Bài tập Homework No. 02:
Digital Signal Processing (Xử lý Tín hiệu Số)

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Biến đổi Z

1. **$x(n) = 2.\delta(n-2) + 3.u(n-3)$**
2. **$x(n) = 3.(0.75)^n \cos(0.3\pi n)u(n) + 4.(0.75)^n \sin(0.3\pi n)u(n)$**
3. **$x(n) = n\sin(\frac{1}{3}\pi n)u(n) + 0.9^n u(n-2)$**
4. **$x(n) = n^2(\frac{2}{3})^{n-2}u(n-1)$**
5. **$x(n)= (n-3)(\frac{1}{4})^{n-2}\cos(\frac{(n-1)\pi}{2})u(n)$**

Lời giải

1. **$x(n) = 2.\delta(n-2) + 3.u(n-3)$**

$X(z) = Z[x(n)] = Z[2.\delta(n-2) + 3.u(n-3)]$

$= 2.X[\delta(n-2)] + 3.X[u(n-3)]$

$= 2.z^{-2}.X[\delta(n)] + 3.z^{-3} X[u(n)]$ (*)

Ta có:

$X_1[\delta(n)] = 1$

$X_2\ [u(n)] = \frac{1}{1-z^{-1}}$

Thay vào (*), ta có:

$X(z) = Z[x(n)] = 2.z^{-2}.1 + 3.z^{-3}.\frac{1}{1-z^{-1}}$

$= \frac{2.z^{-2} - 2.z^{-3} + 3.z^{-3}}{1-z^{-1}} = \mathbf{\frac{2z^{-2} + z^{-3}}{1-z^{-1}}}$

2. **$x(n) = 3.(0.75)^n \cos(0.3\pi n)u(n) + 4.(0.75)^n \sin(0.3\pi n)u(n)$**

Đặt

$x_1(n) = (0.75)^n \cos(0.3\pi n)u(n)$

$x_2(n) = (0.75)^n \sin(0.3\pi n)u(n)$

$\Rightarrow X(z) = Z[x(n)] = 3.Z[x_1(n)] + 4.Z[x_2(n)]$

Z[x_1(n)] = $\frac{1-\ 0.75\cos\ (0.3\pi)z^{-1}}{1-2.(0.75)\ \cos(0.3\pi)z^{-1}+0.75^2z^{-2}} = \frac{1-\ 0.75\cos\ (0.3\pi)z^{-1}}{1-1.5.\cos(0.3\pi)z^{-1}+0.5625z^{-2}}$

Z[x_2(n)] = $\frac{0.75\sin\ (0.3\pi)z^{-1}}{1-2.(0.75)\ \cos(0.3\pi)z^{-1}+0.75^2z^{-2}} = \frac{0.75\sin\ (0.3\pi)z^{-1}}{1-1.5.\cos(0.3\pi)z^{-1}+0.5625\ \ ^{-2}}$

=> X(z) = 3. $\frac{1-\ 0.75\cos\ (0.3\pi)z^{-1}}{1-1.5.\cos(0.3\pi)z^{-1}+0.5625\ \ ^{-2}}$ + 4. $\frac{0.75\sin\ (0.3\pi)z^{-1}}{1-1.5.\cos(0.3\pi)z^{-1}+0.5625\ \ ^{-2}}$

= $\boldsymbol{\frac{3+[3\sin\ (0.3\pi)-2.25\cos(0.3\pi)]z^{-1}}{1-1.5.\cos(0.3\pi)z^{-1}+0.5625z^{-2}}}$

3. **x(n) = nsin($\frac{1}{3}$πn)u(n) + 0.9^nu(n-2)**

   = nsin($\frac{1}{3}$πn)u(n) + $0.9^2.0.9^{n-2}$u(n-2)

   = nsin($\frac{1}{3}$πn)u(n) + (0.81).0.9^{n-2}u(n-2)

Đặt

x_1(n) = n cos($\frac{1}{3}$πn)u(n)

x_2(n) = 0.9^{n-2}u(n-2)

=> X(z) = Z[x(n)] = Z[x_1(n)] + (0.81)Z[x_2(n)] (*)

Biến đổi Z cho :x_1(n)

Z[x_1(n)] = -z. $\frac{dZ\left[\cos\left(\frac{1}{3}\pi n\right)u(n)\right]}{dz}$

Z[cos($\frac{1}{3}$πn)u(n)] = $\frac{\sin\ (\frac{\pi}{3})z^{-1}}{1-2co\ \left(\frac{\pi}{3}\right)z^{-1}+z^{-2}} = \frac{\sqrt{3}}{2}\ .\ \frac{1}{1-z^{-1}+z^{-2}}$

=> Z[x_1(n)] = -z.($\frac{\sqrt{3}}{2}\ .\ \frac{1}{1-z^{-1}+z^{-2}}$)'

= $\frac{-\sqrt{3}}{2}$ z.$\frac{-(1-z^{-1}+z^{-2})'}{(1-z^{-1}+z^{-2})^2} = \frac{\sqrt{3}}{2}$ z.$\frac{z^{-2}-2z^{-3}}{(1+z^{-2}+z^{-4}\ -2z^{-1}+2z^{-2}-2z^{-3})}$

= $\frac{\sqrt{3}}{2}\ \frac{(z^{-1}-2z^{-2})}{(1+z^{-2}+z^{-4}\ -2z^{-1}+2z^{-2}-2z^{-3})} = \frac{\sqrt{3}}{2}\ \frac{(z^{-1}-2z^{-2})}{(1-2z^{-1}+3z^{-2}-2z^{-3}+z^{-4})}$

Z[x_2(n)] = z^{-2}.X[0.9^n.u(n)] = z^{-2} . $\frac{1}{1-\ 0.9z^{-1}}$

Thay vào (*), ta có:

X(z) = Z[x(n)] = $\boldsymbol{\frac{\sqrt{3}}{2}\ .\ \frac{(z^{-1}-2z^{-2})}{(1-2z^{-1}+3z^{-2}-2z^{-3}+z^{-4})}\ +\ \frac{0.81z^{-2}}{1-\ 0.9z^{-1}}}$

4. **x(n) = $n^2(\frac{2}{3})^{n-2}$u(n-1)**

   = $(\frac{2}{3})^{-1}$n.[n$(\frac{2}{3})^{n-1}$u(n-1)] = $\frac{3}{2}$n.[n$(\frac{2}{3})^{n-1}$u(n-1)]

Đặt

$x_1(n) = (\frac{2}{3})^{n-1}u(n-1)$

$x_2(n) = n\, x_1(n)$

⇨ $x(n) = \frac{3}{2}\, nx_2(n)$

⇨ $Z[x(n)] = \frac{3}{2}\, Z[nx_2(n)]$

Biến đổi :

$Z[nx_2(n)] = -z.\frac{dZ[x_2(n)]}{dz}$ **(1)**

$Z[x_2(n)] = Z[nx_1(n)] = -z.\frac{dZ[x_1(n)]}{dz}$ **(2)**

$Z[x_1(n)] = Z[(\frac{2}{3})^{n-1}u(n-1)] = z^{-1}.\left(\frac{1}{1-\frac{2}{3}z^{-1}}\right) = \frac{3z^{-1}}{3-2z^{-2}}$ **(*)**

Thay **(*)** vào **(2)**, ta có:

$Z[x_2(n)] = -z\,(\frac{3z^{-1}}{3-2z^{-2}})' = -z\left[\frac{(3z^{-1})'.(3-2z^{-2}) + (3z^{-1}).(3-2z^{-2})'}{(3-2z^{-2})^2}\right]$

$= -z\left[\frac{(-3z^{-2})\ .(3-2z^{-2}) + (3z^{-1}).(4z^{-3})}{(3-2z^{-2})^2}\right]$

$= \frac{(9z^{-1}-6z^{-3}) - 12z^{-3}}{(3-2z^{-2})^2} = \frac{9z^{-1}-18z^{-3}}{(3-2z^{-2})^2} = \frac{9(z^{-1}-2z^{-3})}{(9-12z^{-2}+4z^{-4})}$ (**)

Thay **(**)** vào **(1)**, ta có:

$Z[nx_2(n)] = -z\,(\frac{9(z^{-1}-2z^{-3})}{9-12z^{-2}+4z^{-4}})' = 9.-z\left[\frac{(z^{-1}-2z^{-3})'.(9-12z^{-2}+4z^{-4}) + (z^{-1}-2z^{-3}).(9-12z^{-2}+4z^{-4})'}{(9-12\ \ ^{-2}+4z^{-4})^2}\right]$

$= 9\left[\frac{-9z^{-1}+18z^{-3}+13\ \ ^{-5}-18z^{-7}}{(9-12z^{-2}+4z^{-4})^2}\right]$

⇨ $Z[x(n)] = \frac{3}{2}.\ 9\left[\frac{-9z^{-1}+18\ \ ^{-3}+132z^{-5}-18\ \ ^{-7}}{(9-12z^{-2}+4z^{-4})^2}\right] = \boldsymbol{\frac{27}{2}.\left[\frac{-9z^{-1}+18z^{-3}+132z^{-5}-18z^{-7}}{(9-12z^{-2}+4z^{-4})^2}\right]}$

5. $\boldsymbol{x(n)= (n-3)(\frac{1}{4})^{n-2}\cos(\frac{(n-1)\pi}{2})u(n)}$

$= 4^2(n-3).(\frac{1}{4})^n \cos[-\left(\frac{\pi}{2}-\frac{n\pi}{2}\right)]u(n)$

$= 4^2(n-3).(\frac{1}{4})^n \cos[\left(\frac{\pi}{2}-\frac{n\pi}{2}\right)]u(n)$

$= 16(n-3)\ (\frac{1}{4})^n \sin(\frac{n\pi}{2})u(n)$

$= 16[n(\frac{1}{4})^n \sin(\frac{n\pi}{2})u(n) - 3.\ (\frac{1}{4})^n \sin(\frac{n\pi}{2})u(n)]$

Đặt

$x_{1(n)} = (\frac{1}{4})^n \sin(\frac{n\pi}{2})u(n)$

=> $X[x_{(n)}] = 16.X[n.x_{1(n)} - 3.x_{1(n)}] = 16.[\{ -z.\frac{dZ[x1(n)]}{dz} \} - 3.\ Z[x1(n)]$ *(*)*

Biến đổi Z cho $x_{1(n)}$

$X_1(z) = Z[x1(n)] = Z[(\frac{1}{4})^n \sin(\frac{n\pi}{2})u(n)] = \frac{(\frac{1}{4})\ \ \sin(\frac{\pi}{2})z^{-1}}{1-2.(\frac{1}{4}).\cos(\frac{\pi}{2})z^{-1}+(\frac{1}{4})^2z^{-2}}$

$$= \frac{(\frac{1}{4})\quad z^{-1}}{1-(\frac{z^{-2}}{16})} = \frac{4z^{-1}}{16- z^{-2}} \textit{ (1)}$$

⇨ $$\frac{dZ[\text{x1(n)}]}{dz} = (\frac{4z^{-1}}{16- z^{-2}})' = \frac{\left(4z^{-1}\right)'.\left(16\quad z^{-2}\right) + \left(4z^{-1}\right).\left(16- z^{-2}\right)'}{(16\quad z^{-2})^2}$$

$$= \frac{\left(-4z^{-2}\right)\quad .\left(16- z^{-2}\right) + \left(4z^{-1}\right).\left(2.z^{-3}\right)}{(16\quad z^{-2})^2}$$

$$= \frac{-64z^{-2}+ 12z^{-4}}{(16\quad z^{-2})^2} \textit{ (2)}$$

Thay (1) , (2) vào (*), ta có:

$$X(z) = Z[x(n)] = 16. [-z\frac{-64z^{-2}+ 12z^{-4}}{(16\quad z^{-2})^2} - 3. \frac{4z^{-1}}{16- z^{-2}}]$$

$$= 16. [\frac{64\quad ^{-1}- 12\quad ^{-3}}{(16- z^{-2})^2} - \frac{12z^{-1}(16\quad z^{-2})}{(16\quad z^{-2})^2}]$$

$$= 16. [\frac{64z^{-1}- 12\quad ^{-3} -192z^{-1} + 12\quad ^{-3}}{(16- z^{-2})^2}]$$

$$= \frac{\mathbf{-2048z^{-1}}}{\mathbf{(16- z^{-2})^2}}$$