## Bài tập Homework No. 02:
## Digital Signal Processing (Xử lý Tín hiệu Số)

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**Biến đổi Z**

1. **$x(n) = 2.\delta(n-2) + 3.u(n-3)$**
2. **$x(n) = 3.(0.75)^n \cos(0.3\pi n)u(n) + 4.(0.75)^n \sin(0.3\pi n)u(n)$**
3. **$x(n) = n\sin(\frac{1}{3}\pi n)u(n) + 0.9^n u(n-2)$**
4. **$x(n) = n^2(\frac{2}{3})^{n-2}u(n-1)$**
5. **$x(n)= (n-3)(\frac{1}{4})^{n-2}\cos(\frac{(n-1)\pi}{2})u(n)$**

**Lời giải**

1. **$x(n) = 2.\delta(n-2) + 3.u(n-3)$**

$X(z) = Z[x(n)] = Z[2.\delta(n-2) + 3.u(n-3)]$

$= 2.X[\delta(n-2)] + 3.X[u(n-3)]$

$= 2.z^{-2}.X[\delta(n)] + 3.z^{-3} X[u(n)]$ (*)

Ta có:

$X_1[\delta(n)] = 1$

$X_2\ [u(n)] = \frac{1}{1-z^{-1}}$

Thay vào (*), ta có:

$X(z) = Z[x(n)] = 2.z^{-2}.1 + 3.z^{-3}.\frac{1}{1-z^{-1}}$

$= \frac{2.z^{-2} - 2.z^{-3} + 3.z^{-3}}{1-z^{-1}} = \mathbf{\frac{2z^{-2} + z^{-3}}{1-z^{-1}}}$

2. **$x(n) = 3.(0.75)^n \cos(0.3\pi n)u(n) + 4.(0.75)^n \sin(0.3\pi n)u(n)$**

Đặt

$x_1(n) = (0.75)^n \cos(0.3\pi n)u(n)$

$x_2(n) = (0.75)^n \sin(0.3\pi n)u(n)$

=> $X(z) = Z[x(n)] = 3.Z[x_1(n)] + 4.Z[x_2(n)]$

Z[$x_1$(n)] = $\frac{0.75\sin(0.3\pi)z^{-1}}{1-2.(0.75)\cos(0.3\pi)z^{-1}+0.75^2z^{-2}} = \frac{0.75\sin(0.3\pi)z^{-1}}{1-1.5.\cos(0.3\pi)z^{-1}+0.5625z^{-2}}$

Z[$x_2$(n)] = $\frac{1-0.75\cos(0.3\pi)z^{-1}}{1-2.(0.75)\cos(0.3\pi)z^{-1}+0.75^2z^{-2}} = \frac{1-0.75\cos(0.3\pi)z^{-1}}{1-1.5.\cos(0.3\pi)z^{-1}+0.5625z^{-2}}$

=> X(z) = 3. $\frac{0.75\sin(0.3\pi)z^{-1}}{1-1.5.\cos(0.3\pi)z^{-1}+0.5625z^{-2}}$ +4. $\frac{1-0.75.z^{-1}\cos(0.3\pi)}{1-1.5.\cos(0.3\pi)z^{-1}+0.5625z^{-2}}$

= $\boldsymbol{\frac{4+[2.25\sin(0.3\pi)-3\cos(0.3\pi)]z^{-1}}{1-1.5.\cos(0.3\pi)z^{-1}+0.5625z^{-2}}}$

3. **x(n) = nsin($\frac{1}{3}\pi$n)u(n) + $0.9^n$u(n-2)**

= nsin($\frac{1}{3}\pi$n)u(n) + $0.9^2.0.9^{n-2}$u(n-2)

= nsin($\frac{1}{3}\pi$n)u(n) + (0.81).$0.9^{n-2}$u(n-2)

Đặt

$x_1$(n) = n cos($\frac{1}{3}\pi$n)u(n)

$x_2$(n) = $0.9^{n-2}$u(n-2)

=> X(z) = Z[x(n)] = Z[$x_1$(n)] + (0.81)Z[$x_2$(n)] (*)

Biến đổi Z cho :$x_1$(n)

Z[$x_1$(n)] = -z. $\frac{dZ\left[\cos\left(\frac{1}{3}\pi n\right)u(n)\right]}{dz}$

Z[cos($\frac{1}{3}\pi$n)u(n)] = $\frac{\sin(\frac{\pi}{3})z^{-1}}{1-2\cos\left(\frac{\pi}{3}\right)z^{-1}+z^{-2}} = \frac{\sqrt{3}}{2} . \frac{1}{1-z^{-1}+z^{-2}}$

=> Z[$x_1$(n)] = -z.( $\frac{\sqrt{3}}{2} . \frac{1}{1-z^{-1}+z^{-2}}$)'

= $\frac{-\sqrt{3}}{2}$ z.$\frac{-(1-z^{-1}+z^{-2})'}{(1-z^{-1}+z^{-2})^2} = \frac{\sqrt{3}}{2}$ z.$\frac{z^{-2}-2z^{-3}}{(1+z^{-2}+z^{-4}-2z^{-1}+2z^{-2}-2z^{-3})}$

= $\frac{\sqrt{3}}{2} \frac{(z^{-1}-2z^{-2})}{(1+z^{-2}+z^{-4}-2z^{-1}+2z^{-2}-2z^{-3})} = \frac{\sqrt{3}}{2} \frac{(z^{-1}-2z^{-2})}{(1-2z^{-1}+3z^{-2}-2z^{-3}+z^{-4})}$

Z[$x_2$(n)] = $z^{-2}$.X[$0.9^n$.u(n)] = $z^{-2}$ . $\frac{1}{1-0.9z^{-1}}$

Thay vào (*), ta có:

X(z) = Z[x(n)] = $\boldsymbol{\frac{\sqrt{3}}{2} . \frac{(z^{-1}-2z^{-2})}{(1-2z^{-1}+3z^{-2}-2z^{-3}+z^{-4})} + \frac{0.81z^{-2}}{1-0.9z^{-1}}}$

4. **x(n) = $n^2(\frac{2}{3})^{n-2}$u(n-1)**

= $(\frac{2}{3})^{-1}$n.[n$(\frac{2}{3})^{n-1}$u(n-1)] = $\frac{3}{2}$n.[n$(\frac{2}{3})^{n-1}$u(n-1)]

Đặt

$x_1(n) = (\frac{2}{3})^{n-1}u(n-1)$

$x_2(n) = n\, x_1(n)$

⇨ $x(n) = \frac{3}{2}\, n x_2(n)$

⇨ $Z[x(n)] = \frac{3}{2}\, Z[n x_2(n)]$

Biến đổi :

$Z[nx_2(n)] = -z.\frac{dZ[x_2(n)]}{dz}$ **(1)**

$Z[x_2(n)] = Z[nx_1(n)] = -z.\frac{dZ[x_1(n)]}{dz}$ **(2)**

$Z[x_1(n)] = Z[(\frac{2}{3})^{n-1}u(n-1)] = z^{-1}.\left(\frac{1}{1-\frac{2}{3}z^{-1}}\right) = \frac{3z^{-1}}{3-2z^{-2}}$ **(*)**

Thay **(*)** vào **(2)**, ta có:

$Z[x_2(n)] = -z\,(\frac{3z^{-1}}{3-2z^{-2}})' = -z\left[\frac{(3z^{-1})'.(3-2z^{-2}) + (3z^{-1}).(3-2z^{-2})'}{(3-2z^{-2})^2}\right]$

$= -z\left[\frac{(-3z^{-2})\;.(3-2z^{-2}) + (3z^{-1}).(4z^{-3})}{(3-2z^{-2})^2}\right]$

$= \frac{(9z^{-1}-6z^{-3}) - 12z^{-3}}{(3-2z^{-2})^2} = \frac{9z^{-1}-18z^{-3}}{(3-2z^{-2})^2} = \frac{9(z^{-1}-2z^{-3})}{(9-12z^{-2}+4z^{-4})}$ (**)

Thay **(**)** vào **(1)**, ta có:

$Z[nx_2(n)] = -z\,(\frac{9(z^{-1}-2z^{-3})}{9-12z^{-2}+4z^{-4}})' = 9.-z\left[\frac{(z^{-1}-2z^{-3})'.(9-12z^{-2}+4z^{-4}) + (z^{-1}-2z^{-3}).(9-12z^{-2}+4z^{-4})'}{(9-12\;\;^{-2}+4z^{-4})^2}\right]$

$= 9\left[\frac{-9z^{-1}+18\;\;^{-3}+132z^{-5}-18z^{-7}}{(9-12\;\;^{-2}+4z^{-4})^2}\right]$

⇨ $Z[x(n)] = \frac{3}{2}.\,9\left[\frac{-9z^{-1}+18z^{-3}+132z^{-5}-18\;\;^{-7}}{(9-12z^{-2}+4z^{-4})^2}\right] = \boldsymbol{\frac{27}{2}.\left[\frac{-9z^{-1}+18z^{-3}+132z^{-5}-18z^{-7}}{(9-12z^{-2}+4z^{-4})^2}\right]}$

**5.** $\boldsymbol{x(n) = (n-3)(\frac{1}{4})^{n-2}\cos(\frac{(n-1)\pi}{2})u(n)}$

$= 4^2(n-3).(\frac{1}{4})^n \cos[-\left(\frac{\pi}{2}-\frac{n\pi}{2}\right)]u(n)$

$= 16(n-3)\,(\frac{1}{4})^n \sin(\frac{n\pi}{2})u(n)$

$= 16[n(\frac{1}{4})^n \sin(\frac{n\pi}{2})u(n) - 3.\,(\frac{1}{4})^n \sin(\frac{n\pi}{2})u(n)]$

Đặt

$x_{1(n)} = (\frac{1}{4})^n \sin(\frac{n\pi}{2})u(n)$

=> $X[x_{(n)}] = 16.X[n.x_{1(n)} - 3.x_{1(n)}] = 16.[\{-z.\frac{dZ[x1(n)]}{dz}\} - 3.\,Z[x1(n)]$ *(*)*

Biến đổi Z cho $x_{1(n)}$

$X_1(z) = Z[x1(n)] = Z[(\frac{1}{4})^n \sin(\frac{n\pi}{2})u(n)] = \frac{(\frac{1}{4})\;\;\sin(\frac{\pi}{2})z^{-1}}{1-2.(\frac{1}{4}).\cos\left(\frac{\pi}{2}\right)z^{-1}+(\frac{1}{4})^2z^{-2}}$

$= \dfrac{(\frac{1}{4})\quad z^{-1}}{1-(\frac{z^{-2}}{16})} = \dfrac{4z^{-1}}{16\quad z^{-2}}$ *(1)*

⇨ $\dfrac{dZ[\text{x1(n)}]}{dz} = (\dfrac{4z^{-1}}{16-z^{-2}})' = \dfrac{(4z^{-1})'.(16-z^{-2})+(4z^{-1}).(16-z^{-2})'}{(16-z^{-2})^2}$

$= \dfrac{(-4z^{-2})\quad .(16-z^{-2})+(4z^{-1}).(2.z^{-3})}{(16-z^{-2})^2}$

$= \dfrac{-64z^{-2}+12z^{-4}}{(16-z^{-2})^2}$ *(2)*

Thay (1) , (2) vào (*), ta có:

X(z) = Z[x(n)] = 16. [-z$\dfrac{-64z^{-2}+12\quad^{-4}}{(16\quad z^{-2})^2}$ – 3. $\dfrac{4z^{-1}}{16\quad z^{-2}}$]

= 16. [$\dfrac{64z^{-1}-12\quad^{-3}}{(16\quad z^{-2})^2}$ – $\dfrac{12\quad^{-1}(16-z^{-2})}{(16-z^{-2})^2}$]

= 16. [$\dfrac{64z^{-1}-12\quad^{-3}-192z^{-1}+12z^{-3}}{(16\quad z^{-2})^2}$]

$= \dfrac{\mathbf{-2048z^{-1}}}{\mathbf{(16-z^{-2})^2}}$