COMPUTING REPORT 7 MACM 316

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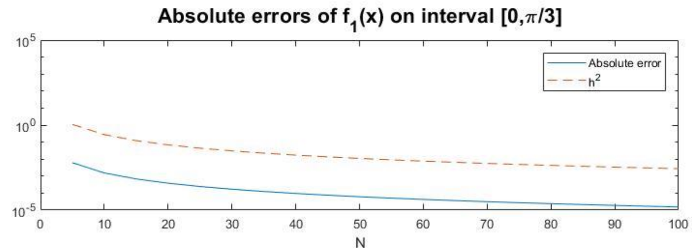
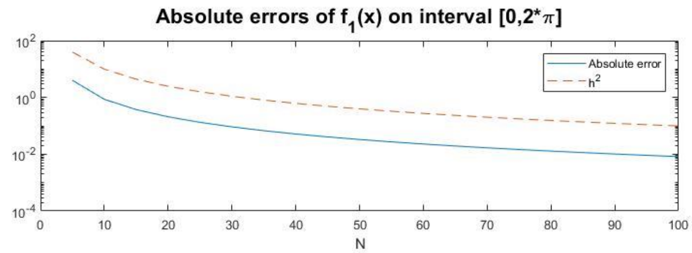
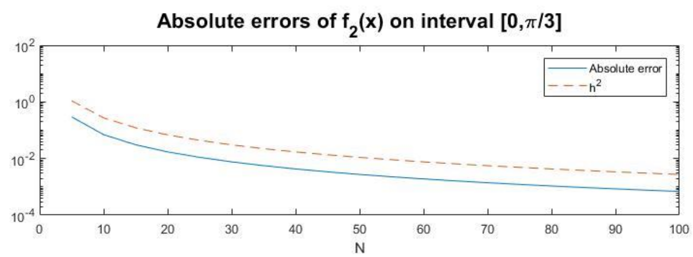
(a) The MATLAB function for trapezoidal rule returns $t = 0.250025000000000$ after computing the integral for $f(x) = x^3$ over [0,1] with $N = 100$.

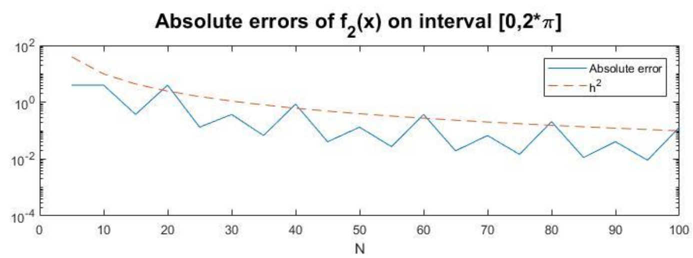




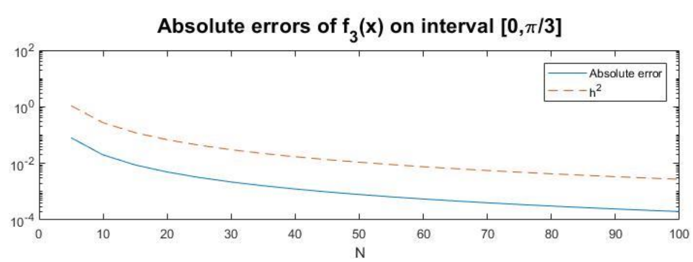
(b) The plots of the absolute errors for all three functions are shown on the right, with $f_1(x) = \sin\left(\frac{1}{2}x\right)$, $f_2(x) = |sin(2x)|$, and $f_3(x) = \cos(x)$. The dotted lines are values of h^2, the order of error for the composite trapezoidal rule, where each value of h is the value returned by the trapezoidal rule MATLAB function.

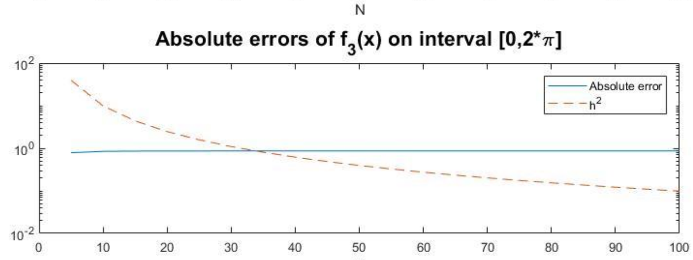
(c) As seen on the graphs, the rates of convergences for the first five calculations follow the expected rate of error for the composite trapezoidal rule, O(h^2). For each graph, the line of h^2 simulates the rate of convergence.





For the last graph, however, the errors for all values of N appear to be close to machine epsilon. According to Anatolii Grinshpan of Drexel University[1], this extreme accuracy is because “for a periodic function over full period there is a great deal of cancellation of error in the trapezoidal sum”. Indeed, f_3 differs from the other two functions in that its integral for one of the intervals, $I_2 = [0,2\pi]$, is computed over its full period.





[1] Link can be found at: http://www.math.drexel.edu/~tolya/301_periodic_integrands.pdf

```
clear

%interval values
I1=pi/3;
I2=2.*pi;

%test case funtion x^3
t=0;
s=0;
ftest=@(x) (x.^3);
[t,s]=mytrapezoidrule(ftest,0,1,100);
t
%the 3 functions to evaluate
f1=@(x) (sin(x.*1/2));
f2=@(x) abs(sin(2.*x));
f3=@(x) cos(x);

N=5:5:100;
%true values of integrals
tru1f1= integral(f1,0,I1);
tru2f1= integral(f1,0,I2);
tru1f2= integral(f2,0,I1);
tru2f2= integral(f2,0,I2);
tru1f3= integral(f3,0,I1);
tru2f3= integral(f3,0,I2);

%trapezoid values for increasing N
for i=1:20
    [trap1f1(i),h1f1(i)]= mytrapezoidrule(f1,0,I1,i);
    [trap2f1(i),h2f1(i)]= mytrapezoidrule(f1,0,I2,i);
    [trap1f2(i),h1f2(i)]= mytrapezoidrule(f2,0,I1,i);
    [trap2f2(i),h2f2(i)]= mytrapezoidrule(f2,0,I2,i);
    [trap1f3(i),h1f3(i)]= mytrapezoidrule(f3,0,I1,i);
    [trap2f3(i),h2f3(i)]= mytrapezoidrule(f3,0,I2,i);
end
for i=1:20
%absolute errors
    error1f1(i)=abs(trap1f1(i)-tru1f1);
    error2f1(i)=abs(trap2f1(i)-tru2f1);
    error1f2(i)=abs(trap1f2(i)-tru1f2);
    error2f2(i)=abs(trap2f2(i)-tru2f2);
    error1f3(i)=abs(trap1f3(i)-tru1f3);
    error2f3(i)=abs(trap1f3(i)-tru2f3);
end
for i=1:20
h1f1(i)=h1f1(i).^2;
h2f1(i)=h2f1(i).^2;
h1f2(i)=h1f2(i).^2;
h2f2(i)=h2f2(i).^2;
h1f3(i)=h1f3(i).^2;
h2f3(i)=h2f3(i).^2;
end
```

```
%plots
%function 1
figure(1)
subplot(2,1,1)
semilogy(N,error1f1)
hold on
semilogy(N,h1f1,'--')
hold off
title('Absolute errors of f_1(x) on interval [0,\pi/3]', 'FontSize', 16)
legend('Absolute error','h^2', 'Location', 'northeast')
xlabel('N')
subplot(2,1,2)
semilogy(N,error2f1)
hold on
semilogy(N,h2f1,'--')
hold off
title('Absolute errors of f_1(x) on interval [0,2*\pi]', 'FontSize', 16)
legend('Absolute error','h^2', 'Location', 'northeast')
xlabel('N')

%function 2
figure(2)
subplot(2,1,1)
semilogy(N,error1f2)
hold on
semilogy(N,h1f2,'--')
hold off
title('Absolute errors of f_2(x) on interval [0,\pi/3]', 'FontSize', 16)
legend('Absolute error','h^2', 'Location', 'northeast')
xlabel('N')
subplot(2,1,2)
semilogy(N,error2f2)
hold on
semilogy(N,h2f2,'--')
hold off
title('Absolute errors of f_2(x) on interval [0,2*\pi]', 'FontSize', 16)
legend('Absolute error','h^2', 'Location', 'northeast')
xlabel('N')

%function 3
figure(3)
subplot(2,1,1)
semilogy(N,error1f3)
hold on
semilogy(N,h1f3,'--')
hold off
title('Absolute errors of f_3(x) on interval [0,\pi/3]', 'FontSize', 16)
legend('Absolute error','h^2', 'Location', 'northeast')
xlabel('N')
subplot(2,1,2)
semilogy(N,error2f3)
hold on
semilogy(N,h2f3,'--')
hold off
title('Absolute errors of f_3(x) on interval [0,2*\pi]', 'FontSize', 16)
legend('Absolute error','h^2', 'Location', 'northeast')
xlabel('N')



function [trap,h] = mytrapezoidrule(fun,a,b,N);
format long
```

```
h=(b-a)/N;
trap = (fun(a)+fun(b))/2;
x = a + h.*(1:N-1);
trap = trap + sum(fun(x));
trap = h*trap;

% for n = 1:N-1
%     trap = trap + fun(a+h*n);
% end
%
% trap = h*trap;
%
%
% format short
end
```