TP1: TÍCH PHÂN HÀM SỐ HỮU TỈ

Dạng 1: Tách phân thức

Câu 1. $I=\int_1^2 \frac{x^2}{x^2-7x+12}dx$

• $I=\int_1^2\left(1+\frac{16}{x-4}-\frac{9}{x-3}\right)dx = \left(x+16\ln|x-4|-9\ln|x-3|\right)\Big|_1^2 = 1+25\ln 2-16\ln 3$.

Câu 2. $I=\int_1^2 \frac{dx}{x^5+x^3}$

• *Ta có:* $\frac{1}{x^3(x^2+1)}=-\frac{1}{x}+\frac{1}{x^3}+\frac{x}{x^2+1}$

$\Rightarrow I=\left[-\ln|x|-\frac{1}{2x^2}+\frac{1}{2}\ln(x^2+1)\right]\Big|_1^2=-\frac{3}{2}\ln 2+\frac{1}{2}\ln 5+\frac{3}{8}$

Câu 3. $I=\int_4^5 \frac{3x^2+1}{x^3-2x^2-5x+6}dx$ • $I=-\frac{2}{3}\ln\frac{4}{3}+\frac{13}{15}\ln\frac{7}{6}+\frac{14}{5}\ln 2$

Câu 4. $I=\int_0^1 \frac{xdx}{(x+1)^3}$

• *Ta có:* $\frac{x}{(x+1)^3}=\frac{x+1-1}{(x+1)^3}=(x+1)^{-2}-(x+1)^{-3} \Rightarrow I=\int_0^1\left[(x+1)^{-2}-(x+1)^{-3}\right]dx=\frac{1}{8}$

Dạng 2: Đổi biến số

Câu 5. $I=\int \frac{(x-1)^2}{(2x+1)^4}dx$ • Ta có: $f(x)=\frac{1}{3}.\left(\frac{x-1}{2x+1}\right)^2.\left(\frac{x-1}{2x+1}\right)' \Rightarrow I=\frac{1}{9}\left(\frac{x-1}{2x+1}\right)^3+C$

Câu 6. $I=\int_0^1 \frac{(7x-1)^{99}}{(2x+1)^{101}}dx$

• $I=\int_0^1\left(\frac{7x-1}{2x+1}\right)^{99}\frac{dx}{(2x+1)^2}=\frac{1}{9}\int_0^1\left(\frac{7x-1}{2x+1}\right)^{99}d\left(\frac{7x-1}{2x+1}\right)$

$=\frac{1}{9}\cdot\frac{1}{100}\left(\frac{7x-1}{2x+1}\right)^{100}\Big|_0^1=\frac{1}{900}\left[2^{100}-1\right]$

Câu 7. $I=\int_0^1 \frac{5x}{(x^2+4)^2}dx$ • *Đặt* $t=x^2+4 \Rightarrow I=\frac{1}{8}$

Câu 8. $I=\int_0^1 \frac{x^7}{(1+x^2)^5}dx$ • *Đặt* $t=1+x^2 \Rightarrow dt=2xdx \Rightarrow I=\frac{1}{2}\int_1^2\frac{(t-1)^3}{t^5}dt=\frac{1}{4}.\frac{1}{2^5}$

Câu 9. $I=\int_0^1 x^5(1-x^3)^6dx$

• *Đặt* $t=1-x^3 \Rightarrow dt=-3x^2dx \Rightarrow dx=\frac{-dt}{3x^2} \Rightarrow I=\frac{1}{3}\int_0^1 t^6(1-t)dt=\frac{1}{3}\left(\frac{t^7}{7}-\frac{t^8}{8}\right)=\frac{1}{168}$

Câu 10. $I=\int_1^{\sqrt[4]{3}} \frac{1}{x(x^4+1)}dx$ • *Đặt* $t=x^2 \Rightarrow I=\frac{1}{2}\int_1^{\sqrt{3}}\left(\frac{1}{t}-\frac{t}{t^2+1}\right)dt=\frac{1}{4}\ln\frac{3}{2}$

Câu 11. $I=\int_1^2 \frac{dx}{x.(x^{10}+1)^2}$ • $I=\int_1^2 \frac{x^4.dx}{x^5.(x^{10}+1)^2}$. *Đặt* $t=x^5 \Rightarrow I=\frac{1}{5}\int_1^{32}\frac{dt}{t(t^2+1)^2}$

Câu 12. $I=\int_1^2 \frac{1-x^7}{x(1+x^7)}dx$ • $I=\int_1^2 \frac{(1-x^7).x^6}{x^7.(1+x^7)}dx$. *Đặt* $t=x^7 \Rightarrow I=\frac{1}{7}\int_1^{128}\frac{1-t}{t(1+t)}dt$

Câu 13. $I=\int_1^{\sqrt{3}} \frac{dx}{x^6(1+x^2)}$

• *Đặt :* $x=\frac{1}{t} \Rightarrow I=-\int_1^{\frac{\sqrt{3}}{3}}\frac{t^6}{t^2+1}dt=\int_{\frac{\sqrt{3}}{3}}^1\left(t^4-t^2+1-\frac{1}{t^2+1}\right)dt = \frac{117-41\sqrt{3}}{135}+\frac{\pi}{12}$

Câu 14. $I=\int_1^2 \frac{x^{2001}}{(1+x^2)^{1002}}.dx$

• $I=\int_1^2 \frac{x^{2004}}{x^3(1+x^2)^{1002}}.dx=\int_1^2 \frac{1}{x^3\left(\frac{1}{x^2}+1\right)^{1002}}.dx$. *Đặt* $t=\frac{1}{x^2}+1 \Rightarrow dt=-\frac{2}{x^3}dx$.

<u>Cách 2</u>: Ta có: $I=\frac{1}{2}\int_0^1\frac{x^{2000}.2xdx}{(1+x^2)^{2000}(1+x^2)^2}$. *Đặt* $t=1+x^2 \Rightarrow dt=2xdx$

$\Rightarrow I=\frac{1}{2}\int_1^2\frac{(t-1)^{1000}}{t^{1000}t^2}dt=\frac{1}{2}\int_1^2\left(1-\frac{1}{t}\right)^{1000}d\left(1-\frac{1}{t}\right)=\frac{1}{2002.2^{1001}}$

Câu 15. $I=\int_1^2 \frac{1+x^2}{1+x^4}dx$

• *Ta có:* $\frac{1+x^2}{1+x^4}=\frac{1+\frac{1}{x^2}}{x^2+\frac{1}{x^2}}$. *Đặt* $t=x-\frac{1}{x} \Rightarrow dt=\left(1+\frac{1}{x^2}\right)dx$

$\Rightarrow I=\int_1^{\frac{3}{2}}\frac{dt}{t^2-2}=\frac{1}{2\sqrt{2}}\int_1^{\frac{3}{2}}\left(\frac{1}{t-\sqrt{2}}-\frac{1}{t+\sqrt{2}}\right)dt=\frac{1}{2\sqrt{2}}.\ln\left|\frac{t-\sqrt{2}}{t+\sqrt{2}}\right|\Bigg|_1^{\frac{3}{2}}=\frac{1}{2\sqrt{2}}\ln\left(\frac{\sqrt{2}-1}{\sqrt{2}+1}\right)$

Câu 16. $I=\int_1^2 \frac{1-x^2}{1+x^4}dx$

• *Ta có:* $\frac{1-x^2}{1+x^4}=\frac{\frac{1}{x^2}-1}{x^2+\frac{1}{x^2}}$. *Đặt* $t=x+\frac{1}{x}\Rightarrow dt=\left(1-\frac{1}{x^2}\right)dx \Rightarrow I=-\int_2^{\frac{5}{2}}\frac{dt}{t^2+2}$.

Đặt $t=\sqrt{2}\tan u\Rightarrow dt=\sqrt{2}\frac{du}{\cos^2 u}$; $\tan u=2\Rightarrow u_1=\arctan 2;\quad \tan u=\frac{5}{2}\Rightarrow u_2=\arctan\frac{5}{2}$

$\Rightarrow I=\frac{\sqrt{2}}{2}\int_{u_1}^{u_2}du=\frac{\sqrt{2}}{2}(u_2-u_1)=\frac{\sqrt{2}}{2}\left(\arctan\frac{5}{2}-\arctan 2\right)$

Câu 17. $I=\int_1^2 \frac{1-x^2}{x+x^3}dx$ • *Ta có:* $I=\int_1^2\frac{\frac{1}{x^2}-1}{\frac{1}{x}+x}dx$. *Đặt* $t=x+\frac{1}{x}\Rightarrow I=\ln\frac{4}{5}$

Câu 18. $I=\int_0^1 \frac{x^4+1}{x^6+1}dx$

• *Ta có:* $\frac{x^4+1}{x^6+1}=\frac{(x^4-x^2+1)+x^2}{x^6+1}=\frac{x^4-x^2+1}{(x^2+1)(x^4-x^2+1)}+\frac{x^2}{x^6+1}=\frac{1}{x^2+1}+\frac{x^2}{x^6+1}$

$\Rightarrow I=\int_0^1\frac{1}{x^2+1}dx+\frac{1}{3}\int_0^1\frac{d(x^3)}{(x^3)^2+1}dx=\frac{\pi}{4}+\frac{1}{3}.\frac{\pi}{4}=\frac{\pi}{3}$

Câu 19. $I=\int_0^{\frac{\sqrt{3}}{3}}\frac{x^2}{x^4-1}dx$

• $I=\int_0^{\frac{\sqrt{3}}{3}}\frac{x^2}{(x^2-1)(x^2+1)}dx=\frac{1}{2}\int_0^{\frac{\sqrt{3}}{3}}\left(\frac{1}{x^2-1}+\frac{1}{x^2+1}\right)dx=\frac{1}{4}\ln(2-\sqrt{3})+\frac{\pi}{12}$

Câu 20. $I=\int_0^1\frac{xdx}{x^4+x^2+1}$. • *Đặt* $t=x^2 \Rightarrow I=\frac{1}{2}\int_0^1\frac{dt}{t^2+t+1}=\frac{1}{2}\int_0^1\frac{dt}{\left(t+\frac{1}{2}\right)^2+\left(\frac{\sqrt{3}}{2}\right)^2}=\frac{\pi}{6\sqrt{3}}$

Câu 21. $I=\int_1^{\frac{1+\sqrt{5}}{2}}\frac{x^2+1}{x^4-x^2+1}dx$

• *Ta có:* $\frac{x^2+1}{x^4-x^2+1}=\frac{1+\frac{1}{x^2}}{x^2+\frac{1}{x^2}-1}$. *Đặt* $t=x-\frac{1}{x}\Rightarrow dt=\left(1+\frac{1}{x^2}\right)dx$

$\Rightarrow I=\int_0^1\frac{dt}{t^2+1}$. *Đặt* $t=\tan u\Rightarrow dt=\frac{du}{\cos^2 u} \Rightarrow I=\int_0^{\frac{\pi}{4}}du=\frac{\pi}{4}$