TP5: TÍCH PHÂN TỔ HỢP NHIỀU HÀM SỐ

Câu 1. $I=\int_0^1\left(x^2e^{x^3}+\dfrac{\sqrt[4]{x}}{1+\sqrt{x}}\right)dx$

• $I=\int_0^1 x^2e^{x^3}dx+\int_0^1\dfrac{\sqrt[4]{x}}{1+\sqrt{x}}dx$.

+ *Tính* $I_1=\int_0^1 x^2e^{x^3}dx$. *Đặt* $t=x^3 \Rightarrow I_1=\dfrac{1}{3}\int_0^1 e^t dt=\dfrac{1}{3}e^t\Big|_0^1=\dfrac{1}{3}e-\dfrac{1}{3}$.

+ *Tính* $I_2=\int_0^1\dfrac{\sqrt[4]{x}}{1+\sqrt{x}}dx$. *Đặt* $t=\sqrt[4]{x} \Rightarrow I_2=4\int_0^1\dfrac{t^4}{1+t^2}dt=4\left(-\dfrac{2}{3}+\dfrac{\pi}{4}\right)$

Vậy: $I=\dfrac{1}{3}e+\pi-3$

Câu 2. $I=\int_1^2 x\left(e^x-\dfrac{\sqrt{4-x^2}}{x^3}\right)dx$

• $I=\int_1^2 xe^x dx+\int_1^2\dfrac{\sqrt{4-x^2}}{x^2}dx$.

+ *Tính* $I_1=\int_1^2 xe^x dx=e^2$ + *Tính* $I_2=\int_1^2\dfrac{\sqrt{4-x^2}}{x^2}dx$. *Đặt* $x=2\sin t$, $t\in\left[0;\dfrac{\pi}{2}\right]$.

$\Rightarrow I_2=\int_{\frac{\pi}{6}}^{\frac{\pi}{2}}\dfrac{\cos^2 t}{\sin^2 t}dt=(-\cot t-t)\Big|_{\frac{\pi}{6}}^{\frac{\pi}{2}}=\sqrt{3}-\dfrac{\pi}{3}$

Vậy: $I=e^2+\sqrt{3}-\dfrac{\pi}{3}$.

Câu 3. $I=\int_0^1\dfrac{x}{\sqrt{4-x^2}}\left(e^{2x}.\sqrt{4-x^2}-x^2\right)dx$.

• $I=\int_0^1 x\,e^{2x}dx-\int_0^1\dfrac{x^3}{\sqrt{4-x^2}}dx=I_1+I_2$

+ *Tính* $I_1=\int_0^1 x\,e^{2x}dx=\dfrac{e^2+1}{4}$

+ *Tính* $I_2=\int_0^1\dfrac{x^3}{\sqrt{4-x^2}}dx$. *Đặt* $t=\sqrt{4-x^2} \Rightarrow I_2=-3\sqrt{3}+\dfrac{16}{3}$

$\Rightarrow I=\dfrac{e^2}{4}+3\sqrt{3}-\dfrac{61}{12}$

Câu 4. $I=\int_0^1\dfrac{x^2+1}{(x+1)^2}e^x dx$

• *Đặt* $t = x+1 \Rightarrow dx = dt$ $I = \int_1^2 \frac{t^2-2t+2}{t^2}e^{t-1}dt = \int_1^2\left(1+\frac{2}{t^2}-\frac{2}{t}\right)e^{t-1}dt = e-1+\frac{2}{e}\left(-\frac{e^2}{2}+e\right)=1$

Câu 5. $I = \int_0^{\sqrt{3}} \frac{x^3.e^{\sqrt{x^2+1}}dx}{\sqrt{1+x^2}}$

• *Đặt* $t = \sqrt{1+x^2} \Rightarrow dx = tdt \Rightarrow I = \int_1^2 (t^2-1)e^t dt = \int_1^2 t^2e^tdt - e^t\Big|_1^2 = J-(e^2-e)$

+ $J = \int_1^2 t^2e^tdt = t^2e^t\Big|_1^2 - \int_1^2 2te^tdt = 4e^2-e-2\left(te^t\Big|_1^2-\int_1^2 e^tdt\right) = 4e^2-e-2(te^t-e^t)\Big|_1^2$

Vậy: $I = e^2$

Câu 6. $I = \int \frac{x\ln(x^2+1)+x^3}{x^2+1}dx$

• *Ta có:* $f(x) = \frac{x\ln(x^2+1)}{x^2+1}+\frac{x(x^2+1)-x}{x^2+1} = \frac{x\ln(x^2+1)}{x^2+1}+x-\frac{x}{x^2+1}$

$\Rightarrow F(x) = \int f(x)dx = \frac{1}{2}\int \ln(x^2+1)d(x^2+1)+\int xdx-\frac{1}{2}\int d\ln(x^2+1)$

$= \frac{1}{4}\ln^2(x^2+1)+\frac{1}{2}x^2-\frac{1}{2}\ln(x^2+1)+C.$

Câu 7. $I = \int_0^4 \frac{\ln\left(x+\sqrt{x^2+9}\right)-3x^3}{\sqrt{x^2+9}}dx$

• $I = \int_0^4 \frac{\ln\left(x+\sqrt{x^2+9}\right)-3x^3}{\sqrt{x^2+9}}dx = \int_0^4 \frac{\ln\left(x+\sqrt{x^2+9}\right)}{\sqrt{x^2+9}}dx - 3\int_0^4 \frac{x^3}{\sqrt{x^2+9}}dx = I_1-3I_2$

+ *Tính* $I_1 = \int_0^4 \frac{\ln\left(x+\sqrt{x^2+9}\right)}{\sqrt{x^2+9}}dx$. *Đặt* $\ln\left(x+\sqrt{x^2+9}\right) = u \Rightarrow du = \frac{1}{\sqrt{x^2+9}}dx$

$\Rightarrow I_1 = \int_{\ln 3}^{\ln 9} udu = \frac{u^2}{2}\Big|_{\ln 3}^{\ln 9} = \frac{\ln^2 9-\ln^2 3}{2}$

+ *Tính* $I_2 = \int_0^4 \frac{x^3}{\sqrt{x^2+9}}dx$. *Đặt* $\sqrt{x^2+9} = v \Rightarrow dv = \frac{x}{\sqrt{x^2+9}}dx,\ \ x^2 = v^2-9$

$\Rightarrow I_2 = \int_3^5 (u^2-9)du = (\frac{u^3}{3}-9u)\Big|_3^5 = \frac{44}{3}$

Vậy $I = \int_0^4 \frac{\ln\left(x+\sqrt{x^2+9}\right)-3x^3}{\sqrt{x^2+9}}dx = I_1-3I_2 = \frac{\ln^2 9-\ln^2 3}{2}-44.$

Câu 8. $I = \int_1^e \frac{(x^3+1)\ln x+2x^2+1}{2+x\ln x}dx$

• $I = \int_1^e x^2dx + \int_1^e \frac{1+\ln x}{2+x\ln x}dx.$ + $\int_1^e x^2dx = \frac{x^3}{3}\Big|_1^e = \frac{e^3-1}{3}$

$+\ \int_1^e \frac{1+\ln x}{2+x\ln x}dx = \int_1^e \frac{d(2+x\ln x)}{2+x\ln x} = \ln\left|2+x\ln x\right|\Big|_1^e = \ln\frac{e+2}{2}$. *Vậy:* $I = \frac{e^3-1}{3}+\ln\frac{e+2}{2}$.

Câu 9. $I = \int_1^{e^3} \frac{\ln^3 x}{x\sqrt{1+\ln x}}dx$

• *Đặt* $t=\sqrt{1+\ln x} \Rightarrow 1+\ln x = t^2 \Rightarrow \frac{dx}{x} = 2tdt$ *và* $\ln^3 x = (t^2-1)^3$

$\Rightarrow I = \int_1^2 \frac{(t^2-1)^3}{t}dt = \int_1^2 \frac{t^6-3t^4+3t^2-1}{t}dt = \int_1^2 (t^5-3t^3+3t-\frac{1}{t})dt = \frac{15}{4}-\ln 2$

Câu 10. $I = \int_0^{\frac{\pi}{4}} \frac{x\sin x}{\cos^2 x}dx$

• *Đặt* $\begin{cases} u = x \\ dv = \frac{\sin x}{\cos^2 x}dx \end{cases} \Rightarrow \begin{cases} du = dx \\ v = \frac{1}{\cos x} \end{cases} \Rightarrow I = \frac{x}{\cos x}\Big|_0^{\frac{\pi}{4}} - \int_0^{\frac{\pi}{4}} \frac{dx}{\cos x} = \frac{\pi\sqrt{2}}{4} - \int_0^{\frac{\pi}{4}} \frac{dx}{\cos x}$

$+\ I_1 = \int_0^{\frac{\pi}{4}} \frac{dx}{\cos x} = \int_0^{\frac{\pi}{4}} \frac{\cos x dx}{1-\sin^2 x}$. *Đặt* $t=\sin x \Rightarrow I_1 = \int_0^{\frac{\sqrt{2}}{2}} \frac{dt}{1-t^2} = \frac{1}{2}\ln\frac{2+\sqrt{2}}{2-\sqrt{2}}$

Vậy: $= \frac{\pi\sqrt{2}}{4} - \frac{1}{2}\ln\frac{2+\sqrt{2}}{2-\sqrt{2}}$

Câu 11. $I = \int_1^4 \frac{\ln(5-x)+x^3.\sqrt{5-x}}{x^2}dx$

• *Ta có:* $I = \int_1^4 \frac{\ln(5-x)}{x^2}dx + \int_1^4 x\sqrt{5-x}.dx = K+H$.

$+\ K = \int_1^4 \frac{\ln(5-x)}{x^2}dx$. *Đặt* $\begin{cases} u = \ln(5-x) \\ dv = \frac{dx}{x^2} \end{cases} \Rightarrow K = \frac{3}{5}\ln 4$

$+\ H = \int_1^4 x\sqrt{5-x}.dx$. *Đặt* $t=\sqrt{5-x} \Rightarrow H = \frac{164}{15}$

Vậy: $I = \frac{3}{5}\ln 4 + \frac{164}{15}$

Câu 12. $I = \int_0^2 \left[\sqrt{x(2-x)}+\ln(4+x^2)\right]dx$

• *Ta có:* $I = \int_0^2 \sqrt{x(2-x)}dx + \int_0^2 \ln(4+x^2)dx = I_1+I_2$

$+\ I_1 = \int_0^2 \sqrt{x(2-x)}dx = \int_0^2 \sqrt{1-(x-1)^2}dx = \frac{\pi}{2}$ *(sử dụng đổi biến:* $x = 1+\sin t$*)*

$+\ I_2 = \int_0^2 \ln(4+x^2)dx = x\ln(4+x^2)\Big|_0^2 - 2\int_0^2 \frac{x^2}{4+x^2}dx$ *(sử dụng tích phân từng phần)*

$= 6\ln 2 + \pi - 4$ *(đổi biến* $x = 2\tan t$*)*

Vậy: $I = I_1 + I_2 = \dfrac{3\pi}{2} - 4 + 6\ln 2$

Câu 13. $I = \int\limits_3^8 \dfrac{\ln x}{\sqrt{x+1}}dx$

• *Đặt* $\begin{cases} u = \ln x \\ dv = \dfrac{dx}{\sqrt{x+1}} \end{cases} \Rightarrow \begin{cases} du = \dfrac{dx}{x} \\ v = 2\sqrt{x+1} \end{cases} \Rightarrow I = 2\sqrt{x+1}\ln x\Big|_3^8 - 2\int\limits_3^8 \dfrac{\sqrt{x+1}}{x}dx$

+ *Tính* $J = \int\limits_3^8 \dfrac{\sqrt{x+1}}{x}dx$. *Đặt* $t = \sqrt{x+1}$ $\Rightarrow J = \int\limits_2^3 \dfrac{2t^2dt}{t^2-1} = 2\int\limits_2^3 \left(1 + \dfrac{1}{t^2-1}\right)dt = 2 + \ln 3 - \ln 2$

$\Rightarrow I = 6\ln 8 - 4\ln 3 - 2(2 + \ln 3 - \ln 2) = 20\ln 2 - 6\ln 3 - 4$

Câu 14. $I = \int\limits_1^2 \dfrac{1+x^2}{x^3}\ln x dx$

• *Ta có:* $I = \int\limits_1^2 \left(\dfrac{1}{x^3} + \dfrac{1}{x}\right)\ln x dx$. *Đặt* $\begin{cases} u = \ln x \\ dv = (\dfrac{1}{x^3} + \dfrac{1}{x})dx \end{cases}$

$\Rightarrow I = \left(\dfrac{-1}{4x^4} + \ln x\right)\ln x\Big|_1^2 - \int\limits_1^2 \left(\dfrac{-1}{4x^5} + \dfrac{1}{x}\ln x\right)dx = -\dfrac{1}{64}\ln 2 + \dfrac{63}{4} + \dfrac{1}{2}\ln^2 2$

Câu 15. $I = \int\limits_1^e \dfrac{x^2 + x\ln x + 1}{x}e^x dx$

• *Ta có:* $I = \int\limits_1^e xe^x dx + \int\limits_1^e e^x \ln x dx + \int\limits_1^e \dfrac{e^x}{x}dx = H + K + J$

+ $H = \int\limits_1^e xe^x dx = xe^x\Big|_1^e - \int\limits_1^e e^x dx = e^e(e-1)$

+ $K = \int\limits_1^e e^x \ln x dx = e^x \ln x\Big|_1^e - \int\limits_1^e \dfrac{e^x}{x}dx = e^e - \int\limits_1^e \dfrac{e^x}{x}dx = e^e - J$

Vậy: $I = H + K + J = e^{e+1} - e^e + e^e - J + J = e^{e+1}$.

Câu 16. $I = \int\limits_{\frac{\pi}{4}}^{\frac{\pi}{2}} \dfrac{x\cos x}{\sin^3 x}dx$

• *Ta có* $\left(\dfrac{1}{\sin^2 x}\right)' = -\dfrac{2\cos x}{\sin^3 x}$. *Đặt* $\begin{cases} u = x \\ dv = \dfrac{\cos x}{\sin^3 x}dx \end{cases} \Rightarrow \begin{cases} du = dx \\ v = -\dfrac{1}{2\sin^2 x} \end{cases}$

$\Rightarrow I = -\dfrac{1}{2}x.\dfrac{1}{\sin^2 x}\Big|_{\frac{\pi}{4}}^{\frac{\pi}{2}} + \dfrac{1}{2}\int\limits_{\frac{\pi}{4}}^{\frac{\pi}{2}} \dfrac{dx}{\sin^2 x} = -\dfrac{1}{2}(\dfrac{\pi}{2} - \dfrac{\pi}{2}) - \dfrac{1}{2}\cot x\Big|_{\frac{\pi}{4}}^{\frac{\pi}{2}} = \dfrac{1}{2}$.

Câu 17. $I = \int\limits_0^{\frac{\pi}{4}} \dfrac{x\sin x}{\cos^3 x}dx$

• *Đặt:* $\begin{cases} u = x \\ dv = \dfrac{\sin x}{\cos^3 x}dx \end{cases} \Rightarrow \begin{cases} du = dx \\ v = \dfrac{1}{2.\cos^2 x} \end{cases} \Rightarrow I = \dfrac{x}{2\cos^2 x}\Bigg|_0^{\frac{\pi}{4}} - \dfrac{1}{2}\displaystyle\int_0^{\frac{\pi}{4}} \dfrac{dx}{\cos^2 x} = \dfrac{\pi}{4} - \dfrac{1}{2}\tan x\Bigg|_0^{\frac{\pi}{4}} = \dfrac{\pi}{4} - \dfrac{1}{2}$

Câu 18. $I = \displaystyle\int_0^{\frac{\pi}{2}} \dfrac{(x + \sin^2 x)}{1 + \sin 2x}dx$

• *Ta có:* $I = \displaystyle\int_0^{\frac{\pi}{2}} \dfrac{x}{1 + \sin 2x}dx + \int_0^{\frac{\pi}{2}} \dfrac{\sin^2 x}{1 + \sin 2x}dx = H + K$

+ $H = \displaystyle\int_0^{\frac{\pi}{2}} \dfrac{x}{1 + \sin 2x}dx = \int_0^{\frac{\pi}{2}} \dfrac{x}{2\cos^2\left(x - \frac{\pi}{4}\right)}dx$. *Đặt:* $\begin{cases} u = x \\ dv = \dfrac{dx}{2\cos^2\left(x - \frac{\pi}{4}\right)} \end{cases} \Rightarrow \begin{cases} du = dx \\ v = \dfrac{1}{2}\tan\left(x - \dfrac{\pi}{4}\right) \end{cases}$

$\Rightarrow H = \dfrac{x}{2}\tan\left(x - \dfrac{\pi}{4}\right)\Bigg|_0^{\frac{\pi}{2}} + \left(\dfrac{1}{2}\ln\left|\cos\left(x - \dfrac{\pi}{4}\right)\right|\right)\Bigg|_0^{\frac{\pi}{2}} = \dfrac{\pi}{4}$

+ $K = \displaystyle\int_0^{\frac{\pi}{2}} \dfrac{\sin^2 x}{1 + \sin 2x}dx$. *Đặt* $t = \dfrac{\pi}{2} - x \Rightarrow K = \displaystyle\int_0^{\frac{\pi}{2}} \dfrac{\cos^2 x}{1 + \sin 2x}dx$

$\Rightarrow 2K = \displaystyle\int_0^{\frac{\pi}{2}} \dfrac{dx}{2\cos^2\left(x - \frac{\pi}{4}\right)} = \dfrac{1}{2}\tan\left(x - \dfrac{\pi}{4}\right)\Bigg|_0^{\frac{\pi}{2}} = 1 \Rightarrow K = \dfrac{1}{2}$

Vậy, $I = H + K = \dfrac{\pi}{4} + \dfrac{1}{2}$.

Câu 19. $I = \displaystyle\int_0^{\pi} \dfrac{x(\cos^3 x + \cos x + \sin x)}{1 + \cos^2 x}dx$

• *Ta có:* $I = \displaystyle\int_0^{\pi} x\left(\dfrac{\cos x(1 + \cos^2 x) + \sin x}{1 + \cos^2 x}\right)dx = \int_0^{\pi} x.\cos x.dx + \int_0^{\pi} \dfrac{x.\sin x}{1 + \cos^2 x}dx = J + K$

+ *Tính* $J = \displaystyle\int_0^{\pi} x.\cos x.dx$. *Đặt* $\begin{cases} u = x \\ dv = \cos x dx \end{cases} \Rightarrow J = (x.\sin x)\Big|_0^{\pi} - \displaystyle\int_0^{\pi} \sin x.dx = 0 + \cos x\Big|_0^{\pi} = -2$

+ *Tính* $K = \displaystyle\int_0^{\pi} \dfrac{x.\sin x}{1 + \cos^2 x}dx$. *Đặt* $x = \pi - t \Rightarrow dx = -dt$

$\Rightarrow K = \displaystyle\int_0^{\pi} \dfrac{(\pi - t).\sin(\pi - t)}{1 + \cos^2(\pi - t)}dt = \int_0^{\pi} \dfrac{(\pi - t).\sin t}{1 + \cos^2 t}dt = \int_0^{\pi} \dfrac{(\pi - x).\sin x}{1 + \cos^2 x}dx$

$\Rightarrow 2K = \displaystyle\int_0^{\pi} \dfrac{(x + \pi - x).\sin x}{1 + \cos^2 x}dx = \pi\int_0^{\pi} \dfrac{\sin x.dx}{1 + \cos^2 x} \Rightarrow K = \dfrac{\pi}{2}\int_0^{\pi} \dfrac{\sin x.dx}{1 + \cos^2 x}$

Đặt $t = \cos x \Rightarrow dt = -\sin x.dx \Rightarrow K = \dfrac{\pi}{2}\displaystyle\int_{-1}^{1} \dfrac{dt}{1 + t^2}$, *đặt* $t = \tan u \Rightarrow dt = (1 + \tan^2 u)du$

$$\Rightarrow K = \frac{\pi}{2}\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{(1+\tan^2 u)du}{1+\tan^2 u} = \frac{\pi}{2}\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} du = \frac{\pi}{2}.u\Big|_{-\frac{\pi}{4}}^{\frac{\pi}{4}} = \frac{\pi^2}{4}$$

Vậy $I = \frac{\pi^2}{4} - 2$

Câu 20. $I = \int_{\frac{\pi}{3}}^{\frac{2\pi}{3}} \frac{x+(x+\sin x)\sin x}{(1+\sin x)\sin^2 x}dx$

• *Ta có:* $I = \int_{\frac{\pi}{3}}^{\frac{2\pi}{3}} \frac{x(1+\sin x)+\sin^2 x}{(1+\sin x)\sin^2 x}dx = \int_{\frac{\pi}{3}}^{\frac{2\pi}{3}} \frac{x}{\sin^2 x}dx + \int_{\frac{\pi}{3}}^{\frac{2\pi}{3}} \frac{dx}{1+\sin x} = H + K$

\+ $H = \int_{\frac{\pi}{3}}^{\frac{2\pi}{3}} \frac{x}{\sin^2 x}dx$. *Đặt* $\begin{cases} u = x \\ dv = \frac{dx}{\sin^2 x} \end{cases} \Rightarrow \begin{cases} du = dx \\ v = -\cot x \end{cases} \Rightarrow H = \frac{\pi}{\sqrt{3}}$

\+ $K = \int_{\frac{\pi}{3}}^{\frac{2\pi}{3}} \frac{dx}{1+\sin x} = \int_{\frac{\pi}{3}}^{\frac{2\pi}{3}} \frac{dx}{1+\cos\left(\frac{\pi}{2}-x\right)} = \int_{\frac{\pi}{3}}^{\frac{2\pi}{3}} \frac{dx}{2\cos^2\left(\frac{\pi}{4}-\frac{x}{2}\right)} = \sqrt{3}-2$

Vậy $I = \frac{\pi}{\sqrt{3}} + \sqrt{3} - 2$

Câu 21. $I = \int_0^{\frac{\pi}{3}} \frac{x+\sin^2 x}{1+\cos 2x}dx$

• *Ta có:* $I = \int_0^{\frac{\pi}{3}} \frac{x+\sin^2 x}{1+\cos 2x}dx = \int_0^{\frac{\pi}{3}} \frac{x}{2\cos^2 x}dx + \int_0^{\frac{\pi}{3}} \frac{\sin^2 x}{2\cos^2 x}dx = H + K$

\+ $H = \int_0^{\frac{\pi}{3}} \frac{x}{2\cos^2 x}dx = \frac{1}{2}\int_0^{\frac{\pi}{3}} \frac{x}{\cos^2 x}dx$. *Đặt* $\begin{cases} u = x \\ dv = \frac{dx}{\cos^2 x} \end{cases} \Rightarrow \begin{cases} du = dx \\ v = \tan x \end{cases}$

$$\Rightarrow H = \frac{1}{2}\left[x\tan x\Big|_0^{\frac{\pi}{3}} - \int_0^{\frac{\pi}{3}} \tan x dx\right] = \frac{\pi}{2\sqrt{3}} + \frac{1}{2}\ln|\cos x|\Big|_0^{\frac{\pi}{3}} = \frac{\pi}{2\sqrt{3}} - \frac{1}{2}\ln 2$$

\+ $K = \int_0^{\frac{\pi}{3}} \frac{\sin^2 x}{2\cos^2 x}dx = \frac{1}{2}\int_0^{\frac{\pi}{3}} \tan^2 x dx = \frac{1}{2}[\tan x - x]\Big|_0^{\frac{\pi}{3}} = \frac{1}{2}\left(\sqrt{3} - \frac{\pi}{3}\right)$

Vậy: $I = H + K = \frac{\pi}{2\sqrt{3}} - \frac{1}{2}\ln 2 + \frac{1}{2}\left(\sqrt{3} - \frac{\pi}{3}\right) = \frac{\pi\left(\sqrt{3}-1\right)}{6} + \frac{1}{2}(\sqrt{3} - \ln 2)$

Câu 22. $I = \int_0^3 \sqrt{x+1}\sin\sqrt{x+1}.dx$

• *Đặt* $t = \sqrt{x+1} \Rightarrow I = \int_1^2 t.\sin t.2tdt = \int_1^2 2t^2 \sin tdt = \int_1^2 2x^2 \sin xdx$

Đặt $\begin{cases} u = 2x^2 \\ dv = \sin xdx \end{cases} \Rightarrow \begin{cases} du = 4xdx \\ v = -\cos x \end{cases} \Rightarrow I = -2x^2\cos x\Big|_1^2 + \int_1^2 4x\cos xdx$

Đặt $\begin{cases} u = 4x \\ dv = \cos xdx \end{cases} \Rightarrow \begin{cases} du = 4dx \\ v = \sin x \end{cases}$. *Từ đó suy ra kết quả.*

Câu 23. $I=\int_0^{\frac{\pi}{2}}\frac{1+\sin x}{1+\cos x}.e^x dx$

• $I=\frac{1}{2}\int_0^{\frac{\pi}{2}}\frac{e^x dx}{\cos^2\frac{x}{2}}+\int_0^{\frac{\pi}{2}}\frac{\sin x}{1+\cos x}e^x dx$

+ *Tính* $I_1=\int_0^{\frac{\pi}{2}}\frac{\sin x}{1+\cos x}e^x dx=\int_0^{\frac{\pi}{2}}\frac{2\sin\frac{x}{2}.\cos\frac{x}{2}}{2\cos^2\frac{x}{2}}e^x dx=\int_0^{\frac{\pi}{2}}\tan\frac{x}{2}e^x dx$

+ *Tính* $I_2=\frac{1}{2}\int_0^{\frac{\pi}{2}}\frac{e^x dx}{\cos^2\frac{x}{2}}$. *Đặt* $\begin{cases} u=e^x \\ dv=\dfrac{dx}{2\cos^2\frac{x}{2}}\end{cases}\Rightarrow\begin{cases} du=e^x dx \\ v=\tan\frac{x}{2}\end{cases}\Rightarrow I_2=e^{\frac{\pi}{2}}-\int_0^{\frac{\pi}{2}}\tan\frac{x}{2}e^x dx$

Do đó: $I=I_1+I_2=e^{\frac{\pi}{2}}$.

Câu 24. $I=\int_0^{\frac{\pi}{2}}\frac{\cos x}{e^x(1+\sin 2x)}dx$

• $I=\int_0^{\frac{\pi}{2}}\frac{\cos x}{e^x(\sin x+\cos x)^2}dx$. *Đặt* $\begin{cases} u=\dfrac{\cos x}{e^x} \\ dv=\dfrac{dx}{(\sin x+\cos x)^2}\end{cases}\Rightarrow\begin{cases} du=\dfrac{-(\sin x+\cos x)dx}{e^x} \\ v=\dfrac{\sin x}{\sin x+\cos x}\end{cases}$

$\Rightarrow I=\frac{\cos x}{e^x}.\frac{\sin x}{\sin x+\cos x}\Big|_0^{\frac{\pi}{2}}+\int_0^{\frac{\pi}{2}}\frac{\sin xdx}{e^x}=\int_0^{\frac{\pi}{2}}\frac{\sin xdx}{e^x}$

Đặt $\begin{cases} u_1=\sin x \\ dv_1=\dfrac{dx}{e^x}\end{cases}\Rightarrow\begin{cases} du_1=\cos xdx \\ v_1=\dfrac{-1}{e^x}\end{cases}\Rightarrow I=\sin x.\frac{-1}{e^x}\Big|_0^{\frac{\pi}{2}}+\int_0^{\frac{\pi}{2}}\frac{\cos xdx}{e^x}=\frac{-1}{e^{\frac{\pi}{2}}}+\int_0^{\frac{\pi}{2}}\frac{\cos xdx}{e^x}$

Đặt $\begin{cases} u_2=\cos x \\ dv_1=\dfrac{dx}{e^x}\end{cases}\Rightarrow\begin{cases} du_2=-\sin xdx \\ v_1=\dfrac{-1}{e^x}\end{cases}$

$\Rightarrow I=\frac{-1}{e^{\frac{\pi}{2}}}+\cos x.\frac{-1}{e^x}\Big|_0^{\frac{\pi}{2}}-\int_0^{\frac{\pi}{2}}\frac{\sin xdx}{e^x}=\frac{-1}{e^{\frac{\pi}{2}}}+1-I\Rightarrow 2I=-e^{\frac{-\pi}{2}}+1\Rightarrow I=\frac{-e^{\frac{-\pi}{2}}}{2}+\frac{1}{2}$

Câu 25. $I=\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}}\frac{\sin^6 x+\cos^6 x}{6^x+1}dx$

• *Đặt* $t=-x \Rightarrow dt=-dx \Rightarrow I=\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} 6^t\frac{\sin^6 t+\cos^6 t}{6^t+1}dt=\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} 6^x\frac{\sin^6 x+\cos^6 x}{6^x+1}dx$

$\Rightarrow 2I=\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}}(6^x+1)\frac{\sin^6 x+\cos^6 x}{6^x+1}dx=\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}}(\sin^6 x+\cos^6 x)dx=\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}}\left(\frac{5}{8}+\frac{3}{8}\cos 4x\right)dx=\frac{5\pi}{16}$

$\Rightarrow I=\frac{5\pi}{32}$.

Câu 26. $I=\int_{-\frac{\pi}{6}}^{\frac{\pi}{6}}\frac{\sin^4 xdx}{2^{-x}+1}$

• *Ta có:* $I=\int_{-\frac{\pi}{6}}^{\frac{\pi}{6}}\frac{2^x\sin^4 xdx}{2^x+1}=\int_{-\frac{\pi}{6}}^{0}\frac{2^x\sin^4 xdx}{2^x+1}+\int_{0}^{\frac{\pi}{6}}\frac{2^x\sin^4 xdx}{2^x+1}=I_1+I_2$

+ *Tính* $I_1=\int_{-\frac{\pi}{6}}^{0}\frac{2^x\sin^4 xdx}{2^x+1}$. *Đặt* $x=-t\Rightarrow I_1=-\int_{\frac{\pi}{6}}^{0}\frac{2^{-t}\sin^4(-t)}{2^{-t}+1}dt=\int_{\frac{\pi}{6}}^{0}\frac{\sin^4 t}{2^t+1}dt=\int_{\frac{\pi}{6}}^{0}\frac{\sin^4 x}{2^x+1}dx$

$\Rightarrow I=\int_0^{\frac{\pi}{6}}\frac{\sin^4 xdx}{2^x+1}+\int_0^{\frac{\pi}{6}}\frac{2^x\sin^4 xdx}{2^x+1}=\int_0^{\frac{\pi}{6}}\sin^4 xdx=\frac{1}{4}\int_0^{\frac{\pi}{6}}(1-\cos 2x)^2dx$

$=\frac{1}{8}\int_0^{\frac{\pi}{6}}(3-4\cos 2x+\cos 4x)dx=\frac{4\pi-7\sqrt{3}}{64}$

Câu 27. $I=\int_1^{e^\pi}\cos(\ln x)dx$

• *Đặt* $t=\ln x\Rightarrow x=e^t\Rightarrow dx=e^tdt$

$\Rightarrow I=\int_0^{\pi}e^t\cos tdt=-\frac{1}{2}(e^\pi+1)$ *(dùng pp tích phân từng phần).*

Câu 28. $I=\int_0^{\frac{\pi}{2}}e^{\sin^2 x}.\sin x.\cos^3 xdx$

• *Đặt* $t=\sin^2 x\Rightarrow I=\frac{1}{2}\int_0^1 e^t(1-t)dt=\frac{1}{2}e$ *(dùng tích phân từng phần)*

Câu 29. $I=\int_0^{\frac{\pi}{4}}\ln(1+\tan x)dx$

• *Đặt* $t=\frac{\pi}{4}-x \Rightarrow I=\int_0^{\frac{\pi}{4}}\ln\left(1+\tan\left(\frac{\pi}{4}-t\right)\right)dt=\int_0^{\frac{\pi}{4}}\ln\left(1+\frac{1-\tan t}{1+\tan t}\right)dt=\int_0^{\frac{\pi}{4}}\ln\frac{2}{1+\tan t}dt$

$=\int_0^{\frac{\pi}{4}}\ln 2dt-\int_0^{\frac{\pi}{4}}\ln(1+\tan t)dt \; = \; t.\ln 2\Big|_0^{\frac{\pi}{4}}-I$

$\Rightarrow 2I=\frac{\pi}{4}\ln 2 \Rightarrow I=\frac{\pi}{8}\ln 2$.

Câu 30. $I=\int_0^{\frac{\pi}{2}}\sin x\ln(1+\sin x)dx$

• *Đặt* $\begin{cases}u=\ln(1+\sin x)\\dv=\sin xdx\end{cases} \Rightarrow \begin{cases}du=\frac{1+\cos x}{1+\sin x}dx\\v=-\cos x\end{cases}$

$\Rightarrow I=-\cos x.\ln(1+\sin x)\Big|_0^{\frac{\pi}{2}}+\int_0^{\frac{\pi}{2}}\cos x.\frac{\cos x}{1+\sin x}dx=0+\int_0^{\frac{\pi}{2}}\frac{1-\sin^2 x}{1+\sin x}dx=\int_0^{\frac{\pi}{2}}(1-\sin x)dx=\frac{\pi}{2}-1$

Câu 31. $I=\int_0^{\frac{\pi}{4}}\frac{\tan x.\ln(\cos x)}{\cos x}dx$

• *Đặt* $t=\cos x \Rightarrow dt=-\sin xdx \Rightarrow I=-\int_1^{\frac{1}{\sqrt{2}}}\frac{\ln t}{t^2}dt=\int_{\frac{1}{\sqrt{2}}}^{1}\frac{\ln t}{t^2}dt$.

Đặt $\begin{cases}u=\ln t\\dv=\frac{1}{t^2}dt\end{cases} \Rightarrow \begin{cases}du=\frac{1}{t}dt\\v=-\frac{1}{t}\end{cases} \Rightarrow I=\sqrt{2}-1-\frac{\sqrt{2}}{2}\ln 2$