Option Pricing Model

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1. Introduction

In this project, I focused on **option pricing** using two models: **the Binomial Tree** and **the Black-Scholes** model. The **underlying asset** chosen for this analysis is **AMD stock**. I conducted a comparative study to demonstrate that the option price calculated using the **Binomial Tree** model **approaches** the **Black-Scholes** price as the number of steps $N \to \infty$. This exploration provides insights into the effectiveness of the **Binomial Tree model** in **approximating** the values predicted by the **Black-Scholes model** for option pricing.

2. Implementation

First, I downloaded the **AMD stock** data from **yfinance** library, with start date is 2015 - 01 - 01. Below is the AMD stock values over time:

Figure 1: AMD Stock Values

a. Binomial Tree model

In the binomial tree model, we focus on the **up** and **down** movements of a stock, as well as the risk-neutral probability. In my project, however, the up and down movements differ from the **usual formulaic** ones often seen in textbooks, which is given by the formula:

$$u = e^{\sigma \times \sqrt{h}}$$
$$d = e^{-\sigma \times \sqrt{h}}$$

where h is the length of 1 period. This is the formula from the **Cox-Ross-Rubinstein Binomial Tree**. This is a very usual approach in practice. However, a problem with this approach is that if h is large or σ is small, it is possible that $e^{rh} > e^{\sigma\sqrt{h}}$, in which case the binomial tree violates the restriction in the equation:

$$d < e^{rh} < u$$

In my project, I construct the stock price movement using the following up and down formula:

$$u = e^{rh + \sigma\sqrt{h}}$$
$$d = e^{rh - \sigma\sqrt{h}}$$

where r is the risk - free rate. There are many ways to model stock price movements, and all methods yield different option prices for finite N, but they approach the same price as $N \to \infty$.

I will take an example using the **European Call option** with strike price $K = \$150$, the initial stock price S_0 using the first price of AMD in 2015, risk - free rate $r = 3.73\%$, time to maturity $T = 1$ and the steps of tree is $N = 5$. The graph of tree is below:

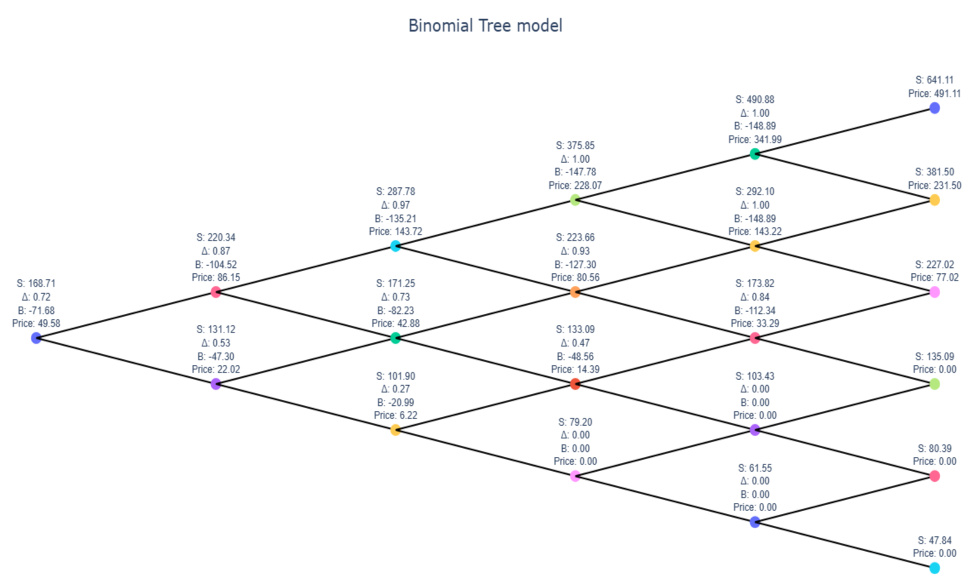

Figure 2: Binomial Tree model

I also compute the replicating portfolio consists of Δ shares and B in lending with the following formula:

$$\Delta = e^{-\delta h}\frac{C_u - C_d}{S(u-d)}$$
$$B = e^{-rh}\frac{uC_d - dC_u}{u-d}$$

b. Black - Scholes model

The **Black - Scholes formula** for European Call & Put option is:

$$C(S,K,\sigma,r,T,\delta) = Se^{-\delta T}N(d_1) - Ke^{-rT}N(d_2)$$
$$P(S,K,\sigma,r,T,\delta) = Ke^{-rT}N(-d_2) - Se^{-\delta T}N(-d_1)$$

where:

$$d_1 = \frac{\ln\left(\frac{S}{K}\right) + \left(r - \delta + \frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}}$$
$$d_2 = d_1 - \sigma\sqrt{T}$$

With the parameters above, our **European Call option** under the BS model has a price of \$49.09. I have also computed the six Greek values, which include: Δ, Γ, Θ, Vega, ρ, and Ψ.

c. Comparison between 2 models.

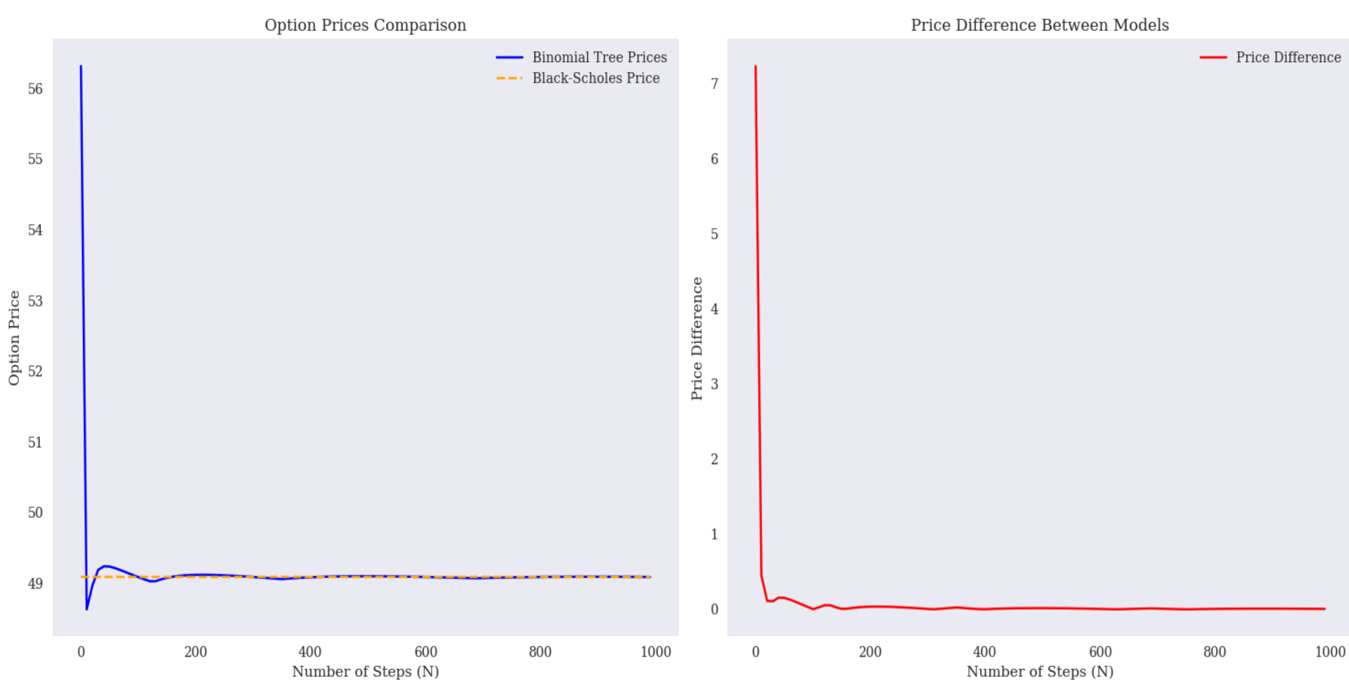

Figure 3: Comparison between 2 models.

When the number of steps is **small**, the binomial model tends to produce **significantly higher** option prices compared to the Black-Scholes model. However, as N **increases** and approaches ∞ , the binomial model's price **converges** to the Black-Scholes price, with the difference between the two approaching 0. Formally, as $N \to \infty$, we can express this relationship as:

$$\lim_{N \to \infty} C_{Binomial} = C_{BS}$$