Use Python and Matlab for all questions below (unless some derivation is required which you can do better by hand). If using Python, use Jupyter notebook to format properly, comment when necessary, and include all required and necessary outputs (Jupyter allows LATEX embedding which would be great if you could use it). Convert the notebook to pdf and upload to blackboard against the assignment. If using MATLAB, use MATLAB publish to convert to pdf. Solution documents should be well commented.

Q1. Consider the vectors

$$v(r) = \sin(r) + r^2 + 1$$

(a) Find the expressions for $\frac{dv}{dr}, \frac{d^2v}{dr^2}, \frac{d^3v}{dr^3}$

(b) Plot v in the range $-\pi \leq r \leq \pi$. v should be on the vertical axis and r should be on the horizontal axis

(c) Through visual inspection of the plot, determine the value of r for which v is minimum

(d) Plot $w = \frac{dv}{dr}$ for $-\pi \leq r \leq \pi$. At what value of r is $w = 0$? How does this answer compare with the answer in part c? If the answers the close to each other, explain why?

Q2. Plot the function $z = \sin(xy) + \cos(xy)$ where x, y are the usual orthogonal axes. You can plot z over a grid of x, y. I am leaving the range of the grid and the discretization open ended in this question. Choose your best judgment and make sure that the plot is clear.

Q3. Consider the function $y = \sin(x) + \cos(x)$

(a) Plot the function for $0 \leq x \leq 2\pi$

(b) Approximate the function with the first 2, 3, 4, 5, and 6 terms of its Taylor series expansion about point $x_0 = 0$. Let's call these approximations y_2, y_3, y_4, y_5, y_6 respectively. Plot $y, y_2, y_3, y_4, y_5, y_6$ for $0 \leq x \leq 2\pi$ showing the approximation improving as more terms are taken into consideration in the Taylor series.

(c) Approximate the function with the first 2, 3, 4, 5, and 6 terms of its Taylor series expansion about point $x_0 = \pi/4$. Let's call these approximations y_2, y_3, y_4, y_5, y_6 respectively. Plot $y, y_2, y_3, y_4, y_5, y_6$ for $0 \leq x \leq 2\pi$ showing the approximation improving as more terms are taken into consideration in the Taylor series.

Q4. Consider the nonlinear function:

$$f(x) = x^3 [\cos(x)]$$

(a) Plot y as a function of x as x is varied between -6π and 6π. In this plot mark all the locations of x where $y = 0$. These locations are some of the roots of the above equation.

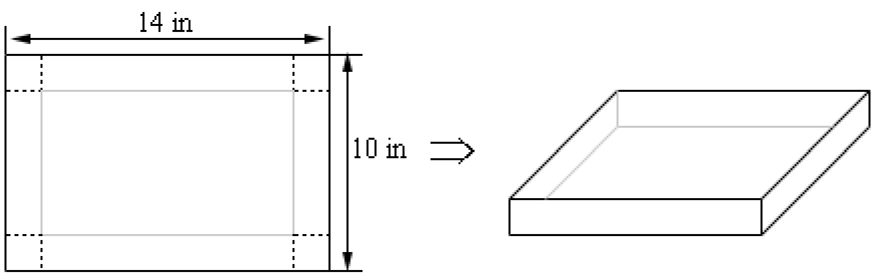
(b) Use fsolve in Python (or fzero in MATLAB) to solve for all the roots of the above equation between -6π and 6π. These should match the results that you get in 'a'.

Q5. Find the solutions of the following system of nonlinear equations:

$$y = -2x - 4$$
$$y = x^2 - 4$$

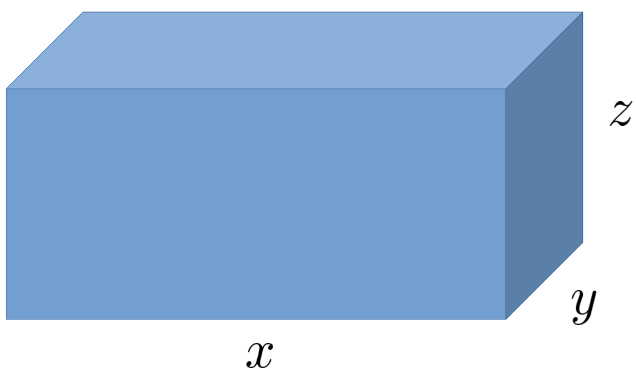
Q6. Determine the dimensions of a cylindrical can whose volume is 1.5 liters and which has the minimum amount of surface area. Find this minimum value of the surface and show that it is, in fact, the minimum and not the maximum. [Hint: The variables are the radius r and height h. Write the expressions for the surface area and volume in terms of r, h. Use the given volume to express the surface area in terms of one variable. Now find the value of that variable by minimizing the surface area.]

Q7. A piece of cardboard needs to be folded in the following fashion:



Determine the height of the box which will maximize its volume. Find this maximum value of the volume and show that it is, in fact, the maximum and not the minimum.

Q8. Consider the following:



You are given that the volume of the above box is $1000cm^3$. Find the dimensions x, y, z such that the total surface area of all the faces of the box is minimum.