Assignment 5
Question 1

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October 26, 2017

Question:
Given g_1 and g_2, and assuming h_1 and h_2 are known, derive a formula to determine f_1 and f_2 assuming that there was no relative motion between the camera and the scene outside while the two pictures were being acquired and there is no change whatsoever to the scene inside or outside.

Answer

Given the equations

$$g_1(x, y) = f_1(x, y) + (h_2 * f_2)(x, y)$$

$$g_2(x, y) = (h_1 * f_1)(x, y) + f_2(x, y)$$

Taking Fourier transform of these equations and using convolution theorem (Fourier transform of a convolution is the pointwise product of the Fourier transforms) we get-

$$G_1(u, v) = F_1(u, v) + H_2(u, v)F_2(u, v) \tag{1}$$

$$G_2(u, v) = H_1(u, v)F_1(u, v) + F_2(u, v) \tag{2}$$

Multiplying (1) by $H_1(u, v)$ and subtracting (2) we get

$$H_1(u, v)G_1(u, v) - G_2(u, v) = H_1(u, v)H_2(u, v)F_2(u, v) - F_2(u, v)$$

$$\hat{F}_2(u, v) = \frac{G_2(u, v) - H_1(u, v)G_1(u, v)}{1 - H_1(u, v)H_2(u, v)} = F_2(u, v)$$

Taking Inverse Fourier Transform we get

$$f_2(x, y) = \mathcal{F}^{-1}(\hat{F}_2(u, v))$$

Similarly we can get f_1. Multiplying (2) by $H_2(u, v)$ and subtracting (1) we get

$$H_2(u, v)G_2(u, v) - G_1(u, v) = H_2(u, v)H_1(u, v)F_1(u, v) - F_1(u, v)$$

$$\hat{F}_1(u, v) = \frac{G_1(u, v) - H_2(u, v)G_2(u, v)}{1 - H_2(u, v)H_1(u, v)} = F_1(u, v)$$

Taking Inverse Fourier Transform we get

$$f_1(x, y) = \mathcal{F}^{-1}(\hat{F}_1(u, v))$$

Even after all the assumptions we do notice something problematic with the formulas. It is that as $h_1(x, y), h_2(x, y)$ are blur kernels hence $H_1(u, v), H_2(u, v)$ are low pass filters and so at low frequencies they will both tend to 1. This will cause $H_1(u, v)H_2(u, v)$ to tend to one and so $1 - H_1(u, v)H_2(u, v)$ will tend to 0.
If the value of $1 - H_1(u, v)H_2(u, v)$ becomes 0, then we cannot use this approach to extract low frequency components of $\hat{F_1}(u, v)$ and $\hat{F_2}(u, v)$. Now if $1 - H_1(u, v)H_2(u, v)$ does not become 0, then we can calculate $f_1(x, y)$ and $f_2(x, y)$ accurately provided there is no noise. If we consider noise then if $1 - H_1(u, v)H_2(u, v)$ tends to 0 ,the value of $\hat{F_1}(u, v)$ and $\hat{F_2}(u, v)$ will blow up amplifying noise in $f_1(x, y)$ and $f_2(x, y)$.