Assignment 5
Question 5

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October 26, 2017

The second statement i.e., $E_L(V) \geq E_N(V)$ is correct.
In the case of linear approximation, we are forced to select only the first k columns of V corresponding to the largest k eigen values for any point x_i. Hence it would give the lowest reconstruction error only in the case when the reconstruction is done using the exact same set of k columns from V for all the points.
On the other hand, the non-linear approximation of V gives us the liberty to choose different subsets of k columns of V for different $x_i s$ and hence the one that minimises the error can be chosen flexibly for all the points.

Algorithm to obtain the order k non-linear approximation of x_i given V:-

- Compute: $\alpha_i = V^T x_i$
- Create a new vector β_i such that β_i contains the largest of the k entries from α_i and rest of the entries are set to 0.

These $\beta_i s$ would be the $\alpha_i s$ for the non-linear approximation.
This method is correct because of the fact that V is orthonormal. Hence,

$$\begin{aligned} ||x_i - V\beta_i||^2 &= (x_i - V\beta_i)(x_i - V\beta_i)^T \\ &= (x_i - V\beta_i)(x_i^T - \beta_i^T V^T) \\ &= ||\alpha_i - \beta_i||^2 \end{aligned}$$

So for the minimum value, in β_i we take the largest k values of α_i and and set rest of the values to zero. This would reduce the contribution to the error by the larger terms and hence the error would be minimised.