COMPUTING REPORT 8 MACM 316

Khang Vu
301255281

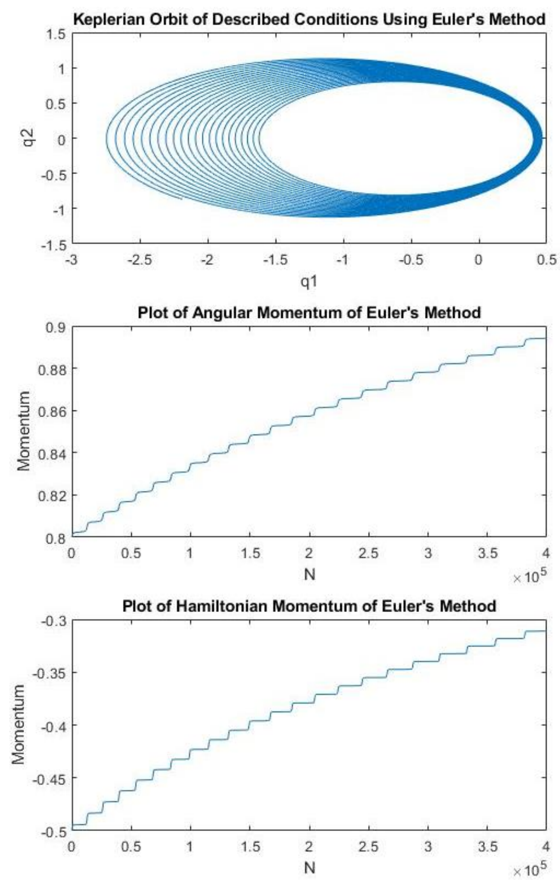
For this assignment, using Kepler's laws,

$$\mathbf{q}(t) = \begin{pmatrix} q_1(t) \\ q_2(t) \end{pmatrix}, \qquad \mathbf{p}(t) = \begin{pmatrix} p_1(t) \\ p_2(t) \end{pmatrix}, \qquad\qquad \mathbf{q}'(t) = \mathbf{p}(t), \qquad \mathbf{p}'(t) = -\frac{1}{(q_1(t)^2 + q_2(t)^2)^{3/2}}\mathbf{q}(t).$$

the initial conditions for time $0 < t < T = 200$ and stepsize h=0.0005, and the initial conditions,

$$q_1(0) = 1 - e, \quad q_2(0) = 0, \quad p_1(0) = 0, \quad p_2(0) = \sqrt{\frac{1+e}{1-e}}, \qquad e = 0$$

Figure (1)



I wrote a code to calculate the orbit using Euler's Method, of which the graph is shown in figure (1). Also shown in figure (1) are two graphs showing angular and Hamiltonian momentum, governed by:

$$A(t) = q_1(t)p_2(t) - q_2(t)p_1(t), \qquad H(t) = \frac{1}{2}(p_1(t)^2 + p_2(t)^2) - \frac{1}{\sqrt{q_1(t)^2 + q_2(t)^2}}.$$

As seen in figure (1), the orbit does not resemble a closed elliptical orbit as expected; but with each revolution of the orbit, it seems that the right focus of the ellipse remains fixed while the left focus changes. This could be partly explained by the graphs for angular and Hamiltonian momentum. Kepler's laws state that these values are conserved, but the graphs indicate that they are not, but increasing over time, which would change the ellipse of the orbit.

Figure (2) shows the same initial problem, but solved with a modified Euler's Method, called the symplectic Euler's Method:

$$\mathbf{q}_{n+1} = \mathbf{q}_n + h\mathbf{p}_n, \qquad \mathbf{p}_{n+1} = \mathbf{p}_n - \frac{h}{(q_{n+1,1}^2 + q_{n+1,2}^2)^{3/2}}\mathbf{q}_{n+1}.$$

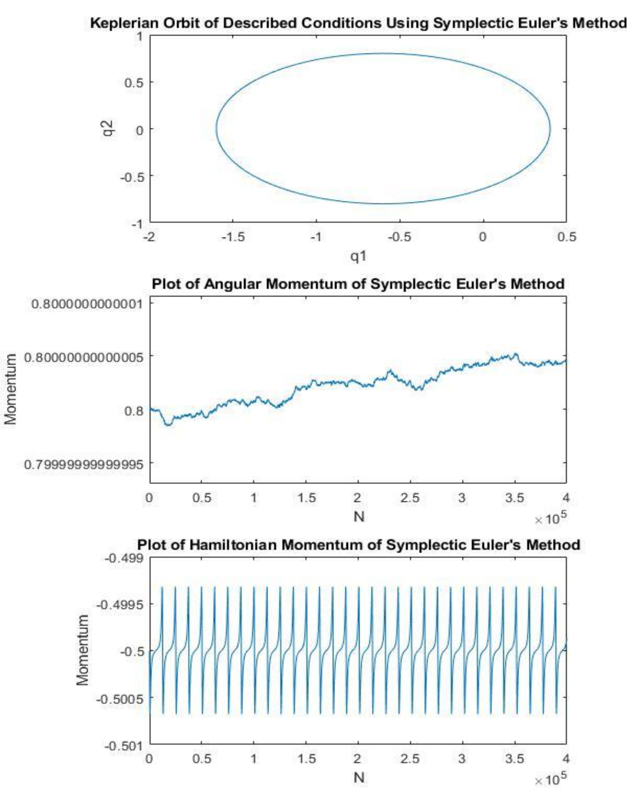


Figure (2)

The solution now resembles a closed elliptical orbit, instead of the previous changing orbit. The angular momentum is still increasing, but not as quickly as that of the standard Euler's Method. The Hamiltonian momentum, however, seems to be conserved with an error margin of about 0.005.

Lastly, as a visual aid, figure (3) shows the two solutions compared to each other.

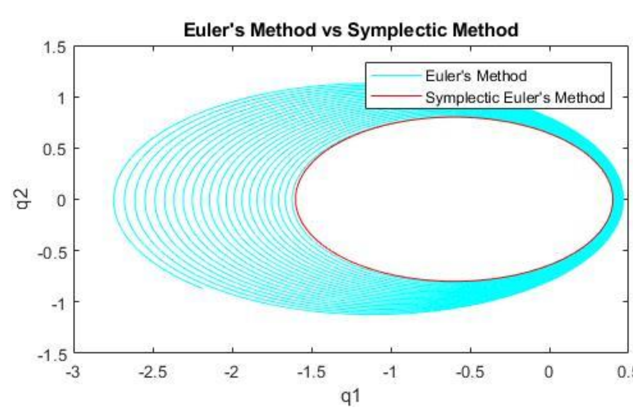


Figure (3)

```matlab
clear

T = 200;
h = 0.0005;
t = 0:h:200;
N = T/h;

e = 0.6;
%Normal Euler's Methd
q1(1)=1-e;
q2(1)=0;
p1(1)=0;
p2(1)=sqrt((1+e)/(1-e));

q(1,1)=q1(1);
q(2,1)=q2(1);
p(1,1)=p1(1);
p(2,1)=p2(1);
qprime(1,1)=p(1,1);
qprime(2,1)=p(2,1);
pprime(1,1)=-(q(1,1)/((q(1,1)^2) + (q(2,1)^2))^(3/2));
pprime(2,1)=-(q(2,1)/((q(1,1)^2) + (q(2,1)^2))^(3/2));
A(1)=q(1,1)*p(2,1)-q(2,1)*p(1,1);
H(1)=0.5*(p(1,1)^2+(p(2,1)^2))-1/sqrt(q(1,1)^2+q(2,1)^2);

for i = 2:N
    p(1,i)=p(1,i-1)+h*pprime(1,i-1);
    p(2,i)=p(2,i-1)+h*pprime(2,i-1);
    q(1,i)=q(1,i-1)+h*qprime(1,i-1);
    q(2,i)=q(2,i-1)+h*qprime(2,i-1);
    qprime(1,i)=p(1,i);
    qprime(2,i)=p(2,i);
    pprime(1,i)=-(q(1,i)/((q(1,i)^2) + (q(2,i)^2))^(3/2));
    pprime(2,i)=-(q(2,i)/((q(1,i)^2) + (q(2,i)^2))^(3/2));
    A(i)=q(1,i)*p(2,i)-q(2,i)*p(1,i);
    H(i)=0.5*(p(1,i)^2+(p(2,i)^2))-1/sqrt(q(1,i)^2+q(2,i)^2);
end

%Symplectic Euler's Method
qq(1:2,1) = [q1(1);q2(1)];
pp(1:2,1) = [p1(1);p2(1)];

for i = 1:N
    qq(1:2,i+1) = qq(1:2,i) + h*pp(1:2,i);
    pp(1:2,i+1) = pp(1:2,i) - ( ( h*qq(1:2,i+1) )/( qq(1,i+1)^2 + qq(2,i+1)^2 )^(3/2) );

    %angular and Ham. momenti
    symA(i)=qq(1,i)*pp(2,i)-qq(2,i)*pp(1,i);
    symH(i)=0.5*(pp(1,i)^2+(pp(2,i)^2))-1/sqrt(qq(1,i)^2+qq(2,i)^2);


end


figure(1)
subplot(3,1,1)
plot(q(1,1:N),q(2,1:N))
title ("Keplerian Orbit of Described Conditions Using Euler's Method")
xlabel('q1')
ylabel('q2')
subplot(3,1,2)
```

```
plot(1:N,A)
title ("Plot of Angular Momentum of Euler's Method")
xlabel('N')
ylabel('Momentum')
subplot(3,1,3)
plot(1:N,H)
title ("Plot of Hamiltonian Momentum of Euler's Method")
xlabel('N')
ylabel('Momentum')

figure(2)
subplot(3,1,1)
plot(qq(1,1:N),qq(2,1:N))
title ("Keplerian Orbit of Described Conditions Using Symplectic Euler's Method")
xlabel('q1')
ylabel('q2')
subplot(3,1,2)
plot(1:N,symA)
title ("Plot of Angular Momentum of Symplectic Euler's Method")
xlabel('N')
ylabel('Momentum')
subplot(3,1,3)
plot(1:N,symH)
title ("Plot of Hamiltonian Momentum of Symplectic Euler's Method")
xlabel('N')
ylabel('Momentum')

figure(3)
plot(q(1,1:N),q(2,1:N), "cyan")
hold on
plot(qq(1,1:N),qq(2,1:N),"red")
hold off
title("Euler's Method vs Symplectic Method")
legend("Euler's Method", "Symplectic Euler's Method")
xlabel('q1')
ylabel('q2')
```