EE 120 Cheatsheet

Time-Freq Relationships (Lecture 9)

1. $$H(\omega) = \sum_{k=-\infty}^{\infty} h(k) e^{-i\omega k}$$

$$h(n) \to h(n-N) \implies H(\omega) \to e^{-i\omega N} H(\omega)$$

2. Given $h(n)$ real, $\overline{H(\omega)} = H(-\omega)$
3. Time-reversal

$$g(n) = h(-n) \implies G(\omega) = H(-\omega)$$

- If $h(n)$ is real and symmetric/even, $H(\omega)$ is also real and symmetric/even.
- If $h(n)$ is real and odd, $H(\omega)$ is imaginary.

Causality (Lecture 9-10)

At any time n, $y(n)$ does not depend on any future value of $x(n)$.
For LTI system, $\forall n < 0, h(n) = 0 \iff$ causality

Real Definition

Upto and including some time n, make x_1 and x_2 the same, if this always results in y having the same property, then the system is called causal.

Bounded Input Bounded Output (BIBO) Stability (Lecture 10)

$$\sum_{k=-\infty}^{\infty} |h(k)| < \infty \iff \text{BIBO Stability}$$

The Dirac $\delta(t)$ function (Lecture 11-12)

$$\delta(t) = \begin{cases} \infty, & t = 0 \\ 0, & t \neq 0 \end{cases}$$

$$\int_a^b \delta(t) dt = \begin{cases} 1, & \text{if } 0 \in [a, b] \\ 0, & \text{otherwise} \end{cases}$$

Not a well-defined function.

Properties

1. Sifting

$$\int_{\tau=-\infty}^{\infty} x(\tau)\delta(\tau - T) d\tau = x(T)$$

2. $$u(t) = \int_{\tau=-\infty}^{t} \delta(\tau) d\tau \iff \frac{d}{dt} u(t) = \delta(t)$$

3. $$x(t)\delta(t-T) = x(T)\delta(t-T)$$

Linear Constant-Coefficient Differential Equations (Lecture 13)

$$a_N \frac{d^N}{dt^N} y(t) + a_{N-1} \frac{d^{N-1}}{dt^{N-1}} y(t) + \cdots + a_0 y(t) =$$
$$b_M \frac{d^M}{dt^M} x(t) + b_{M-1} \frac{d^{M-1}}{dt^{M-1}} x(t) + \cdots + b_0 x(t)$$

$$H(\omega) = \frac{(i\omega)^M b_M + (i\omega)^{M-1} b_{M-1} + \cdots + b_0}{(i\omega)^N a_N + (i\omega)^{N-1} a_{N-1} + \cdots + a_0}$$

$H(\omega)$ is a rational function in $i\omega$.

IAG Diagrams

Solve second order using state-space form

Graphs (Lecture 14)

$$\lambda_{1,2} = \frac{-D \pm \sqrt{D^2 - 4MK}}{2M}$$

- Overdamped
$$D^2 - 4MK > 0$$
- Critical damping
$$D^2 - 4MK = 0$$
- Underdamping
$$D^2 - 4MK < 0$$
- Undamped
$$D = 0$$

Impulse Response

$$h(t) = Ce^{At} Bu(t)$$
$$= \frac{e^{\frac{-D}{2M}t}\left(e^{\frac{\sqrt{D^2-4KM}}{2M}t} - e^{-\frac{\sqrt{D^2-4KM}}{2M}t}\right)}{\sqrt{D^2 - 4KM}} u(t)$$

Frequency Response

$$H(\omega) = \frac{1}{M(i\omega)^2 + D(i\omega) + K}$$

Feedback

Black's Formula

- P: Plant
- K: Controller

$$H(\omega) = \frac{P(\omega)}{1 - K(\omega)P(\omega)}$$