

1. In this problem, you will implement gradient descent to find the minima of a function. The function that we are looking at is  $f(x) = x^2 + 3x - 30$ 
  - Plot the function between  $x = -10, 10$  and determine where the minima is using either the `minimize` command or by converting the problem to a roots problem and solving it.
  - Define a function `f` which takes one argument:  $x$  The function returns  $x^2 + 3x - 30$ .
  - Define a function `gradMinimize` which takes two arguments:  $\alpha, x_0$  where  $\alpha$  is the learning rate and  $x_0$  is the initial guess. This function returns the minima of `f` using gradient descent. Inside this function, you need to create a loop which implements the gradient descent algorithm:  $x_{i+1} = x_i - \alpha f'(x_i)$ . You can create this loop using the `while` function in python. The loop should end when either one of the two conditions are satisfied: a.  $|x_{i+1} - x_i| < \text{tol}$  or b. the number of iterations  $> N$ . Here `tol` is some small number you choose (such as  $10^{-6}$ ) and it indicates that the solution is not changing much with more iterations. Here `N` is some integer that you choose and it indicates how many maximum iterations the loop should run for - it should not run forever.
  - Use `gradMinimize` to find the minima of  $f(x)$  and make sure that you get the same answer as that found in the first part of this question. You can try different values of  $\alpha$  and  $x_0$  and you can even plot how the solution converges by saving  $f(x_i)$  inside the loop into an evolving vector and then plotting it as a function of iteration number after the problem is solved.
2. Use `gradMinimize` to find all the roots of  $y = f(x) = x^2 - 10x + 12$ . As discussed in class, you can do this by minimizing  $[f(x)]^2$ . Verify that the roots are indeed correct.
3. Create a function `gradMinimize2var` to find the minima of  $f(x, y) = (x-1)^2 + y^2$  using the gradient descent algorithm:  $(x_{i+1}, y_{i+1}) = (x_i, y_i) - \alpha \nabla f(x_i, y_i)$ . Here,  $\nabla f(x_i, y_i) = \nabla f(x, y)|_{(x_i, y_i)}$  is the gradient of the function ( $\partial f / \partial x, \partial f / \partial y$ ) evaluated at  $(x_i, y_i)$ . This function will take inputs  $\alpha$  and the initial guess  $(x_0, y_0)$  (you can pass this as a vector in the function). And it will return  $(x_{min}, y_{min})$  as the minima of the function.