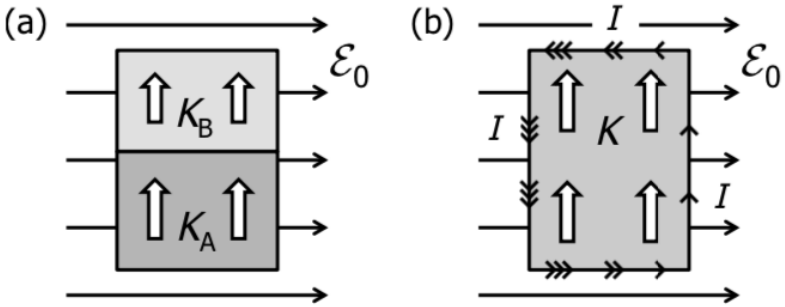
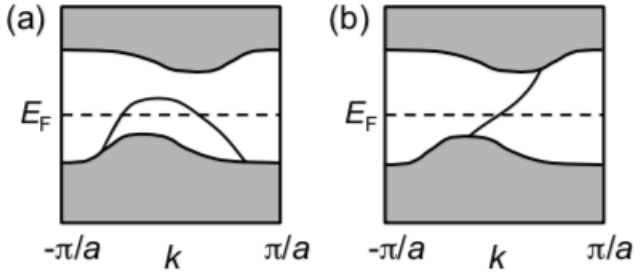
Quantum anomalous Hall insulators in 2D

- Review Ch. 1
- Review E-field perturbation
- Analogy with 1D quantum charge pump
- Models: Haldane etc.
- Search for physical realizations

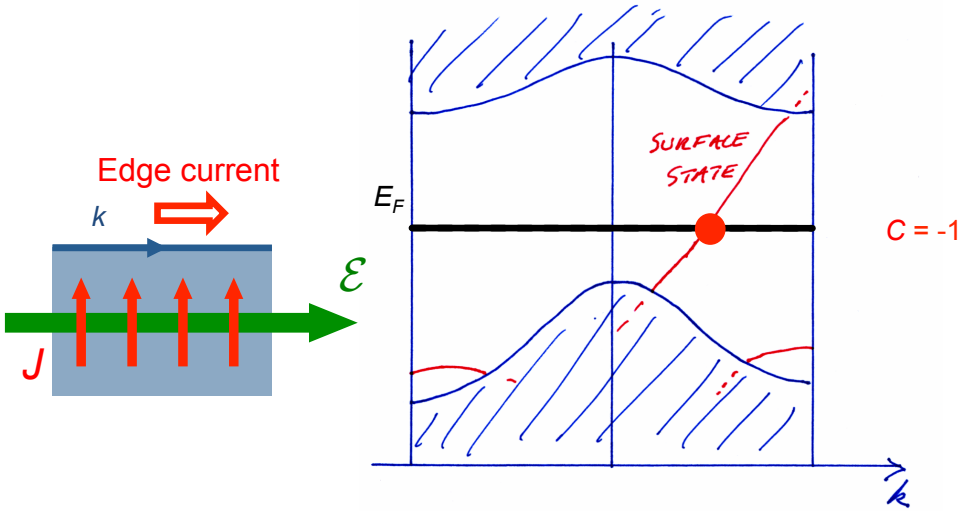
Review Ch 1



$$\sigma_{\mathrm{AHC}} = C\,\frac{e^2}{h}$$



$$n_{\mathrm{up}} - n_{\mathrm{down}} = -C$$



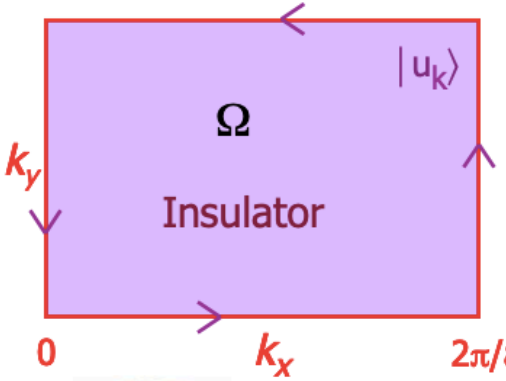
$$\sigma_{\mathrm{AHC}} = C\,\frac{e^2}{h}$$

Conservation of charge ⇒ chiral surface state

E-field perturbation

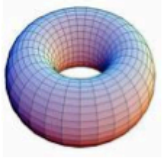
$$\sigma_{\nu\mu} = \frac{e^2}{\hbar}\frac{1}{(2\pi)^3}\sum_n \int d^3k\, f_{nk}\, \Omega_{n,\mu\nu}$$

$f_{occupation}$, 0 or 1



$$\Omega_z(\mathbf{k}) = -2\mathrm{Im}\left\langle \frac{du}{dk_x} \middle| \frac{du}{dk_y} \right\rangle$$

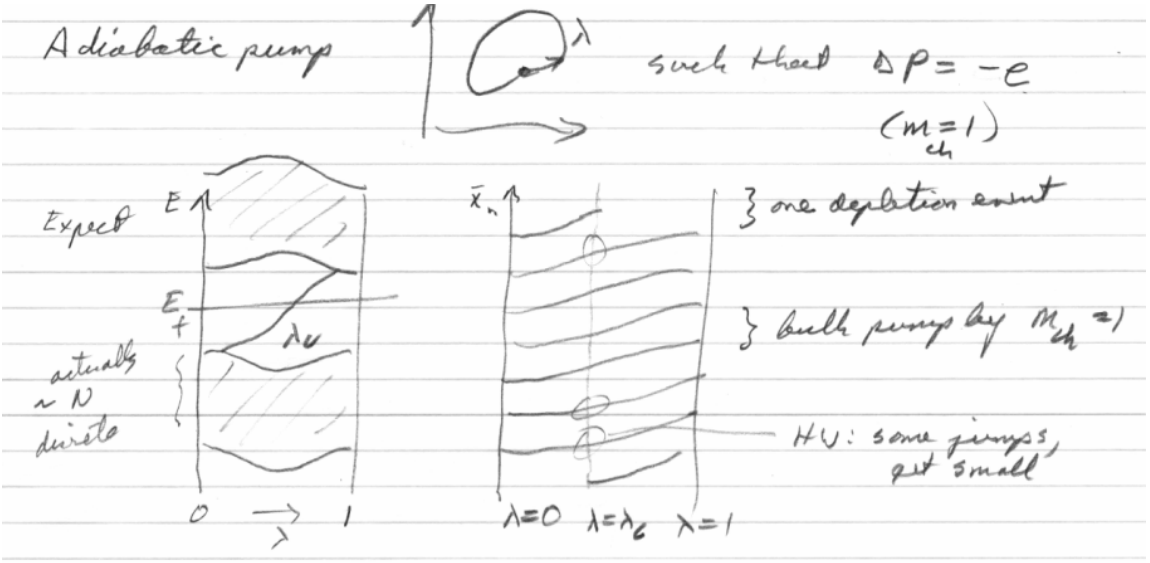
$$\int_{BZ} \Omega_z(\mathbf{k})\, d^2k = 2\pi C$$



Quantum Anomalous Hall: $\sigma_{xy} = \frac{-e^2}{h} C$

Chern number

Analogy with 1D quantum charge pump



∴ # of up-crossings of surface states

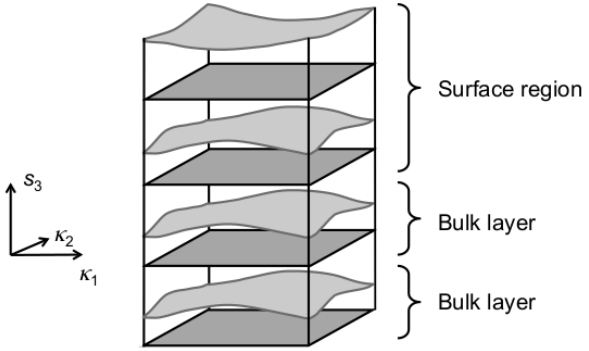
must = Chern # (Wannier winding #) of bulk

in order for charge cons. to hold

"Bulk-surface correspondence"

Hybrid Wannier representation

In 3D, with surface



In 2D, only showing in bulk

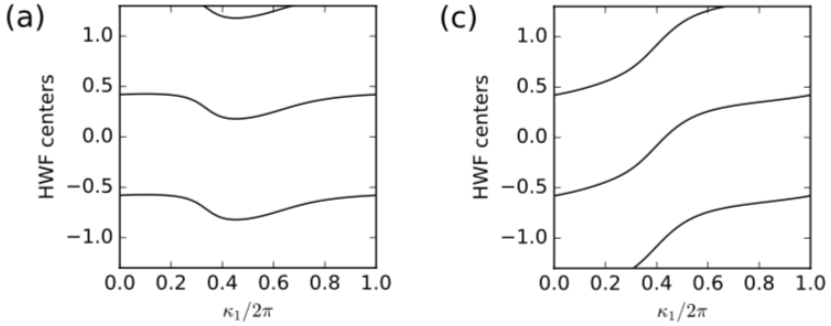
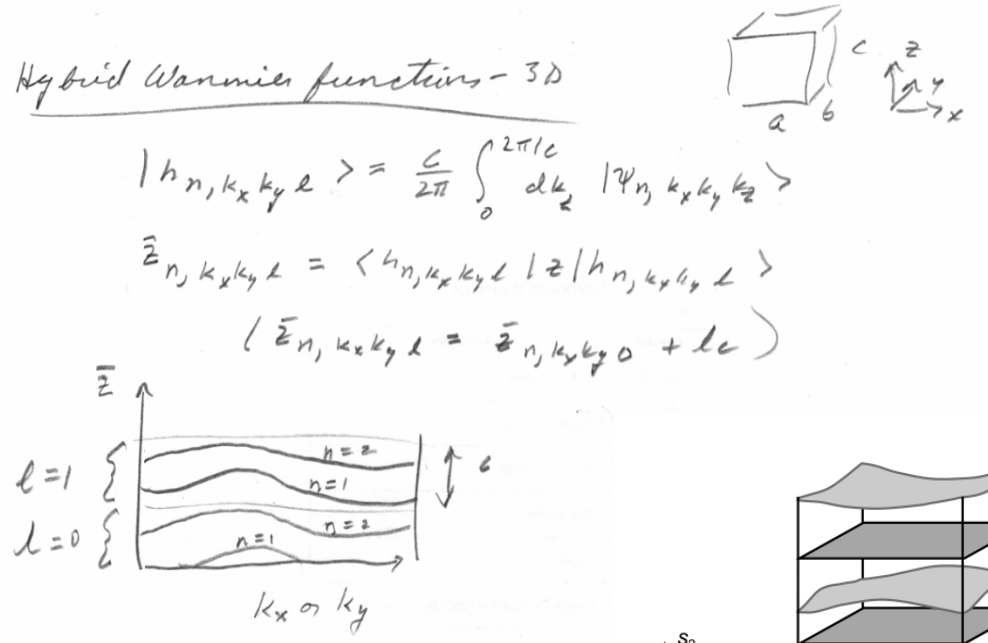
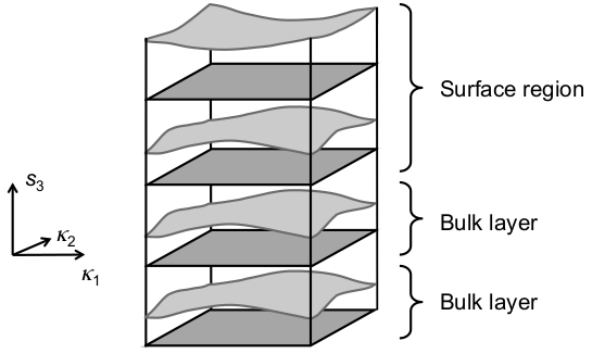


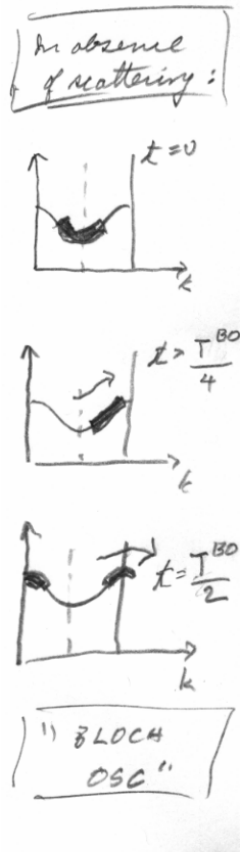
Figure 5.4

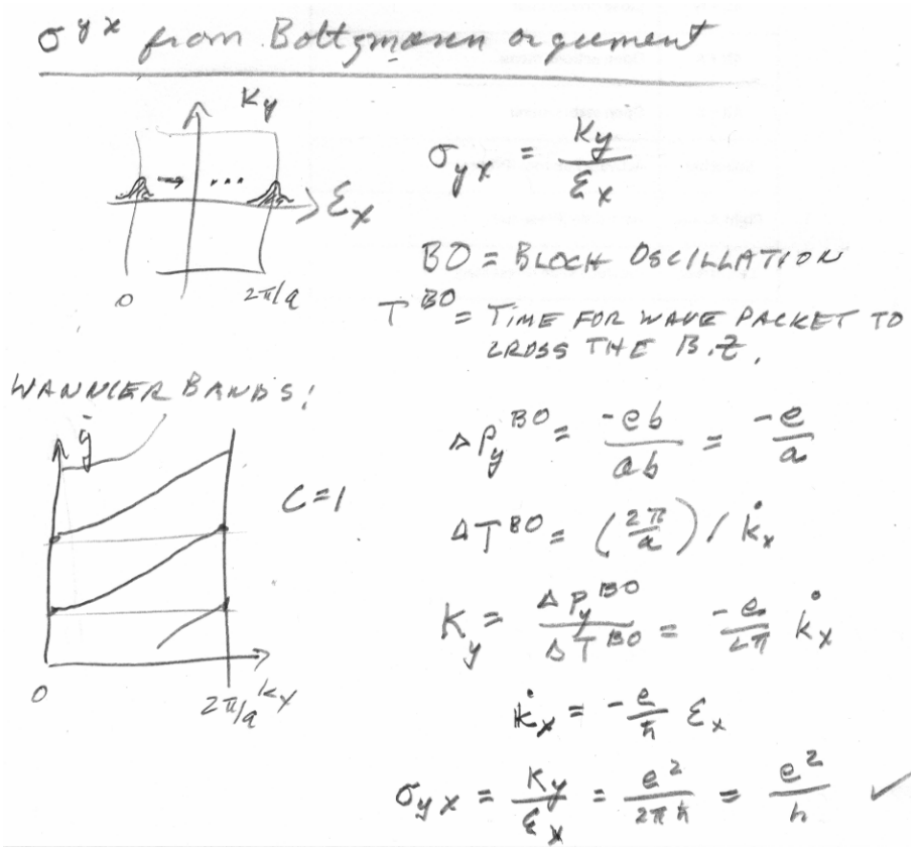
Hybrid Wannier representation





Elementary derivation of quantized AHC





More generally, $\sigma_{\mathrm{AHC}} = C\,\dfrac{e^2}{h}$

Haldane model

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Model for a Quantum Hall Effect without Landau Levels: Condensed-Matter Realization of the "Parity Anomaly"

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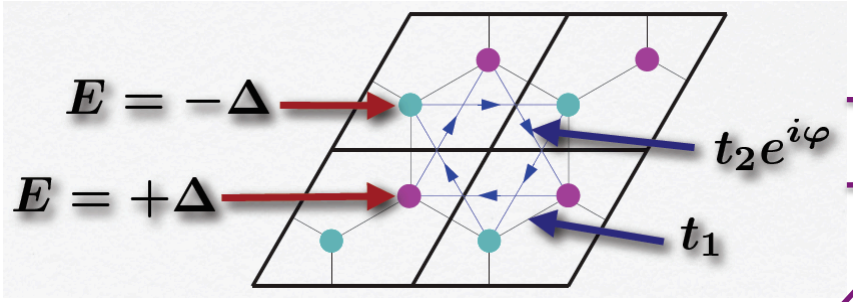
(Received 16 September 1987)

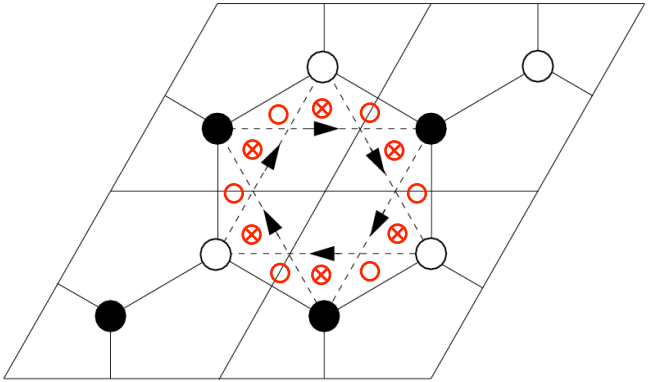
A two-dimensional condensed-matter lattice model is presented which exhibits a nonzero quantization of the Hall conductance σ^{xy} in the *absence* of an external magnetic field. Massless fermions *without spectral doubling* occur at critical values of the model parameters, and exhibit the so-called "parity anomaly" of (2+1)-dimensional field theories.



Usually just it_2

$$H = \Delta \sum_i (-)^{\tau_i} c_i^\dagger c_i + t_1 \sum_{\langle ij \rangle} (c_i^\dagger c_j + \text{h.c.}) + t_2 \sum_{\langle\langle ij \rangle\rangle} (i c_i^\dagger c_j + \text{h.c.})$$

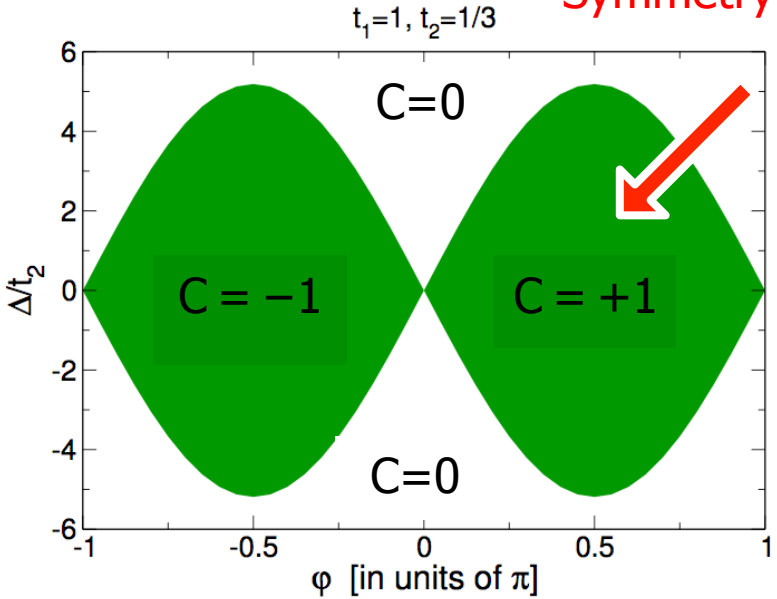
Flux tubes in Haldane model

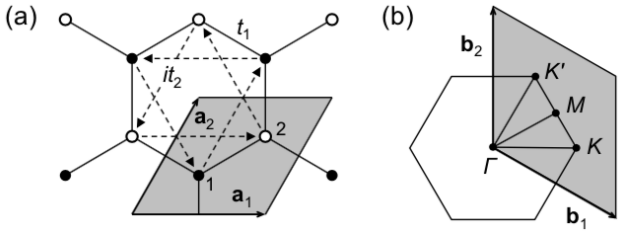


(Real materials: spin-orbit interaction gives similar effects)

Topological phase transition

No change of Symmetry!





Increasing t_2

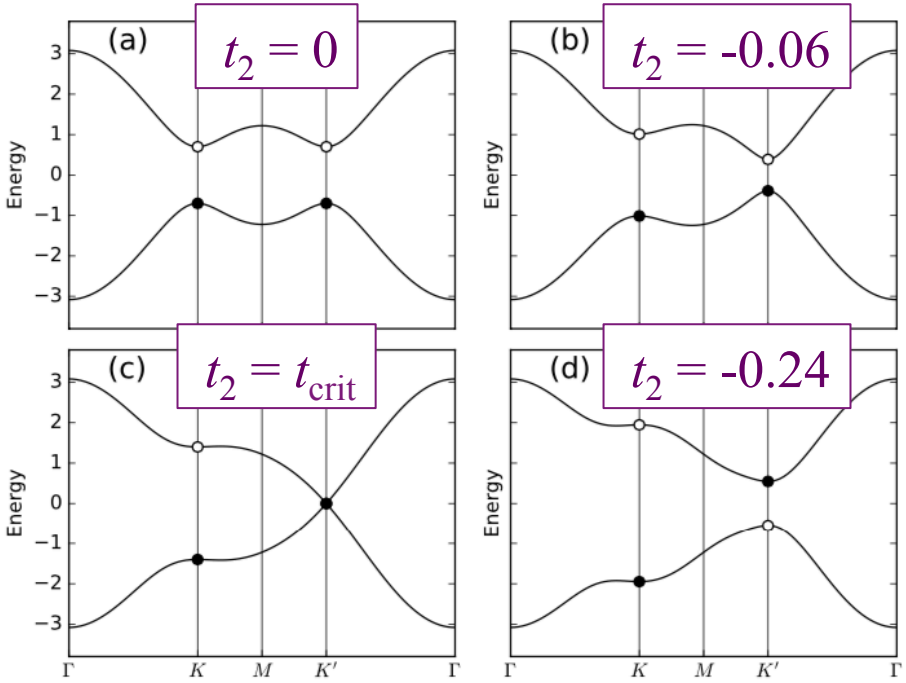
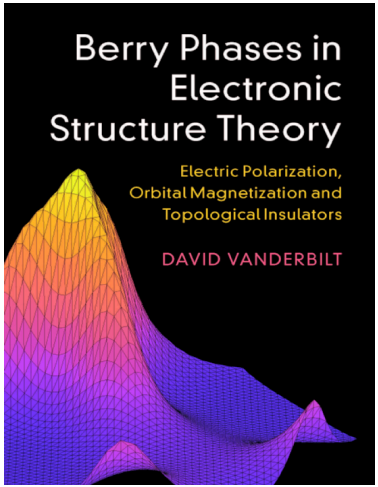


Figure 5.2 Haldane model band structures for the Haldane model of



haldane_bcurv.py

```
#!/usr/bin/env python
from __future__ import print_function # python3 style print

# Berry curvature of Haldane model

from pythtb import * # import TB model class
import matplotlib.pyplot as plt

# define setup of Haldane model
def set_model(delta,t,t2):
  lat=[[1.0,0.0],[0.5,np.sqrt(3.0)/2.0]]
  orb=[[1./3.,1./3.],[2./3.,2./3.]]
  model=tb_model(2,2,lat,orb)
  model.set_onsite([-delta,delta])
  for lvec in ([ 0, 0], [-1, 0], [ 0,-1]):
    model.set_hop(t, 0, 1, lvec)
  for lvec in ([ 1, 0], [-1, 1], [ 0,-1]):
    model.set_hop(t2*1.j, 0, 0, lvec)
  for lvec in ([-1, 0], [ 1,-1], [ 0, 1]):
    model.set_hop(t2*1.j, 1, 1, lvec)
  return model
```

etc

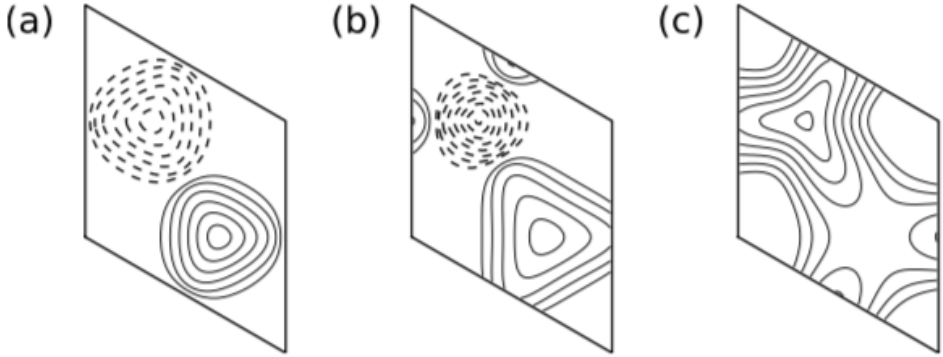
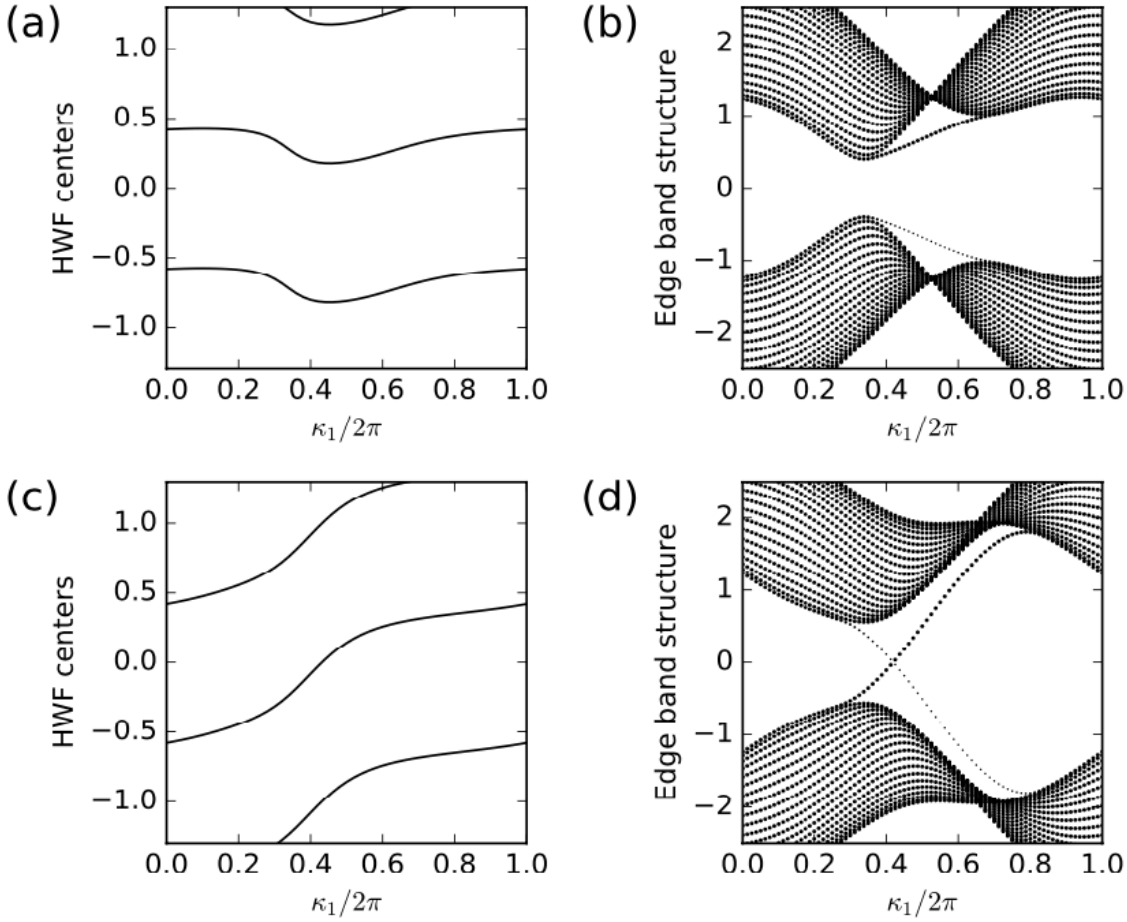


Figure 5.3 Contour plots of Berry curvature $\Omega(\mathbf{k})$ for the Haldane model


(a)
HWF centers
1.0
0.5
0.0
−0.5
−1.0
0.0 0.2 0.4 0.6 0.8 1.0
$\kappa_1/2\pi$
(b)
Edge band structure
2
1
0
−1
−2
0.0 0.2 0.4 0.6 0.8 1.0
$\kappa_1/2\pi$
(c)
HWF centers
1.0
0.5
0.0
−0.5
−1.0
0.0 0.2 0.4 0.6 0.8 1.0
$\kappa_1/2\pi$
(d)
Edge band structure
2
1
0
−1
−2
0.0 0.2 0.4 0.6 0.8 1.0
$\kappa_1/2\pi$

Search for physical realizations

(Next presentation)