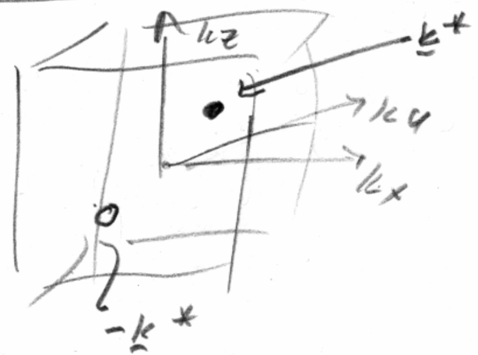
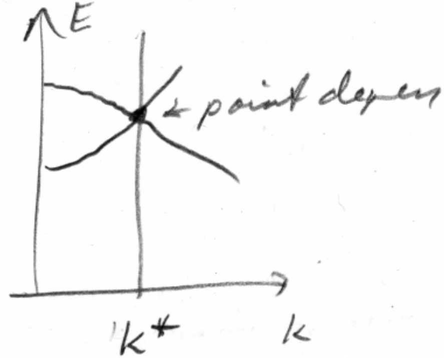
Generic Weyl nodes!

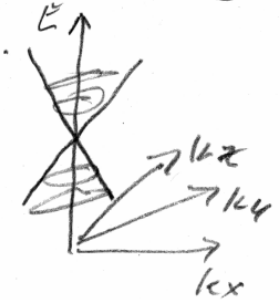




$$H_{2\times 2}(\underline{k}) = \overset{\swarrow \text{drop}}{f_0(\underline{k})} + \underline{f}(\underline{k}) \cdot \underline{\sigma}$$

Really $\underline{k}' = \underline{k} - \underline{k}^*$

Example: $\underline{f}(\underline{k}) = v_0(k_x, k_y, k_z)$ $H = v_0(k_x\sigma_x + k_y\sigma_y + k_z\sigma_z)$



Hard to draw on 2D paper!

$E = \pm v_0 k$ $v_0 =$ "Fermi veloc."

Chirality:

Let $A_{ij} = \frac{\partial f_i}{\partial k_j}$

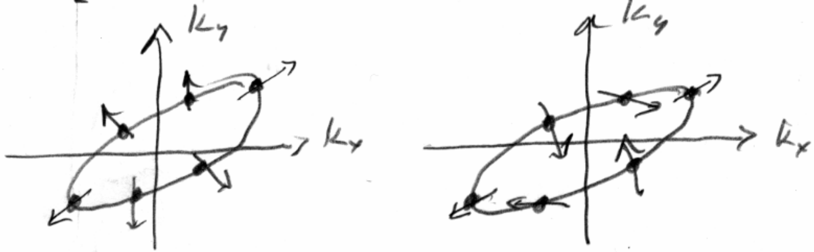
$$A = \begin{pmatrix} \partial f_1/\partial k_x & \partial f_1/\partial k_y & \partial f_1/\partial k_z \\ \partial f_2/\partial k_x & \cdots & \cdots \\ \partial f_3/\partial k_x & \cdots & \cdots \end{pmatrix}$$

(above: $A = v_0 \mathbb{I}_{3\times 3}$)

$$\boxed{\chi = \operatorname{sgn} \det A}$$

Intuition for Weyl chirality

at $k_z = +\Delta$: spins of upper band go like



$\chi = +1$ $\qquad\qquad$ $\chi = -1$

another way:

R-handed $\quad +1$
or
L-handed $\quad -1$
triad?

Examples: $\underline{f} = v_0 (k_x, k_y, k_z) \quad \chi = +1$

$= v_0 (k_y, k_x, k_z) \quad \chi = -1$

$= v_0 (-k_x, -k_y, -k_z) \quad \chi = -1$

$= v_0 (k_y, -k_x, k_z) \quad \chi = +1$

$= (100 k_x, 0.01 k_y, 3 k_z) \quad \chi = +1$

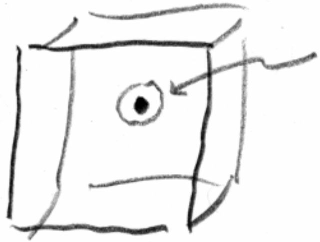
If $\chi = +1$:

States in band n+1 see -2π Berry flux
" " " n see $+2\pi$ Berry flux

"Magnetic monopoles in k-space"

band n

band n+1



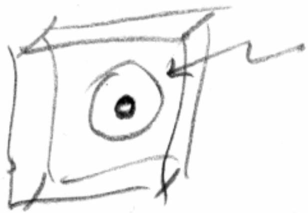
$C_n = +1$

$C_{n+1} = -1$

If $\chi = -1$:

band n

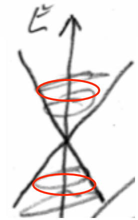
band n+1



$C_n = -1$

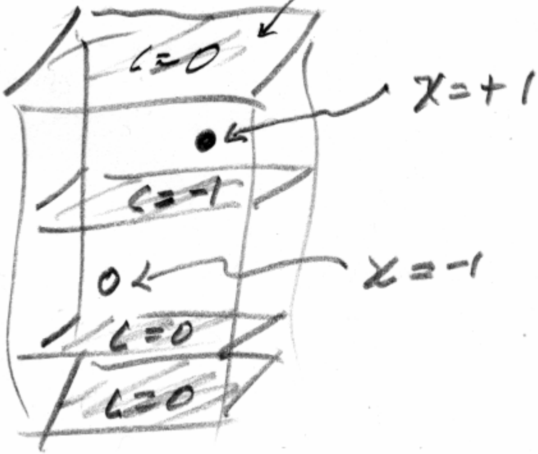
$C_{n+1} = +1$

Note $\displaystyle\int_{S_F} \boldsymbol{\Omega} \cdot \hat{\mathbf{v}}_F \, dS = -2\pi\chi$ for both elec. and hole pockets



Nielsen-Ninomiya theorem

c = Chern # of band n (or sum over 1 … n)



Slice Chern number must return

$$\boxed{\sum_i \chi_i = 0}$$

to all Weyl points between n and n+1 in 3D BZ

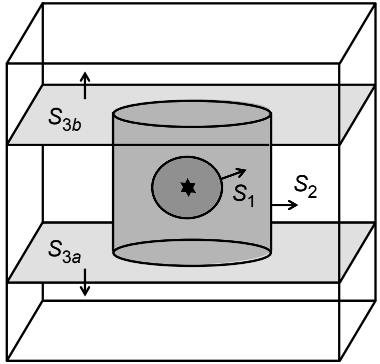


Figure 5.24 Sketch of three surfaces surrounding a Weyl point (star) in a region that is otherwise free of band touchings in the 3D BZ. Arrows show surface normal directions $\hat{\mathbf{n}}$ used for computing the Chern index on each surface.

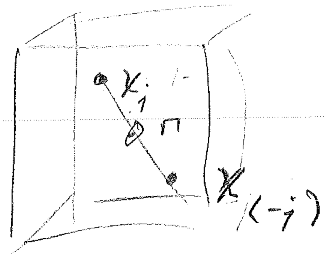
$$\sum_j \chi_j = 0 \tag{5.36}$$

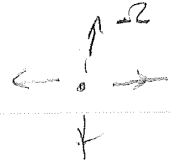
where index j runs over the Weyl points in the BZ.[32] We can see why this must be so by considering the sketch in Fig. 5.24, which shows a Weyl point (denoted by the star) in a region of the BZ that is otherwise free of band touchings. As discussed earlier, the Chern index of the lower band on the inner sphere S_1 is $-\chi$. Now imagine gradually deforming this surface until it adopts the shape of S_2 (a cylinder including its end caps), and then increasing the radius of the cylinder until it merges at the side boundaries of the BZ and fills the entire region between planes S_{3a} and S_{3b}. If no other conical intersections were encountered during these deformations, the Chern number on surface S_3, computed as the sum of contributions from S_{3a} and S_{3b} with downward and upward unit normals $\hat{\mathbf{n}}$, respectively, as shown in the sketch, must still equal $-\chi$. Actually, each of these "planes" is a 2D BZ, a torus. It then follows that $\chi = C_a - C_b$, where the slice Chern numbers C_a and C_b are now both defined with $\hat{\mathbf{n}}$ pointing upward.

Erratum: Reverse signs

Recall	Inv Sym	TR Sym
	$E(-\underline{k})=E(\underline{k})$	$E(-\underline{k})=E(\underline{k})$
	$\underline{\Omega}(-\underline{k})=\underline{\Omega}(\underline{k})$	$\underline{\Omega}(-\underline{k})=-\underline{\Omega}(\underline{k})$
	$\chi_{(-j)}=-\chi_{(j)}$	$\chi_{(-j)}=+\chi_{(j)}$

$1^{st} \Rightarrow$ Pairs

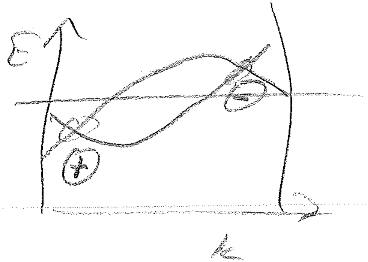




This config is odd under inv
even " TR

So if either I or TR is present, WPs come in sym pairs.

If neither is present, then normally

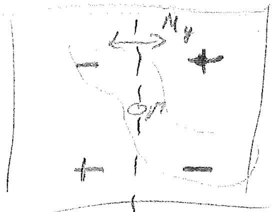


E_f won't lie at both sim'ly

(unless sym?)

TR symm (I is broken)

Need ≥ 4 WPs



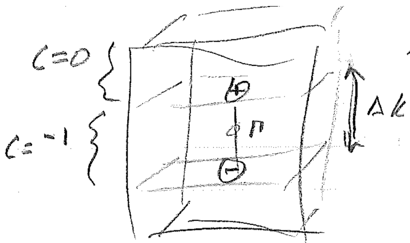
← sum to 0

— related by $k \to -k$

Can be at common E_f if related by a symm, eg, M_y

TaP, TaAs, NbP, NbAs 24 WP in BZ! 2 diff en.

I symm (TR is broken)



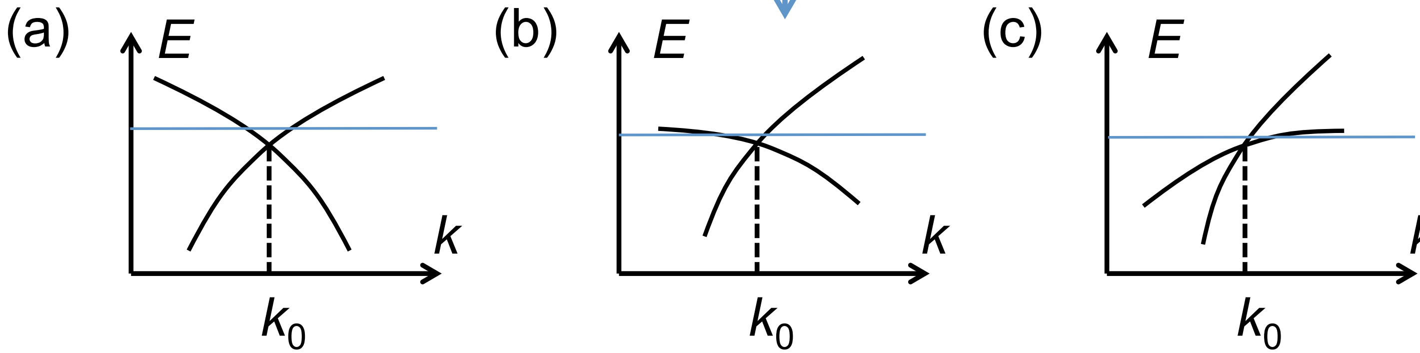
too, eg, proposal of Burkov + Balents, PRL 2011

Comment!

$$\sigma_{yx} = \frac{-e^2}{hc} \frac{\Delta k}{G_z}$$

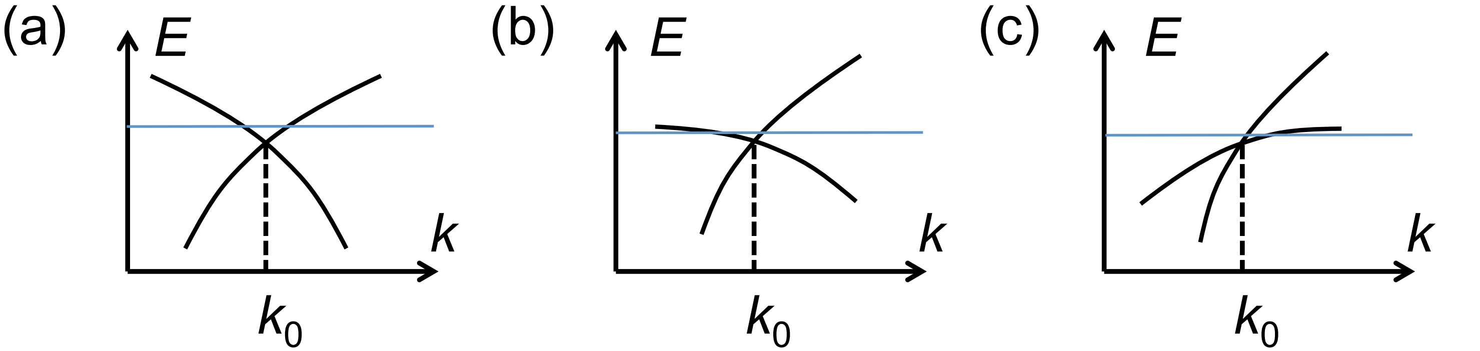
$$H_{2\times 2} = \left(E_0 + \frac{\hbar^2}{2m^*}k^2 + \bar{v}k_x\right)\mathbb{1}_{2\times 2} + v_0\, \underline{k}\cdot\underline{\sigma}$$

$\bar{v}=0$ $\qquad$ $\bar{v}<v_0$ $\qquad$ $\bar{v}>v_0$



"Type 1" $\qquad$ "Type 1" $\qquad$ "Type 2"

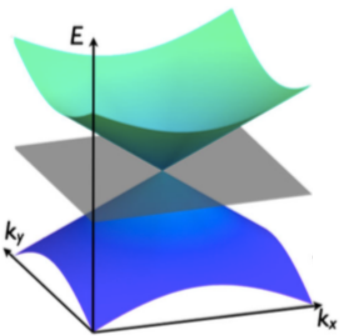
Soluyanov *et al.*, 2015

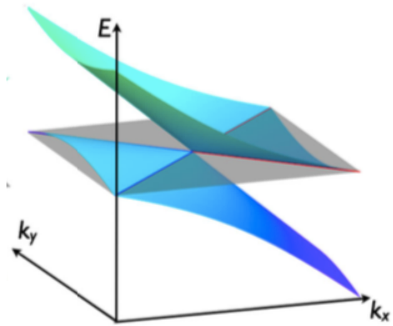


"Type 1" "Type 1" "Type 2"

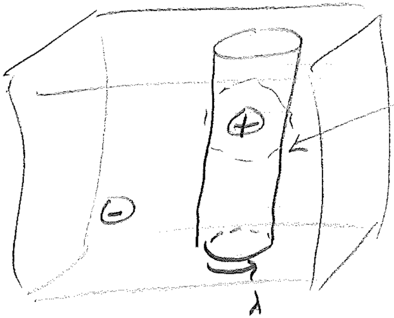
Soluyanov *et al.*, 2015)

N. P. Armitage, E. J. Mele, and Ashvin Vishwanath: Weyl and Dirac semimetals in three- …



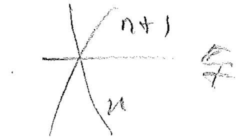


Fermi Arcs

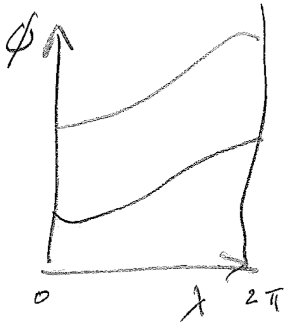


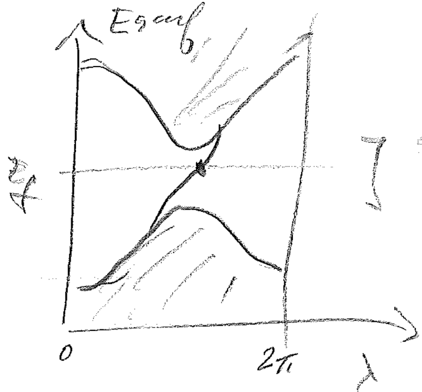
construct cyl..

It is really  so has
Chern # $C = \chi = +1$
(in band n)

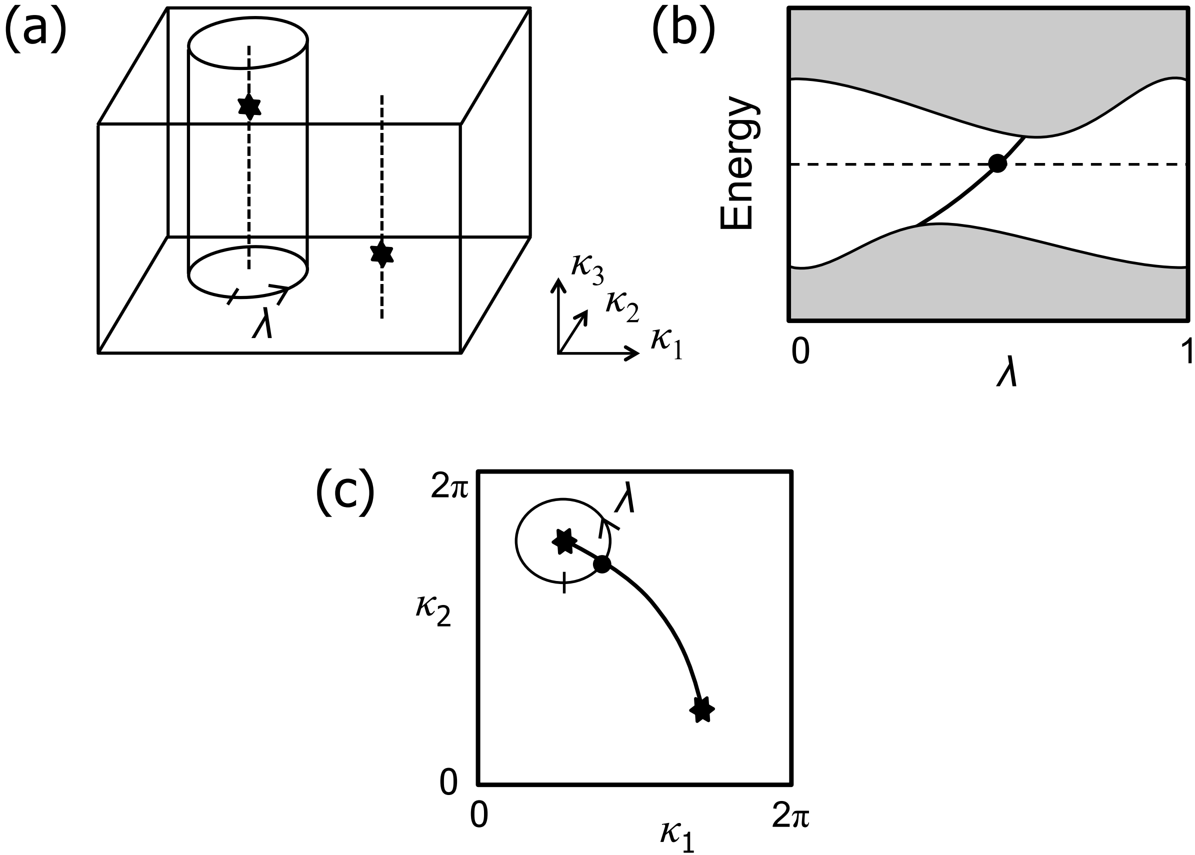


k-space





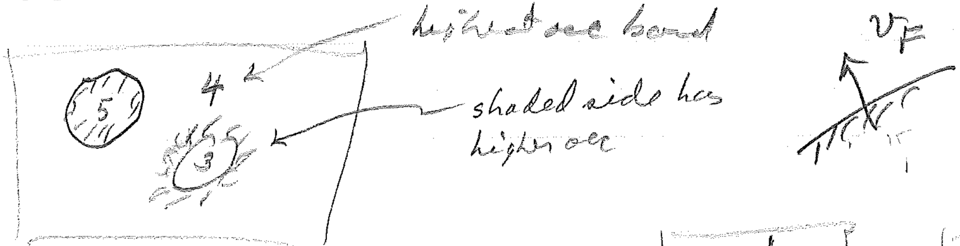
smallest gap
∝ radius of
cyl


(a)
λ
κ_3
κ_2
κ_1
(b)
Energy
0
1
λ
(c)
2π
λ
κ_2
0
0
2π
κ_1

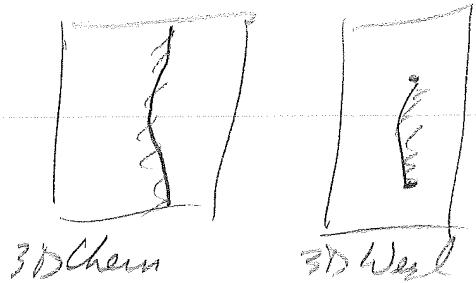
Metallic surf! Let's see:

Again, physics of isolated 2D metal ≠ physics at surf of 3D

Fermi arcs not allowed.
Can count states.



On surf, cannot count bands



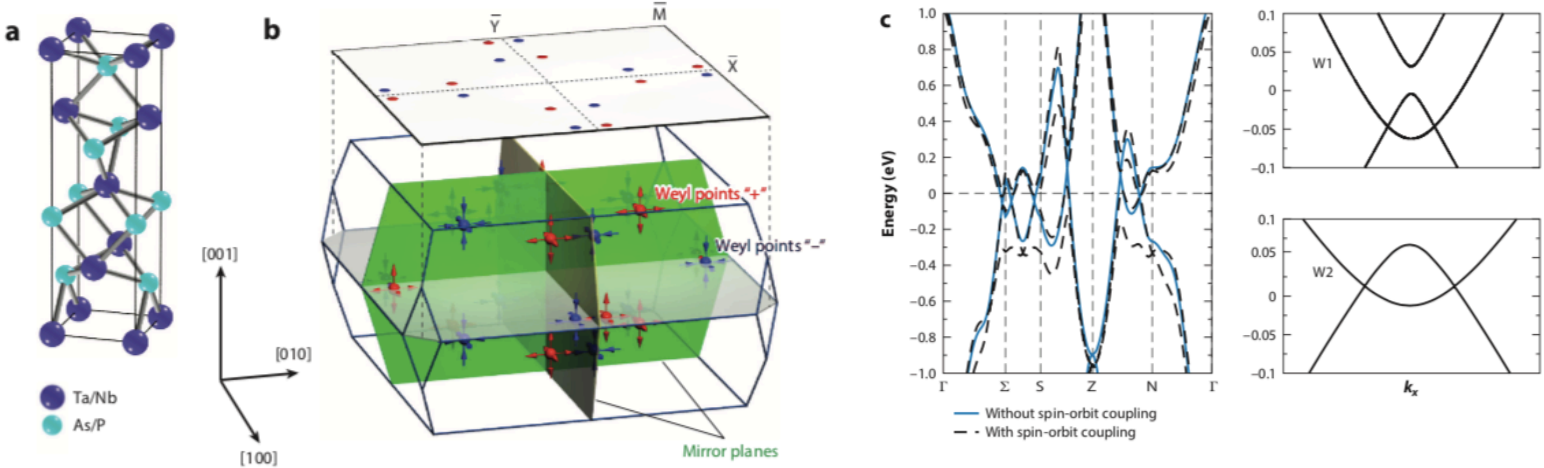
Annu. Rev. Condens. Matter Phys. 2017. 8:337–54

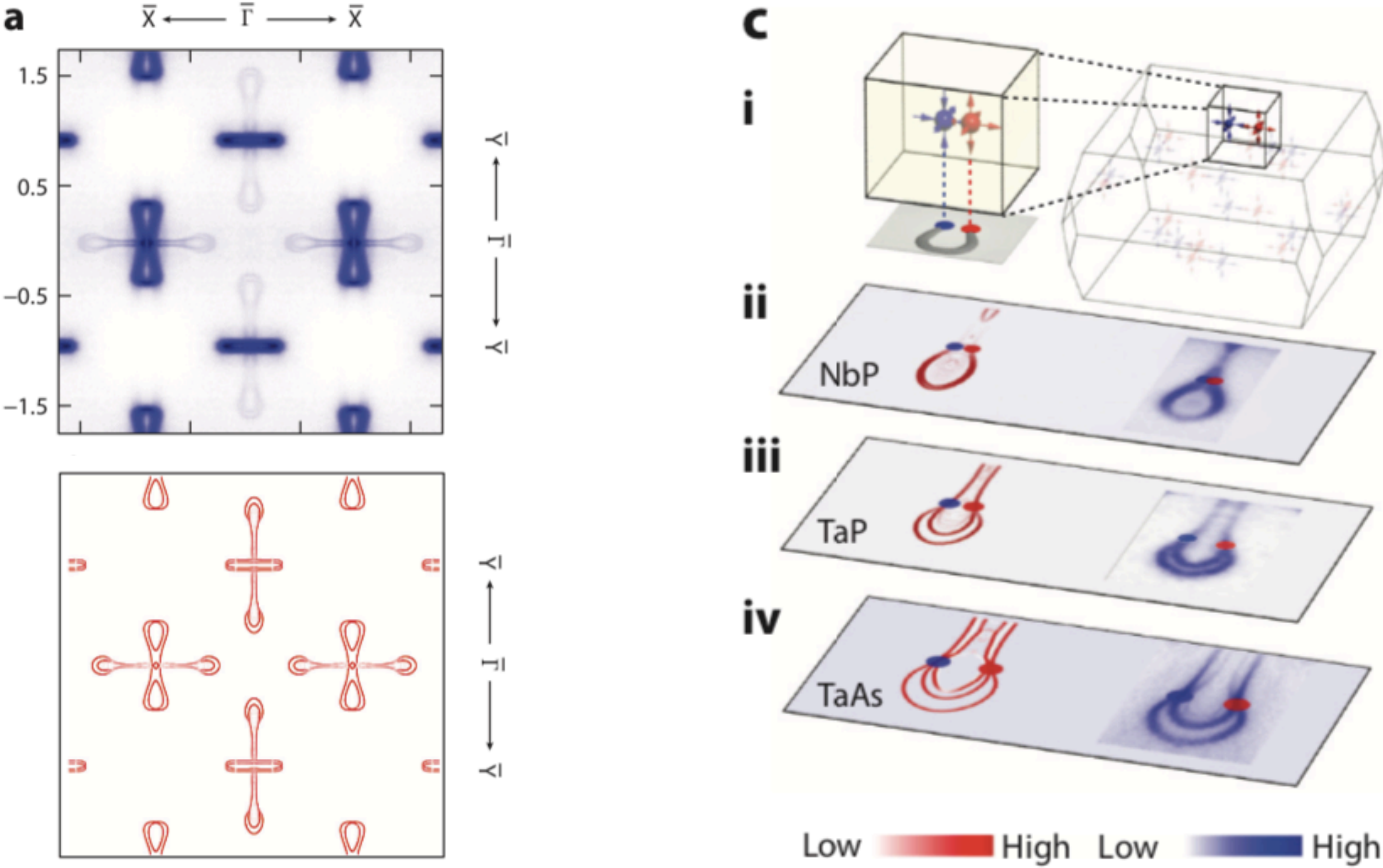
Topological Materials: Weyl Semimetals

Binghai Yan and Claudia Felser

Department of Solid State Chemistry, Max Planck Institute for Chemical Physics of Solids, 01187 Dresden, Germany; email: yan@cpfs.mpg.de, felser@cpfs.mpg.de

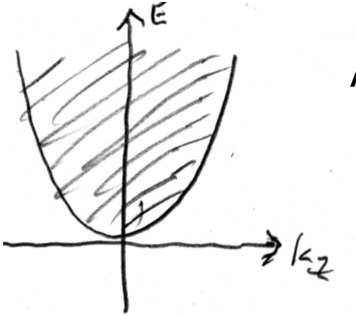
WEYL SEMIMETALS: THE TaAs FAMILY



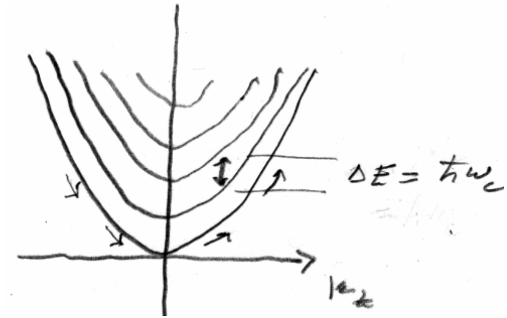

a
X̄ Γ̄ X̄
1.5
0.5
−0.5
−1.5
Ȳ Γ̄ Ȳ
Ȳ Γ̄ Ȳ
c
i
ii
iii
iv
NbP
TaP
TaAs
Low High Low High

Chiral anomaly: First viewpoint

Quadratic band minimum

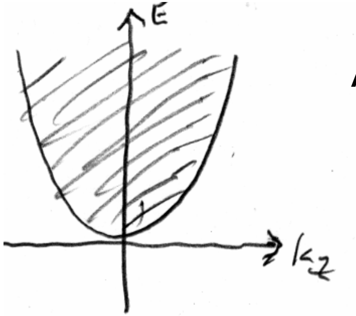


Apply B_z, then E_z :

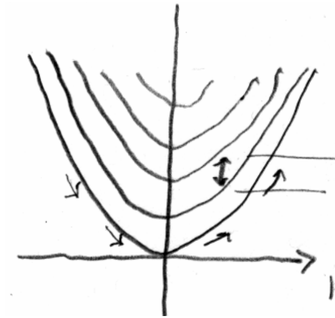


Chiral anomaly: First viewpoint

Quadratic band minimum

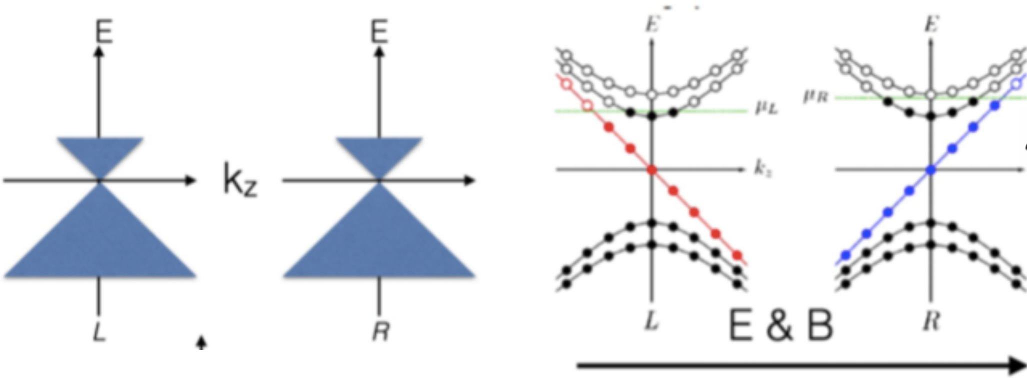


Apply B_z, then E_z :



$\Delta E = \hbar\omega_c$

Weyl point



$\Delta E \propto |E|^{-1/2}$

N. P. Armitage, E. J. Mele, and Ashvin Vishwanath: Weyl and Dirac semimetals in three- …

Note transport of electrons from L to R; proportional to **E** · **B**

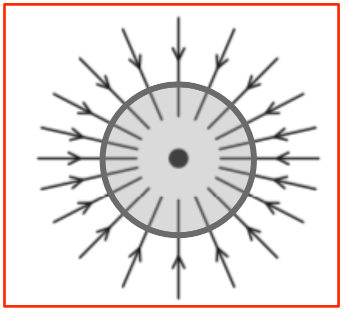
Chiral anomaly: Second viewpoint (Text 5.4.3)

$$\dot{\mathbf{r}} = \mathbf{v}_{\mathrm{g}} - \dot{\mathbf{k}} \times \boldsymbol{\Omega}\,, \tag{5.11a}$$

$$\dot{\mathbf{k}} = -\frac{e}{\hbar}\boldsymbol{\mathcal{E}} - \frac{e}{\hbar c}\,\dot{\mathbf{r}} \times \mathbf{B}\,, \tag{5.11b}$$

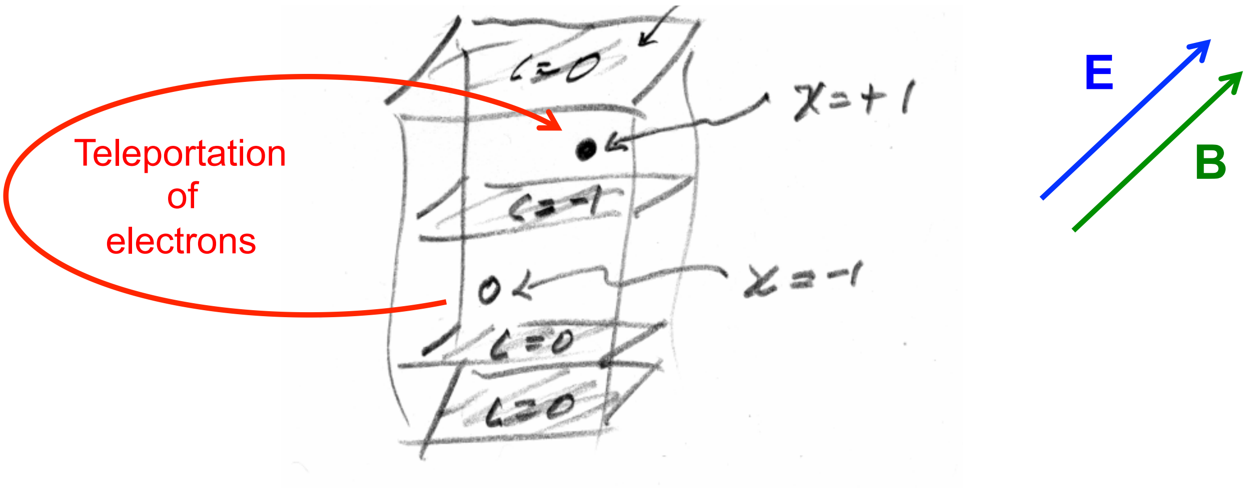
$$\left(1 + \frac{e}{\hbar c}\mathbf{B}\cdot\boldsymbol{\Omega}\right)\dot{\mathbf{r}} = \mathbf{v}_{\mathrm{g}} + \frac{e}{\hbar}\,\boldsymbol{\mathcal{E}} \times \boldsymbol{\Omega} + \frac{e}{\hbar c}\,(\mathbf{v}_{\mathrm{g}}\cdot\boldsymbol{\Omega})\,\mathbf{B}\,, \tag{5.39a}$$

$$\left(1 + \frac{e}{\hbar c}\mathbf{B}\cdot\boldsymbol{\Omega}\right)\boxed{\dot{\mathbf{k}}} = -\frac{e}{\hbar}\boldsymbol{\mathcal{E}} - \frac{e}{\hbar c}\mathbf{v}_{\mathrm{g}} \times \mathbf{B} - \frac{e^2}{\hbar^2 c}\,(\boldsymbol{\mathcal{E}}\cdot\mathbf{B})\,\boxed{\boldsymbol{\Omega}}\,. \tag{5.39b}$$

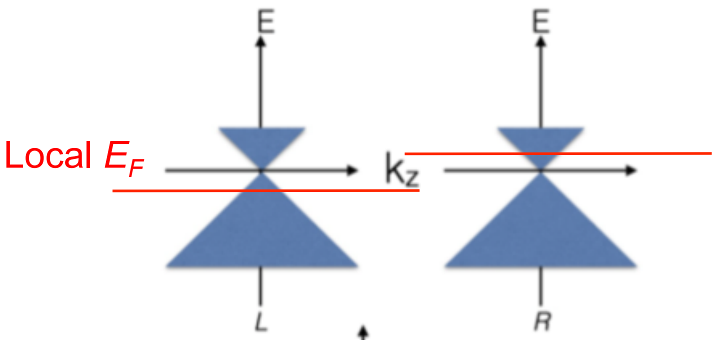


$$\frac{dn}{dt} = \frac{-1}{(2\pi)^3}\,\frac{e^2}{\hbar^2 c}\,(\boldsymbol{\mathcal{E}}\cdot\mathbf{B})\int_{S_{\mathrm{F}}} \boldsymbol{\Omega}\cdot\hat{\mathbf{v}}_{\mathrm{F}}\,d^2k$$

$$= \frac{e^2}{h^2 c}\,(\boldsymbol{\mathcal{E}}\cdot\mathbf{B})\,\chi_i$$



One consequence:

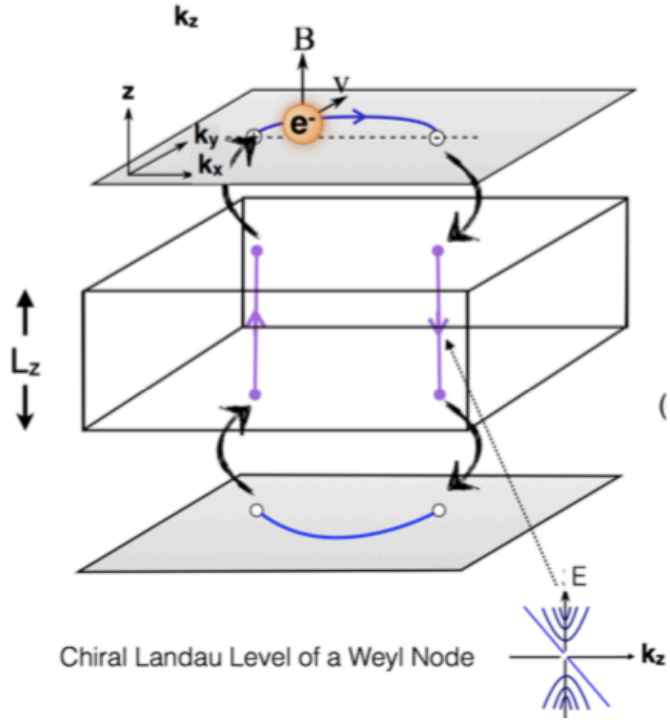


Chiral magnetic effect

$$\mathbf{J} = \left(\frac{e^2}{h^2 c} \sum_i \mu_i \chi_i \right) \mathbf{B}$$

Local E_F
of i^{th} Weyl node

Landau orbits



Chiral Landau Level of a Weyl Node

<u>More on Weyl semimetals</u>

See reviews by

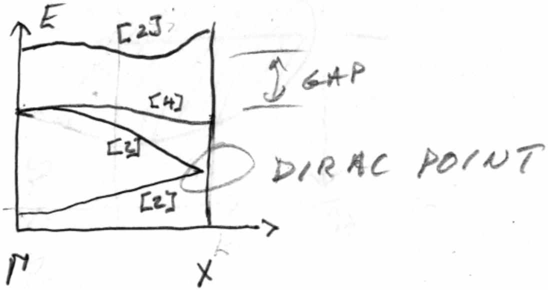
- Hasan et al. (2017)
- Yan and Felser (2017)
- Armitage et al. (2018)

Dirac semimetals

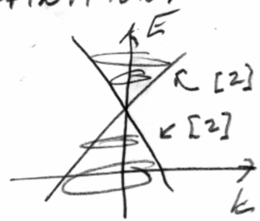
IF I AND TR ARE BOTH GOOD SYMMETRIES,

$\underline{k} \xrightarrow[I]{} -\underline{k} \xrightarrow[TR]{} \underline{k}$ SO $H_{\underline{k}}$ = HAM OF TR-INVAR 0D SYSTEM FOR ANY $\underline{k}$

Si:



DEFINITION:



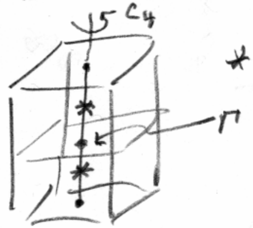
$k = k_x, k_y$, or k_z

MAPS TO MASSLESS DIRAC THEORY

CAN IT OCCUR AT E_F, AS TOUCHING OF VB & CB ?

YES, E.G., Na_3Bi, Cd_3As_2, ETC.

OCCURS ON ROTATIONAL SYMMETRY AXIS:



$*$ = DIRAC POINTS

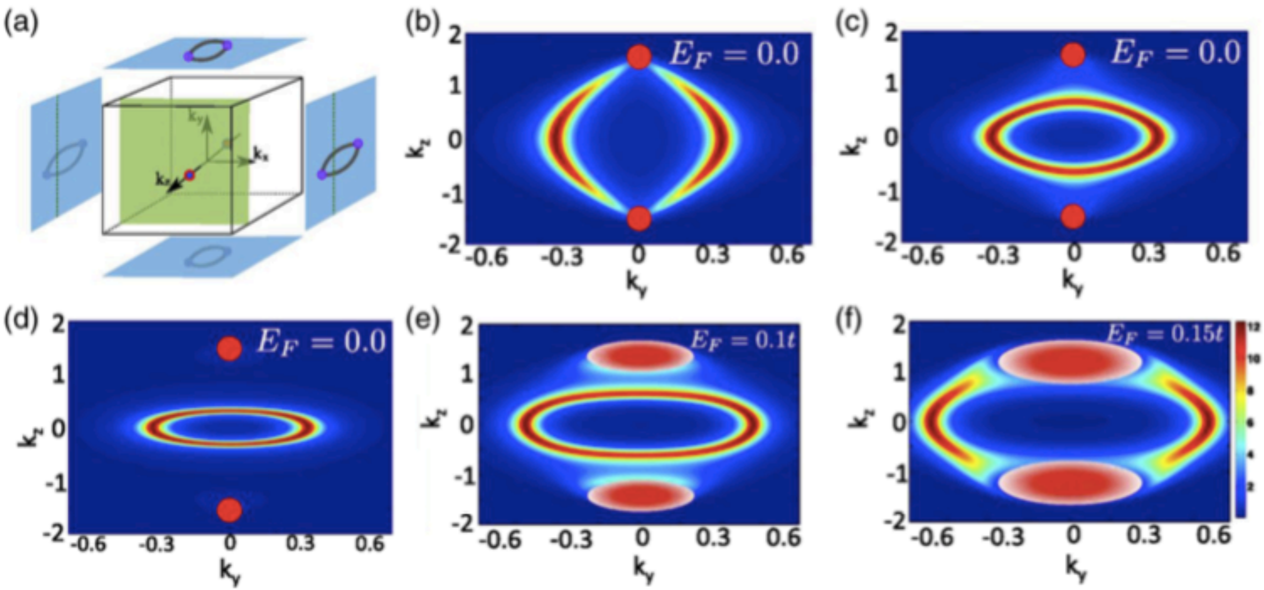


FIG. 13. Fermi arcs on the surface of DSMs. (a) A schematic of a DSM showing Dirac nodes along the k_z axis in the bulk BZ and double Fermi arcs on the surface BZs. Note that surfaces perpendicular to the z axis have no arcs. A 2D slice of the bulk BZ perpendicular to the k_z axis is shown as a shaded (green) plane, which projects to the dashed (green) line on the surface BZ. (b)–(d) A symmetry-allowed mass term at the surface admits backscattering between these branches at the contact point which dissociates the surface band from the projected Dirac point. These surface branches can be deformed but not removed from the time reversal symmetric plane at $k_z = 0$. If the chemical potential is not aligned with the bulk Dirac points the surface Fermi arcs disappear by merging with the bulk continuum (e), (f). Adapted from Kargarian, Randeria, and Lu, 2016.

Chin. Phys. B Vol. 25, No. 11 (2016) 117106

TOPICAL REVIEW — Topological electronic states

Topological nodal line semimetals*

Chen Fang(方辰)[1,†], Hongming Weng(翁红明)[1,2,‡], Xi Dai(戴希)[1,2], and Zhong Fang(方忠)[1,2]

2.2. Nodal lines protected by inversion, time-reversal, and SU(2) spin-rotation symmetries

Here we first assume that all the three symmetries are present in our system. Since SU(2) is a symmetry, we can redefine time-reversal operator, combining it with a π spin rotation about the y-axis,

$$T \to T\,\mathrm{e}^{\mathrm{i}s_y\pi}, \tag{7}$$

after which we have $T^2 = +1$ instead of -1 for fermions. Since both inversion, P and T, reverse the momentum $\boldsymbol{k} \to -\boldsymbol{k}$, $P*T$ is an anti-unitary symmetry that preserves the momentum. Since $[P,T] = 0$, we have

$$(P*T)^2 = P^2T^2 = 1. \tag{8}$$

Equation (8) dictates that it can be represented as

$$P*T = K, \tag{9}$$

where K is the complex conjugation, in a proper orbital basis. In this basis, $P*T$-symmetry ensures that

$$H(\boldsymbol{k}) = H^*(\boldsymbol{k}), \tag{10}$$

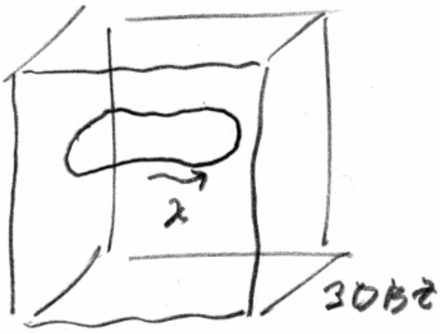
or that $H(\boldsymbol{k})$ is real at each $\boldsymbol{k}$.

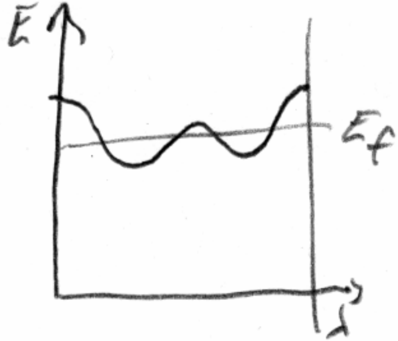
$$H_{2\times 2} = \begin{pmatrix} a & c \\ c & b \end{pmatrix} = \frac{a+b}{2}\mathbb{I} + \frac{a-b}{2}\underline{\underline{\sigma_3}} + c\,\underline{\underline{\sigma_1}}$$

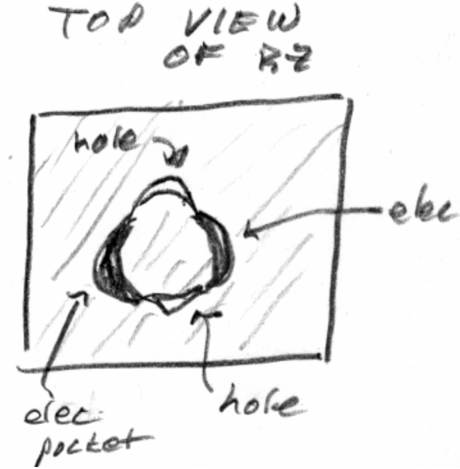
Codimension of 2 is enough!

Nodal lines can occur in 3D BZ.

(also can occur due to symmetries)


λ
3DBZ


E
E_f
λ


TOP VIEW
OF BZ
hole
elec
elec
pocket
hole