Axion EM

Maxwell equations

$$\boldsymbol{\nabla} \cdot \boldsymbol{\mathcal{E}} = 4\pi\rho$$
$$\boldsymbol{\nabla} \cdot \mathbf{B} = 0$$
$$\boldsymbol{\nabla} \times \boldsymbol{\mathcal{E}} = -\frac{1}{c}\frac{\partial \mathbf{B}}{\partial t}$$
$$\boldsymbol{\nabla} \times \mathbf{B} = \frac{4\pi}{c}\mathbf{J} + \frac{1}{c}\frac{\partial \boldsymbol{\mathcal{E}}}{\partial t}$$

With bound charges and currents

$$\boldsymbol{\nabla} \cdot \boldsymbol{\mathcal{E}} = 4\pi(\rho_{\mathrm{f}} + \rho_{\mathrm{b}})$$
$$\boldsymbol{\nabla} \cdot \mathbf{B} = 0$$
$$\boldsymbol{\nabla} \times \boldsymbol{\mathcal{E}} = -\frac{1}{c}\frac{\partial \mathbf{B}}{\partial t}$$
$$\boldsymbol{\nabla} \times \mathbf{B} = \frac{4\pi}{c}(\mathbf{J}_{\mathrm{f}} + \mathbf{J}_{\mathrm{b}} + \mathbf{J}_{\mathrm{p}}) + \frac{1}{c}\frac{\partial \boldsymbol{\mathcal{E}}}{\partial t}$$

$$\rho_{\mathrm{b}} = -\boldsymbol{\nabla} \cdot \mathbf{P}$$
$$\mathbf{J}_{\mathrm{b}} = \boldsymbol{\nabla} \times \mathbf{M}$$
$$\mathbf{J}_{\mathrm{p}} = \frac{\partial \mathbf{P}}{\partial t}$$

Constitutive relations (isotropic)

$$\underline{P} = \underline{P}_0 + \cancel{\chi_e} \, \underline{\mathcal{E}} + \alpha \, \underline{B}$$
$$\underline{M} = \underline{M}_0 + \alpha \, \underline{\mathcal{E}} + \cancel{\chi_m} \, \underline{B}$$

Drop χ_e, χ_m (no good reason):

$$
\mathbf{P} = \mathbf{P}_0 + \alpha\,\mathbf{B}
$$

$$
\mathbf{M} = \mathbf{M}_0 + \alpha\,\boldsymbol{\mathcal{E}}
$$

$$
\tilde{\rho}_{\mathrm{b}} = -\boldsymbol{\nabla}\cdot\mathbf{P}_0
$$

$$
\tilde{\mathbf{J}}_{\mathrm{b}} = \boldsymbol{\nabla}\times\mathrm{M}_0
$$

$$
\tilde{\mathbf{J}}_{\mathrm{p}} = \frac{\partial \mathbf{P}_0}{\partial t}
$$

Then Maxwell equations become, for $\alpha(\underline{x}, t)$:

$$
\boldsymbol{\nabla}\cdot\boldsymbol{\mathcal{E}} = 4\pi\left(\rho_{\mathrm{f}} + \tilde{\rho}_{\mathrm{b}} - (\boldsymbol{\nabla}\alpha)\cdot\mathbf{B}\right)
$$

$$
\boldsymbol{\nabla}\cdot\mathbf{B} = 0
$$

$$
\boldsymbol{\nabla}\times\boldsymbol{\mathcal{E}} = -\frac{1}{c}\,\frac{\partial \mathbf{B}}{\partial t}
$$

$$
\boldsymbol{\nabla}\times\mathbf{B} = \frac{4\pi}{c}\left(\mathbf{J}_{\mathrm{f}} + \tilde{\mathbf{J}}_{\mathrm{b}} + \tilde{\mathbf{J}}_{\mathrm{p}} + c\,(\boldsymbol{\nabla}\alpha)\times\boldsymbol{\mathcal{E}} + \frac{\partial\alpha}{\partial t}\,\mathbf{B}\right) + \frac{1}{c}\,\frac{\partial\boldsymbol{\mathcal{E}}}{\partial t}
$$

In the context of elementary particle physics:

(I) α = fixed background field, independent of $(\underline{x}, t)$

(II) $\alpha(\underline{x}, t)$, or equivalently $\theta(\underline{x}, t)$, is a dynamical field. Its vacuum expectation value is zero, and its quantized fluctuations are "axions"

Lagrangian has a term proportional to E • B

$$\Delta\mathcal{L}_{EM} = \frac{\theta e^2}{2\pi h}\mathbf{E}\cdot\mathbf{B} = \frac{\theta e^2}{16\pi h}\epsilon^{\alpha\beta\gamma\delta}F_{\alpha\beta}F_{\gamma\delta}$$

$$F^{\mu\nu} = \begin{bmatrix} 0 & -E_x/c & -E_y/c & -E_z/c \\ E_x/c & 0 & -B_z & B_y \\ E_y/c & B_z & 0 & -B_x \\ E_z/c & -B_y & B_x & 0 \end{bmatrix}$$

Suggested for the fundamental Lagrangian of our universe:

- Static field θ: Some bizarre consequences
  - CP violation
    - Known to be broken in weak sector
    - Apparently not broken in QCD
  - Electric charges acquire magnetic monopole
- Dynamic field θ(r,t): Quantum is "axion"
  - Possible dark matter candidate

Here, θ(r) ≠ 0 inside a magnetoelectric insulator

- Emergent property of insulating ground state

In the context of elementary particle physics:

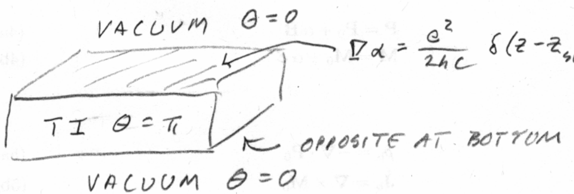
$$\boldsymbol{\nabla}\cdot\boldsymbol{\mathcal{E}} = 4\pi\Big(\rho - (\boldsymbol{\nabla}\alpha)\cdot\mathbf{B}\Big)$$
$$\boldsymbol{\nabla}\cdot\mathbf{B} = 0$$
$$\boldsymbol{\nabla}\times\boldsymbol{\mathcal{E}} = -\frac{1}{c}\frac{\partial\mathbf{B}}{\partial t}$$
$$\boldsymbol{\nabla}\times\mathbf{B} = \frac{4\pi}{c}\Big(\mathbf{J} + c\,(\boldsymbol{\nabla}\alpha)\times\boldsymbol{\mathcal{E}} + \frac{\partial\alpha}{\partial t}\mathbf{B}\Big) + \frac{1}{c}\frac{\partial\boldsymbol{\mathcal{E}}}{\partial t}$$

In materials, $\alpha = \alpha(\underline{r})$ independent of t.
Static case, no bound ρ or $\underline{J}$, $\underline{P}_0 = 0$, $\underline{M}_0 = 0$:

$$\boldsymbol{\nabla}\cdot\boldsymbol{\mathcal{E}} = -4\pi(\boldsymbol{\nabla}\alpha)\cdot\mathbf{B}$$
$$\boldsymbol{\nabla}\cdot\mathbf{B} = 0$$
$$\boldsymbol{\nabla}\times\boldsymbol{\mathcal{E}} = 0$$
$$\boldsymbol{\nabla}\times\mathbf{B} = 4\pi(\boldsymbol{\nabla}\alpha)\times\boldsymbol{\mathcal{E}}$$

which we can think of as

$$\rho_\alpha = -(\underline{\nabla}\alpha)\cdot\underline{B} \qquad \text{"STREDA"}$$
$$\underline{J}_\alpha = c\,(\underline{\nabla}\alpha)\times\underline{\mathcal{E}} \qquad \text{"ANOMALOUS HALL"}$$



$$\underline{\nabla}\alpha = \frac{e^2}{2hc}\,\delta(z - z_{SURF})\,\hat{z}$$

$$\sigma_\alpha = -\frac{e^2}{2hc}\,B_z$$
$$\underset{\sim}{K}_\alpha = \frac{e^2}{2h}\,\hat{z}\times\underline{\mathcal{E}}$$