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a) To compute the 2nd order formula, I first generated the matrices A and B as described in the guideline. Their plots are on the right.

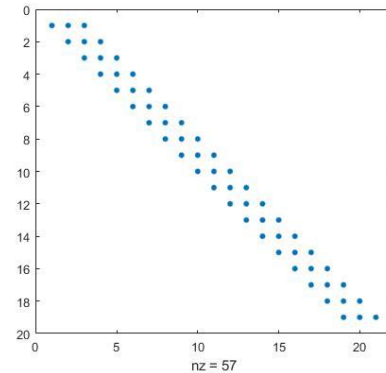
These matrices resemble tridiagonal matrices except the fact that they are non-square with dimensions of (n-1) by (n+1), possibly to accommodate the two endpoints into the calculation of the middle points.

Then I used A and B and the approximated equations for y' and y'' to obtain the matrix C as follows:

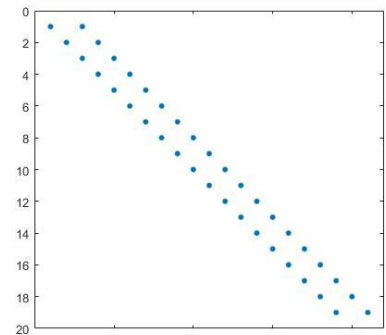
$$C = \left(\frac{1}{h^2}\right) A + 0.5 \left(\frac{1}{2h}\right) B \quad \text{then, } Cy(x) = \sin(x).$$

Using the backslash command gives me the approximated values of $y(x)$ at points on the grid.

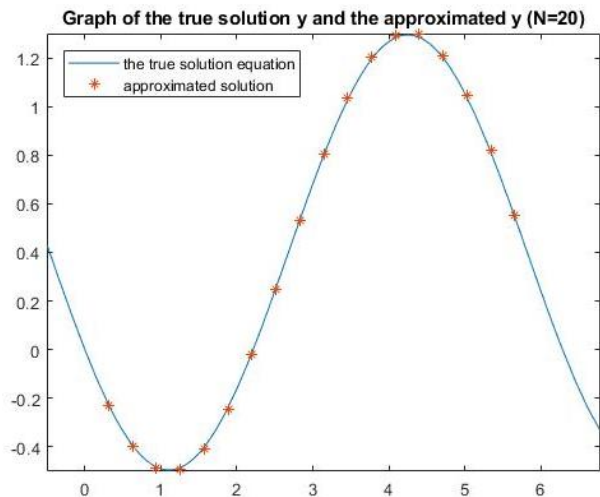
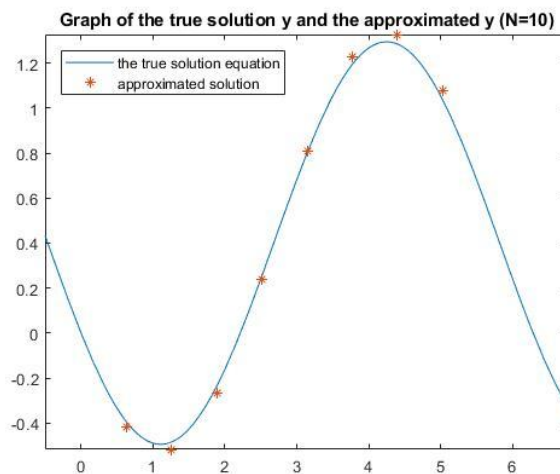
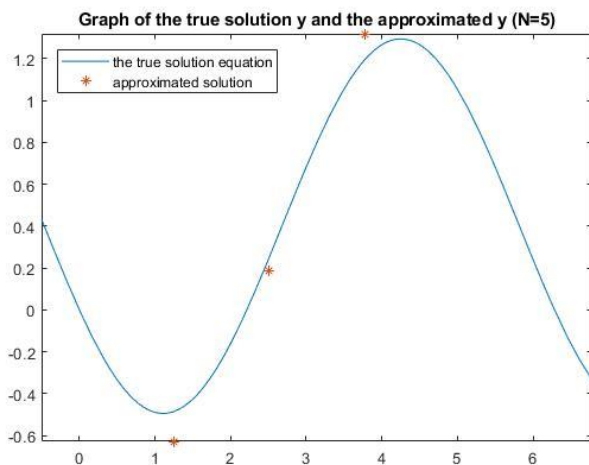
These are the approximated y for values of $N=5,10,20$ (without the two endpoints of 0):



Plot of Matrix A



Plot of Matrix B



As expected, the values of the errors decrease as N increases.

```
%Numerical Differentiation
```

```
n=20;  
A=zeros(n-1,n+1);  
B=zeros(n-1,n+1);  
sinx=zeros(n-1,1);  
dy=zeros(n,1);  
ddy=zeros(n,1);  
x=zeros(n-1,1);  
step=2*pi/n;  
ogx=zeros(n+1,1);
```

```
%Generate A and B matrices
```

```
k=1;  
i=1;  
while i<n  
    B(i,k)=-1;  
    B(i,k+2)=1;  
    A(i,k)=1;  
    A(i,k+1)=-2;  
    A(i,k+2)=1;  
    k=k+1;  
    i=i+1;  
end
```

```
%Generating grid points x
```

```
i=1;  
while i<(n)  
    ogx(i+1)=step*i;  
    x(i)=step*i;  
    i=i+1;  
end
```

```
%Compute sin(x) at grid points x
```

```
i=1;  
while i<n  
    sinx(i)=sin(x(i));  
    i=i+1;  
end
```

```
%Calculating dy and ddy
```

```
dy=(1/(2*step))*B;  
ddy=(1/(step^2))*A;
```

```
%C=ddy+0.5*dy  
C=ddy+0.5*dy;
```

```
%Computing y(xi)  
y=C\sinx;
```

```
%computing true values
```

```
tru=@(t) 0.4*(1-cos(t))-0.8*sin(t);  
fplot(tru, [-0.5 2*pi+0.5])  
title('Graph of the true solution y and the approximated y (N=20)')  
hold on  
plot(ogx(2:n-1),y(2:n-1) , '*')  
legend('the true solution equation','approximated solution', 'Location', 'northwest')
```