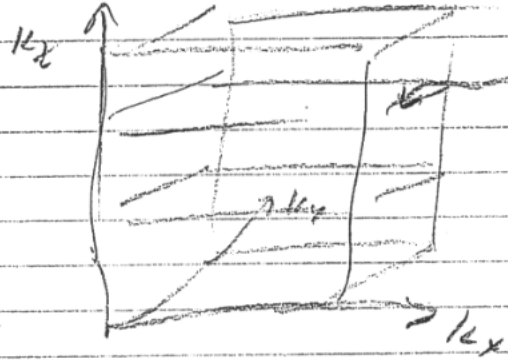
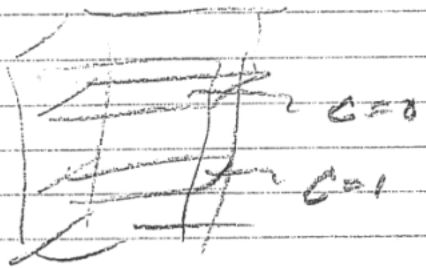
$\boxed{\text{3D QAH ins}}$

$C \Rightarrow m^{ch}$




For given $k_z$, $H_{k_z}(k_x, k_y)$ acts like Ham of 2D system
- Ins
- $C = 0, \neq 0$ ?

So it goes




?

No!

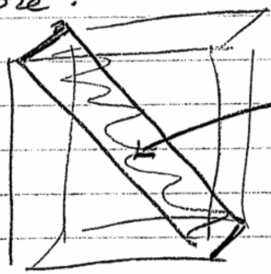
Imagine smoothly changing $k_z$ ...

No gap in 3D k-space, by assump!

So clearly there are 3 top. invariants :

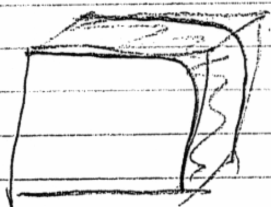
$C_x$ , $C_y$ , $C_z$ (Triplet of integers).

Any more?



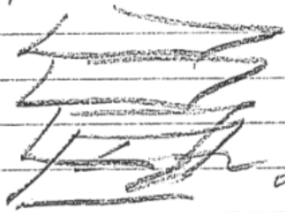
$C = ?$ $= C_x + C_z$ !

Why? Deform to



$= C_x + C_z$ !

Simplest case:




$\Longrightarrow$ $C_x = C_y = 0$

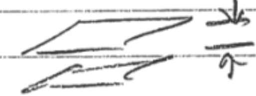
$C_z = 1$

If $C_x = C_y = 1$,


So any 3D QAH ins is equiv to 2D QAH insulator stacked up.

Nothing really new in 3D.

Also $\sigma_{yx}^{(3D)} = \frac{e^2}{h}\frac{1}{d} C_z$



$d$ = latt const along z

No longer universal

2D QAH = "Strong topo ins"
3D QAH = "Weak topo ins"
} discuss

# General 3D QAH insulator

Note $\underline{\underline{\sigma}}^{AHC} = \begin{pmatrix} 0 & a & -c \\ -a & 0 & b \\ c & -b & 0 \end{pmatrix} \iff \underline{\sigma}^{AHC} = \begin{pmatrix} a \\ b \\ c \end{pmatrix}$

Statement without proof here:
3D insulator is characterized by a recip. latt. vec.

$$\underline{G}^{\text{Chern}} = m_1 \underline{b}_1 + m_2 \underline{b}_2 + m_3 \underline{b}_3$$

Chern indices are $(m_1, m_2, m_3)$.

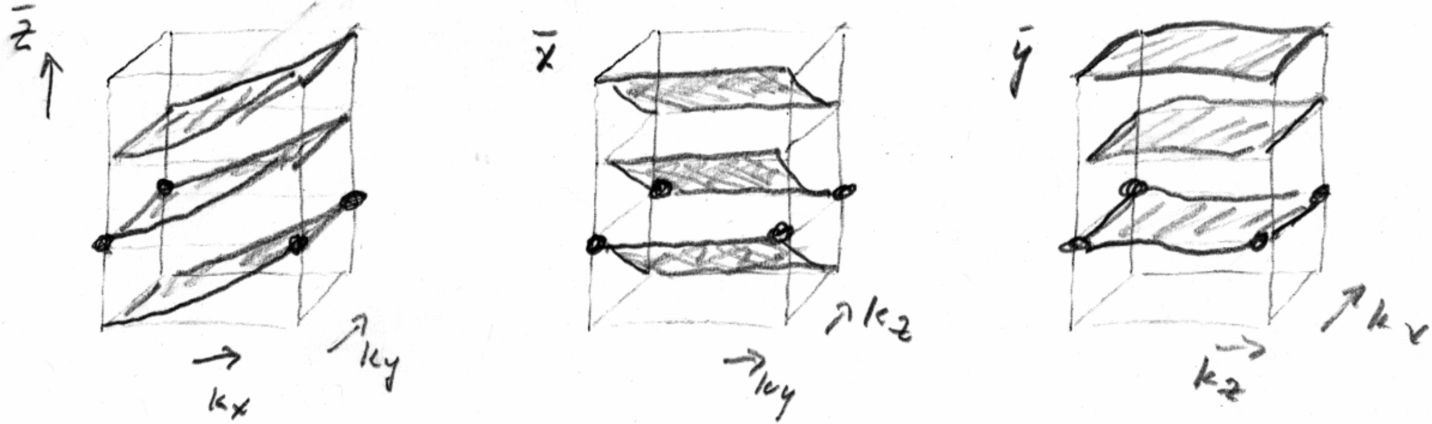
Claim:

$$\underline{\sigma}^{AHC} = \frac{e^2}{2\pi h} \underline{G}^{\text{Chern}} = \frac{e^2}{2\pi h} \left( m_1 \underline{b}_1 + m_2 \underline{b}_2 + m_3 \underline{b}_3 \right)$$

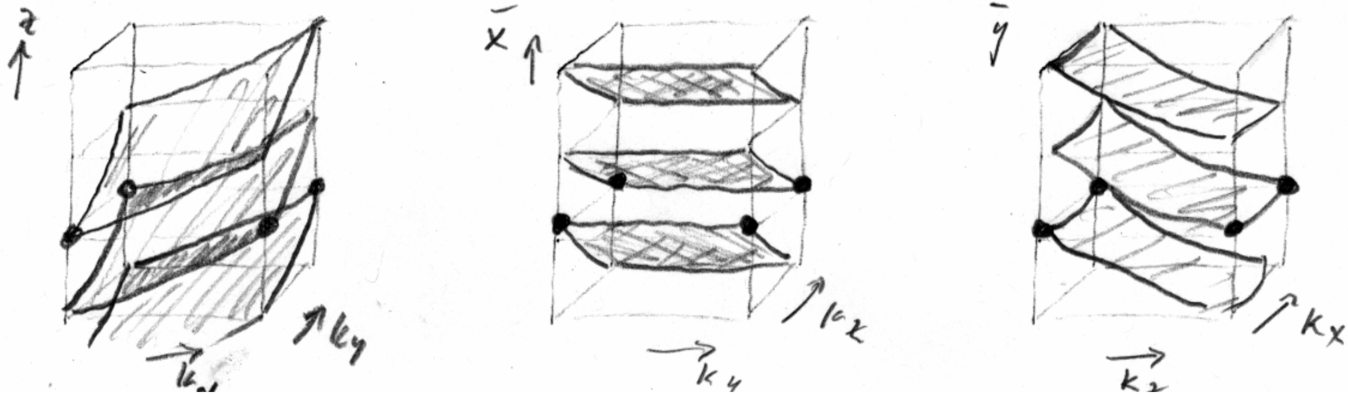
Ex: $\underline{a}_1 = a\hat{x}$, $\underline{a}_2 = a\hat{y}$, $\underline{a}_3 = d\hat{z}$ $\quad (m_1, m_2, m_3) = (0,0,1)$ $\quad \underline{G}^{\text{Chern}} = \frac{2\pi}{d}\hat{z}$

$$\underline{\sigma}^{AHC} = \frac{e^2}{2\pi h} \frac{2\pi}{d} \hat{z} = \frac{e^2}{hd} \hat{z} \qquad \left( = \frac{e^2}{h} \text{ per layer} \right)$$

$\underline{m} = (0, 1, 0) \qquad \underline{G} = (0, \frac{2\pi}{b}, 0) \qquad \sigma_{xz} = \frac{e^2}{bh}$



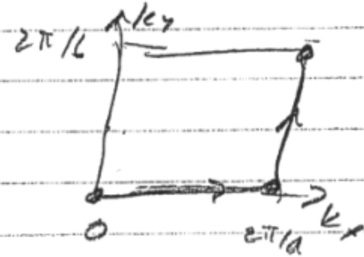

$\underline{m} = (1, 1, 0)$

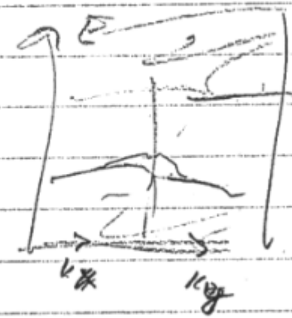


# Surface of 3D QAH

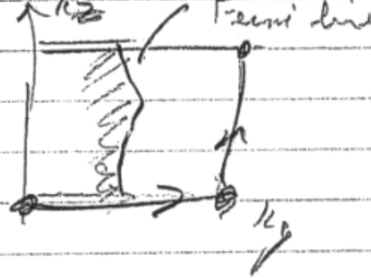
Suppose $C_x = C_y = 0$, $C_z = 1$

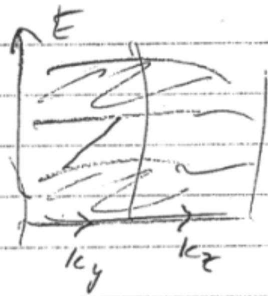
Top surface, $\hat{n} = \hat{z}$




Side surf, $\hat{n} = \hat{x}$




Metallic edge states on some surfaces.

# Analogy with 1D quantum charge pump

Adiabatic pump


such that $\Delta P = -e$

$(m_{ch} = 1)$

Expect


actually ~ N discrete


} one depletion event

} bulk pumps by $m_{ch}=1$

HV: some jumps, get small

∴ # of up-crossings of surface states
must = Chern # (Wannier winding #) of bulk
in order for charge cons. to hold

"Bulk-surface correspondence"

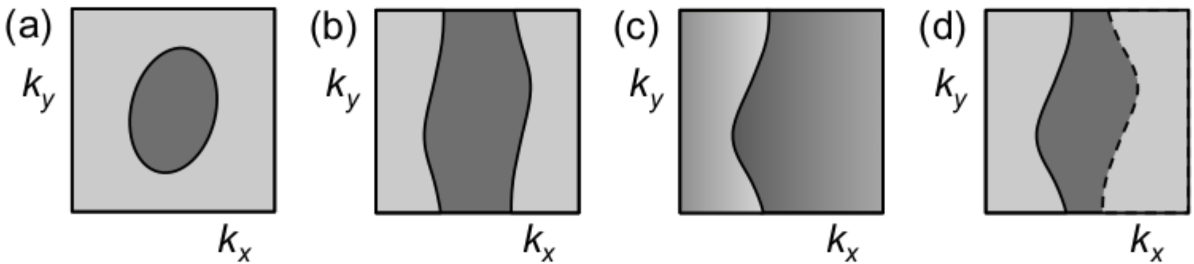


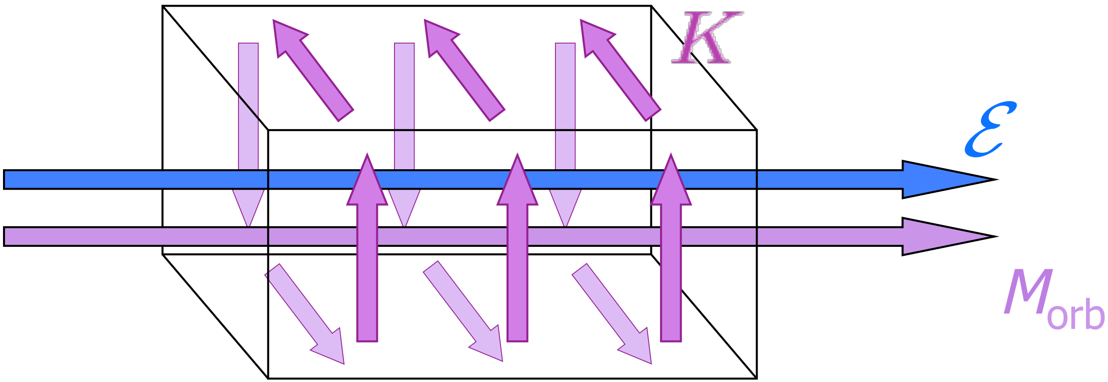

Figure 5.6 (a-b) Sketch of possible Fermi loop structures in the 2D BZ for a metallic 2D layer or a metallic 2D surface of a normal 3D insulator. The heavily shaded side of a Fermi loop is the side with one more surface band occupied. (c) Same, but for the (001) surface of a 3D Chern insulator with indices $(C_x, C_y, C_z) = (0, 1, 0)$, showing a band filling anomaly. (d) Same as (c), but for an (001) slab of finite thickness in the $z$ direction; solid and dashed lines indicate Fermi loops on the top and bottom surfaces respectively.

## Overview

Up to now, models of spinless elec. w/ broken T symm. (Complex hoppings.)

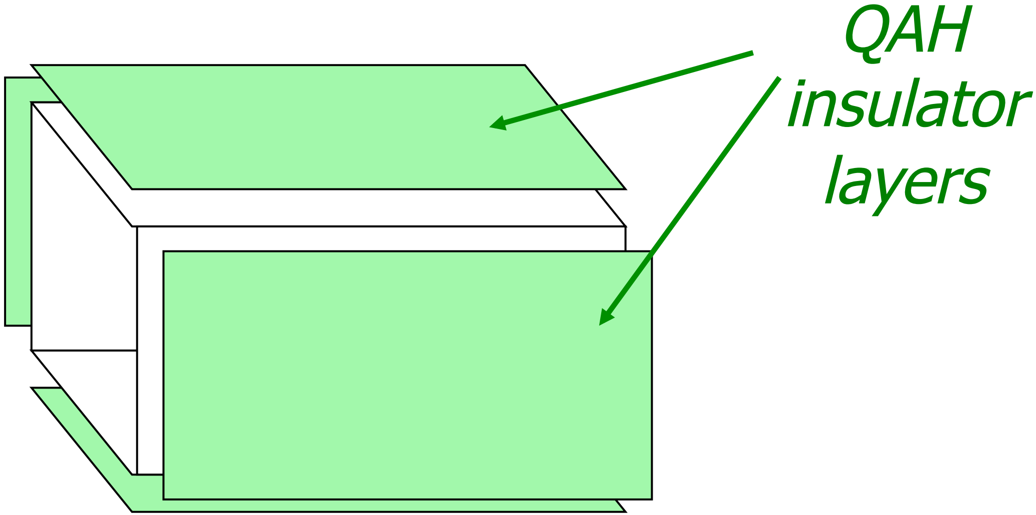
Plan:
- restore spin.
- study systems with T symm.
- but also with SO int so that spin and orbital deg of freedom are coupled
- New kind of topo ins: "$Z_2$ topo ins"

# Orbital MEC ↔ Surface $\sigma_{yx}$

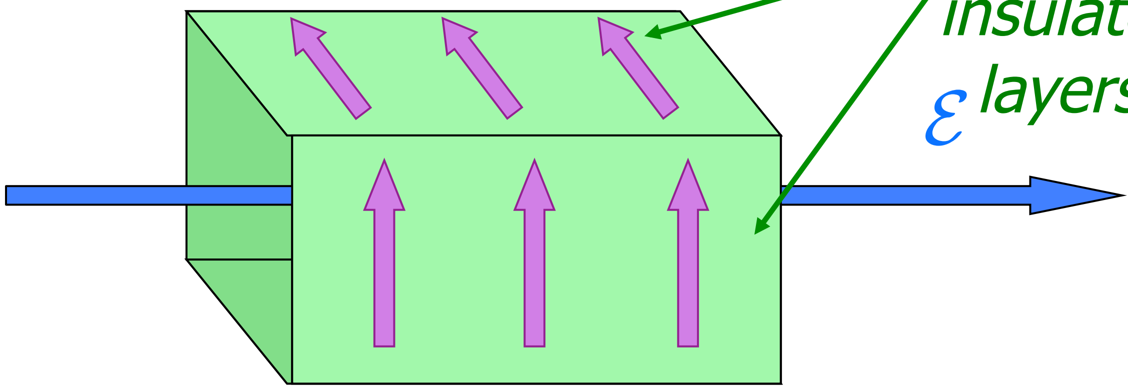



$$\alpha_{\mathrm{orb}} = \frac{dM_{\mathrm{orb}}}{d\mathcal{E}} = \frac{dK}{d\mathcal{E}} = \sigma_{yx}^{\mathrm{surf}}$$

# How to build a magnetoelectric coupler



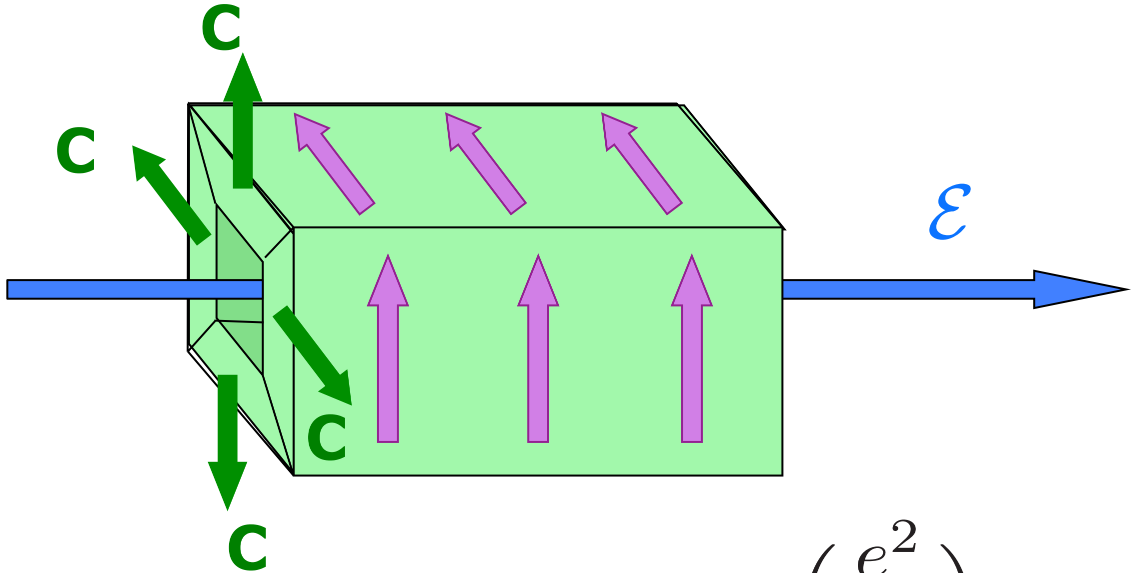
How to build a magnetoelectric coupler




$$\alpha_{\rm orb} = \frac{dK}{d\mathcal{E}} = \frac{e^2}{h} = \frac{1}{2\pi}\frac{1}{137}\ \text{g.u.}$$

For comparison, $Cr_2O_3$ has $\alpha \simeq 10^{-4}$ g.u.

# How to build a magnetoelectric coupler




$$\alpha_{\mathrm{orb}} = N_{\mathrm{layers}} \left( \frac{e^2}{h} \right)$$

This can easily be $10^8$ times that of $Cr_2O_3$ !