approximation-pattern

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Consider the problem of minimizing a function $f : D \to \mathbb{R}$

In many scenarios, it is hard to find an optimal, or it is even also hard to compute the value of f. The method below was inspired from the work of Isaac Vandermeulen, Roderich Groß, Andreas Kolling [1].

For a domain $X \subseteq D$ of the minimization problem, let $f_1 : X \to \mathbb{R}$ be the proxy function such that

$$(1) : f(x) = c(f_1(x)) + v(x) \; \forall x \in x$$

Where c is a monotonically increasing function and $v : X \to \mathbb{R}$ is function on X.

Let x^* and x_1^* be the optimal values for f and f_1 in the domain $X \subseteq D$. Let $t_{\max} = \max_{x\in X} v(x)$ and $t_{\min} = \min_{x\in X} v(x)$ be the maximum value and minimum value of v over the domain X. [1]

Consider 3 inequalities:

$$(A) : f(x_1^*) \leq f(x^*) + t_{\max} - t_{\min}$$
$$(B) : f(x_1^*) \leq c(f_1(x_1^*)) + t_{\max}$$
$$(C) : f(x^*) \geq c(f_1(x^*)) + t_{\min}$$

We have $f(x^*) \leq f(x_1^*)$ and $f_1(x_1^*) \leq f_1(x^*)$. Since c is a monotonically increasing function, so that $c(f_1(x_1^*)) \leq c(f_1(x^*))$.

By definition of $t_{\max}$, (B) holds,

$$f(x_1^*) \leq c(f_1(x_1^*)) + t_{\max} \leq c(f_1(x^*)) + t_{\max}$$

By definition of $t_{\min}$, (C) holds,

$$f(x^*) \geq c(f_1(x^*)) + t_{\min} = (c(f_1(x^*)) + t_{\max}) - (t_{\max} - t_{\min})$$

Hence,

$$(A) : f(x_1^*) \leq f(x^*) + (t_{\max} - t_{\min})$$

[1] Here, we used the maximum value and minimum value for v since in the original work [1], the authors did not make it clear why x^* and x_1^* are independent from v and further more, their proof does not make a clear statement on the feasibility of the method to any problem but rather most problems.

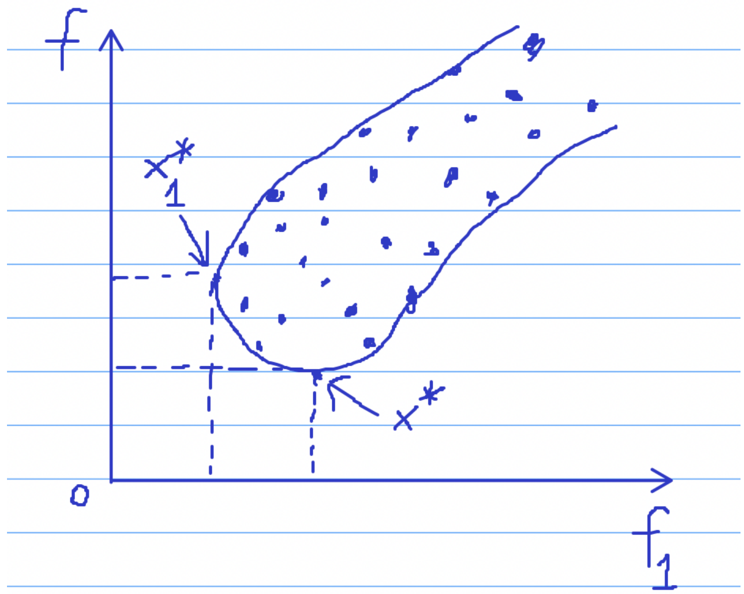

Figure 1: $f_1 approximation$

By choosing an appropriate proxy function f_1, we can approximate the solution of f.

In the derivation, we have set $t_{\max}$ and $t_{\min}$ be the maximum and minimum value of v in the domain of X. However, the bound can be even better if we have some methods to approximate the maximum and minimum value of v in a subset of X that contains both x^* and x_1^*

References

[1] Isaac Vandermeulen, Roderich Groß, and Andreas Kolling. Balanced task allocation by partitioning the multiple traveling salesperson problem. In *2019 International Conference on Autonomous Agents and Multiagent Systems*, pages 1479–1487. ACM, 2019.