CIS 521:
ARTIFICIAL INTELLIGENCE

Alpha-Beta Pruning

Professor Chris Callison-Burch
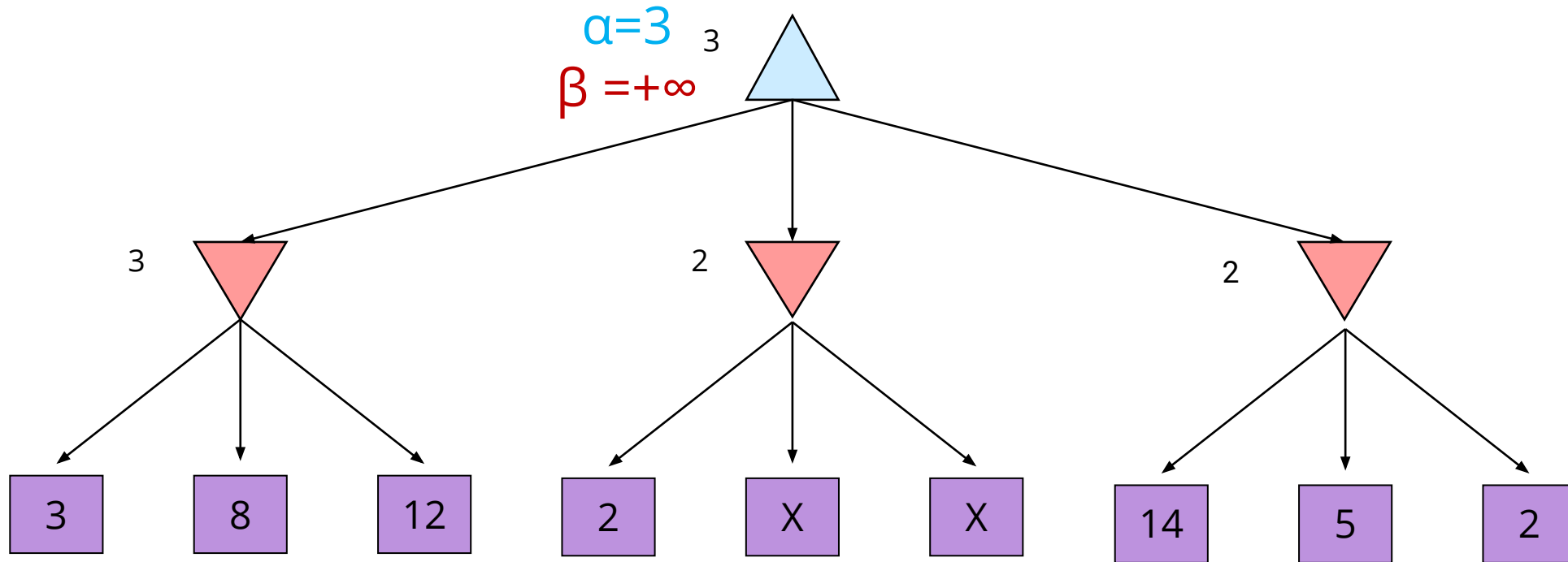
Many of today's slides are courtesy of Dan Klein and
Pieter Abbeel of University of California, Berkeley

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Alpha-Beta Pruning

○ During Minimax, keep track of two additional values:

- **α: MAX's current *lower* bound on MAX's outcome**
- **β: MIN's current *upper* bound on MIN's outcome**

○ MAX will never allow a move that could lead to a worse score (for MAX) than α

○ MIN will never allow a move that could lead to a better score (for MAX) than β

○ Therefore, stop evaluating a branch whenever:

- When evaluating a MAX node: a value $v \geq \beta$ is backed-up
  - MIN will never select that MAX node
- When evaluating a MIN node: a value $v \leq \alpha$ is found
  - MAX will never select that MIN node
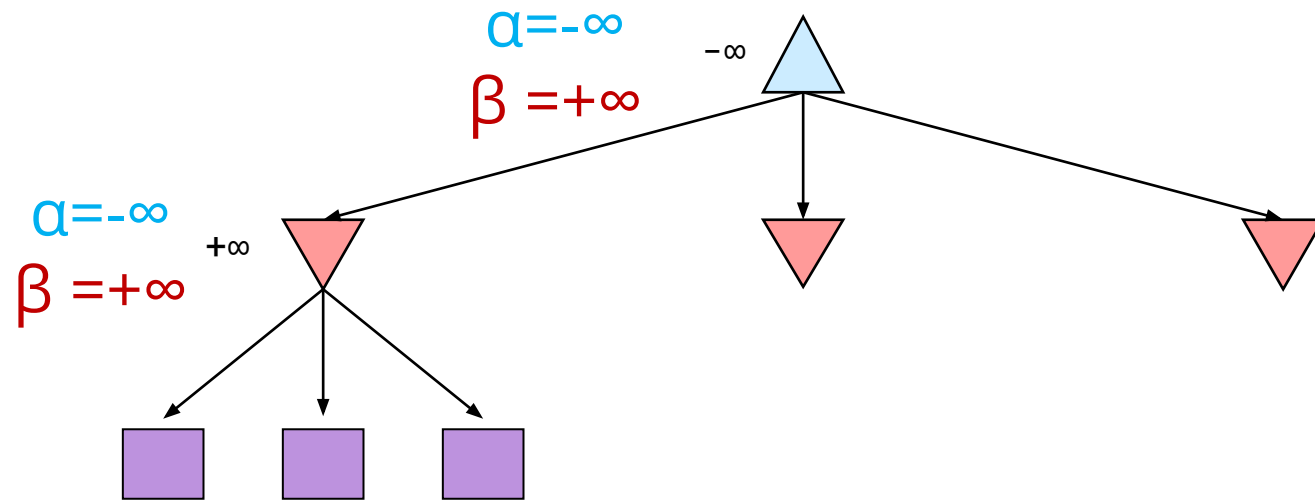
Alpha-Beta Pruning

α=3
β =+∞

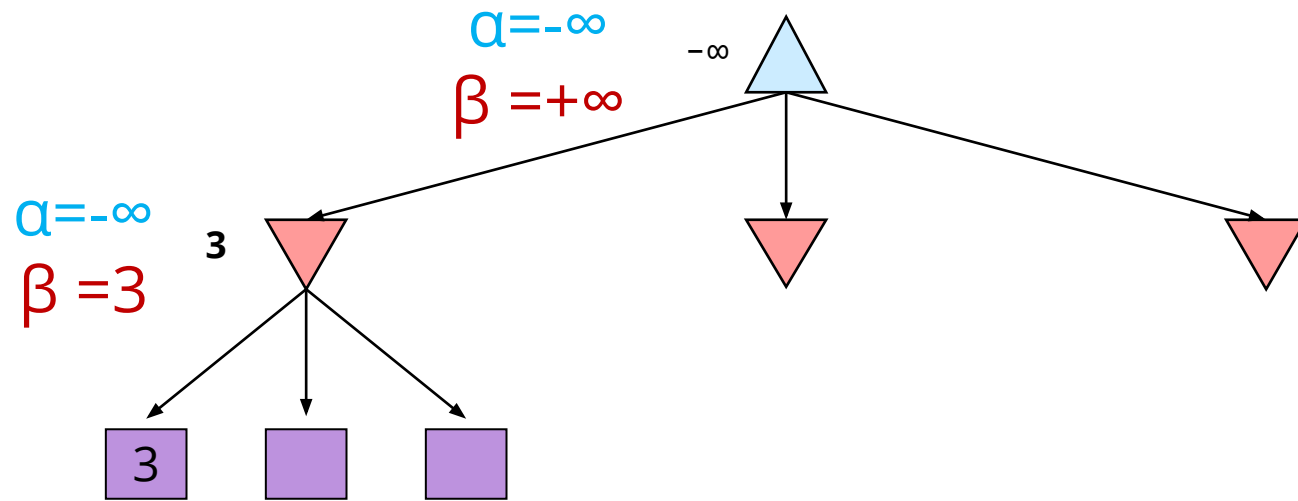α: MAX's best option on path to root
β: MIN's best option on path to root

Alpha-Beta Pruning Example

$\alpha = -\infty$

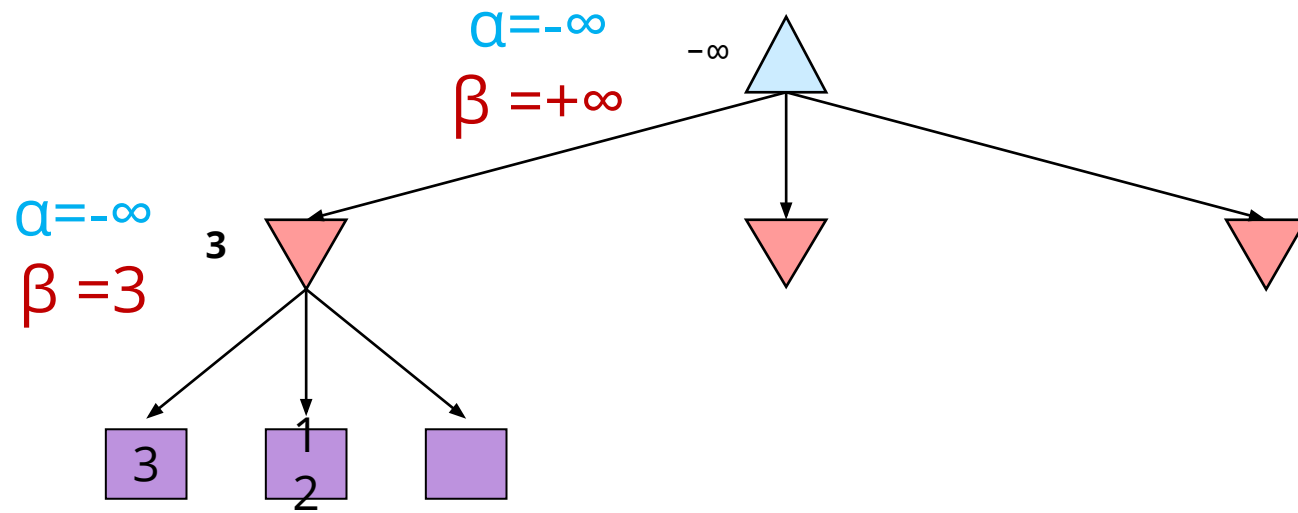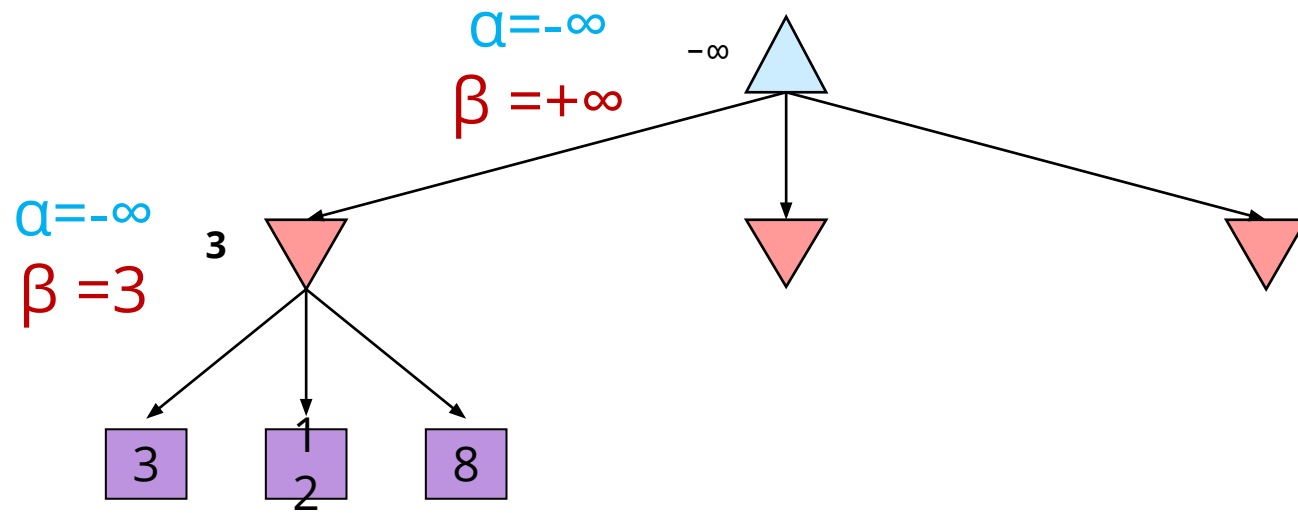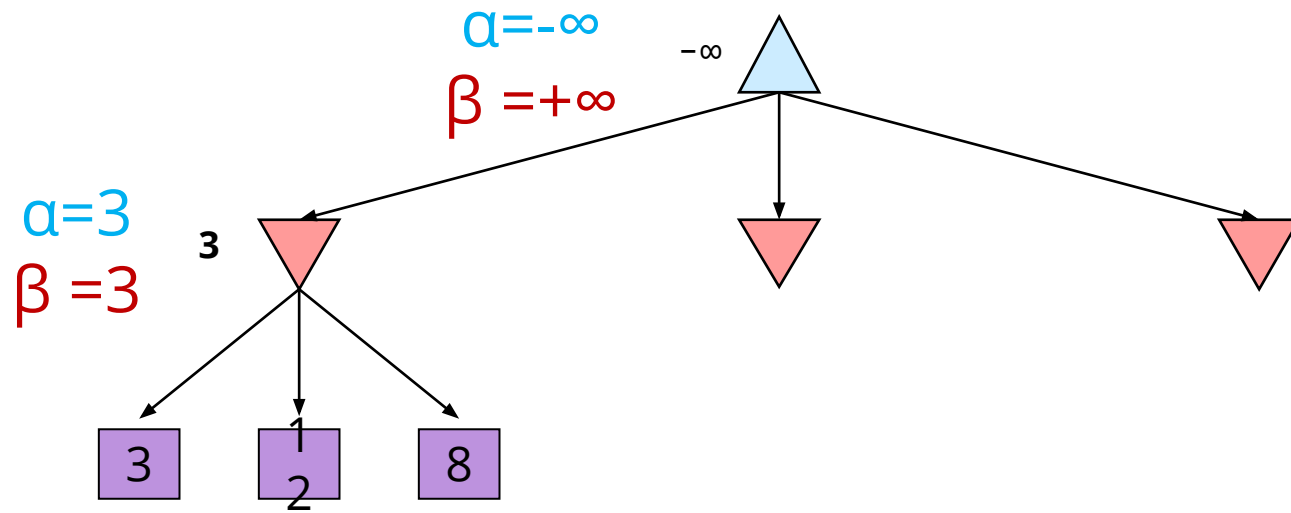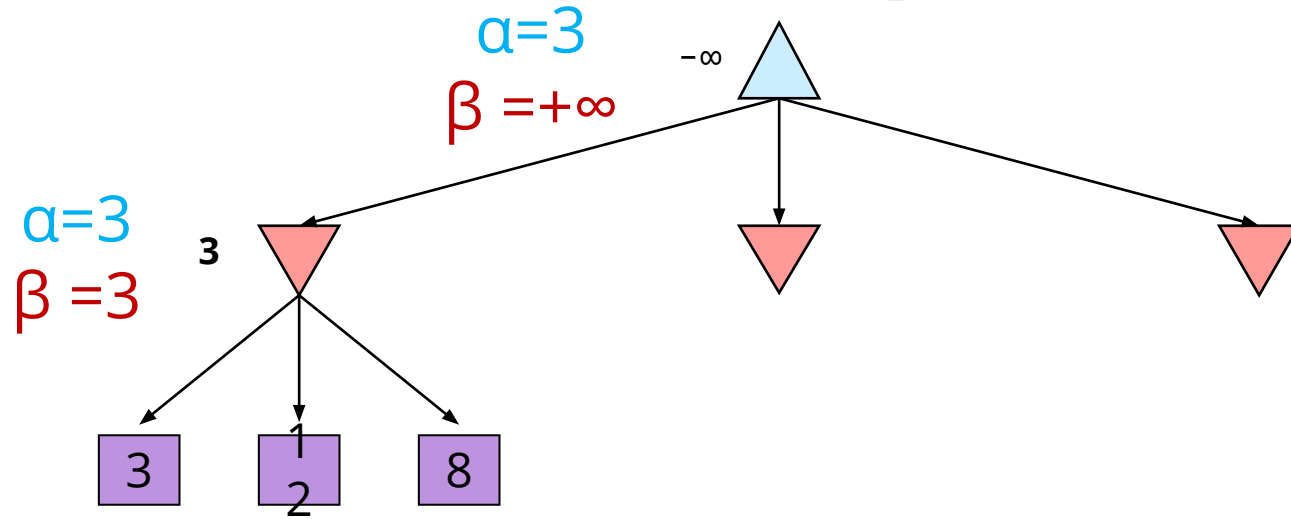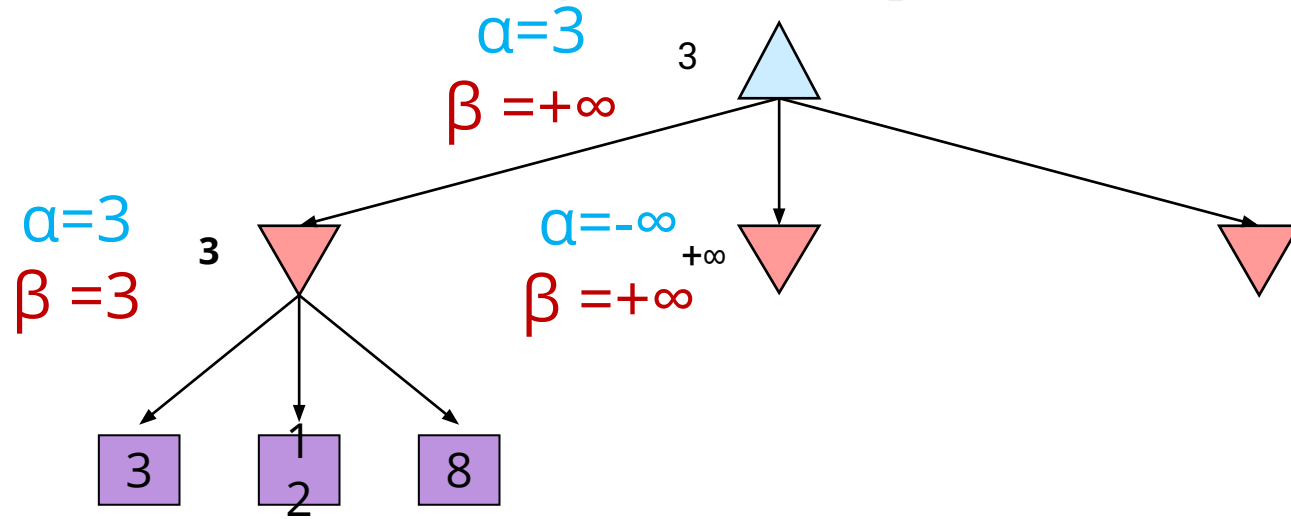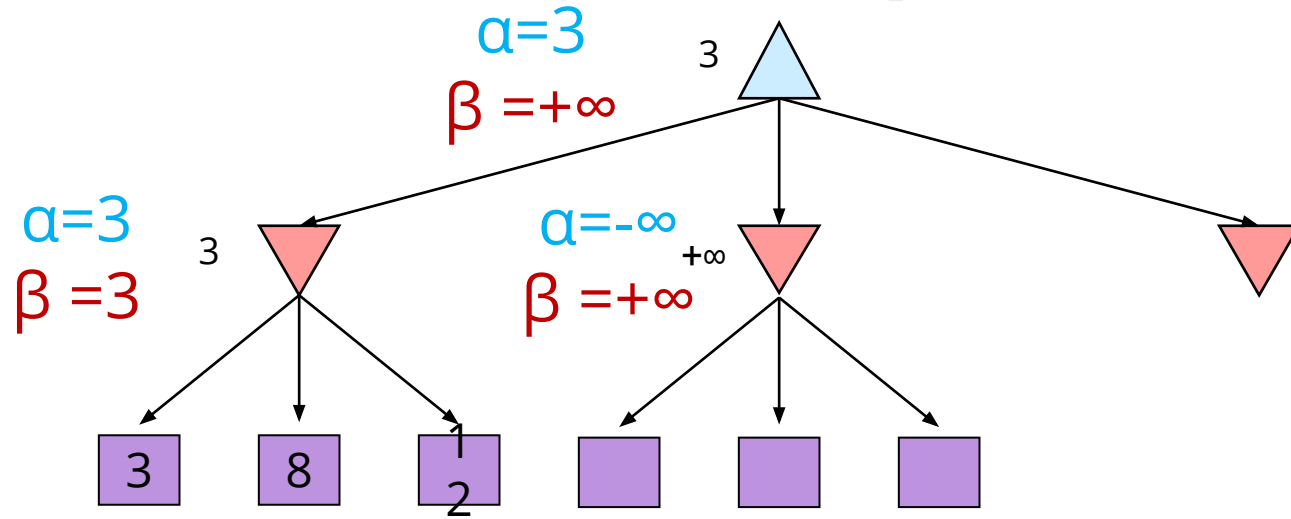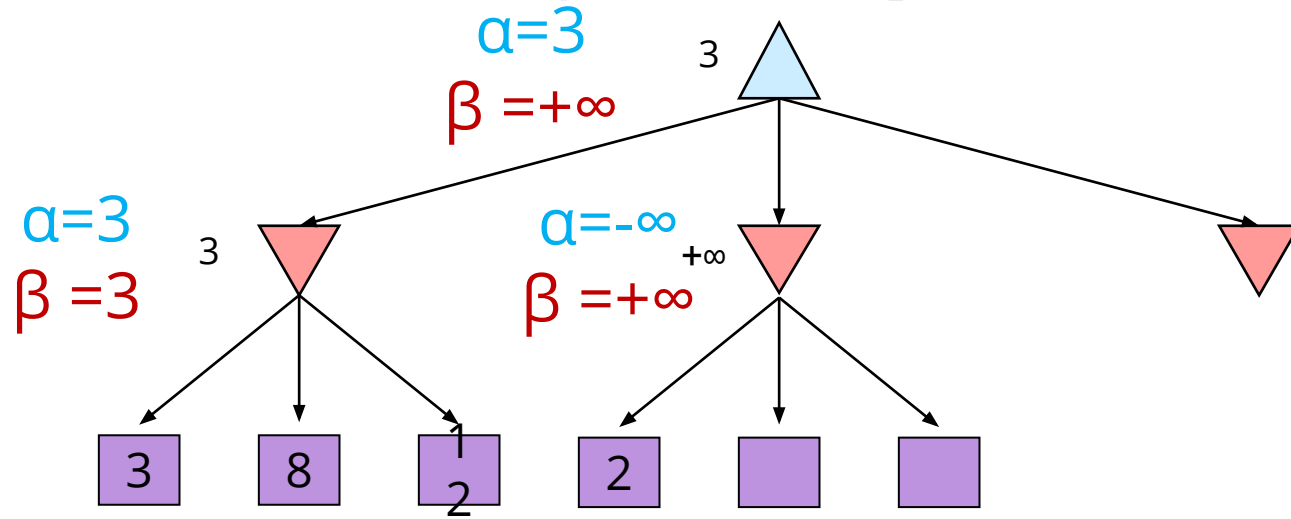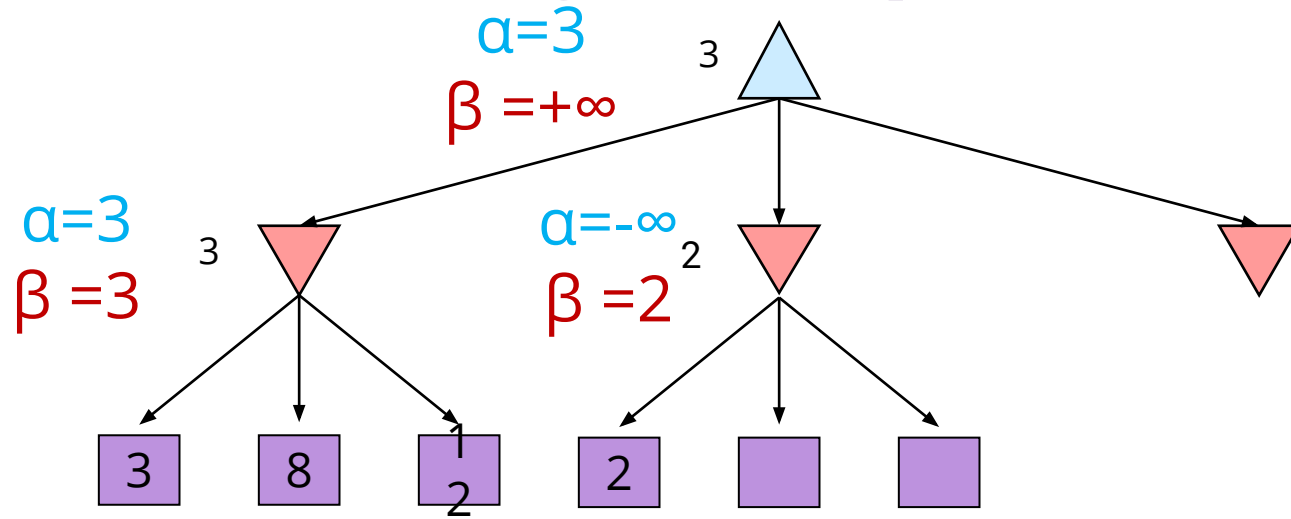$\beta = +\infty$

$-\infty$

Alpha-Beta Pruning Example

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Alpha-Beta Pruning Example

Alpha-Beta Pruning Example

α=3
β =+∞

3

α=3
β =3
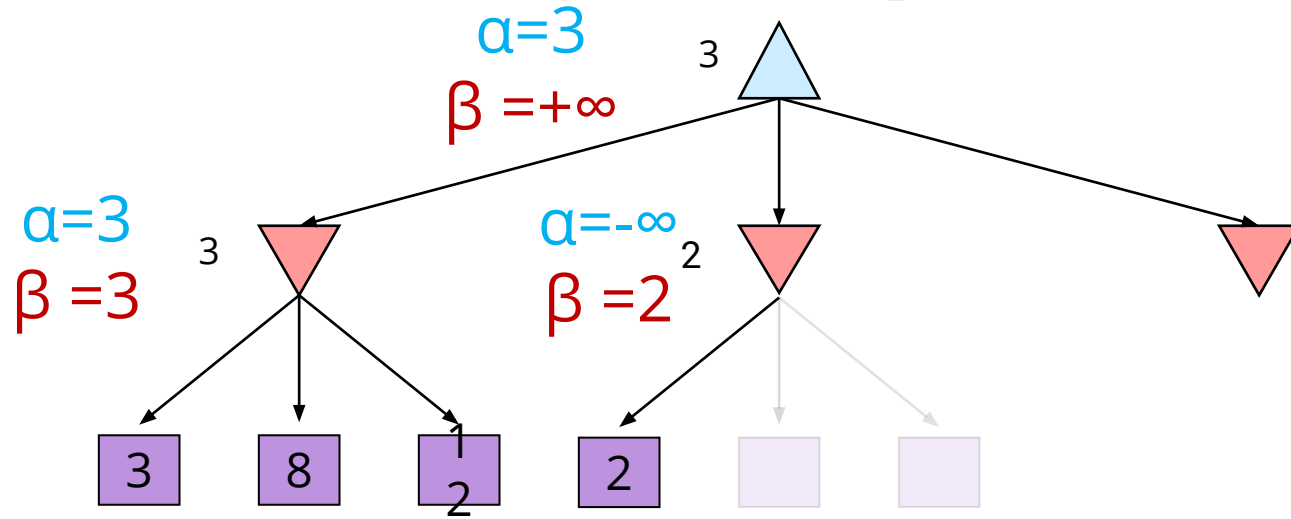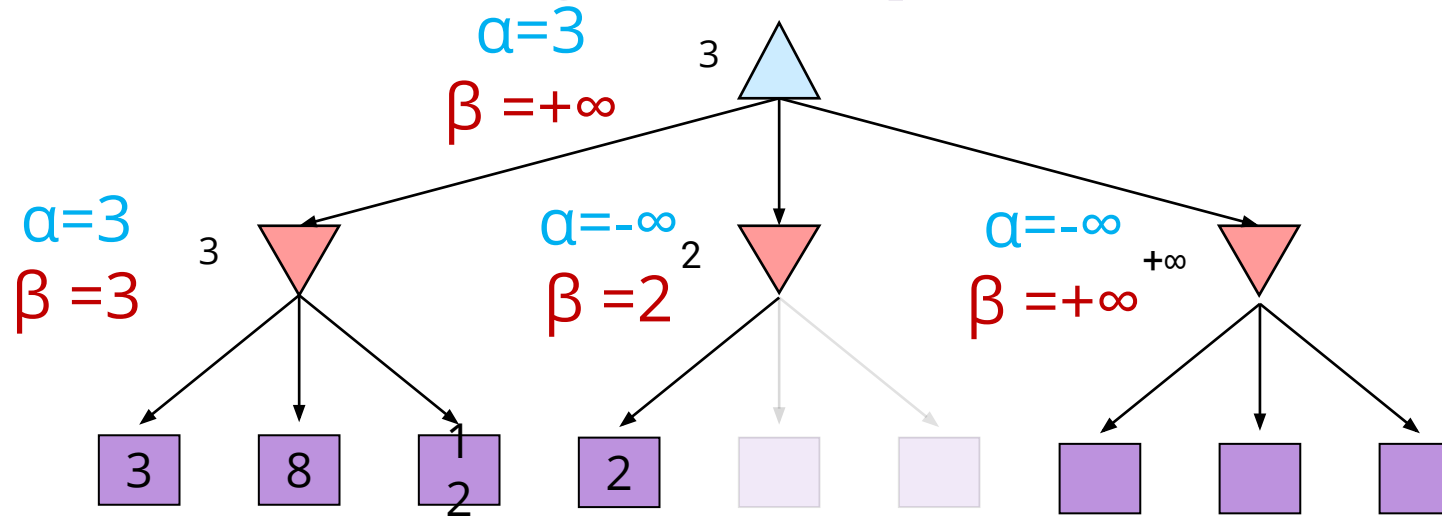
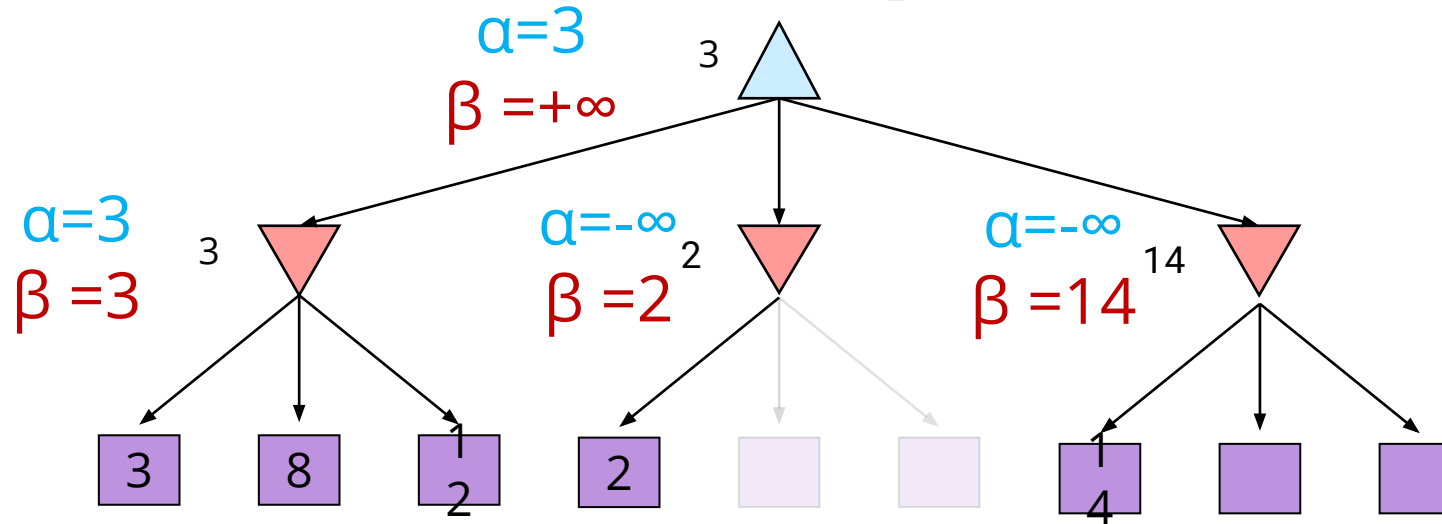3

α=-∞
β =+∞

+∞

3   1   8
     2

Alpha-Beta Pruning Example

Alpha-Beta Pruning Example

Alpha-Beta Pruning Example

Alpha-Beta Pruning Example

Alpha-Beta Pruning Example

α=3
β =+∞
3

α=3
β =3
3

α=-∞
β =2
2

α=-∞
β =+∞
+∞

3   8   12   2

Alpha-Beta Pruning Example

Alpha-Beta Pruning Example

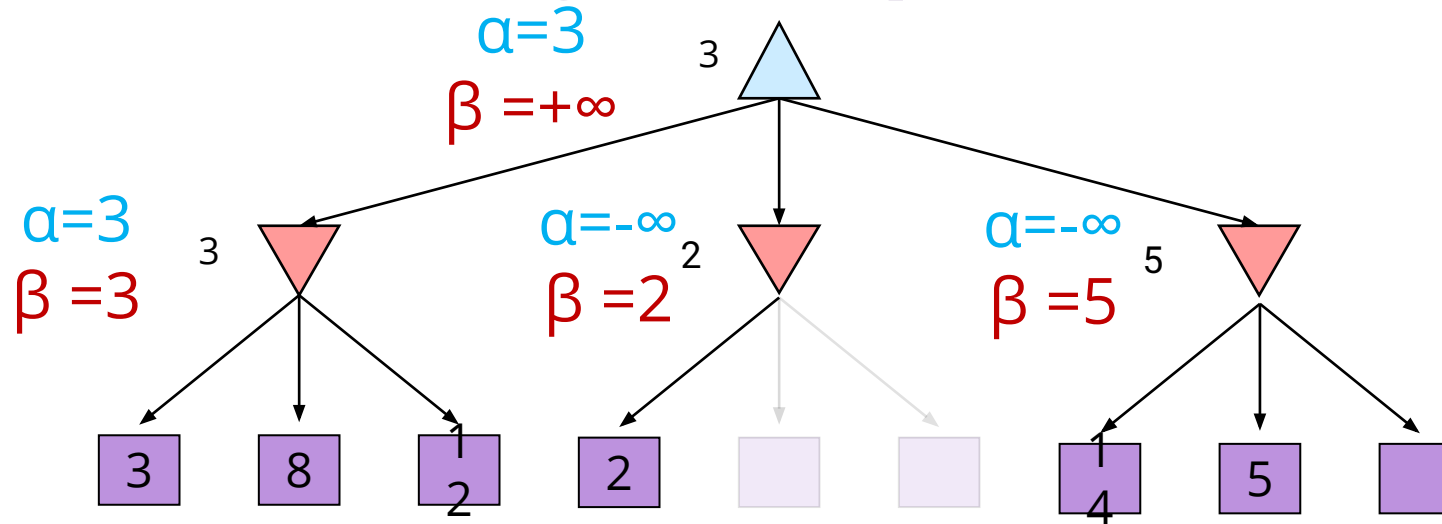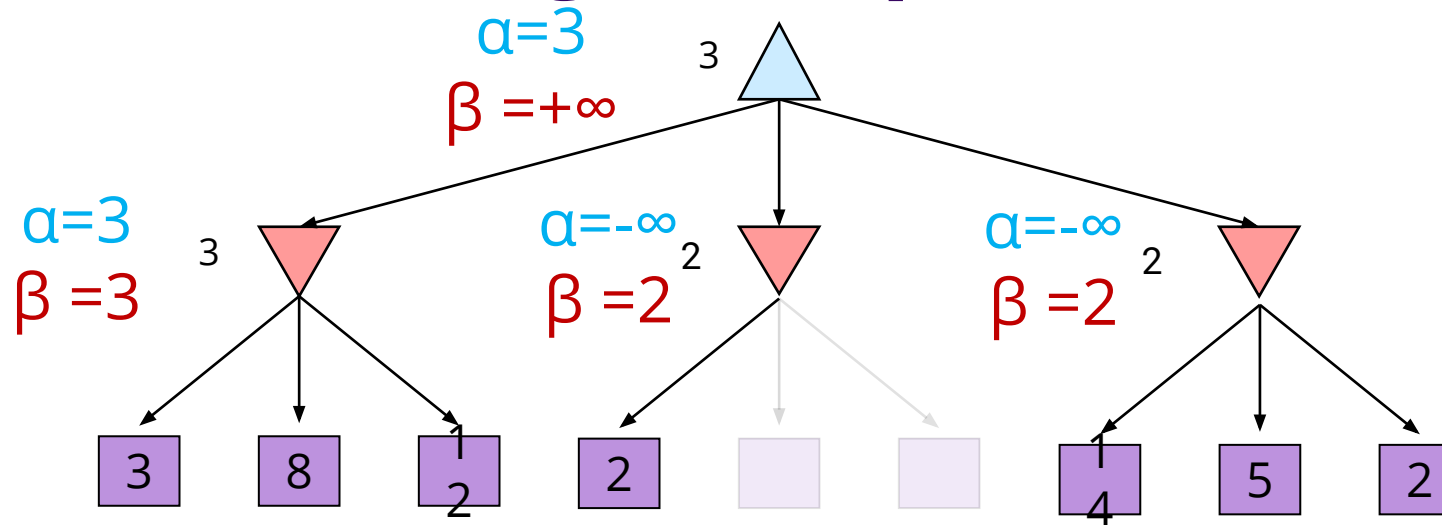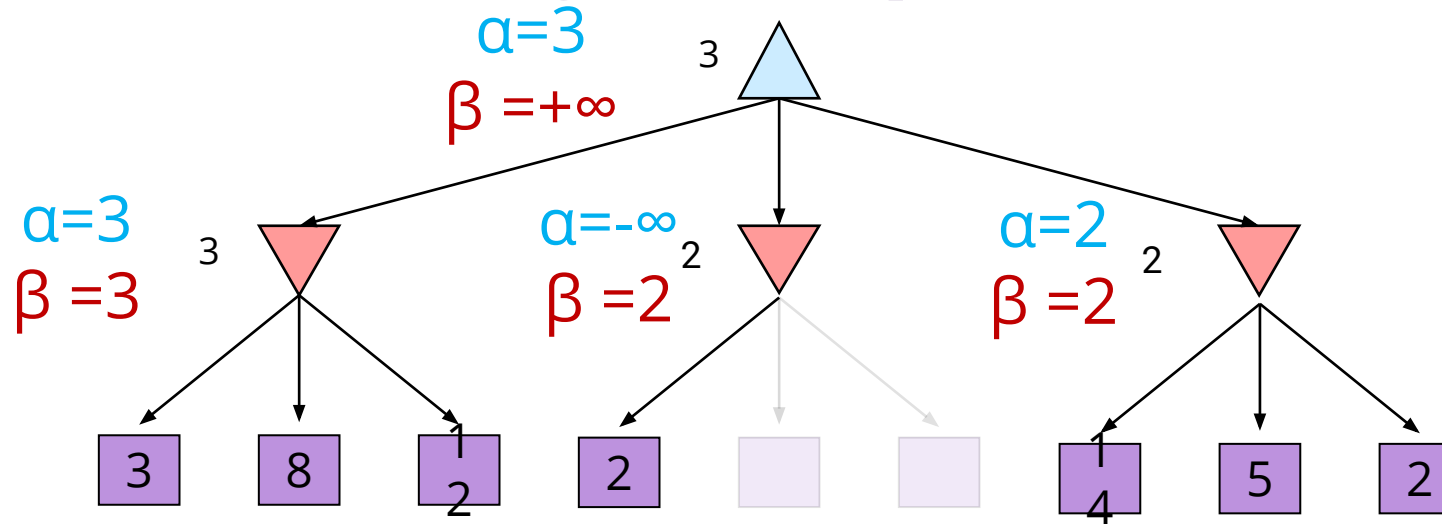# Alpha-Beta Pruning Example

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