


Lecturer: Phan Thi Khanh Van	Date: ...	Approved by: Nguyen Tien Dung	Date ..
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 University of Technology Fuculty of AS	MIDTERM	Semester/A.year		I	2023-2024
		Date	11/03/2024		
	Course title	Linear Algebra - No 1			
	Course ID	MT1007			
	Duration	50 minus	Q.sheet code	1101	
Notes: - There are 20 questions/4 pages. - <i>This is a closed book exam.</i> - <i>For each wrong answer of a multiple-choice question, students will have a penalty of one-fifth of the score for that question. If students do not choose any answer, no penalty will be applied.</i>					

EXAM ĐỀ THI

(Question 1 through 4)

Let $A = \begin{bmatrix} 0 & 1 & 2 & 1 \\ 1 & 2 & 1 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & m \\ 0 & 2 \end{bmatrix}$, where $m \in \mathbb{R}$.

Question 1 (L.O.1, L.O.2). Which of the following statements is CORRECT?

- A. None of the others. B. $A + B = \begin{bmatrix} 2 & m+1 \\ 1 & 4 \end{bmatrix}$. C. $AB = \begin{bmatrix} 0 & 2 \\ 2 & m+4 \\ 4 & 2m+2 \\ 2 & m+4 \end{bmatrix}$.
- D. $BA = \begin{bmatrix} m & 2m+2 & m+4 & 2m+2 \\ 2 & 4 & 2 & 4 \end{bmatrix}$. E. BA does not exist.

Question 2 (L.O.1, L.O.2). Let $f(x) = 2x^2 - 3x + 5$ be a polynomial. Find $f(B)$.

- A. $\begin{bmatrix} 7 & 5m \\ 0 & 7 \end{bmatrix}$. B. None of the others. C. $f(B)$ does not exist.
- D. $\begin{bmatrix} 7 & 5m+5 \\ 5 & 7 \end{bmatrix}$. E. $\begin{bmatrix} 7 & 8m \\ -3m & 7 \end{bmatrix}$.

Question 3 (L.O.1, L.O.2). $m = 0$. Find the matrix X such that $BX = A - 2X$.

- A. X does not exist. B. $X = \begin{bmatrix} 0 & \frac{5}{4} & \frac{5}{2} & \frac{5}{4} \\ \frac{5}{4} & \frac{5}{2} & \frac{5}{4} & \frac{5}{2} \end{bmatrix}$. C. $X = \begin{bmatrix} 0 & \frac{1}{4} & \frac{1}{2} & \frac{1}{4} \\ \frac{1}{4} & \frac{1}{2} & \frac{1}{4} & \frac{1}{2} \end{bmatrix}$.
- D. None of the others. E. $X = \begin{bmatrix} 0 & \frac{1}{4} \\ \frac{1}{4} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{4} \\ \frac{1}{4} & \frac{1}{2} \end{bmatrix}$.

Question 4 (L.O.1, L.O.2). Let $m = 1$ and C be a 2×2 matrix with the determinant 3. Evaluate $\det(2B \cdot (3C)^{-1})$.

- A. $\frac{8}{3}$. B. $\frac{16}{27}$. C. $\frac{8}{27}$.
- D. $\frac{16}{3}$. E. None of the others.

(Question 5 through 7)

Let $A = \begin{bmatrix} 1 & 2 & -1 & -1 \\ 2 & 1 & 1 & 1 \\ 1 & 5 & -4 & -4 \\ -3 & 0 & -3 & m \end{bmatrix}$ and $B = \begin{bmatrix} -3 \\ 4 \\ -13 \\ -12 \end{bmatrix}$ be two matrices.

Question 5 (L.O.1, L.O.2). Find all values of m such that the rank of A is 2.

- A. $m = -3$. B. m does not exist. C. $m = 3$.
D. $m = 1$. E. None of the others.

Question 6 (L.O.1, L.O.2). Find all real values of m such that the linear system $AX = B$ has no solution.

- A. None of the others. B. $m = 3$. C. $\forall m \in \mathbb{R}$.
D. $\nexists m$. E. $m = -3$.

Question 7 (L.O.1, L.O.2). Let $m = 1$. In the vector space \mathbb{R}_4 , let V be a subspace, which is the set of all solutions of the homogeneous linear system $AX = 0$. Find one basis of V .

- A. $\{(1,2,-2,0),(0,-3,3,3),(0,0,0,4)\}$. B. V does not have any basis.
C. None of the others. D. $\{(-3,3,3,0)\}$. E. $\{(0,-1,1,-2)\}$.

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(Question 8 through 9)

Consider a population, which is divided into 3 age classes: Class I: from 0 to 6 months, Class II: from 6 to 12 months and Class III: older than 12 months. Suppose that after each 6 months, the average numbers of offsprings that each individual in Class I, Class II and Class III produces are 1, 4 and 4, respectively. The survival rates of Class I, Class II and Class III after each 6 months are 70%, 90% and 80%, respectively. Given that at the beginning, there are only 100 individuals in Class I (there is no individual in Class II and Class III).

.....
Question 8 (L.O.1, L.O.2). Find the Leslie matrix.

- A. None of the others. B. $\begin{bmatrix} 4 & 5 & 6 \\ 0.8 & 0 & 0.7 \\ 0 & 0.9 & 0.1 \end{bmatrix}$. C. $\begin{bmatrix} 1 & 4 & 4 \\ 0.7 & 0 & 0 \\ 0 & 0.9 & 0.8 \end{bmatrix}$.
D. $\begin{bmatrix} 0 & 2 & 5 \\ 0.8 & 0 & 0 \\ 0 & 0.9 & 1 \end{bmatrix}$. E. $\begin{bmatrix} 0 & 1 & 4 \\ 0.8 & 0 & 0 \\ 0 & 0.9 & 0.7 \end{bmatrix}$.

Question 9 (L.O.1, L.O.2). After 2 years, how many individuals are there in Class III? (Round the answer to the nearest integer).

- A. 3398. B. 2430. C. 330.
D. None of the others. E. 638.

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(Question 10 to Question 12)

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Consider an economy with three industries: I, II and III with the input-output matrix $A = \begin{bmatrix} 0.13 & 0.01 & 0.08 \\ 0.025 & 0.18 & 0.075 \\ 0.12 & 0.08 & 0.09 \end{bmatrix}$. In 2023, the total productions of the three industries are 4, 5 and 6 billions of dollars, respectively.

.....
Question 10 (L.O.1, L.O.2). What does the value 0.08 in the row 1, column 3 of A mean?

- A. The industry III provides 8% of its total production to the industry I.
B. None of the others.
C. The industry I provides 8% of its total production to the industry III.
D. In order to produce \$1 production value in the industry III, we need \$0.08 from the industry I.
E. In order to produce \$1 production value in the industry I, we need \$0.08 from the industry III.

Question 11 (L.O.1, L.O.2). In 2023, what is the external demand of the industry II? (Round the answer to 3 decimal places).

- A. 6.987. B. None of the others. C. 4.580.
D. 3.550. E. 5.406.

Question 12 (L.O.1, L.O.2). In 2023, what is the input demand value (in billions of dollars of the industry I(the sum of production values that all three industries have provided to the industry I)?

- A. 1.365. B. None of the others. C. 1.05.
D. 1.1. E. 0.88.

Question 13 (L.O.1, L.O.2). In the vector space \mathbb{R}_3 , let m be a real number and $M = \{(1, 2, -1), (1, 1, -1), (-1, 3, m)\}$ be a vector set. Find all values of m such that M is a basis of \mathbb{R}_3 .

- A. $m \neq 3$. B. $m \neq 1$. C. $m \neq 2$.
D. $m \neq 0$. E. None of the others.

Question 14 (L.O.1, L.O.2). In the vector space $M_2(R)$, let $M = \left\{ \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}, \begin{bmatrix} 2 & 1 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix} \right\}$ be a vector set. Find all real values of m such that $X = \begin{bmatrix} -3 & -2 \\ -1 & m \end{bmatrix}$ is a linear combination of M .

- A. m does not exist. B. $m = 1$. C. $m = 0$.
D. None of the others. E. $m = -1$.

Question 15 (L.O.1, L.O.2). In the vector space \mathbb{R}_4 , let $F = \text{span}\{(1, 1, -3, 2), (2, 1, -1, 2), (-4, -1, -3, -2)\}$ be a subspace. Find one basis of F .

- A. None of the others.
B. $\{(1, 1, -3, 2), (0, -1, 5, -2), (0, 0, 0, 0)\}$.
C. $\{(1, 1, -3, 2), (2, 1, -1, 2), (-4, -1, -3, -2)\}$.
D. F does not have any basis.
E. $\{(1, 1, -3, 2), (2, 1, -1, 2)\}$.

Question 16 (L.O.1, L.O.2). Let V be a vector space with a spanning set $\{x, y, z\}$. Given that $\dim(V) = 2$. Which of the following statement is CORRECT?

- A. z is a linear combination of $\{x, y\}$.
B. $\text{Rank}(\{x, y\}) = 2$.
C. None of the others.
D. $\{x, y, z\}$ is a basis of V .
E. $\{x, y, z\}$ is linearly independent.

(Question 17 through 18)

In the vector space \mathbb{R}_3 , let $E = \{(1, 1, 2), (2, 1, -1), (4, 1, -6)\}$
and $F = \{(2, 1, 1), (3, 2, 2), (1, 1, 2)\}$ be two bases.

Question 17 (L.O.1, L.O.2). Find the transition matrix (change of basis matrix) from E to F.

- A. $P_{F \leftarrow E} = \begin{bmatrix} -11 & -8 & -5 \\ 21 & 15 & 10 \\ -4 & -3 & -2 \end{bmatrix}$.
B. $P_{F \leftarrow E} = \begin{bmatrix} 4 & 3 & 1 \\ -5 & -2 & 0 \\ 2 & 1 & 0 \end{bmatrix}$.
C. None of the others.
D. $P_{F \leftarrow E} = \begin{bmatrix} 0 & -1 & -2 \\ 0 & 2 & 5 \\ 1 & -2 & -7 \end{bmatrix}$.
E. $P_{F \leftarrow E} = \begin{bmatrix} 0 & 1 & 5 \\ -2 & -2 & -5 \\ 3 & 1 & -3 \end{bmatrix}$.

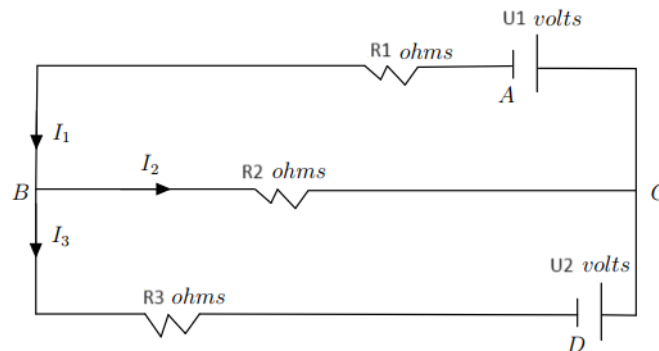
Question 18 (L.O.1, L.O.2). Let $u \in \mathbb{R}_3$ be a vector, whose $[u]_E = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$. Find $[u]_F$.

- A. $[u]_F = \begin{bmatrix} -8 \\ 19 \\ -24 \end{bmatrix}$. B. $[u]_F = \begin{bmatrix} 17 \\ -21 \\ -4 \end{bmatrix}$. C. $[u]_F = \begin{bmatrix} 13 \\ -9 \\ 4 \end{bmatrix}$.
- D. $[u]_F = \begin{bmatrix} -42 \\ 81 \\ -16 \end{bmatrix}$. E. None of the others.

Question 19 (L.O.1, L.O.2). Let $A = \begin{bmatrix} 1 & 2 & -3 & 1 \\ 2 & 1 & -1 & 3 \\ 3 & 1 & -4 & -2 \\ x & x^2 & x^3 & x^4 \end{bmatrix}$ be a matrix and $f(x) = \det(A)$ be a polynomial. The coefficient of the third degree term for $f(x)$ is

- A. None of the others. B. 20. C. -25.
- D. -13. E. -20.

Question 20 (L.O.1, L.O.2). A circuit is given in the following figure




Given that the resistances are $R_1 = 4(\Omega)$, $R_2 = 3(\Omega)$, $R_3 = 4(\Omega)$, and the voltage sources are $U_1 = U_2 = 100(V)$. Find the absolute value of the current flowing through R_2 .

- A. 18. B. 21. C. None of the others.
- D. 20. E. 10.

===== The end =====

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	Course ID	MT1007			
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Notes: - There are 20 questions/4 pages. - <i>This is a closed book exam.</i> - <i>For each wrong answer of a multiple-choice question, students will have a penalty of one-fifth of the score for that question. If students do not choose any answer, no penalty will be applied.</i>					

EXAM ĐỀ THI

(Question 1 through 4)

Let $A = \begin{bmatrix} -1 & 1 & 1 & 1 \\ 0 & 2 & 1 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & m \\ -1 & 1 \end{bmatrix}$, where $m \in \mathbb{R}$.

Question 1 (L.O.1, L.O.2). Which of the following statements is CORRECT?

- A. $A + B = \begin{bmatrix} 1 & m+1 \\ -1 & 3 \end{bmatrix}$. B. $BA = \begin{bmatrix} -2 & 2m+2 & m+2 & m+2 \\ 1 & 1 & 0 & 0 \end{bmatrix}$.
- C. $AB = \begin{bmatrix} -2 & -m \\ 0 & m+2 \\ 1 & m+1 \\ 1 & m+1 \end{bmatrix}$. D. None of the others. E. BA does not exist.

Question 2 (L.O.1, L.O.2). Let $f(x) = 2x^2 - 3x + 5$ be a polynomial. Find $f(B)$.

- A. $\begin{bmatrix} 7-2m & 3m \\ -3 & 4-2m \end{bmatrix}$. B. $\begin{bmatrix} 7-2m & 3m+5 \\ 2 & 4-2m \end{bmatrix}$. C. None of the others.
- D. $\begin{bmatrix} 7-2m & 6m+3 \\ -3m-6 & 4-2m \end{bmatrix}$. E. $f(B)$ does not exist.

Question 3 (L.O.1, L.O.2). $m = 0$. Find the matrix X such that $BX = A - 2X$.

- A. $X = \begin{bmatrix} -\frac{5}{4} & \frac{5}{4} & \frac{5}{4} & \frac{5}{4} \\ -\frac{1}{12} & \frac{11}{4} & \frac{17}{12} & \frac{17}{12} \end{bmatrix}$. B. $X = \begin{bmatrix} -\frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \\ -\frac{1}{12} & \frac{3}{4} & \frac{5}{12} & \frac{5}{12} \end{bmatrix}$.
- C. None of the others. D. $X = \begin{bmatrix} -\frac{1}{4} & 0 \\ \frac{5}{12} & \frac{2}{3} \\ \frac{1}{3} & \frac{1}{3} \\ \frac{1}{3} & \frac{1}{3} \end{bmatrix}$. E. X does not exist.

Question 4 (L.O.1, L.O.2). Let $m = 1$ and C be a 2×2 matrix with the determinant 3. Evaluate $\det(2B \cdot (3C)^{-1})$.

- A. $\frac{2}{9}$. B. 2. C. None of the others.
- D. 4. E. $\frac{4}{9}$.

(Question 5 through 7)

Let $A = \begin{bmatrix} 1 & 2 & 4 & -1 \\ 2 & 1 & 1 & 4 \\ 1 & 5 & 11 & -7 \\ -3 & 0 & 2 & m \end{bmatrix}$ and $B = \begin{bmatrix} 7 \\ 7 \\ 14 \\ -8 \end{bmatrix}$ be two matrices.

Question 5 (L.O.1, L.O.2). Find all values of m such that the rank of A is 2.

- A. $m = 4$. B. m does not exist. C. None of the others.
D. $m = -9$. E. $m = 15$.

Question 6 (L.O.1, L.O.2). Find all real values of m such that the linear system $AX = B$ has no solution.

- A. $m = -9$. B. $\forall m \in \mathbb{R}$. C. None of the others.
D. $\nexists m$. E. $m = 9$.

Question 7 (L.O.1, L.O.2). Let $m = 1$. In the vector space \mathbb{R}_4 , let V be a subspace, which is the set of all solutions of the homogeneous linear system $AX = 0$. Find one basis of V .

- A. $\{(1,2,3,0), (0,-3,-7,6), (0,0,0,10)\}$. B. V does not have any basis.
C. None of the others. D. $\{(5,4,1,-2)\}$. E. $\{(2,-7,3,0)\}$.

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(Question 8 through 9)

Consider a population, which is divided into 3 age classes: Class I: from 0 to 6 months, Class II: from 6 to 12 months and Class III: older than 12 months. Suppose that after each 6 months, the average numbers of offsprings that each individual in Class I, Class II and Class III produces are 1, 4 and 3, respectively. The survival rates of Class I, Class II and Class III after each 6 months are 70%, 90% and 80%, respectively. Given that at the beginning, there are only 100 individuals in Class I (there is no individual in Class II and Class III).

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Question 8 (L.O.1, L.O.2). Find the Leslie matrix.

- A. $\begin{bmatrix} 0 & 2 & 4 \\ 0.8 & 0 & 0 \\ 0 & 0.9 & 1 \end{bmatrix}$. B. $\begin{bmatrix} 0 & 1 & 3 \\ 0.8 & 0 & 0 \\ 0 & 0.9 & 0.7 \end{bmatrix}$. C. None of the others.
D. $\begin{bmatrix} 1 & 4 & 3 \\ 0.7 & 0 & 0 \\ 0 & 0.9 & 0.8 \end{bmatrix}$. E. $\begin{bmatrix} 4 & 5 & 5 \\ 0.8 & 0 & 0.7 \\ 0 & 0.9 & 0.1 \end{bmatrix}$.

Question 9 (L.O.1, L.O.2). After 2 years, how many individuals are there in Class III? (Round the answer to the nearest integer).

- A. 594. B. 3178. C. 330.
D. None of the others. E. 2253.

.....
(Question 10 to Question 12)

.....
Consider an economy with three industries: I, II and III with the input-output matrix

$A = \begin{bmatrix} 0.13 & 0.01 & 0.08 \\ 0.05 & 0.18 & 0.025 \\ 0.12 & 0.08 & 0.09 \end{bmatrix}$. In 2023, the total productions of the three industries are 4, 5 and 6 billions of dollars, respectively.

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Question 10 (L.O.1, L.O.2). What does the value 0.08 in the row 1, column 3 of A mean?

- A. The industry III provides 8% of its total production to the industry I.
B. None of the others.
C. In order to produce \$1 production value in the industry III, we need \$0.08 from the industry I.
D. The industry I provides 8% of its total production to the industry III.
E. In order to produce \$1 production value in the industry I, we need \$0.08 from the industry III.

Question 11 (L.O.1, L.O.2). In 2023, what is the external demand of the industry II? (Round the answer to 3 decimal places).

- A. 4.580. B. 5.400. C. 6.667.
D. 3.750. E. None of the others.

Question 12 (L.O.1, L.O.2). In 2023, what is the input demand value (in billions of dollars of the industry I(the sum of producion values that all three industries have provided to the industry I)?

- A. None of the others. B. 0.88. C. 1.05.
D. 1.2. E. 1.49.

Question 13 (L.O.1, L.O.2). In the vector space \mathbb{R}_3 , let m be a real number and $M = \{(1, 2, -1), (1, 1, 3), (-1, 3, m)\}$ be a vector set. Find all values of m such that M is a basis of \mathbb{R}_3 .

- A. $m \neq -19$. B. $m \neq -18$. C. $m \neq -20$.
D. $m \neq -17$. E. None of the others.

Question 14 (L.O.1, L.O.2). In the vector space $M_2(R)$, let $M = \left\{ \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}, \begin{bmatrix} 2 & 1 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix} \right\}$ be a vector set. Find all real values of m such that $X = \begin{bmatrix} -3 & -2 \\ -1 & m \end{bmatrix}$ is a linear combination of M .

- A. $m = 1$. B. m does not exist. C. $m = 0$.
D. None of the others. E. $m = -1$.

Question 15 (L.O.1, L.O.2). In the vector space \mathbb{R}_4 , let $F = \text{span}\{(1, 1, 1, 2), (2, 1, -1, 2), (-4, -1, 5, -2)\}$ be a subspace. Find one basis of F .

- A. None of the others.
B. $\{(1, 1, 1, 2), (2, 1, -1, 2), (-4, -1, 5, -2)\}$.
C. $\{(1, 1, 1, 2), (0, -1, -3, -2), (0, 0, 0, 0)\}$.
D. F does not have any basis.
E. $\{(1, 1, 1, 2), (2, 1, -1, 2)\}$.

Question 16 (L.O.1, L.O.2). Let V be a vector space with a spanning set $\{x, y, z\}$. Given that $\dim(V) = 2$. Which of the following statement is CORRECT?

- A. $\text{Rank}(\{x, y\}) = 2$.
B. $\{x, y, z\}$ is a basis of V .
C. z is a linear combination of $\{x, y\}$.
D. $\{x, y, z\}$ is linearly independent.
E. None of the others.

(Question 17 through 18)

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In the vector space \mathbb{R}_3 , let $E = \{(1, 1, 2), (2, 1, -1), (4, 1, -6)\}$
and $F = \{(2, 1, 1), (3, 2, 2), (0, 1, 2)\}$ be two bases.
.....

Question 17 (L.O.1, L.O.2). Find the transition matrix (change of basis matrix) from E to F.

- A. None of the others.
B. $P_{F \leftarrow E} = \begin{bmatrix} -14 & -8 & -5 \\ 26 & 15 & 10 \\ -5 & -3 & -2 \end{bmatrix}$.
C. $P_{F \leftarrow E} = \begin{bmatrix} 4 & 3 & -4 \\ -5 & -2 & 8 \\ 2 & 1 & -3 \end{bmatrix}$.
D. $P_{F \leftarrow E} = \begin{bmatrix} 2 & -5 & -16 \\ -1 & 4 & 12 \\ 1 & -2 & -7 \end{bmatrix}$.
E. $P_{F \leftarrow E} = \begin{bmatrix} 0 & 1 & 5 \\ -2 & -3 & -10 \\ 3 & 2 & 2 \end{bmatrix}$.

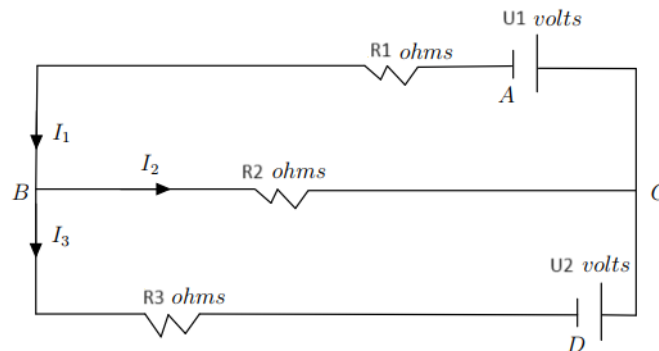
Question 18 (L.O.1, L.O.2). Let $u \in \mathbb{R}_3$ be a vector, whose $[u]_E = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$. Find $[u]_F$.

- A. $[u]_F = \begin{bmatrix} -56 \\ 43 \\ -24 \end{bmatrix}$. B. None of the others. C. $[u]_F = \begin{bmatrix} -2 \\ 15 \\ -5 \end{bmatrix}$.
- D. $[u]_F = \begin{bmatrix} -45 \\ 86 \\ -17 \end{bmatrix}$. E. $[u]_F = \begin{bmatrix} 17 \\ -38 \\ 13 \end{bmatrix}$.

Question 19 (L.O.1, L.O.2). Let $A = \begin{bmatrix} 1 & 2 & 1 & 1 \\ 2 & 1 & -1 & 3 \\ 3 & 1 & 0 & -2 \\ x & x^2 & x^3 & x^4 \end{bmatrix}$ be a matrix and $f(x) = \det(A)$ be a polynomial. The coefficient of the third degree term for $f(x)$ is

- A. 20. B. -13. C. None of the others.
D. -25. E. -20.

Question 20 (L.O.1, L.O.2). A circuit is given in the following figure




Given that the resistances are $R_1 = 4(\Omega)$, $R_2 = 3(\Omega)$, $R_3 = 5(\Omega)$, and the voltage sources are $U_1 = U_2 = 100(V)$. Find the absolute value of the current flowing through R_2 .

- A. $\frac{900}{47}$. B. None of the others. C. $\frac{500}{47}$.
D. $\frac{400}{47}$. E. $\frac{947}{47}$.

===== The end =====

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EXAM ĐỀ THI

(Question 1 through 4)

Let $A = \begin{bmatrix} 0 & 1 & -1 & 1 \\ 1 & 2 & 1 & -1 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & m \\ 0 & -1 \end{bmatrix}$, where $m \in \mathbb{R}$.

Question 1 (L.O.1, L.O.2). Which of the following statements is CORRECT?

- A.** $BA = \begin{bmatrix} m & 2m+2 & m-2 & 2-m \\ -1 & -2 & -1 & 1 \end{bmatrix}$. **B.** $AB = \begin{bmatrix} 0 & -1 \\ 2 & m-2 \\ -2 & -m-1 \\ 2 & m+1 \end{bmatrix}$.
C. $A+B = \begin{bmatrix} 2 & m+1 \\ 1 & 1 \end{bmatrix}$. **D.** BA does not exist. **E.** None of the others.

Question 2 (L.O.1, L.O.2). Let $f(x) = 2x^2 - 3x + 5$ be a polynomial. Find $f(B)$.

- A.** $f(B)$ does not exist. **B.** $\begin{bmatrix} 7 & 5-m \\ 5 & 10 \end{bmatrix}$. **C.** $\begin{bmatrix} 7 & -m \\ 0 & 10 \end{bmatrix}$.
D. $\begin{bmatrix} 7 & 2m \\ -3m & 10 \end{bmatrix}$. **E.** None of the others.

Question 3 (L.O.1, L.O.2). $m = 0$. Find the matrix X such that $BX = A - 2X$.

- A.** X does not exist. **B.** $X = \begin{bmatrix} 0 & \frac{1}{4} & -\frac{1}{4} & \frac{1}{4} \\ 1 & 2 & 1 & -1 \end{bmatrix}$. **C.** $X = \begin{bmatrix} 0 & \frac{5}{4} & -\frac{5}{4} & \frac{5}{4} \\ 2 & 4 & 2 & -2 \end{bmatrix}$.
D. $X = \begin{bmatrix} 0 & 1 \\ \frac{1}{4} & 2 \\ -\frac{1}{4} & 1 \\ \frac{1}{4} & -1 \end{bmatrix}$. **E.** None of the others.

Question 4 (L.O.1, L.O.2). Let $m = 1$ and C be a 2×2 matrix with the determinant 3. Evaluate $\det(2B \cdot (3C)^{-1})$.

- A.** $-\frac{8}{3}$. **B.** $-\frac{4}{27}$. **C.** None of the others.
D. $-\frac{8}{27}$. **E.** $-\frac{4}{3}$.

(Question 5 through 7)

Let $A = \begin{bmatrix} 1 & 2 & -2 & -1 \\ 2 & 1 & 1 & 2 \\ 1 & 5 & -7 & -5 \\ -3 & 0 & -4 & m \end{bmatrix}$ and $B = \begin{bmatrix} -5 \\ 5 \\ -20 \\ -16 \end{bmatrix}$ be two matrices.

Question 5 (L.O.1, L.O.2). Find all values of m such that the rank of A is 2.

- A. None of the others. B. $m = 2$. C. m does not exist.
D. $m = -5$. E. $m = 7$.

Question 6 (L.O.1, L.O.2). Find all real values of m such that the linear system $AX = B$ has no solution.

- A. None of the others. B. $m = -5$. C. $\nexists m$.
D. $m = 5$. E. $\forall m \in \mathbb{R}$.

Question 7 (L.O.1, L.O.2). Let $m = 1$. In the vector space \mathbb{R}_4 , let V be a subspace, which is the set of all solutions of the homogeneous linear system $AX = 0$. Find one basis of V .

- A. $\{(1,2,-3,0),(0,-3,5,4),(0,0,0,6)\}$.
B. $\{(-4,5,3,0)\}$.
C. None of the others.
D. V does not have any basis.
E. $\{(-1,-2,1,-2)\}$.

.....
(Question 8 through 9)

Consider a population, which is divided into 3 age classes: Class I: from 0 to 6 months, Class II: from 6 to 12 months and Class III: older than 12 months. Suppose that after each 6 months, the average numbers of offsprings that each individual in Class I, Class II and Class III produces are 1, 4 and 3, respectively. The survival rates of Class I, Class II and Class III after each 6 months are 70%, 90% and 80%, respectively. Given that at the beginning, there are only 100 individuals in Class I (there is no individual in Class II and Class III).

.....
Question 8 (L.O.1, L.O.2). Find the Leslie matrix.

- A. None of the others. B. $\begin{bmatrix} 0 & 1 & 3 \\ 0.8 & 0 & 0 \\ 0 & 0.9 & 0.7 \end{bmatrix}$. C. $\begin{bmatrix} 4 & 5 & 5 \\ 0.8 & 0 & 0.7 \\ 0 & 0.9 & 0.1 \end{bmatrix}$.
D. $\begin{bmatrix} 0 & 2 & 4 \\ 0.8 & 0 & 0 \\ 0 & 0.9 & 1 \end{bmatrix}$. E. $\begin{bmatrix} 1 & 4 & 3 \\ 0.7 & 0 & 0 \\ 0 & 0.9 & 0.8 \end{bmatrix}$.

Question 9 (L.O.1, L.O.2). After 2 years, how many individuals are there in Class III? (Round the answer to the nearest integer).

- A. 594. B. 2253. C. 330.
D. None of the others. E. 3178.

(Question 10 to Question 12)

.....
Consider an economy with three industries: I, II and III with the input-output matrix

$A = \begin{bmatrix} 0.13 & 0.01 & 0.08 \\ 0.025 & 0.18 & 0.075 \\ 0.12 & 0.08 & 0.09 \end{bmatrix}$. In 2023, the total productions of the three industries are 4, 5 and 6 billions of dollars, respectively.

.....
Question 10 (L.O.1, L.O.2). What does the value 0.08 in the row 1, column 3 of A mean?

- A. None of the others.
B. The industry III provides 8% of its total production to the industry I.
C. The industry I provides 8% of its total production to the industry III.
D. In order to produce \$1 production value in the industry III, we need \$0.08 from the industry I.
E. In order to produce \$1 production value in the industry I, we need \$0.08 from the industry III.

Question 11 (L.O.1, L.O.2). In 2023, what is the external demand of the industry II? (Round the answer to 3 decimal places).

- A. 6.987. B. 3.550. C. 5.406.
D. None of the others. E. 4.580.

Question 12 (L.O.1, L.O.2). In 2023, what is the input demand value (in billions of dollars of the industry I(the sum of production values that all three industries have provided to the industry I)?

- A. 0.88. B. 1.365. C. None of the others.
D. 1.1. E. 1.05.

Question 13 (L.O.1, L.O.2). In the vector space \mathbb{R}_3 , let m be a real number and $M = \{(1, 2, -1), (1, 1, -1), (-1, 3, m)\}$ be a vector set. Find all values of m such that M is a basis of \mathbb{R}_3 .

- A. $m \neq 0$. B. $m \neq 3$. C. $m \neq 2$.
D. $m \neq 1$. E. None of the others.

Question 14 (L.O.1, L.O.2). In the vector space $M_2(R)$, let $M = \left\{ \begin{bmatrix} 1 & -3 \\ 1 & 1 \end{bmatrix}, \begin{bmatrix} 2 & 1 \\ 1 & -3 \end{bmatrix}, \begin{bmatrix} 1 & -10 \\ 2 & 6 \end{bmatrix} \right\}$ be a vector set. Find all real values of m such that $X = \begin{bmatrix} -3 & -5 \\ -1 & m \end{bmatrix}$ is a linear combination of M .

- A. m does not exist. B. $m = 7$. C. $m = 5$.
D. $m = 6$. E. None of the others.

Question 15 (L.O.1, L.O.2). In the vector space \mathbb{R}_4 , let $F = \text{span}\{(1, 1, 1, 2), (2, 1, -1, 2), (-4, -1, 5, -2)\}$ be a subspace. Find one basis of F .

- A. F does not have any basis.
B. None of the others.
C. $\{(1, 1, 1, 2), (0, -1, -3, -2), (0, 0, 0, 0)\}$.
D. $\{(1, 1, 1, 2), (2, 1, -1, 2)\}$.
E. $\{(1, 1, 1, 2), (2, 1, -1, 2), (-4, -1, 5, -2)\}$.

Question 16 (L.O.1, L.O.2). Let V be a vector space with a spanning set $\{x, y, z\}$. Given that $\dim(V) = 2$. Which of the following statement is CORRECT?

- A. $\{x, y, z\}$ is linearly independent.
B. None of the others.
C. z is a linear combination of $\{x, y\}$.
D. $\text{Rank}(\{x, y\}) = 2$.
E. $\{x, y, z\}$ is a basis of V .

(Question 17 through 18)

In the vector space \mathbb{R}_3 , let $E = \{(1, 1, 2), (2, 1, -1), (4, 1, -6)\}$
and $F = \{(2, 1, 1), (3, 2, 2), (-2, 1, 2)\}$ be two bases.

Question 17 (L.O.1, L.O.2). Find the transition matrix (change of basis matrix) from E to F.

- A. $P_{F \leftarrow E} = \begin{bmatrix} 0 & 1 & 5 \\ -2 & -5 & -20 \\ 3 & 4 & 12 \end{bmatrix}$.
B. None of the others.
C. $P_{F \leftarrow E} = \begin{bmatrix} -20 & -8 & -5 \\ 36 & 15 & 10 \\ -7 & -3 & -2 \end{bmatrix}$.

D. $P_{F \leftarrow E} = \begin{bmatrix} 6 & -13 & -44 \\ -3 & 8 & 26 \\ 1 & -2 & -7 \end{bmatrix}.$

E. $P_{F \leftarrow E} = \begin{bmatrix} 4 & 3 & -14 \\ -5 & -2 & 24 \\ 2 & 1 & -9 \end{bmatrix}.$

Question 18 (L.O.1, L.O.2). Let $u \in \mathbb{R}_3$ be a vector, whose $[u]_E = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}.$ Find $[u]_F.$

A. $[u]_F = \begin{bmatrix} -152 \\ 91 \\ -24 \end{bmatrix}.$

B. None of the others.

C. $[u]_F = \begin{bmatrix} -32 \\ 63 \\ -23 \end{bmatrix}.$

D. $[u]_F = \begin{bmatrix} 17 \\ -72 \\ 47 \end{bmatrix}.$

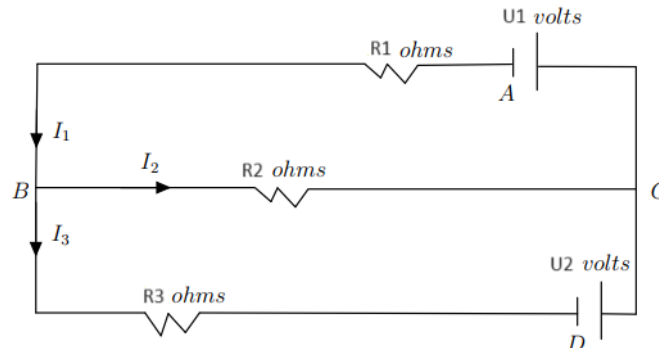
E. $[u]_F = \begin{bmatrix} -51 \\ 96 \\ -19 \end{bmatrix}.$

Question 19 (L.O.1, L.O.2). Let $A = \begin{bmatrix} 1 & 2 & 3 & 1 \\ 2 & 1 & -1 & 3 \\ 3 & 1 & 2 & -2 \\ x & x^2 & x^3 & x^4 \end{bmatrix}$ be a matrix and $f(x) = \det(A)$ be a

polynomial. The coefficient of the third degree term for $f(x)$ is

- A. -25. B. None of the others. C. -20.
D. 20. E. -13.

Question 20 (L.O.1, L.O.2). A circuit is given in the following figure




Given that the resistances are $R_1 = 5(\Omega)$, $R_2 = 3(\Omega)$, $R_3 = 2(\Omega)$, and the voltage sources are $U_1 = U_2 = 100(V)$. Find the absolute value of the current flowing through R_2 .

- A. $\frac{731}{31}.$ B. $\frac{200}{31}.$ C. $\frac{700}{31}.$
D. None of the others. E. $\frac{500}{31}.$

===== The end =====

Lecturer: Phan Thi Khanh Van	Date: ...	Approved by: Nguyen Tien Dung	Date ..
.....

 University of Technology Fuculty of AS	MIDTERM	Semester/A.year		I	2023-2024
		Date	11/03/2024		
	Course title	Linear Algebra - No 1			
	Course ID	MT1007			
	Duration	50 minus	Q.sheet code	1104	
Notes: - There are 20 questions/4 pages. - <i>This is a closed book exam.</i> - <i>For each wrong answer of a multiple-choice question, students will have a penalty of one-fifth of the score for that question. If students do not choose any answer, no penalty will be applied.</i>					

EXAM ĐỀ THI

(Question 1 through 4)

Let $A = \begin{bmatrix} -3 & 1 & 4 & 1 \\ -2 & 2 & 1 & 4 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & m \\ -3 & 4 \end{bmatrix}$, where $m \in \mathbb{R}$.

Question 1 (L.O.1, L.O.2). Which of the following statements is CORRECT?

A. None of the others.

B. BA does not exist.

C. $BA = \begin{bmatrix} -2m-6 & 2m+2 & m+8 & 4m+2 \\ 1 & 5 & -8 & 13 \end{bmatrix}$.

D. $AB = \begin{bmatrix} 0 & -3m-8 \\ -4 & m+8 \\ 5 & 4m+4 \\ -10 & m+16 \end{bmatrix}$.

E. $A+B = \begin{bmatrix} -1 & m+1 \\ -5 & 6 \end{bmatrix}$.

Question 2 (L.O.1, L.O.2). Let $f(x) = 2x^2 - 3x + 5$ be a polynomial. Find $f(B)$.

A. $\begin{bmatrix} 7-6m & 9m \\ -27 & 25-6m \end{bmatrix}$.

B. $\begin{bmatrix} 7-6m & 9m+5 \\ -22 & 25-6m \end{bmatrix}$.

C. $\begin{bmatrix} 7-6m & 12m+9 \\ -3m-36 & 25-6m \end{bmatrix}$.

D. $f(B)$ does not exist.

E. None of the others.

Question 3 (L.O.1, L.O.2). $m = 0$. Find the matrix X such that $BX = A - 2X$.

A. $X = \begin{bmatrix} -1 & -\frac{1}{3} \\ \frac{1}{2} & \frac{1}{3} \\ \frac{9}{8} & \frac{1}{6} \\ \frac{3}{4} & \frac{2}{3} \end{bmatrix}$.

B. X does not exist.

C. $X = \begin{bmatrix} -\frac{15}{4} & \frac{5}{4} & 5 & \frac{5}{4} \\ -\frac{65}{24} & \frac{59}{24} & \frac{5}{3} & \frac{115}{24} \end{bmatrix}$.

D. $X = \begin{bmatrix} -\frac{3}{4} & \frac{1}{4} & 1 & \frac{1}{4} \\ -\frac{17}{24} & \frac{11}{24} & \frac{2}{3} & \frac{19}{24} \end{bmatrix}$.

E. None of the others.

Question 4 (L.O.1, L.O.2). Let $m = 1$ and C be a 2×2 matrix with the determinant 3. Evaluate $\det(2B \cdot (3C)^{-1})$.

A. $\frac{22}{27}$.

B. $\frac{44}{3}$.

C. $\frac{44}{27}$.

D. $\frac{22}{3}$.

E. None of the others.

(Question 5 through 7)

Let $A = \begin{bmatrix} 1 & 2 & -2 & -1 \\ 2 & 1 & 1 & 3 \\ 1 & 5 & -7 & -6 \\ -3 & 0 & -4 & m \end{bmatrix}$ and $B = \begin{bmatrix} -5 \\ 6 \\ -21 \\ -18 \end{bmatrix}$ be two matrices.

Question 5 (L.O.1, L.O.2). Find all values of m such that the rank of A is 2.

- A. $m = 3$. B. $m = -7$. C. None of the others.
D. m does not exist. E. $m = 11$.

Question 6 (L.O.1, L.O.2). Find all real values of m such that the linear system $AX = B$ has no solution.

- A. $\forall m \in \mathbb{R}$. B. $m = 7$. C. None of the others.
D. $\nexists m$. E. $m = -7$.

Question 7 (L.O.1, L.O.2). Let $m = 1$. In the vector space \mathbb{R}_4 , let V be a subspace, which is the set of all solutions of the homogeneous linear system $AX = 0$. Find one basis of V .

- A. $\{(-4, 5, 3, 0)\}$.
B. V does not have any basis.
C. $\{(-1, -2, 1, -2)\}$.
D. $\{(1, 2, -3, 0), (0, -3, 5, 5), (0, 0, 0, 8)\}$.
E. None of the others.

(Question 8 through 9)

Consider a population, which is divided into 3 age classes: Class I: from 0 to 6 months, Class II: from 6 to 12 months and Class III: older than 12 months. Suppose that after each 6 months, the average numbers of offsprings that each individual in Class I, Class II and Class III produces are 2, 5 and 3, respectively. The survival rates of Class I, Class II and Class III after each 6 months are 70%, 90% and 80%, respectively. Given that at the beginning, there are only 100 individuals in Class I (there is no individual in Class II and Class III).

Question 8 (L.O.1, L.O.2). Find the Leslie matrix.

- A. $\begin{bmatrix} 0 & 3 & 4 \\ 0.8 & 0 & 0 \\ 0 & 0.9 & 1 \end{bmatrix}$. B. $\begin{bmatrix} 2 & 5 & 3 \\ 0.7 & 0 & 0 \\ 0 & 0.9 & 0.8 \end{bmatrix}$. C. $\begin{bmatrix} 0 & 2 & 3 \\ 0.8 & 0 & 0 \\ 0 & 0.9 & 0.7 \end{bmatrix}$.
D. None of the others. E. $\begin{bmatrix} 5 & 6 & 5 \\ 0.8 & 0 & 0.7 \\ 0 & 0.9 & 0.1 \end{bmatrix}$.

Question 9 (L.O.1, L.O.2). After 2 years, how many individuals are there in Class III? (Round the answer to the nearest integer).

- A. None of the others. B. 1672. C. 614.
D. 10218. E. 7932.

(Question 10 to Question 12)

Consider an economy with three industries: I, II and III with the input-output matrix $A = \begin{bmatrix} 0.13 & 0.01 & 0.08 \\ 0.05 & 0.18 & 0.075 \\ 0.12 & 0.08 & 0.09 \end{bmatrix}$. In 2023, the total productions of the three industries are 4, 5 and 6 billions of dollars, respectively.

Question 10 (L.O.1, L.O.2). What does the value 0.08 in the row 1, column 3 of A mean?

- A. In order to produce \$1 production value in the industry I, we need \$0.08 from the industry III.
B. None of the others.

- E. The industry I provides 8% of its total production to the industry III.

Question 11 (L.O.1, L.O.2). In 2023, what is the external demand of the industry II? (Round the answer to 3 decimal places).

- A. 4.580. B. 7.153. C. 3.450.
D. 5.410. E. None of the others.

Question 12 (L.O.1, L.O.2). In 2023, what is the input demand value (in billions of dollars of the industry I(the sum of production values that all three industries have provided to the industry I)?

- A.** 0.88. **B.** None of the others. **C.** 1.49.
D. 1.05. **E.** 1.2.

Question 13 (L.O.1, L.O.2). In the vector space \mathbb{R}_3 , let m be a real number and $M = \{(1, 2, -1), (1, 1, 2), (-1, 3, m)\}$ be a vector set. Find all values of m such that M is a basis of \mathbb{R}_3 .

- A.** $m \neq -13$. **B.** $m \neq -14$. **C.** $m \neq -12$.
D. None of the others. **E.** $m \neq -15$.

Question 14 (L.O.1, L.O.2). In the vector space $M_2(R)$, let $M = \left\{ \begin{bmatrix} 1 & -2 \\ 1 & 1 \end{bmatrix}, \begin{bmatrix} 2 & 1 \\ 1 & -2 \end{bmatrix}, \begin{bmatrix} 1 & -7 \\ 2 & 5 \end{bmatrix} \right\}$ be a vector set. Find all real values of m such that $X = \begin{bmatrix} -3 & -4 \\ -1 & m \end{bmatrix}$ is a linear combination of M .

- A. $m = 5$. B. $m = 4$. C. None of the others.
D. m does not exist. E. $m = 3$.

Question 15 (L.O.1, L.O.2). In the vector space \mathbb{R}_4 , let $F = \text{span}\{(1, 1, 1, 2), (2, 1, -1, 2), (-4, -1, 5, -2)\}$ be a subspace. Find one basis of F .

- A. $\{(1, 1, 1, 2), (2, 1, -1, 2), (-4, -1, 5, -2)\}$.
 B. F does not have any basis.
 C. $\{(1, 1, 1, 2), (0, -1, -3, -2), (0, 0, 0, 0)\}$.
 D. $\{(1, 1, 1, 2), (2, 1, -1, 2)\}$.
 E. None of the others.

Question 16 (L.O.1, L.O.2). Let V be a vector space with a spanning set $\{x, y, z\}$. Given that $\dim(V) = 2$. Which of the following statement is CORRECT?

- A.** $\{x, y, z\}$ is a basis of V .
B. None of the others.
C. z is a linear combination of $\{x, y\}$.
D. $\text{Rank}(\{x, y\}) = 2$.
E. $\{x, y, z\}$ is linearly independent.

(Question 17 through 18)

In the vector space \mathbb{R}_3 , let $E = \{(1, 1, 2), (2, 1, -1), (4, 1, -6)\}$ and $F = \{(2, 1, 1), (3, 2, 2), (1, 1, 2)\}$ be two bases.

Question 17 (L.O.1, L.O.2). Find the transition matrix (change of basis matrix) from E to F.

- $$\mathbf{A}. \ P_{F \leftarrow E} = \begin{bmatrix} 0 & 1 & 5 \\ -2 & -2 & -5 \\ 3 & 1 & -3 \end{bmatrix}.$$

- B.** None of the others.

C. $P_{F \leftarrow E} = \begin{bmatrix} -11 & -8 & -5 \\ 21 & 15 & 10 \\ -4 & -3 & -2 \end{bmatrix}.$

D. $P_{F \leftarrow E} = \begin{bmatrix} 4 & 3 & 1 \\ -5 & -2 & 0 \\ 2 & 1 & 0 \end{bmatrix}.$

E. $P_{F \leftarrow E} = \begin{bmatrix} 0 & -1 & -2 \\ 0 & 2 & 5 \\ 1 & -2 & -7 \end{bmatrix}.$

Question 18 (L.O.1, L.O.2). Let $u \in \mathbb{R}_3$ be a vector, whose $[u]_E = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$. Find $[u]_F$.

A. $[u]_F = \begin{bmatrix} -8 \\ 19 \\ -24 \end{bmatrix}.$

B. $[u]_F = \begin{bmatrix} 17 \\ -21 \\ -4 \end{bmatrix}.$

C. $[u]_F = \begin{bmatrix} 13 \\ -9 \\ 4 \end{bmatrix}.$

D. $[u]_F = \begin{bmatrix} -42 \\ 81 \\ -16 \end{bmatrix}.$

E. None of the others.

Question 19 (L.O.1, L.O.2). Let $A = \begin{bmatrix} 1 & 2 & 0 & 1 \\ 2 & 1 & -1 & 3 \\ 3 & 1 & -1 & -2 \\ x & x^2 & x^3 & x^4 \end{bmatrix}$ be a matrix and $f(x) = \det(A)$ be a polynomial. The coefficient of the third degree term for $f(x)$ is

A. -13.

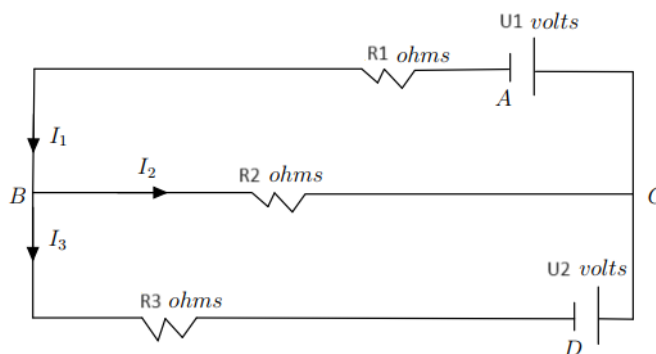
B. 20.

C. -25.

D. None of the others.

E. -20.

Question 20 (L.O.1, L.O.2). A circuit is given in the following figure



Given that the resistances are $R_1 = 4(\Omega)$, $R_2 = 3(\Omega)$, $R_3 = 3(\Omega)$, and the voltage sources are $U_1 = U_2 = 100(V)$. Find the absolute value of the current flowing through R_2 .

A. $\frac{100}{733}.$

B. $\frac{700}{33}.$

C. $\frac{400}{33}.$

D. $\frac{733}{33}.$

E. None of the others.

===== The end =====

ĐÁP ÁN đề 1101 - ngày thi 11/03/2024

1 D	3 C	5 A	7 D	9 C	11 D	13 B	15 E	17 D	19 E
2 A	4 B	6 E	8 C	10 D	12 D	14 B	16 C	18 A	20 D

ĐÁP ÁN đề 1102

1 B	3 B	5 D	7 E	9 C	11 D	13 A	15 E	17 D	19 E
2 A	4 E	6 A	8 D	10 C	12 D	14 A	16 E	18 A	20 A

ĐÁP ÁN đề 1103

1 A	3 B	5 D	7 B	9 C	11 B	13 D	15 D	17 D	19 C
2 C	4 D	6 B	8 E	10 D	12 D	14 B	16 B	18 A	20 C

ĐÁP ÁN đề 1104

1 C	3 D	5 B	7 A	9 C	11 C	13 B	15 D	17 E	19 E
2 A	4 C	6 E	8 B	10 C	12 E	14 A	16 B	18 A	20 B