1. Find the eigenvalues and eigenvectors of the following matrix:

$$A = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & 4 & 7 & 1 & 2 \\ 3 & 7 & 8 & 5 & 6 \\ 4 & 1 & 5 & 8 & 10 \\ 5 & 2 & 6 & 10 & 3 \end{bmatrix}$$

   - Explicitly show that the eigenvalue equation $Au = \lambda u$ holds for each of the eigenvalue-eigenvector pair (λ_i, u_i).
   - Show that the eigenvalues of A^{-1} are $1/\lambda_i$.

2. Consider the following system of equations for the motion of a $BeCl_2$ molecule:

$$\begin{aligned} -\omega^2 A_1 &= -\frac{k}{m_1} A_1 + \frac{k}{m_1} A_2 \\ -\omega^2 A_2 &= -\frac{2k}{m_2} A_2 + \frac{k}{m_2} A_1 + \frac{k}{m_2} A_3 \\ -\omega^2 A_3 &= \frac{k}{m_1} A_2 - \frac{k}{m_1} A_3 \end{aligned}$$

   where M_1 the mass of Cl atom and M_2 the mass of Be atom and ω the frequency of vibration of the molecule.

   - Rewrite the above as an eigenvalue problem and show that the quantity ω^2 is the eigenvalue.
   - Solve for the different frequencies ω when $k = 1.8 \times 10^2$ kg/s^2, $m_1 = 35.45 \times 1.6605 \times 10^{-27}$ kg, $m_2 = 9.01 \times 1.6605 \times 10^{-27}$. These frequencies will be derived from the various eigenvalues of the problem.
   - Find the eigenvectors corresponding to the frequencies calculated above.

3. Three dimensional state of stress at a point is given by:

$$\begin{bmatrix} \sigma_{xx} & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \sigma_{yy} & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \sigma_{zz} \end{bmatrix} = \begin{bmatrix} 40 & 20 & -18 \\ 20 & 28 & 12 \\ -18 & 12 & 4 \end{bmatrix}$$

   The principal stresses at the point are given by the eigenvalues of the stress matrix above. The associated eigenvectors of the above are pointing along the normal directions to the principal planes. Shear stresses are zero over principal planes as you have learned in 202.

   - Find the principal stresses at the point.

- Calculate the angles that the normal to the principal plane which has the maximum principal stress makes with the x, y, z axes.
- Calculate the angles that the normal to the principal plane which has the minimum principal stress makes with the x, y, z axes.