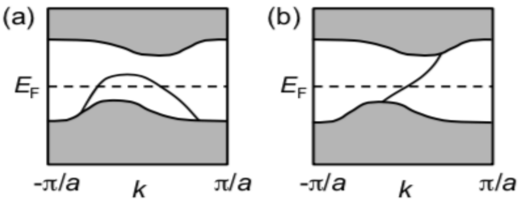
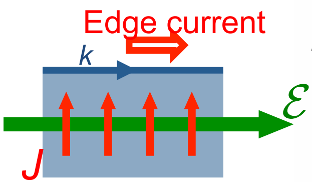
Quantum anomalous Hall insulators in 2D

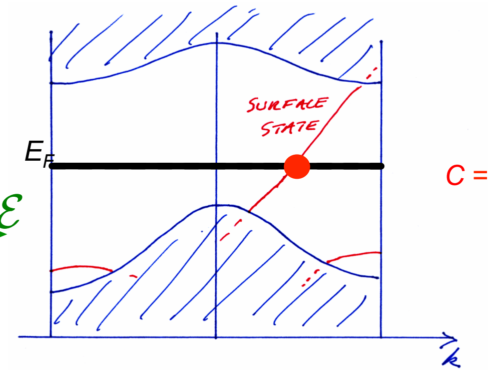
Last time:



$$n_{\text{up}} - n_{\text{down}} = -C$$

$$\sigma_{\text{AHC}} = C\,\frac{e^2}{h}$$





C = -1

Conservation of charge $\Rightarrow$ chiral surface state

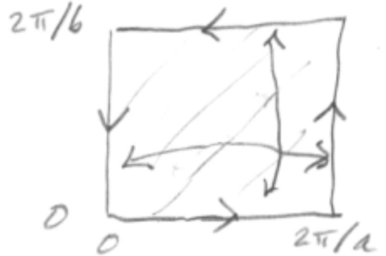
LECTURE 3/30/20

Can we construct 2D Wannier functions (WF's) for QAH insulator? No!

Suppose yes: Let $|\psi_{n\underline{k}}\rangle = \sum_{\underline{R}} e^{i\underline{k}\cdot\underline{R}} |w_{n\underline{R}}\rangle$

This is smooth in interior of BZ and periodic on boundary:

$$|\psi_{n,\underline{k}+\underline{G}}\rangle = \sum_{\underline{R}} e^{i(\underline{k}+\underline{G})\cdot\underline{R}} |w_{n\underline{R}}\rangle = |\psi_{n\underline{k}}\rangle$$



Sides cancel
Top & bottom cancel

"Smooth and periodic gauge"

Apply Stokes: $\phi_{\text{boundary}} = \int_{BZ} \Omega \, d^2k$

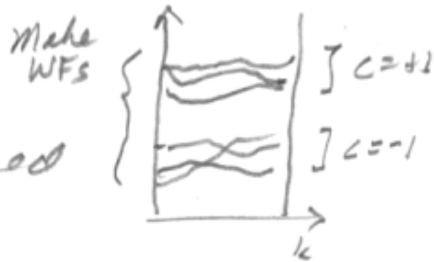
But $\phi_{\text{boundary}} = 0$

$\Rightarrow \boxed{C=0}$ (zero Chern #)

This is an example of a "topological obstruction".

"A non-zero Chern index presents a topological obstruction to the construction of a set of Wannier functions."

Note:



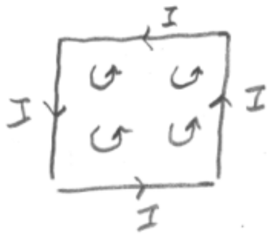
① Often can construct WFs in enlarged space of bands.

③ No obstruction for hybrid Wannier construction.

② TB model at full filling always has C=0.

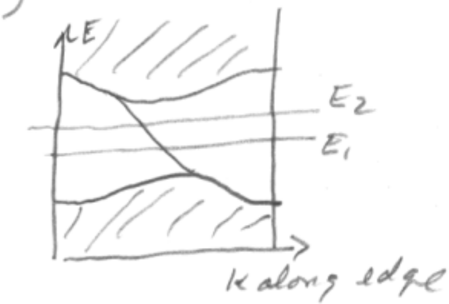
ORBITAL MAGNETIZATION & STREDA FORMULA p. 33 in book.

Orbital magnetization in 2D:



$\boxed{M = \frac{1}{c} I}$ (Gaussian units)

If Chern = +1:

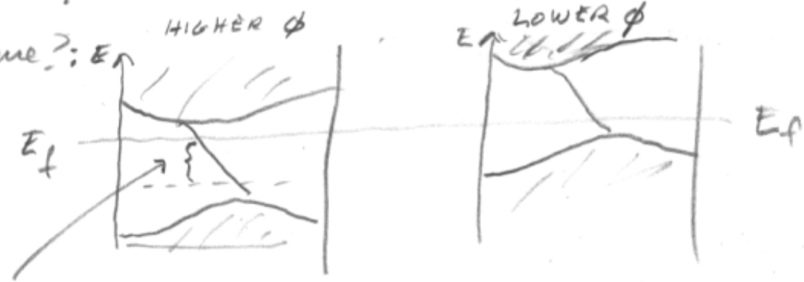


apply change of elec pot ϕ:

$$\Delta E = -e\Delta\phi$$

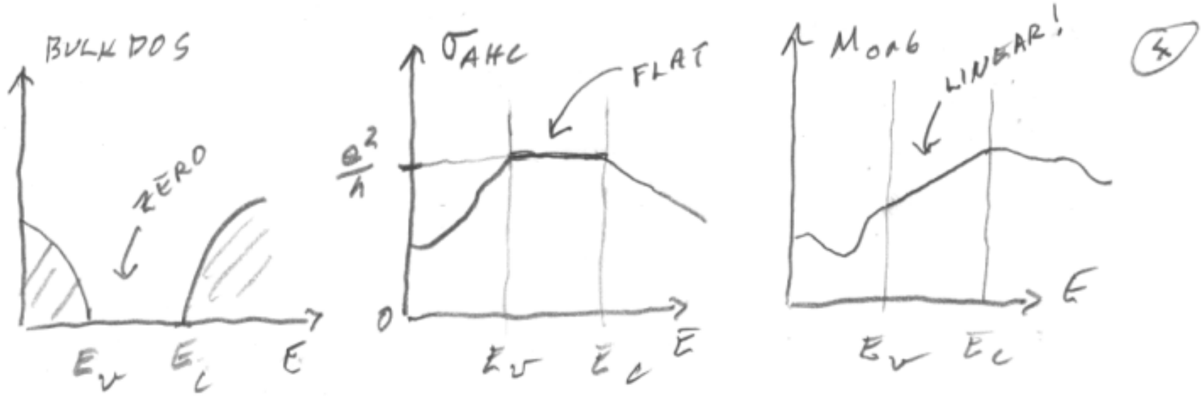
$$c\,\frac{dM}{d\phi} = \frac{dI}{d\phi} = \text{"conductance"} = \sigma = \frac{e^2}{h} \quad \left(\text{generally, } C\frac{e^2}{h}\right)$$

Better picture?:



POPULATE MORE STATES CARRYING CURRENT TO RIGHT.

when you add electrostatic potential on the system and since the slope of the edge state is negative meaning electron velcotity is negative as well as the group velocity and the electron are moving in negativedirection and since electronis carry -ve charge the current is moving in right (positive) direction.

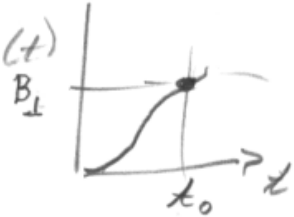


$$\boxed{\frac{dM}{d\phi} = \frac{1}{c}\sigma_{AHC} = \frac{1}{c}\frac{e^2}{h}C}$$ in gap of QAH insulator in 2D.

Streda effect in 2D

QAH sample, $C = +1$

$\underline{B} = B(t)\,\hat{z}$, $B(t)$



$$\underline{\nabla} \times \underline{\mathcal{E}} = -\frac{1}{c}\frac{d\underline{B}}{dt}$$

$$\underline{K} = \sigma^{xy}\,\hat{z} \times \underline{\mathcal{E}} = \frac{e^2}{h}\,\hat{z} \times \underline{\mathcal{E}}$$

$$\frac{dQ_S}{dt} = -\oint_P K_\perp\, dl = \frac{e^2}{h}\oint \mathcal{E}_\parallel\, dl$$

$$= \frac{e^2}{h}\left(-\frac{1}{c}\right)\frac{d\Phi}{dt}$$

CHARGE DENS.!

$$\frac{d\sigma}{dt} = \frac{-e^2}{hc}\frac{dB}{dt} \Rightarrow \boxed{\Delta\sigma = -\frac{e^2}{hc}\,\Delta B} \quad \text{or} \quad \boxed{\Delta n = \frac{e}{hc}\, B_\perp}$$

2D particle density

In general, $\boxed{\Delta n = \frac{eC}{hc}\, B_\perp = \frac{\sigma_{AHE}}{ec}\, B_\perp}$ STREDA FORMULA (1.43)

STREDA - CONTINUED

Earlier we discussed a general rule:

"In an insulator, the number of electrons per unit cell contributed by one band is 1 (or 2 for spin)."

Wrong in general; only true if $\underline{B} = 0$!

(orbital B)

$$\left[\; H = \frac{1}{2m}\left(\underline{p} + \frac{e}{c}\underline{A}\right)^2 - \mu \cdot B \; \right]$$

orbital term
$\underline{B} = \nabla \times \underline{A}$

Zeeman term

Revised Liouville's theorem

2D: $$\frac{d^2N}{d^2r\, d^2k} = \frac{1}{(2\pi)^2}\left[1 + \frac{e}{\hbar c} B_\perp \Omega(k)\right]$$

3D: $$\frac{d^2N}{d^3r\, d^3k} = \frac{1}{(2\pi)^3}\left[1 + \underbrace{\frac{e}{\hbar c}\underline{B}\cdot\underline{\Omega}}_{\text{correction}}\right]$$

(p. 264 OF TEXT)

Usual Liouville

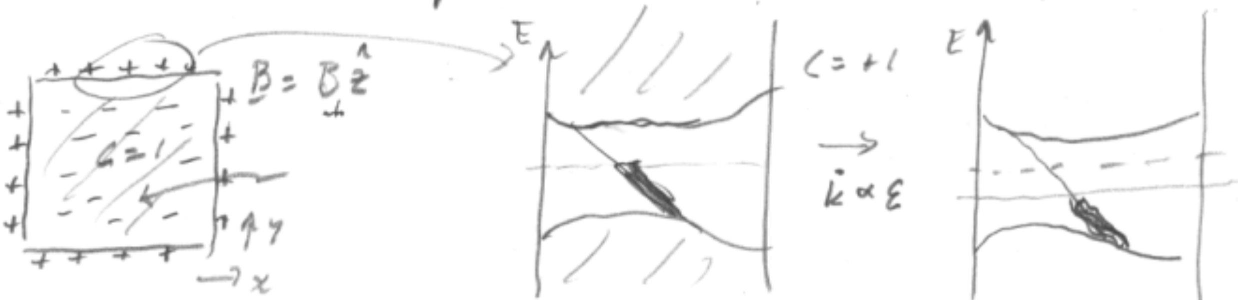
See Xiao, Shi & Niu, 2005.

Highly recommended

STREDA – CONT:

Where does charge come from? (Or go to.)

- at fixed $\mu = E_f$, from reservoir.
- Open boundary conditions: Must be edge channel!



Before pulse

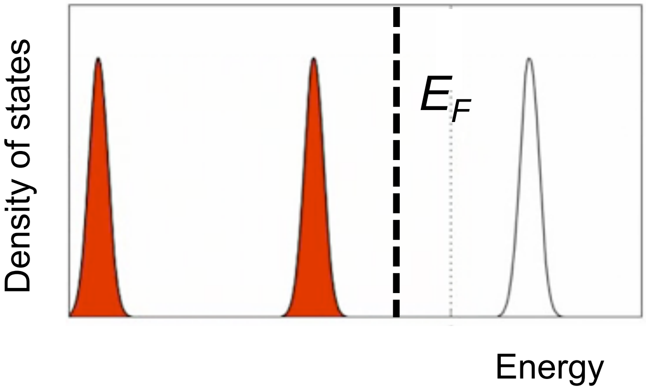
After pulse

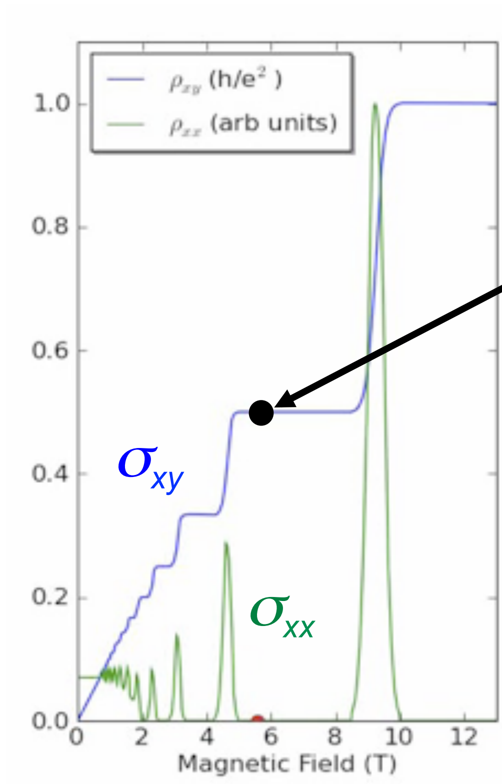
Yes, it is consistent!

<u>Final comment</u>: Q Hall, one Landau level: $n = \frac{eB}{hc}$

Here it is obvious n changes with B.

Quantum Hall effect





$$\sigma_{xy} = 2\ \frac{e^2}{h}$$

Final Comment

$$\frac{dM}{d\phi} = \text{const} \iff \text{Streda}$$

see (1.35) in book.

$$\frac{\partial M_{orb}}{\partial \phi} = \frac{1}{c}\sigma_{AHC}$$

$$\frac{\partial M_{orb}}{\partial \phi} = -\frac{\partial}{\partial \phi}\left(\frac{\partial E}{\partial B_\perp}\right) = -\frac{\partial}{\partial B_\perp}\left(\frac{\partial E}{\partial \phi}\right) = -\frac{\partial}{\partial B_\perp}\overbrace{(-en)}^{\sigma_{surf}}$$

$$= e\frac{\partial n}{\partial B_\perp} = \frac{e^2 C}{hc} = \frac{1}{c}\sigma_{AHC} \checkmark$$

"Equality of mixed partials" or "thermodynamic relation"