)$

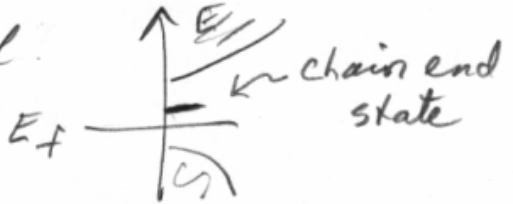
Theorem: $Q_{surf} = P \quad (\text{mod } e)$

do for example



Q_{surf} same mode!

Fermi level



vs.

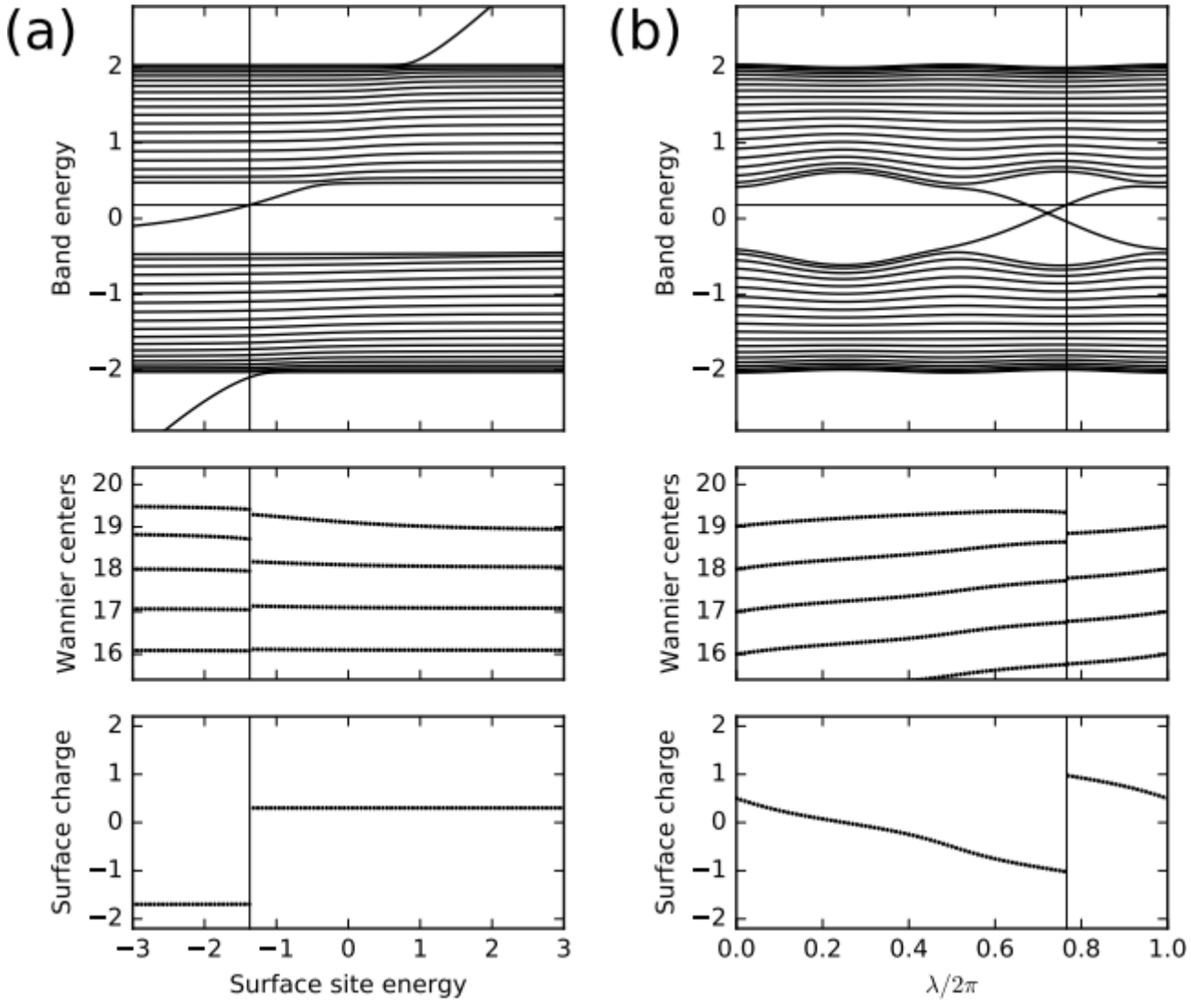
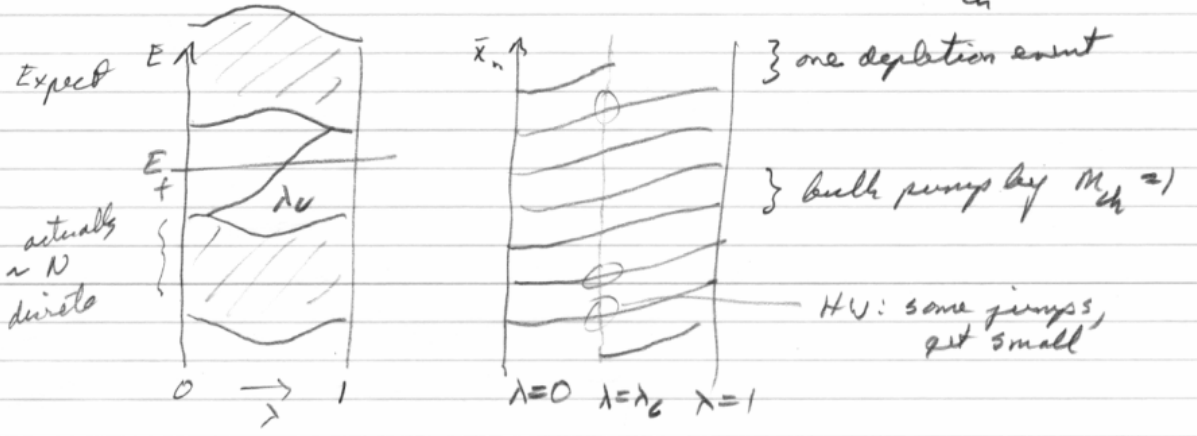

(a)
(b)
Band energy
Wannier centers
Surface charge
Surface site energy
$\lambda/2\pi$

Fig. 4.16

Adiabatic pump

such that $\Delta P = -e$

$(n_{ch} = 1)$



∴ # of up-crossings of surface states

must = Chern # (Wannier winding #) of bulk

in order for charge cons. to hold

"Bulk-surface correspondence"

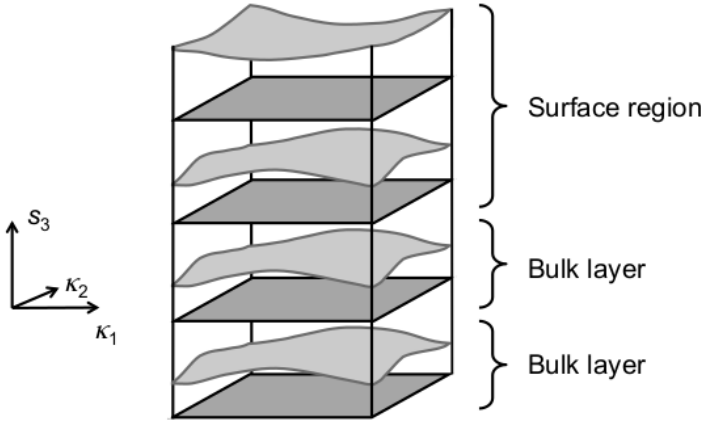
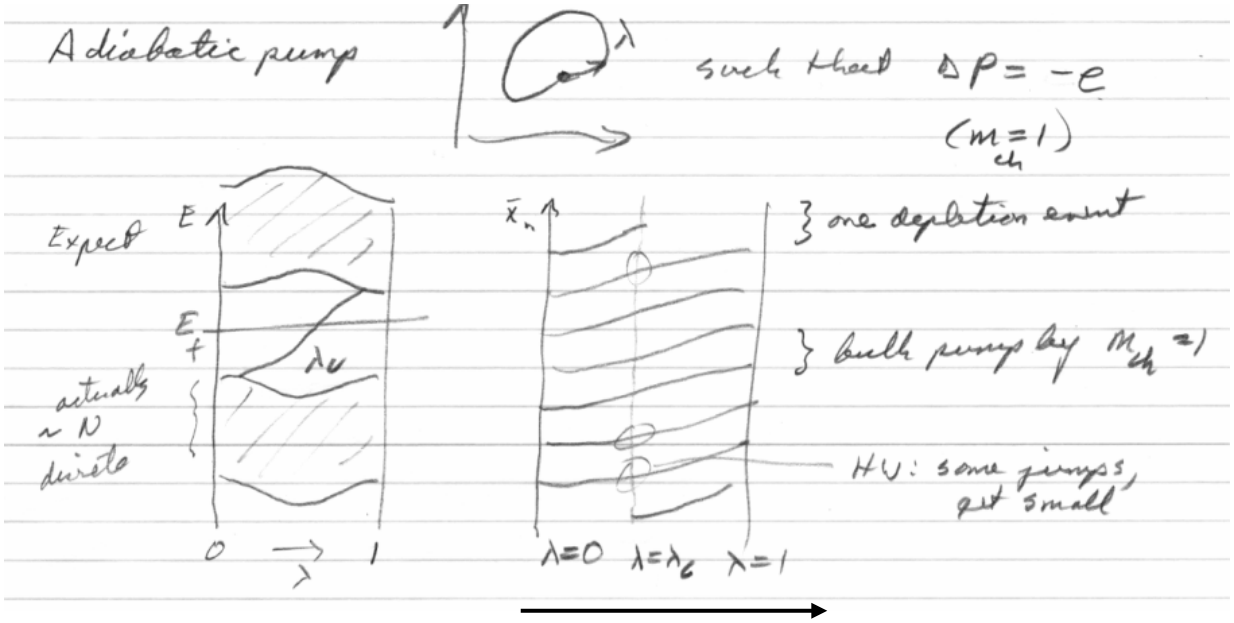


Figure 4.17

$$\sigma_{\text{surf}} := \frac{1}{A_{\text{surf}}}\, p_3$$

$$p_{\text{elec},3} = \frac{1}{(2\pi)^2} \sum_n \iint p_{n3}(\kappa_1, \kappa_2)\, d\kappa_1 d\kappa_2\,, \tag{4.99}$$



Could be k_y or k_z instead