Lecurer: ThS. Phan Thị Khánh Vân	*Date* *Oct. 16^{th}, 2023*	**Approved by:**	*Date* *Oct. 16^{th}, 2023*

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UNIVERSITY OF TECHNOLOGY - VNUHCM **Faculty of Applied Science**	**Midterm Exam**	Academic year	2023-2024	Semester	1
		Exam date	October 28^{th}, 2023		
	Course title	Linear Algebra			**Score**
	Course ID	MT1007	Sheet code	**3131**	
	Duration	50 minutes	Shift	12:00	

Instructions to students:

- *This is a closed book exam. Only your calculator is allowed. Total available score: 10.*
- *At the beginning of the working time, you MUST fill in your full name and student ID on this question sheet. There are 20 questions on 4 pages.*
- *Do not round between steps. Round your final answers to 4 decimal places.*

Student's full name:	Invigilator 1: . . . .
Student Id: . . . . Group:	Invigilator 2: . . . .

1. (L.O.1,L.O.2) Given two matrices $A \in M_{2\times 4}$ and $B \in M_{5\times 5}$. Let X and Y satisfy $Y = AXB$. Then, the size of X is:

   (A) None of the others (B) $X \in M_{4\times 5}$ (C) $X \in M_{4\times 2}$ (D) $X \in M_{3\times 5}$ (E) $X \in M_{5\times 4}$

2. (L.O.1,L.O.2) Find the rank of the matrix $A = \begin{bmatrix} 1 & 2 & 0 & 2 \\ 2 & -1 & -1 & -4 \\ 1 & 2 & -4 & 1 \\ 1 & -1 & 3 & 4 \end{bmatrix}$.

   (A) 1 (B) None of the others (C) 3 (D) 2 (E) 4

3. (L.O.1,L.O.2) Let A be the matrix $\begin{bmatrix} 1 & 1 & 1 \\ 1 & -4 & 3 \\ 3 & 2 & 2 \end{bmatrix}$ with the determinant 7. Evaluate $\det(2A^3)$.

   (A) 512 (B) None of the others (C) 2744 (D) 1000 (E) 1728

4. (L.O.1,L.O.2) In the vector space $\mathbb{R}_3$, let m be a real number and

   $$M = \{(1, -4, 1), (2, -7, 2), (1, 2, m), (3, -11, 3)\}$$

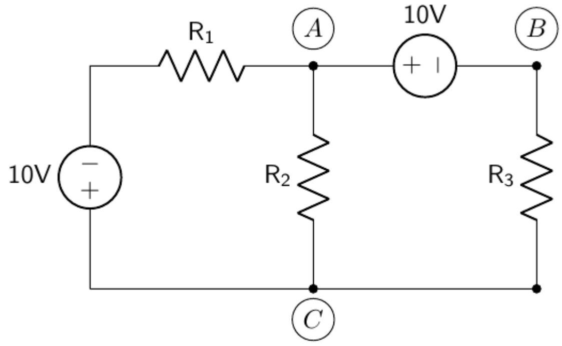
   be a vector set. Find m such that M is a spanning set of $\mathbb{R}_3$.

   (A) $m \neq 3$ (B) $m \neq -2$ (C) $m \neq 1$ (D) None of the others (E) $m \neq 0$

5. (L.O.1,L.O.2) In the vector space $P_2[x]$, let m be a real number and $M = \{-4x^2 + 2x + 1, x^2 + 2, 2x + m\}$ be a vector set. Find m such that M is linearly independent.

   (A) $m \neq 6$ (B) $m \neq 9$ (C) $m \neq 11$ (D) $m \neq 8$ (E) None of the others

6. (L.O.1,L.O.2) A circuit is given in the following figure.



Given that $R_1 = 2\Omega$, $R_2 = 5\Omega$, $R_3 = 4\Omega$. Find the current which flows through R_1 (the result is rounded to 2 decimal digits.).

Ⓐ 3.68 Ⓑ 3.67 Ⓒ 3.23 Ⓓ 4.0 Ⓔ None of the others

Questions 7 through 9

Let m be a real number such that the system $\begin{cases} x_1 + 2x_2 - x_3 = 2 \\ 2x_1 + 3x_2 + (m-2)x_3 = 2 \\ 3x_1 + 4x_2 + mx_3 = 2, \end{cases}$ has infinitely many solutions.

7. (L.O.1,L.O.2) Find m.

   Ⓐ There is no such m Ⓑ $m = 5$ Ⓒ $m = 4$ Ⓓ $m = 3$ Ⓔ None of the others

8. (L.O.1,L.O.2) Let (x_1, x_2, x_3) be a solution of the above system satisfying $x_1 = x_2$.

   Find x_3.

   Ⓐ $x_3 = -1$ Ⓑ $x_3 = -1/2$ Ⓒ None of the others Ⓓ $x_3 = -3/7$ Ⓔ $x_3 = -5$

9. (L.O.1,L.O.2) In $\mathbb{R}_3$, let $x = (x_1, x_2, x_3)$ be a solution of the above system (a vector in $\mathbb{R}_3$), given that x is a linear combination of $M = \{(1,1,0), (1,3,2)\}$. Find x_3.

   Ⓐ $x_3 = -3/7$ Ⓑ $x_3 = -5$ Ⓒ $x_3 = -5/6$ Ⓓ $x_3 = -4/7$ Ⓔ None of the others

Questions 10 through 11

In the vector space $\mathbb{R}_3$, given that $E = \{(1,2,-4), (2,5,-3), (-4,-3,m)\}$

and $F = \{(1,0,0), (1,1,0), (1,1,1)\}$ are two bases.

10. (L.O.1,L.O.2) Determine all values $m \in \mathbb{R}$ to make sure that E is a basis of $\mathbb{R}_3$.

    Ⓐ $m \neq -2$ Ⓑ None of the others Ⓒ $m \neq 41$ Ⓓ $m \neq 13$ Ⓔ $m \neq 40$

11. (L.O.1,L.O.2) Let $m = 0$. Find the coordinate vector of a vector u with respect to the basis F, given that the coordinate vector of u with respect to the basis E is $[u]_E = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$.

    Ⓐ $\begin{bmatrix} -13 \\ 14 \\ -9 \end{bmatrix}$ Ⓑ $\begin{bmatrix} -7 \\ 10 \\ -13 \end{bmatrix}$ Ⓒ None of the others Ⓓ $\begin{bmatrix} -10 \\ 23 \\ -23 \end{bmatrix}$ Ⓔ $\begin{bmatrix} 0 \\ 1 \\ -19 \end{bmatrix}$

Questions 12 through 13

In the vector space $M_{3\times2}(R)$ (*the set of all real matrices size of* 3×2),

let $V = \{X \in M_{3\times2}(R) | XA = 0\}$ be a subspace of $M_{3\times2}(R)$ where A=$\begin{bmatrix} 1 & 2 \\ 3 & 6 \end{bmatrix}$.

12. (L.O.1,L.O.2) Find all values $m \in \mathbb{R}$ such that $\begin{bmatrix} -6 & 2 \\ m & 2 \\ -m-6 & 0 \end{bmatrix} \in V$.

Ⓐ $m=-6$ Ⓑ None of the others Ⓒ $m=0$ Ⓓ $m=-4$ Ⓔ $m=1$

13. (L.O.1,L.O.2) Find the dimension of V.

Ⓐ 5 Ⓑ 1 Ⓒ None of the others Ⓓ 3 Ⓔ 6

Questions 14 through 17

Assume that the input-output matrix of a economic system with 3 sectors: Industry, Agriculture, Service is given as below: $A = \begin{bmatrix} 0.1 & 0.1 & 0.1 \\ 0.1 & 0.15 & 0.1 \\ 0.15 & 0.2 & m \end{bmatrix}$, $m \in \mathbb{R}$. Let $B = 20A$. Given that the output values (in the order: Industry, Agriculture, Service) of each sector is 60, 40, 50 (billion USD).

14. (L.O.1,L.O.2) Find all values $m \in \mathbb{R}$ such that A is invertible.

Ⓐ $m \neq 3/20$ Ⓑ $m \neq 11/40$ Ⓒ $m \neq 1/5$ Ⓓ $m \neq 1/4$ Ⓔ $m \neq 1/3$

15. (L.O.1,L.O.2) What is the total value of products supplied by the industry to the agricultural sector in billions of USD?

Ⓐ 4 Ⓑ 6.5 Ⓒ None of the others Ⓓ 6 Ⓔ 5.5

16. (L.O.1,L.O.2) Find the total input values (billion USD) of the industry sector (*the total value that all sectors supply to the industry*)?

Ⓐ 14.0 Ⓑ None of the others Ⓒ 18.5 Ⓓ 23.2 Ⓔ 21.0

17. (L.O.1,L.O.2) For a sector, let out := the total output value and in := the total input value. Then the profit margin is defined by: $\boxed{\text{roe} = \dfrac{\text{out -in}}{\text{in}} \cdot 100\%}$. Compute the profit margin of the industry sector.

Ⓐ 162.71% Ⓑ None of the others Ⓒ 185.71% Ⓓ 241.71% Ⓔ 96.71%

Questions 18 through 20

Given the life span of a population of species is 9 months (after 9 months it will be sold). This population is divided into 3 classes: $0 < \text{age} \leq 3$ (I), $3 \leq \text{age} < 6$ (II), $6 \leq \text{age} < 9$ (III). The average numbers of offsprings produced in 3 months by the age class I, II, III are: 1.0, 6, 2, respectively. The probabilities of survival after 3 months of the age classes I and II are 80% and 90%, respectively. Suppose that at the initial moment, one has 100 individuals in age class I while there is not any individual in classes II and III.

18. (L.O.1,L.O.2) Find the Leslie matrix L.

Ⓐ $\begin{bmatrix} 1.0 & 6.0 & 2.0 \\ 0.8 & 0.0 & 0.0 \\ 0.0 & 0.9 & 0.0 \end{bmatrix}$ Ⓑ $\begin{bmatrix} 0.0 & 2.0 & 2.0 \\ 0.8 & 0.0 & 0.0 \\ 0.0 & 0.9 & 0.0 \end{bmatrix}$ Ⓒ $\begin{bmatrix} 0.0 & 6.0 & 2.0 \\ 0.8 & 0.0 & 0.0 \\ 0.0 & 0.9 & 1.0 \end{bmatrix}$ Ⓓ $\begin{bmatrix} 1.0 & 6.0 & 2.0 \\ 0.8 & 0.0 & 0.2 \\ 0.0 & 0.9 & 0.1 \end{bmatrix}$

Ⓔ None of the others

19. (L.O.1,L.O.2) After 1 year, how many individuals are there in the age class III? (Round the result to the nearest integer).

Ⓐ 418.0 Ⓑ 441.0 Ⓒ 404.0 Ⓓ 286.0 Ⓔ None of the others

20. (L.O.1,L.O.2) Each newborn individual will get a vaccination dose. After one year, what is the total number of doses will have been used? (Round the result to the neareast integer).

Ⓐ 5193.0 Ⓑ None of the others Ⓒ 1367 Ⓓ 6016.0 Ⓔ 9542.0

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Student's full name:	Invigilator 1: ....
Student Id: .... Group:	Invigilator 2: ....

1. (L.O.1,L.O.2) Given two matrices $A \in M_{2\times 3}$ and $B \in M_{5\times 5}$. Let X and Y satisfy $Y = AXB$. Then, the size of X is:

   (A) None of the others (B) $X \in M_{4\times 4}$ (C) $X \in M_{3\times 2}$ (D) $X \in M_{3\times 5}$ (E) $X \in M_{2\times 5}$

2. (L.O.1,L.O.2) Find the rank of the matrix $A = \begin{bmatrix} 1 & 2 & 0 & 2 \\ 2 & -1 & -1 & 2 \\ 1 & 2 & 2 & 1 \\ 1 & -1 & 3 & 4 \end{bmatrix}$.

   (A) 2 (B) 3 (C) None of the others (D) 1 (E) 4

3. (L.O.1,L.O.2) Let A be the matrix $\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 3 & 2 & 2 \end{bmatrix}$ with the determinant 1. Evaluate $\det(2A^3)$.

   (A) -8 (B) None of the others (C) -64 (D) 8 (E) 0

4. (L.O.1,L.O.2) In the vector space $\mathbb{R}_3$, let m be a real number and

$$M = \{(1, 2, 1), (2, 5, 2), (1, 2, m), (3, 7, 3)\}$$

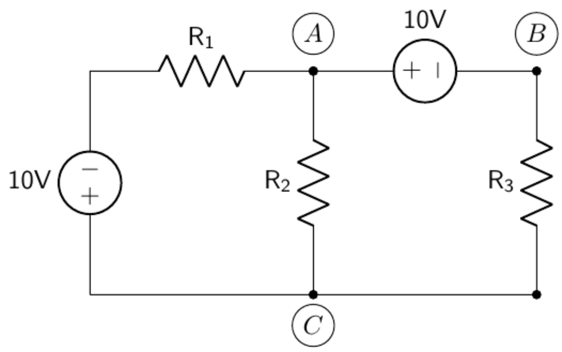
   be a vector set. Find m such that M is a spanning set of $\mathbb{R}_3$.

   (A) $m \neq -2$ (B) None of the others (C) $m \neq 0$ (D) $m \neq 3$ (E) $m \neq 1$

5. (L.O.1,L.O.2) In the vector space $P_2[x]$, let m be a real number and $M = \{2x^2 + 2x + 1, x^2 + 2, 2x + m\}$ be a vector set. Find m such that M is linearly independent.

   (A) $m \neq -3$ (B) $m \neq -6$ (C) $m \neq -4$ (D) None of the others (E) $m \neq -1$

6. (L.O.1,L.O.2) A circuit is given in the following figure.



Given that $R_1 = 1\Omega$, $R_2 = 4\Omega$, $R_3 = 3\Omega$. Find the current which flows through R_1 (the result is rounded to 2 decimal digits.).

Ⓐ 8.0 Ⓑ 5.62 Ⓒ None of the others Ⓓ 3.23 Ⓔ 5.79

Questions 7 through 9

Let m be a real number such that the system $\begin{cases} x_1 + 2x_2 - x_3 = 1 \\ 2x_1 + 3x_2 + (m-1)x_3 = 2 \\ 3x_1 + 4x_2 + mx_3 = 3, \end{cases}$ has infinitely many solutions.

7. (L.O.1,L.O.2) Find m.

   Ⓐ $m = 1$ Ⓑ There is no such m Ⓒ None of the others Ⓓ $m = 3$ Ⓔ $m = 2$

8. (L.O.1,L.O.2) Let (x_1, x_2, x_3) be a solution of the above system satisfying $x_1 = x_2$.

   Find x_3.

   Ⓐ $x_3 = 1/2$ Ⓑ $x_3 = 1/5$ Ⓒ None of the others Ⓓ $x_3 = 0$ Ⓔ $x_3 = 2$

9. (L.O.1,L.O.2) In $\mathbb{R}_3$, let $x = (x_1, x_2, x_3)$ be a solution of the above system (a vector in $\mathbb{R}_3$), given that x is a linear combination of $M = \{(1, 1, 0), (1, 2, 1)\}$. Find x_3.

   Ⓐ $x_3 = 0$ Ⓑ $x_3 = 1/2$ Ⓒ $x_3 = 2$ Ⓓ $x_3 = 1/4$ Ⓔ None of the others

Questions 10 through 11

In the vector space $\mathbb{R}_3$, given that $E = \{(1, 2, 2), (2, 5, 3), (2, 3, m)\}$

and $F = \{(1, 0, 0), (1, 1, 0), (1, 1, 1)\}$ are two bases.

10. (L.O.1,L.O.2) Determine all values $m \in \mathbb{R}$ to make sure that E is a basis of $\mathbb{R}_3$.

    Ⓐ None of the others Ⓑ $m \neq -2$ Ⓒ $m \neq 5$ Ⓓ $m \neq 1$ Ⓔ $m \neq 4$

11. (L.O.1,L.O.2) Let $m = 0$. Find the coordinate vector of a vector u with respect to the basis F, given that the coordinate vector of u with respect to the basis E is $[u]_E = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$.

    Ⓐ $\begin{bmatrix} 11 \\ 10 \\ -13 \end{bmatrix}$ Ⓑ $\begin{bmatrix} 0 \\ 1 \\ -7 \end{bmatrix}$ Ⓒ None of the others Ⓓ $\begin{bmatrix} -13 \\ 14 \\ -3 \end{bmatrix}$ Ⓔ $\begin{bmatrix} -10 \\ 23 \\ -5 \end{bmatrix}$

Questions 12 through 13

In the vector space $M_{3\times2}(R)$ (*the set of all real matrices size of* 3×2),

let $V = \{X \in M_{3\times2}(R) | XA = 0\}$ be a subspace of $M_{3\times2}(R)$ where A=$\begin{bmatrix} 1 & 2 \\ 3 & 6 \end{bmatrix}$.

12. (L.O.1,L.O.2) Find all values $m \in \mathbb{R}$ such that $\begin{bmatrix} -3 & 1 \\ m & 2 \\ -m-3 & -1 \end{bmatrix} \in V$.

    (A) $m = -4$ (B) $m = -6$ (C) $m = 0$ (D) None of the others (E) $m = -2$

13. (L.O.1,L.O.2) Find the dimension of V.

    (A) 1 (B) 3 (C) 6 (D) 5 (E) None of the others

Questions 14 through 17

Assume that the input-output matrix of a economic system with 3 sectors: Industry, Agriculture, Service is given as below: $A = \begin{bmatrix} 0.05 & 0.05 & 0.1 \\ 0.1 & 0.15 & 0.1 \\ 0.15 & 0.2 & m \end{bmatrix}$, $m \in \mathbb{R}$. Let $B = 20A$. Given that the output values (in the order: Industry, Agriculture, Service) of each sector is 60, 40, 50 (billion USD).

14. (L.O.1,L.O.2) Find all values $m \in \mathbb{R}$ such that A is invertible.

    (A) $m \neq 1/5$ (B) $m \neq 1/4$ (C) $m \neq 9/20$ (D) $m \neq 2/5$ (E) $m \neq 8/15$

15. (L.O.1,L.O.2) What is the total value of products supplied by the industry to the agricultural sector in billions of USD?

    (A) None of the others (B) 3.5 (C) 4 (D) 2 (E) 3

16. (L.O.1,L.O.2) Find the total input values (billion USD) of the industry sector (*the total value that all sectors supply to the industry*)?

    (A) 18.5 (B) None of the others (C) 23.2 (D) 12.0 (E) 18.0

17. (L.O.1,L.O.2) For a sector, let out := the total output value and in := the total input value. Then the profit margin is defined by: $\boxed{\text{roe} = \frac{\text{out -in}}{\text{in}} \cdot 100\%}$. Compute the profit margin of the industry sector.

    (A) 144.33% (B) 289.33% (C) 210.33% (D) None of the others (E) 233.33%

Questions 18 through 20

Given the life span of a population of species is 9 months (after 9 months it will be sold). This population is divided into 3 classes: $0 < \text{age} \leq 3$ (I), $3 \leq \text{age} < 6$ (II), $6 \leq \text{age} < 9$ (III). The average numbers of offsprings produced in 3 months by the age class I, II, III are: 0.8, 5, 2, respectively. The probabilities of survival after 3 months of the age classes I and II are 80% and 90%, respectively. Suppose that at the initial moment, one has 100 individuals in age class I while there is not any individual in classes II and III.

18. (L.O.1,L.O.2) Find the Leslie matrix L.

    (A) $\begin{bmatrix} 0.8 & 5.0 & 2.0 \\ 0.8 & 0.0 & 0.2 \\ 0.0 & 0.9 & 0.1 \end{bmatrix}$ (B) $\begin{bmatrix} 0.0 & 1.0 & 2.0 \\ 0.8 & 0.0 & 0.0 \\ 0.0 & 0.9 & 0.0 \end{bmatrix}$ (C) $\begin{bmatrix} 0.0 & 5.0 & 2.0 \\ 0.8 & 0.0 & 0.0 \\ 0.0 & 0.9 & 1.0 \end{bmatrix}$ (D) $\begin{bmatrix} 0.8 & 5.0 & 2.0 \\ 0.8 & 0.0 & 0.0 \\ 0.0 & 0.9 & 0.0 \end{bmatrix}$

    (E) None of the others

19. (L.O.1,L.O.2) After 1 year, how many individuals are there in the age class III? (Round the result to the nearest integer).

    (A) None of the others (B) 228.0 (C) 307.0 (D) 334.0 (E) 308.0

20. (L.O.1,L.O.2) Each newborn individual will get a vaccination dose. After one year, what is the total number of doses will have been used? (Round the result to the neareast integer).

(A) None of the others (B) 4019.0 (C) 7545.0 (D) 1367 (E) 3196.0

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		Exam date	October 28th, 2023		
	Course title	Linear Algebra		**Score**	
	Course ID	MT1007	Sheet code	**3133**	
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Student's full name:	Invigilator 1: . . . .
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1. (L.O.1,L.O.2) Given two matrices $A \in M_{2\times3}$ and $B \in M_{4\times5}$. Let X and Y satisfy $Y = AXB$. Then, the size of X is:

   (A) $X \in M_{3\times1}$ (B) $X \in M_{2\times4}$ (C) $X \in M_{4\times3}$ (D) None of the others (E) $X \in M_{3\times4}$

2. (L.O.1,L.O.2) Find the rank of the matrix $A = \begin{bmatrix} 1 & 2 & 0 & 2 \\ 2 & -1 & -1 & -5 \\ 1 & 2 & -5 & 1 \\ 1 & -1 & 3 & 4 \end{bmatrix}$.

   (A) 1 (B) 3 (C) 2 (D) 4 (E) None of the others

3. (L.O.1,L.O.2) Let A be the matrix $\begin{bmatrix} 1 & 1 & 1 \\ 1 & -5 & 3 \\ 3 & 1 & 2 \end{bmatrix}$ with the determinant 10. Evaluate $\det(2A^3)$.

   (A) None of the others (B) 8000 (C) 2744 (D) 4096 (E) 5832

4. (L.O.1,L.O.2) In the vector space $\mathbb{R}_3$, let m be a real number and

$$M = \{(1, -5, 1), (2, -9, 1), (1, 2, m), (3, -14, 2)\}$$

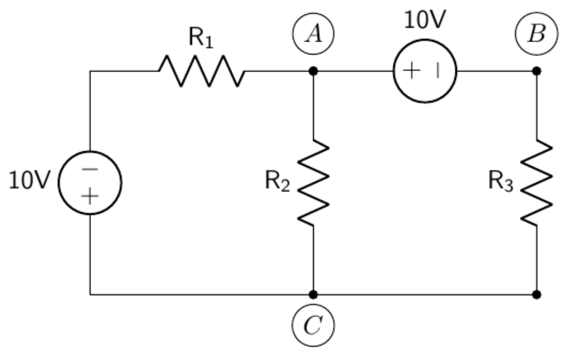
   be a vector set. Find m such that M is a spanning set of $\mathbb{R}_3$.

   (A) $m \neq -6$ (B) $m \neq -4$ (C) None of the others (D) $m \neq -7$ (E) $m \neq -9$

5. (L.O.1,L.O.2) In the vector space $P_2[x]$, let m be a real number and $M = \{-5x^2 + 2x + 1, x^2 + 1, 2x + m\}$ be a vector set. Find m such that M is linearly independent.

   (A) $m \neq 5$ (B) $m \neq 8$ (C) $m \neq 3$ (D) $m \neq 6$ (E) None of the others

6. (L.O.1,L.O.2) A circuit is given in the following figure.



Given that $R_1 = 1\Omega$, $R_2 = 4\Omega$, $R_3 = 3\Omega$. Find the current which flows through R_1 (the result is rounded to 2 decimal digits.).

Ⓐ None of the others Ⓑ 5.62 Ⓒ 8.0 Ⓓ 5.79 Ⓔ 3.23

Questions 7 through 9

Let m be a real number such that the system $\begin{cases} x_1 + 2x_2 - x_3 = 1 \\ 2x_1 + 3x_2 + (m-1)x_3 = 1 \\ 3x_1 + 4x_2 + mx_3 = 1, \end{cases}$ has infinitely many solutions.

7. (L.O.1,L.O.2) Find m.

   Ⓐ There is no such m Ⓑ $m = 3$ Ⓒ None of the others Ⓓ $m = 2$ Ⓔ $m = 1$

8. (L.O.1,L.O.2) Let (x_1, x_2, x_3) be a solution of the above system satisfying $x_1 = x_2$.
   Find x_3.

   Ⓐ None of the others Ⓑ $x_3 = -3/2$ Ⓒ $x_3 = -3$ Ⓓ $x_3 = -1/4$ Ⓔ $x_3 = -2/5$

9. (L.O.1,L.O.2) In $\mathbb{R}_3$, let $x = (x_1, x_2, x_3)$ be a solution of the above system (a vector in $\mathbb{R}_3$), given that x is a linear combination of $M = \{(1,1,0),(1,2,1)\}$. Find x_3.

   Ⓐ None of the others Ⓑ $x_3 = -1/4$ Ⓒ $x_3 = -1$ Ⓓ $x_3 = -1/2$ Ⓔ $x_3 = -3$

Questions 10 through 11

In the vector space $\mathbb{R}_3$, given that $E = \{(1,2,-5),(2,5,-4),(-5,-4,m)\}$
and $F = \{(1,0,0),(1,1,0),(1,1,1)\}$ are two bases.

10. (L.O.1,L.O.2) Determine all values $m \in \mathbb{R}$ to make sure that E is a basis of $\mathbb{R}_3$.

    Ⓐ $m \neq 61$ Ⓑ $m \neq 60$ Ⓒ $m \neq -2$ Ⓓ None of the others Ⓔ $m \neq 22$

11. (L.O.1,L.O.2) Let $m = 0$. Find the coordinate vector of a vector u with respect to the basis F, given that the coordinate vector of u with respect to the basis E is $[u]_E = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$.

    Ⓐ None of the others Ⓑ $\begin{bmatrix} -10 \\ 10 \\ -13 \end{bmatrix}$ Ⓒ $\begin{bmatrix} -10 \\ 23 \\ -26 \end{bmatrix}$ Ⓓ $\begin{bmatrix} -13 \\ 14 \\ -10 \end{bmatrix}$ Ⓔ $\begin{bmatrix} 0 \\ 1 \\ -21 \end{bmatrix}$

Questions 12 through 13

In the vector space $M_{3\times2}(R)$ (*the set of all real matrices size of* 3×2),

let $V = \{X \in M_{3\times2}(R) | XA = 0\}$ be a subspace of $M_{3\times2}(R)$ where A=$\begin{bmatrix} 1 & 2 \\ 3 & 6 \end{bmatrix}$.

12. (L.O.1,L.O.2) Find all values $m \in \mathbb{R}$ such that $\begin{bmatrix} -3 & 1 \\ m & 1 \\ -m-3 & 0 \end{bmatrix} \in V$.

Ⓐ $m = -2$ Ⓑ $m = 3$ Ⓒ $m = -1$ Ⓓ None of the others Ⓔ $m = -3$

13. (L.O.1,L.O.2) Find the dimension of V.

Ⓐ 6 Ⓑ 1 Ⓒ 3 Ⓓ None of the others Ⓔ 5

Questions 14 through 17

Assume that the input-output matrix of a economic system with 3 sectors: Industry, Agriculture, Service is given as below: $A = \begin{bmatrix} 0.05 & 0.05 & 0.05 \\ 0.1 & 0.15 & 0.1 \\ 0.15 & 0.2 & m \end{bmatrix}$, $m \in \mathbb{R}$. Let $B = 20A$. Given that the output values (in the order: Industry, Agriculture, Service) of each sector is 60, 40, 50 (billion USD).

14. (L.O.1,L.O.2) Find all values $m \in \mathbb{R}$ such that A is invertible.

Ⓐ $m \neq 1/4$ Ⓑ $m \neq 3/10$ Ⓒ $m \neq 1/3$ Ⓓ $m \neq 3/20$ Ⓔ $m \neq 1/5$

15. (L.O.1,L.O.2) What is the total value of products supplied by the industry to the agricultural sector in billions of USD?

Ⓐ None of the others Ⓑ 3.5 Ⓒ 3 Ⓓ 4.5 Ⓔ 2

16. (L.O.1,L.O.2) Find the total input values (billion USD) of the industry sector (*the total value that all sectors supply to the industry*)?

Ⓐ 23.2 Ⓑ 18.5 Ⓒ 12.0 Ⓓ 18.0 Ⓔ None of the others

17. (L.O.1,L.O.2) For a sector, let out := the total output value and in := the total input value. Then the profit margin is defined by: $\boxed{\text{roe} = \frac{\text{out -in}}{\text{in}} \cdot 100\%}$. Compute the profit margin of the industry sector.

Ⓐ None of the others Ⓑ 210.33% Ⓒ 144.33% Ⓓ 233.33% Ⓔ 289.33%

Questions 18 through 20

Given the life span of a population of species is 9 months (after 9 months it will be sold). This population is divided into 3 classes: $0 < \text{age} \leq 3$ (I), $3 \leq \text{age} < 6$ (II), $6 \leq \text{age} < 9$ (III). The average numbers of offsprings produced in 3 months by the age class I, II, III are: 0.8, 5, 1, respectively. The probabilities of survival after 3 months of the age classes I and II are 80% and 90%, respectively. Suppose that at the initial moment, one has 100 individuals in age class I while there is not any individual in classes II and III.

18. (L.O.1,L.O.2) Find the Leslie matrix L.

Ⓐ None of the others Ⓑ $\begin{bmatrix} 0.0 & 1.0 & 1.0 \\ 0.8 & 0.0 & 0.0 \\ 0.0 & 0.9 & 0.0 \end{bmatrix}$ Ⓒ $\begin{bmatrix} 0.0 & 5.0 & 1.0 \\ 0.8 & 0.0 & 0.0 \\ 0.0 & 0.9 & 1.0 \end{bmatrix}$ Ⓓ $\begin{bmatrix} 0.8 & 5.0 & 1.0 \\ 0.8 & 0.0 & 0.0 \\ 0.0 & 0.9 & 0.0 \end{bmatrix}$

Ⓔ $\begin{bmatrix} 0.8 & 5.0 & 1.0 \\ 0.8 & 0.0 & 0.2 \\ 0.0 & 0.9 & 0.1 \end{bmatrix}$

19. (L.O.1,L.O.2) After 1 year, how many individuals are there in the age class III? (Round the result to the nearest integer).

Ⓐ None of the others Ⓑ 334.0 Ⓒ 307.0 Ⓓ 228.0 Ⓔ 308.0

20. (L.O.1,L.O.2) Each newborn individual will get a vaccination dose. After one year, what is the total number of doses will have been used? (Round the result to the neareast integer).

(A) 3831.0 (B) 3008.0 (C) 1367 (D) None of the others (E) 7357.0

Lecturer: ThS. Phan Thị Khánh Vân	*Date* *Oct. 16th, 2023*	**Approved by:**	*Date* *Oct. 16th, 2023*

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UNIVERSITY OF TECHNOLOGY - VNUHCM Faculty of Applied Science	**Midterm Exam**	Academic year	2023-2024	Semester	1
		Exam date	October 28th, 2023		
	Course title	Linear Algebra			**Score**
	Course ID	MT1007	Sheet code	**3134**	
	Duration	50 minutes	Shift	12:00	

Instructions to students:

- *This is a closed book exam. Only your calculator is allowed. Total available score: 10.*
- *At the beginning of the working time, you MUST fill in your full name and student ID on this question sheet. There are 20 questions on 4 pages.*
- *Do not round between steps. Round your final answers to 4 decimal places.*

Student's full name:	Invigilator 1: . . . .
Student Id: . . . . Group:	Invigilator 2: . . . .

1. (L.O.1,L.O.2) Given two matrices $A \in M_{2\times 6}$ and $B \in M_{5\times 5}$. Let X and Y satisfy $Y = AXB$. Then, the size of X is:

   Ⓐ $X \in M_{7\times 4}$ Ⓑ $X \in M_{6\times 5}$ Ⓒ $X \in M_{5\times 5}$ Ⓓ $X \in M_{6\times 2}$ Ⓔ None of the others

2. (L.O.1,L.O.2) Find the rank of the matrix $A = \begin{bmatrix} 1 & 2 & 0 & 2 \\ 2 & -1 & -1 & 4 \\ 1 & 2 & 4 & 1 \\ 1 & -1 & 3 & 4 \end{bmatrix}$.

   Ⓐ 2 Ⓑ 3 Ⓒ 4 Ⓓ None of the others Ⓔ 1

3. (L.O.1,L.O.2) Let A be the matrix $\begin{bmatrix} 1 & 1 & 1 \\ 1 & 4 & 3 \\ 3 & 1 & 2 \end{bmatrix}$ with the determinant 1. Evaluate $\det(2A^3)$.

   Ⓐ -8 Ⓑ None of the others Ⓒ 8 Ⓓ -64 Ⓔ 0

4. (L.O.1,L.O.2) In the vector space $\mathbb{R}_3$, let m be a real number and

   $$M = \{(1,4,1),(2,9,1),(1,2,m),(3,13,2)\}$$

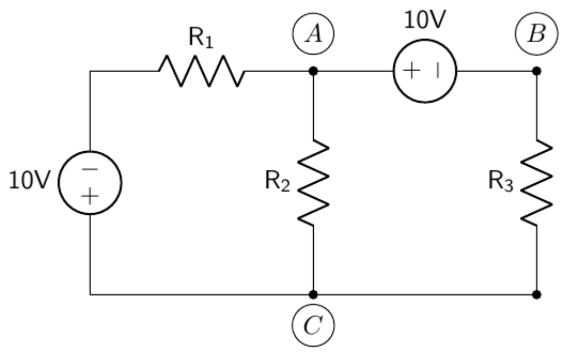
   be a vector set. Find m such that M is a spanning set of $\mathbb{R}_3$.

   Ⓐ $m \neq 3$ Ⓑ None of the others Ⓒ $m \neq 2$ Ⓓ $m \neq 5$ Ⓔ $m \neq 0$

5. (L.O.1,L.O.2) In the vector space $P_2[x]$, let m be a real number and $M = \{4x^2+2x+1, x^2+1, 2x+m\}$ be a vector set. Find m such that M is linearly independent.

   Ⓐ $m \neq -1$ Ⓑ $m \neq -4$ Ⓒ $m \neq -3$ Ⓓ None of the others Ⓔ $m \neq -6$

6. (L.O.1,L.O.2) A circuit is given in the following figure.



Given that $R_1 = 4\Omega$, $R_2 = 7\Omega$, $R_3 = 6\Omega$. Find the current which flows through R_1 (the result is rounded to 2 decimal digits.).

Ⓐ 1.82 Ⓑ 2.14 Ⓒ None of the others Ⓓ 3.23 Ⓔ 2.13

Questions 7 through 9

Let m be a real number such that the system $\begin{cases} x_1 + 2x_2 - x_3 = 4 \\ 2x_1 + 3x_2 + (m-4)x_3 = 2 \\ 3x_1 + 4x_2 + mx_3 = 0, \end{cases}$ has infinitely many solutions.

7. (L.O.1,L.O.2) Find m.

   Ⓐ $m = 9$ Ⓑ There is no such m Ⓒ None of the others Ⓓ $m = 7$ Ⓔ $m = 8$

8. (L.O.1,L.O.2) Let (x_1, x_2, x_3) be a solution of the above system satisfying $x_1 = x_2$.

   Find x_3.

   Ⓐ $x_3 = -15$ Ⓑ None of the others Ⓒ $x_3 = -15/11$ Ⓓ $x_3 = -3$ Ⓔ $x_3 = -1$

9. (L.O.1,L.O.2) In $\mathbb{R}_3$, let $x = (x_1, x_2, x_3)$ be a solution of the above system (a vector in $\mathbb{R}_3$), given that x is a linear combination of $M = \{(1,1,0), (1,5,4)\}$. Find x_3.

   Ⓐ $x_3 = -1$ Ⓑ $x_3 = -14/13$ Ⓒ None of the others Ⓓ $x_3 = -5/4$ Ⓔ $x_3 = -15$

Questions 10 through 11

In the vector space $\mathbb{R}_3$, given that $E = \{(1,2,4), (2,5,5), (4,5,m)\}$

and $F = \{(1,0,0), (1,1,0), (1,1,1)\}$ are two bases.

10. (L.O.1,L.O.2) Determine all values $m \in \mathbb{R}$ to make sure that E is a basis of $\mathbb{R}_3$.

    Ⓐ $m \neq 25$ Ⓑ $m \neq 24$ Ⓒ $m \neq -2$ Ⓓ $m \neq 13$ Ⓔ None of the others

11. (L.O.1,L.O.2) Let $m = 0$. Find the coordinate vector of a vector u with respect to the basis F, given that the coordinate vector of u with respect to the basis E is $[u]_E = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$.

    Ⓐ None of the others Ⓑ $\begin{bmatrix} 17 \\ 10 \\ -13 \end{bmatrix}$ Ⓒ $\begin{bmatrix} 0 \\ 1 \\ -3 \end{bmatrix}$ Ⓓ $\begin{bmatrix} -13 \\ 14 \\ -1 \end{bmatrix}$ Ⓔ $\begin{bmatrix} -10 \\ 23 \\ 1 \end{bmatrix}$

Questions 12 through 13

In the vector space $M_{3\times2}(R)$ (*the set of all real matrices size of* 3×2),

let $V = \{X \in M_{3\times2}(R) | XA = 0\}$ be a subspace of $M_{3\times2}(R)$ where A=$\begin{bmatrix} 1 & 2 \\ 3 & 6 \end{bmatrix}$.

12. (L.O.1,L.O.2) Find all values $m \in \mathbb{R}$ such that $\begin{bmatrix} -12 & 4 \\ m & 2 \\ -m-12 & 2 \end{bmatrix} \in V$.

   (A) None of the others (B) $m = 7$ (C) $m = 0$ (D) $m = -6$ (E) $m = -4$

13. (L.O.1,L.O.2) Find the dimension of V.

   (A) 5 (B) None of the others (C) 3 (D) 1 (E) 6

Questions 14 through 17

Assume that the input-output matrix of a economic system with 3 sectors: Industry, Agriculture, Service is given as below: $A = \begin{bmatrix} 0.2 & 0.2 & 0.1 \\ 0.1 & 0.15 & 0.1 \\ 0.15 & 0.2 & m \end{bmatrix}$, $m \in \mathbb{R}$. Let $B = 20A$. Given that the output values (in the order: Industry, Agriculture, Service) of each sector is 60, 40, 50 (billion USD).

14. (L.O.1,L.O.2) Find all values $m \in \mathbb{R}$ such that A is invertible.

   (A) $m \neq 3/16$ (B) $m \neq 1/8$ (C) $m \neq 7/30$ (D) $m \neq 7/40$ (E) $m \neq 7/50$

15. (L.O.1,L.O.2) What is the total value of products supplied by the industry to the agricultural sector in billions of USD?

   (A) 9.5 (B) 12 (C) None of the others (D) 12.5 (E) 8

16. (L.O.1,L.O.2) Find the total input values (billion USD) of the industry sector (*the total value that all sectors supply to the industry*)?

   (A) 27.0 (B) 23.2 (C) 18.5 (D) 18.0 (E) None of the others

17. (L.O.1,L.O.2) For a sector, let out := the total output value and in := the total input value. Then the profit margin is defined by: $\boxed{\text{roe} = \frac{\text{out -in}}{\text{in}} \cdot 100\%}$. Compute the profit margin of the industry sector.

   (A) None of the others (B) 99.22% (C) 178.22% (D) 33.22% (E) 122.22%

Questions 18 through 20

Given the life span of a population of species is 9 months (after 9 months it will be sold). This population is divided into 3 classes: $0 < \text{age} \leq 3$ (I), $3 \leq \text{age} < 6$ (II), $6 \leq \text{age} < 9$ (III). The average numbers of offsprings produced in 3 months by the age class I, II, III are: 1.4, 8, 2, respectively. The probabilities of survival after 3 months of the age classes I and II are 80% and 90%, respectively. Suppose that at the initial moment, one has 100 individuals in age class I while there is not any individual in classes II and III.

18. (L.O.1,L.O.2) Find the Leslie matrix L.

   (A) $\begin{bmatrix} 0.0 & 8.0 & 2.0 \\ 0.8 & 0.0 & 0.0 \\ 0.0 & 0.9 & 1.0 \end{bmatrix}$ (B) $\begin{bmatrix} 0.0 & 4.0 & 2.0 \\ 0.8 & 0.0 & 0.0 \\ 0.0 & 0.9 & 0.0 \end{bmatrix}$ (C) $\begin{bmatrix} 1.4 & 8.0 & 2.0 \\ 0.8 & 0.0 & 0.2 \\ 0.0 & 0.9 & 0.1 \end{bmatrix}$ (D) $\begin{bmatrix} 1.4 & 8.0 & 2.0 \\ 0.8 & 0.0 & 0.0 \\ 0.0 & 0.9 & 0.0 \end{bmatrix}$

   (E) None of the others

19. (L.O.1,L.O.2) After 1 year, how many individuals are there in the age class III? (Round the result to the nearest integer).

   (A) 615.0 (B) None of the others (C) 421.0 (D) 722.0 (E) 602.0

20. (L.O.1,L.O.2) Each newborn individual will get a vaccination dose. After one year, what is the total number of doses will have been used? (Round the result to the neareast integer).

Ⓐ 11010.0 Ⓑ 1367 Ⓒ 11833.0 Ⓓ 15359.0 Ⓔ None of the others

Lecturer:	*Date*	**Approved by:**	*Date*
ThS. Phan Thị Khánh Vân	*Oct. 16th, 2023*		*Oct. 16th, 2023*

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UNIVERSITY OF TECHNOLOGY - VNUHCM **Faculty of Applied Science**	**Midterm Exam**		Academic year	2023-2024	Semester	1
			Exam date	October 28th, 2023		
	Course title	Linear Algebra			**Score**	
	Course ID	MT1007	Sheet code	**3135**		
	Duration	50 minutes	Shift	12:00		

Instructions to students:

- *This is a closed book exam. Only your calculator is allowed. Total available score: 10.*
- *At the beginning of the working time, you MUST fill in your full name and student ID on this question sheet. There are 20 questions on 4 pages.*
- *Do not round between steps. Round your final answers to 4 decimal places.*

Student's full name: . . . . Invigilator 1: . . . .

Student Id: . . . . Group: . . . . Invigilator 2: . . . .

1. (L.O.1,L.O.2) Given two matrices $A \in M_{2\times 6}$ and $B \in M_{4\times 5}$. Let X and Y satisfy $Y = AXB$. Then, the size of X is:

   (A) None of the others (B) $X \in M_{5\times 4}$ (C) $X \in M_{6\times 4}$ (D) $X \in M_{6\times 1}$ (E) $X \in M_{7\times 3}$

2. (L.O.1,L.O.2) Find the rank of the matrix $A = \begin{bmatrix} 1 & 2 & 0 & 2 \\ 2 & -1 & -1 & -1 \\ 1 & 2 & -1 & 1 \\ 1 & -1 & 3 & 4 \end{bmatrix}$.

   (A) 2 (B) 4 (C) None of the others (D) 3 (E) 1

3. (L.O.1,L.O.2) Let A be the matrix $\begin{bmatrix} 1 & 1 & 1 \\ 1 & -1 & 3 \\ 3 & 4 & 2 \end{bmatrix}$ with the determinant 0. Evaluate $\det(2A^3)$.

   (A) -8 (B) -216 (C) 0 (D) -64 (E) None of the others

4. (L.O.1,L.O.2) In the vector space $\mathbb{R}_3$, let m be a real number and

$$M = \{(1, -1, 1), (2, -1, 4), (1, 2, m), (3, -2, 5)\}$$

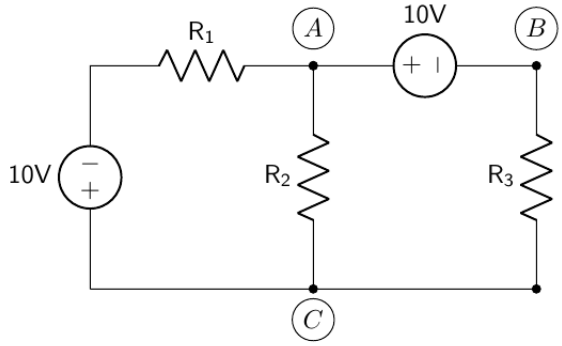
   be a vector set. Find m such that M is a spanning set of $\mathbb{R}_3$.

   (A) $m \neq 6$ (B) $m \neq 7$ (C) $m \neq 4$ (D) $m \neq 9$ (E) None of the others

5. (L.O.1,L.O.2) In the vector space $P_2[x]$, let m be a real number and $M = \{-x^2 + 2x + 1, x^2 + 4, 2x + m\}$ be a vector set. Find m such that M is linearly independent.

   (A) None of the others (B) $m \neq 7$ (C) $m \neq 5$ (D) $m \neq 4$ (E) $m \neq 2$

6. (L.O.1,L.O.2) A circuit is given in the following figure.



Given that $R_1 = 4\Omega$, $R_2 = 7\Omega$, $R_3 = 6\Omega$. Find the current which flows through R_1 (the result is rounded to 2 decimal digits.).

(A) 3.23 (B) None of the others (C) 1.82 (D) 2.13 (E) 2.14

Questions 7 through 9

Let m be a real number such that the system $\begin{cases} x_1 + 2x_2 - x_3 = 4 \\ 2x_1 + 3x_2 + (m-4)x_3 = 1 \\ 3x_1 + 4x_2 + mx_3 = -2, \end{cases}$ has infinitely many solutions.

7. (L.O.1,L.O.2) Find m.

(A) $m = 7$ (B) $m = 9$ (C) $m = 8$ (D) None of the others (E) There is no such m

8. (L.O.1,L.O.2) Let (x_1, x_2, x_3) be a solution of the above system satisfying $x_1 = x_2$.

Find x_3.

(A) None of the others (B) $x_3 = -17/14$ (C) $x_3 = -18$ (D) $x_3 = -18/11$ (E) $x_3 = -16/13$

9. (L.O.1,L.O.2) In $\mathbb{R}_3$, let $x = (x_1, x_2, x_3)$ be a solution of the above system (a vector in $\mathbb{R}_3$), given that x is a linear combination of $M = \{(1, 1, 0), (1, 5, 4)\}$. Find x_3.

(A) $x_3 = -3/2$ (B) None of the others (C) $x_3 = -17/13$ (D) $x_3 = -18$ (E) $x_3 = -16/13$

Questions 10 through 11

In the vector space $\mathbb{R}_3$, given that $E = \{(1, 2, -1), (2, 5, 0), (-1, 0, m)\}$
and $F = \{(1, 0, 0), (1, 1, 0), (1, 1, 1)\}$ are two bases.

10. (L.O.1,L.O.2) Determine all values $m \in \mathbb{R}$ to make sure that E is a basis of $\mathbb{R}_3$.

(A) $m \neq -2$ (B) $m \neq -3$ (C) $m \neq 4$ (D) None of the others (E) $m \neq 5$

11. (L.O.1,L.O.2) Let $m = 0$. Find the coordinate vector of a vector u with respect to the basis F, given that the coordinate vector of u with respect to the basis E is $[u]_E = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$.

(A) $\begin{bmatrix} -10 \\ 23 \\ -14 \end{bmatrix}$ (B) $\begin{bmatrix} 0 \\ 1 \\ -13 \end{bmatrix}$ (C) $\begin{bmatrix} -13 \\ 14 \\ -6 \end{bmatrix}$ (D) $\begin{bmatrix} 2 \\ 10 \\ -13 \end{bmatrix}$ (E) None of the others

Questions 12 through 13

In the vector space $M_{3\times2}(R)$ (*the set of all real matrices size of* 3×2),

let $V = \{X \in M_{3\times2}(R) | XA = 0\}$ be a subspace of $M_{3\times2}(R)$ where A=$\begin{bmatrix} 1 & 2 \\ 3 & 6 \end{bmatrix}$.

12. (L.O.1,L.O.2) Find all values $m \in \mathbb{R}$ such that $\begin{bmatrix} -12 & 4 \\ m & 1 \\ -m-12 & 3 \end{bmatrix} \in V$.

Ⓐ $m = 3$ Ⓑ None of the others Ⓒ $m = 7$ Ⓓ $m = -3$ Ⓔ $m = -1$

13. (L.O.1,L.O.2) Find the dimension of V.

Ⓐ 6 Ⓑ 1 Ⓒ 5 Ⓓ None of the others Ⓔ 3

Questions 14 through 17

Assume that the input-output matrix of a economic system with 3 sectors: Industry, Agriculture, Service is given as below: $A = \begin{bmatrix} 0.2 & 0.2 & 0.05 \\ 0.1 & 0.15 & 0.1 \\ 0.15 & 0.2 & m \end{bmatrix}$, $m \in \mathbb{R}$. Let $B = 20A$. Given that the output values (in the order: Industry, Agriculture, Service) of each sector is 60, 40, 50 (billion USD).

14. (L.O.1,L.O.2) Find all values $m \in \mathbb{R}$ such that A is invertible.

Ⓐ $m \neq 11/80$ Ⓑ $m \neq 13/80$ Ⓒ $m \neq 9/80$ Ⓓ $m \neq 11/60$ Ⓔ $m \neq 3/20$

15. (L.O.1,L.O.2) What is the total value of products supplied by the industry to the agricultural sector in billions of USD?

Ⓐ 12 Ⓑ 12.5 Ⓒ 8 Ⓓ None of the others Ⓔ 9.5

16. (L.O.1,L.O.2) Find the total input values (billion USD) of the industry sector (*the total value that all sectors supply to the industry*)?

Ⓐ None of the others Ⓑ 18.0 Ⓒ 18.5 Ⓓ 27.0 Ⓔ 23.2

17. (L.O.1,L.O.2) For a sector, let out := the total output value and in := the total input value. Then the profit margin is defined by: $\boxed{\text{roe} = \frac{\text{out -in}}{\text{in}} \cdot 100\%}$. Compute the profit margin of the industry sector.

Ⓐ 33.22% Ⓑ 122.22% Ⓒ None of the others Ⓓ 99.22% Ⓔ 178.22%

Questions 18 through 20

Given the life span of a population of species is 9 months (after 9 months it will be sold). This population is divided into 3 classes: $0 < \text{age} \leq 3$ (I), $3 \leq \text{age} < 6$ (II), $6 \leq \text{age} < 9$ (III). The average numbers of offsprings produced in 3 months by the age class I, II, III are: 1.4, 8, 1, respectively. The probabilities of survival after 3 months of the age classes I and II are 80% and 90%, respectively. Suppose that at the initial moment, one has 100 individuals in age class I while there is not any individual in classes II and III.

18. (L.O.1,L.O.2) Find the Leslie matrix L.

Ⓐ $\begin{bmatrix} 1.4 & 8.0 & 1.0 \\ 0.8 & 0.0 & 0.0 \\ 0.0 & 0.9 & 0.0 \end{bmatrix}$ Ⓑ None of the others Ⓒ $\begin{bmatrix} 1.4 & 8.0 & 1.0 \\ 0.8 & 0.0 & 0.2 \\ 0.0 & 0.9 & 0.1 \end{bmatrix}$ Ⓓ $\begin{bmatrix} 0.0 & 8.0 & 1.0 \\ 0.8 & 0.0 & 0.0 \\ 0.0 & 0.9 & 1.0 \end{bmatrix}$

Ⓔ $\begin{bmatrix} 0.0 & 4.0 & 1.0 \\ 0.8 & 0.0 & 0.0 \\ 0.0 & 0.9 & 0.0 \end{bmatrix}$

19. (L.O.1,L.O.2) After 1 year, how many individuals are there in the age class III? (Round the result to the nearest integer).

Ⓐ 722.0 Ⓑ None of the others Ⓒ 615.0 Ⓓ 421.0 Ⓔ 602.0

20. (L.O.1,L.O.2) Each newborn individual will get a vaccination dose. After one year, what is the total number of doses will have been used? (Round the result to the neareast integer).

(A) 10736.0 (B) 15085.0 (C) 1367 (D) None of the others (E) 11559.0

Lecturer:	*Date*	**Approved by:**	*Date*
ThS. Phan Thị Khánh Vân	*Oct. 16^{th}, 2023*		*Oct. 16^{th}, 2023*

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UNIVERSITY OF TECHNOLOGY - VNUHCM
Faculty of Applied Science

Midterm Exam

Academic year	2023-2024	Semester	1
Exam date	October 28^{th}, 2023		
Course title	Linear Algebra		
Course ID	MT1007	Sheet code	**3136**
Duration	50 minutes	Shift	12:00

Score

Instructions to students:
- *This is a closed book exam. Only your calculator is allowed. Total available score: 10.*
- *At the beginning of the working time, you MUST fill in your full name and student ID on this question sheet. There are 20 questions on 4 pages.*
- *Do not round between steps. Round your final answers to 4 decimal places.*

Student's full name: . . . . Invigilator 1: . . . .

Student Id: . . . . Group: . . . . Invigilator 2: . . . .

1. (L.O.1,L.O.2) Given two matrices $A \in M_{2\times 6}$ and $B \in M_{4\times 5}$. Let X and Y satisfy $Y = AXB$. Then, the size of X is:

   (A) $X \in M_{7\times 3}$ (B) $X \in M_{5\times 4}$ (C) None of the others (D) $X \in M_{6\times 1}$ (E) $X \in M_{6\times 4}$

2. (L.O.1,L.O.2) Find the rank of the matrix $A = \begin{bmatrix} 1 & 2 & 0 & 2 \\ 2 & -1 & -1 & 2 \\ 1 & 2 & 2 & 1 \\ 1 & -1 & 3 & 4 \end{bmatrix}$.

   (A) 2 (B) 4 (C) 3 (D) 1 (E) None of the others

3. (L.O.1,L.O.2) Let A be the matrix $\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 3 & -2 & 2 \end{bmatrix}$ with the determinant 9. Evaluate $\det(2A^3)$.

   (A) 1728 (B) 5832 (C) None of the others (D) 4096 (E) 2744

4. (L.O.1,L.O.2) In the vector space $\mathbb{R}_3$, let m be a real number and

$$M = \{(1,2,1),(2,5,-2),(1,2,m),(3,7,-1)\}$$

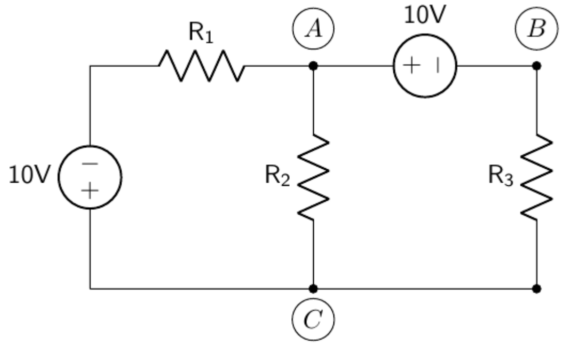
   be a vector set. Find m such that M is a spanning set of $\mathbb{R}_3$.

   (A) $m \neq 1$ (B) $m \neq 0$ (C) $m \neq -2$ (D) None of the others (E) $m \neq 3$

5. (L.O.1,L.O.2) In the vector space $P_2[x]$, let m be a real number and $M = \{2x^2+2x+1, x^2-2, 2x+m\}$ be a vector set. Find m such that M is linearly independent.

   (A) $m \neq 4$ (B) $m \neq 7$ (C) None of the others (D) $m \neq 5$ (E) $m \neq 2$

6. (L.O.1,L.O.2) A circuit is given in the following figure.



Given that $R_1 = 4\Omega$, $R_2 = 7\Omega$, $R_3 = 6\Omega$. Find the current which flows through R_1 (the result is rounded to 2 decimal digits.).

(A) 2.14 (B) 2.13 (C) 3.23 (D) None of the others (E) 1.82

Questions 7 through 9

Let m be a real number such that the system $\begin{cases} x_1 + 2x_2 - x_3 = 4 \\ 2x_1 + 3x_2 + (m-4)x_3 = 1 \\ 3x_1 + 4x_2 + mx_3 = -2, \end{cases}$ has infinitely many solutions.

7. (L.O.1,L.O.2) Find m.

   (A) There is no such m (B) $m = 7$ (C) $m = 9$ (D) $m = 8$ (E) None of the others

8. (L.O.1,L.O.2) Let (x_1, x_2, x_3) be a solution of the above system satisfying $x_1 = x_2$.

   Find x_3.

   (A) $x_3 = -16/13$ (B) None of the others (C) $x_3 = -18$ (D) $x_3 = -18/11$ (E) $x_3 = -17/14$

9. (L.O.1,L.O.2) In $\mathbb{R}_3$, let $x = (x_1, x_2, x_3)$ be a solution of the above system (a vector in $\mathbb{R}_3$), given that x is a linear combination of $M = \{(1,1,0), (1,5,4)\}$. Find x_3.

   (A) $x_3 = -17/13$ (B) None of the others (C) $x_3 = -3/2$ (D) $x_3 = -18$ (E) $x_3 = -16/13$

Questions 10 through 11

In the vector space $\mathbb{R}_3$, given that $E = \{(1,2,2), (2,5,3), (2,3,m)\}$

and $F = \{(1,0,0), (1,1,0), (1,1,1)\}$ are two bases.

10. (L.O.1,L.O.2) Determine all values $m \in \mathbb{R}$ to make sure that E is a basis of $\mathbb{R}_3$.

    (A) $m \neq -2$ (B) None of the others (C) $m \neq 4$ (D) $m \neq 5$ (E) $m \neq 1$

11. (L.O.1,L.O.2) Let $m = 0$. Find the coordinate vector of a vector u with respect to the basis F, given that the coordinate vector of u with respect to the basis E is $[u]_E = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$.

    (A) $\begin{bmatrix} -13 \\ 14 \\ -3 \end{bmatrix}$ (B) $\begin{bmatrix} 11 \\ 10 \\ -13 \end{bmatrix}$ (C) $\begin{bmatrix} -10 \\ 23 \\ -5 \end{bmatrix}$ (D) None of the others (E) $\begin{bmatrix} 0 \\ 1 \\ -7 \end{bmatrix}$

Questions 12 through 13

In the vector space $M_{3\times 2}(R)$ (*the set of all real matrices size of* 3×2),

let $V = \{X \in M_{3\times 2}(R) | XA = 0\}$ be a subspace of $M_{3\times 2}(R)$ where A=$\begin{bmatrix} 1 & 2 \\ 3 & 6 \end{bmatrix}$.

12. (L.O.1,L.O.2) Find all values $m \in \mathbb{R}$ such that $\begin{bmatrix} -12 & 4 \\ m & 1 \\ -m-12 & 3 \end{bmatrix} \in V$.

Ⓐ $m = 7$ Ⓑ $m = 3$ Ⓒ None of the others Ⓓ $m = -1$ Ⓔ $m = -3$

13. (L.O.1,L.O.2) Find the dimension of V.

Ⓐ 1 Ⓑ 3 Ⓒ 5 Ⓓ None of the others Ⓔ 6

Questions 14 through 17

Assume that the input-output matrix of a economic system with 3 sectors: Industry, Agriculture, Service is given as below: $A = \begin{bmatrix} 0.2 & 0.2 & 0.05 \\ 0.1 & 0.15 & 0.1 \\ 0.15 & 0.2 & m \end{bmatrix}$, $m \in \mathbb{R}$. Let $B = 20A$. Given that the output values (in the order: Industry, Agriculture, Service) of each sector is 60, 40, 50 (billion USD).

14. (L.O.1,L.O.2) Find all values $m \in \mathbb{R}$ such that A is invertible.

Ⓐ $m \neq 3/20$ Ⓑ $m \neq 11/60$ Ⓒ $m \neq 11/80$ Ⓓ $m \neq 13/80$ Ⓔ $m \neq 9/80$

15. (L.O.1,L.O.2) What is the total value of products supplied by the industry to the agricultural sector in billions of USD?

Ⓐ 12 Ⓑ 12.5 Ⓒ None of the others Ⓓ 8 Ⓔ 9.5

16. (L.O.1,L.O.2) Find the total input values (billion USD) of the industry sector (*the total value that all sectors supply to the industry*)?

Ⓐ 27.0 Ⓑ 23.2 Ⓒ 18.0 Ⓓ None of the others Ⓔ 18.5

17. (L.O.1,L.O.2) For a sector, let out := the total output value and in := the total input value. Then the profit margin is defined by: $\boxed{\text{roe} = \dfrac{\text{out -in}}{\text{in}} \cdot 100\%}$. Compute the profit margin of the industry sector.

Ⓐ 33.22% Ⓑ 178.22% Ⓒ 99.22% Ⓓ None of the others Ⓔ 122.22%

Questions 18 through 20

Given the life span of a population of species is 9 months (after 9 months it will be sold). This population is divided into 3 classes: $0 < \text{age} \leq 3$ (I), $3 \leq \text{age} < 6$ (II), $6 \leq \text{age} < 9$ (III). The average numbers of offsprings produced in 3 months by the age class I, II, III are: 1.4, 8, 1, respectively. The probabilities of survival after 3 months of the age classes I and II are 80% and 90%, respectively. Suppose that at the initial moment, one has 100 individuals in age class I while there is not any individual in classes II and III.

18. (L.O.1,L.O.2) Find the Leslie matrix L.

Ⓐ $\begin{bmatrix} 0.0 & 8.0 & 1.0 \\ 0.8 & 0.0 & 0.0 \\ 0.0 & 0.9 & 1.0 \end{bmatrix}$ Ⓑ $\begin{bmatrix} 1.4 & 8.0 & 1.0 \\ 0.8 & 0.0 & 0.0 \\ 0.0 & 0.9 & 0.0 \end{bmatrix}$ Ⓒ $\begin{bmatrix} 0.0 & 4.0 & 1.0 \\ 0.8 & 0.0 & 0.0 \\ 0.0 & 0.9 & 0.0 \end{bmatrix}$ Ⓓ None of the others

Ⓔ $\begin{bmatrix} 1.4 & 8.0 & 1.0 \\ 0.8 & 0.0 & 0.2 \\ 0.0 & 0.9 & 0.1 \end{bmatrix}$

19. (L.O.1,L.O.2) After 1 year, how many individuals are there in the age class III? (Round the result to the nearest integer).

Ⓐ 722.0 Ⓑ 421.0 Ⓒ None of the others Ⓓ 602.0 Ⓔ 615.0

20. (L.O.1,L.O.2) Each newborn individual will get a vaccination dose. After one year, what is the total number of doses will have been used? (Round the result to the neareast integer).

    (A) 15085.0 (B) 1367 (C) None of the others (D) 10736.0 (E) 11559.0

Lecturer: ThS. Phan Thị Khánh Vân	*Date* *Oct. 16th, 2023*	**Approved by:**	*Date* *Oct. 16th, 2023*

. . . .

UNIVERSITY OF TECHNOLOGY - VNUHCM Faculty of Applied Science	**Midterm Exam**	Academic year	2023-2024	Semester	1
		Exam date	October 28th, 2023		
	Course title	Linear Algebra		**Score**	
	Course ID	MT1007	Sheet code	**3137**	
	Duration	50 minutes	Shift	12:00	

Instructions to students:
- *This is a closed book exam. Only your calculator is allowed. Total available score: 10.*
- *At the beginning of the working time, you MUST fill in your full name and student ID on this question sheet. There are 20 questions on 4 pages.*
- *Do not round between steps. Round your final answers to 4 decimal places.*

Student's full name:	Invigilator 1: . . . .
Student Id: . . . . Group:	Invigilator 2: . . . .

1. (L.O.1,L.O.2) Given two matrices $A \in M_{2\times 6}$ and $B \in M_{4\times 5}$. Let X and Y satisfy $Y = AXB$. Then, the size of X is:

   (A) $X \in M_{6\times 4}$ (B) $X \in M_{5\times 4}$ (C) $X \in M_{6\times 1}$ (D) $X \in M_{7\times 3}$ (E) None of the others

2. (L.O.1,L.O.2) Find the rank of the matrix $A = \begin{bmatrix} 1 & 2 & 0 & 2 \\ 2 & -1 & -1 & 2 \\ 1 & 2 & 2 & 1 \\ 1 & -1 & 3 & 4 \end{bmatrix}$.

   (A) None of the others (B) 1 (C) 4 (D) 3 (E) 2

3. (L.O.1,L.O.2) Let A be the matrix $\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 3 & 0 & 2 \end{bmatrix}$ with the determinant 5. Evaluate $\det(2A^3)$.

   (A) 64 (B) 1000 (C) None of the others (D) 512 (E) 216

4. (L.O.1,L.O.2) In the vector space $\mathbb{R}_3$, let m be a real number and

$$M = \{(1, 2, 1), (2, 5, 0), (1, 2, m), (3, 7, 1)\}$$

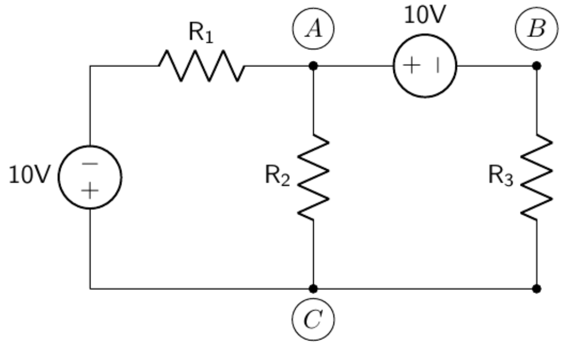
   be a vector set. Find m such that M is a spanning set of $\mathbb{R}_3$.

   (A) $m \neq 0$ (B) $m \neq 1$ (C) None of the others (D) $m \neq -2$ (E) $m \neq 3$

5. (L.O.1,L.O.2) In the vector space $P_2[x]$, let m be a real number and $M = \{2x^2 + 2x + 1, x^2, 2x + m\}$ be a vector set. Find m such that M is linearly independent.

   (A) $m \neq 1$ (B) None of the others (C) $m \neq -2$ (D) $m \neq 3$ (E) $m \neq 0$

6. (L.O.1,L.O.2) A circuit is given in the following figure.



Given that $R_1 = 4\Omega$, $R_2 = 7\Omega$, $R_3 = 6\Omega$. Find the current which flows through R_1 (the result is rounded to 2 decimal digits.).

Ⓐ 1.82 Ⓑ 3.23 Ⓒ 2.13 Ⓓ None of the others Ⓔ 2.14

Questions 7 through 9

Let m be a real number such that the system $\begin{cases} x_1 + 2x_2 - x_3 = 4 \\ 2x_1 + 3x_2 + (m-4)x_3 = 1 \\ 3x_1 + 4x_2 + mx_3 = -2, \end{cases}$ has infinitely many solutions.

7. (L.O.1,L.O.2) Find m.

   Ⓐ There is no such m Ⓑ $m = 8$ Ⓒ None of the others Ⓓ $m = 7$ Ⓔ $m = 9$

8. (L.O.1,L.O.2) Let (x_1, x_2, x_3) be a solution of the above system satisfying $x_1 = x_2$.

   Find x_3.

   Ⓐ $x_3 = -16/13$ Ⓑ $x_3 = -17/14$ Ⓒ None of the others Ⓓ $x_3 = -18$ Ⓔ $x_3 = -18/11$

9. (L.O.1,L.O.2) In $\mathbb{R}_3$, let $x = (x_1, x_2, x_3)$ be a solution of the above system (a vector in $\mathbb{R}_3$), given that x is a linear combination of $M = \{(1,1,0),(1,5,4)\}$. Find x_3.

   Ⓐ $x_3 = -16/13$ Ⓑ $x_3 = -17/13$ Ⓒ $x_3 = -18$ Ⓓ None of the others Ⓔ $x_3 = -3/2$

Questions 10 through 11

In the vector space $\mathbb{R}_3$, given that $E = \{(1,2,2),(2,5,3),(2,3,m)\}$

and $F = \{(1,0,0),(1,1,0),(1,1,1)\}$ are two bases.

10. (L.O.1,L.O.2) Determine all values $m \in \mathbb{R}$ to make sure that E is a basis of $\mathbb{R}_3$.

    Ⓐ $m \neq 1$ Ⓑ $m \neq -2$ Ⓒ None of the others Ⓓ $m \neq 5$ Ⓔ $m \neq 4$

11. (L.O.1,L.O.2) Let $m = 0$. Find the coordinate vector of a vector u with respect to the basis F, given that the coordinate vector of u with respect to the basis E is $[u]_E = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$.

    Ⓐ $\begin{bmatrix} -13 \\ 14 \\ -3 \end{bmatrix}$ Ⓑ $\begin{bmatrix} -10 \\ 23 \\ -5 \end{bmatrix}$ Ⓒ $\begin{bmatrix} 11 \\ 10 \\ -13 \end{bmatrix}$ Ⓓ None of the others Ⓔ $\begin{bmatrix} 0 \\ 1 \\ -7 \end{bmatrix}$

Questions 12 through 13

In the vector space $M_{3\times 2}(R)$ (*the set of all real matrices size of* 3×2),

let $V = \{X \in M_{3\times 2}(R) | XA = 0\}$ be a subspace of $M_{3\times 2}(R)$ where A=$\begin{bmatrix} 1 & 2 \\ 3 & 6 \end{bmatrix}$.

12. (L.O.1,L.O.2) Find all values $m \in \mathbb{R}$ such that $\begin{bmatrix} -12 & 4 \\ m & 1 \\ -m-12 & 3 \end{bmatrix} \in V$.

   Ⓐ $m = -3$ Ⓑ $m = 3$ Ⓒ None of the others Ⓓ $m = 7$ Ⓔ $m = -1$

13. (L.O.1,L.O.2) Find the dimension of V.

   Ⓐ None of the others Ⓑ 6 Ⓒ 1 Ⓓ 3 Ⓔ 5

Questions 14 through 17

Assume that the input-output matrix of a economic system with 3 sectors: Industry, Agriculture, Service is given as below: $A = \begin{bmatrix} 0.2 & 0.2 & 0.05 \\ 0.1 & 0.15 & 0.1 \\ 0.15 & 0.2 & m \end{bmatrix}$, $m \in \mathbb{R}$. Let $B = 20A$. Given that the output values (in the order: Industry, Agriculture, Service) of each sector is 60, 40, 50 (billion USD).

14. (L.O.1,L.O.2) Find all values $m \in \mathbb{R}$ such that A is invertible.

   Ⓐ $m \neq 13/80$ Ⓑ $m \neq 9/80$ Ⓒ $m \neq 11/60$ Ⓓ $m \neq 11/80$ Ⓔ $m \neq 3/20$

15. (L.O.1,L.O.2) What is the total value of products supplied by the industry to the agricultural sector in billions of USD?

   Ⓐ 8 Ⓑ 12 Ⓒ 9.5 Ⓓ None of the others Ⓔ 12.5

16. (L.O.1,L.O.2) Find the total input values (billion USD) of the industry sector (*the total value that all sectors supply to the industry*)?

   Ⓐ 18.5 Ⓑ 27.0 Ⓒ 18.0 Ⓓ None of the others Ⓔ 23.2

17. (L.O.1,L.O.2) For a sector, let out := the total output value and in := the total input value. Then the profit margin is defined by: $\boxed{\text{roe} = \dfrac{\text{out -in}}{\text{in}} \cdot 100\%}$. Compute the profit margin of the industry sector.

   Ⓐ 99.22% Ⓑ 122.22% Ⓒ None of the others Ⓓ 178.22% Ⓔ 33.22%

Questions 18 through 20

Given the life span of a population of species is 9 months (after 9 months it will be sold). This population is divided into 3 classes: $0 < \text{age} \leq 3$ (I), $3 \leq \text{age} < 6$ (II), $6 \leq \text{age} < 9$ (III). The average numbers of offsprings produced in 3 months by the age class I, II, III are: 1.4, 8, 1, respectively. The probabilities of survival after 3 months of the age classes I and II are 80% and 90%, respectively. Suppose that at the initial moment, one has 100 individuals in age class I while there is not any individual in classes II and III.

18. (L.O.1,L.O.2) Find the Leslie matrix L.

   Ⓐ $\begin{bmatrix} 1.4 & 8.0 & 1.0 \\ 0.8 & 0.0 & 0.2 \\ 0.0 & 0.9 & 0.1 \end{bmatrix}$ Ⓑ $\begin{bmatrix} 0.0 & 4.0 & 1.0 \\ 0.8 & 0.0 & 0.0 \\ 0.0 & 0.9 & 0.0 \end{bmatrix}$ Ⓒ $\begin{bmatrix} 0.0 & 8.0 & 1.0 \\ 0.8 & 0.0 & 0.0 \\ 0.0 & 0.9 & 1.0 \end{bmatrix}$ Ⓓ None of the others Ⓔ $\begin{bmatrix} 1.4 & 8.0 & 1.0 \\ 0.8 & 0.0 & 0.0 \\ 0.0 & 0.9 & 0.0 \end{bmatrix}$

19. (L.O.1,L.O.2) After 1 year, how many individuals are there in the age class III? (Round the result to the nearest integer).

   Ⓐ 722.0 Ⓑ 602.0 Ⓒ 615.0 Ⓓ None of the others Ⓔ 421.0

20. (L.O.1,L.O.2) Each newborn individual will get a vaccination dose. After one year, what is the total number of doses will have been used? (Round the result to the neareast integer).

    Ⓐ None of the others Ⓑ 11559.0 Ⓒ 1367 Ⓓ 10736.0 Ⓔ 15085.0

Lecturer:	*Date*	**Approved by:**	*Date*
ThS. Phan Thị Khánh Vân	*Oct. 16th, 2023*		*Oct. 16th, 2023*

UNIVERSITY OF TECHNOLOGY - VNUHCM **Faculty of Applied Science**	**Midterm Exam**	Academic year	2023-2024	Semester	1
		Exam date	October 28$^{\text{th}}$, 2023		
	Course title	Linear Algebra		**Score**	
	Course ID	MT1007	Sheet code	**3138**	
	Duration	50 minutes	Shift	12:00	

Instructions to students:

- *This is a closed book exam. Only your calculator is allowed. Total available score: 10.*
- *At the beginning of the working time, you MUST fill in your full name and student ID on this question sheet. There are 20 questions on 4 pages.*
- *Do not round between steps. Round your final answers to 4 decimal places.*

Student's full name: .... Invigilator 1: ....

Student Id: .... Group: .... Invigilator 2: ....

1. (L.O.1,L.O.2) Given two matrices $A \in M_{2\times 6}$ and $B \in M_{7\times 5}$. Let X and Y satisfy $Y = AXB$. Then, the size of X is:

   (A) $X \in M_{6\times 4}$ (B) $X \in M_{5\times 7}$ (C) None of the others (D) $X \in M_{6\times 7}$ (E) $X \in M_{7\times 6}$

2. (L.O.1,L.O.2) Find the rank of the matrix $A = \begin{bmatrix} 1 & 2 & 0 & 2 \\ 2 & -1 & -1 & -1 \\ 1 & 2 & -1 & 1 \\ 1 & -1 & 3 & 4 \end{bmatrix}$.

   (A) 4 (B) 3 (C) None of the others (D) 2 (E) 1

3. (L.O.1,L.O.2) Let A be the matrix $\begin{bmatrix} 1 & 1 & 1 \\ 1 & -1 & 3 \\ 3 & 2 & 2 \end{bmatrix}$ with the determinant 4. Evaluate $\det(2A^3)$.

   (A) None of the others (B) 8 (C) 512 (D) 64 (E) 216

4. (L.O.1,L.O.2) In the vector space $\mathbb{R}_3$, let m be a real number and

$$M = \{(1,-1,1),(2,-1,2),(1,2,m),(3,-2,3)\}$$

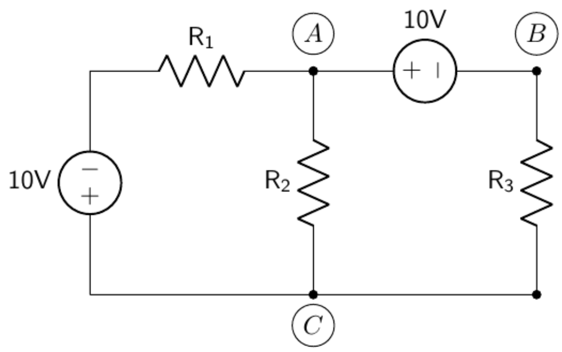
   be a vector set. Find m such that M is a spanning set of $\mathbb{R}_3$.

   (A) $m \neq 1$ (B) $m \neq 3$ (C) None of the others (D) $m \neq -2$ (E) $m \neq 0$

5. (L.O.1,L.O.2) In the vector space $P_2[x]$, let m be a real number and $M = \{-x^2+2x+1, x^2+2, 2x+m\}$ be a vector set. Find m such that M is linearly independent.

   (A) $m \neq 0$ (B) $m \neq 3$ (C) None of the others (D) $m \neq 2$ (E) $m \neq 5$

6. (L.O.1,L.O.2) A circuit is given in the following figure.



Given that $R_1 = 4\Omega$, $R_2 = 7\Omega$, $R_3 = 6\Omega$. Find the current which flows through R_1 (the result is rounded to 2 decimal digits.).

(A) 2.13 (B) None of the others (C) 2.14 (D) 1.82 (E) 3.23

Questions 7 through 9

Let m be a real number such that the system $\begin{cases} x_1 + 2x_2 - x_3 = 4 \\ 2x_1 + 3x_2 + (m-4)x_3 = 4 \\ 3x_1 + 4x_2 + mx_3 = 4, \end{cases}$ has infinitely many solutions.

7. (L.O.1,L.O.2) Find m.

   (A) $m = 9$ (B) None of the others (C) There is no such m (D) $m = 8$ (E) $m = 7$

8. (L.O.1,L.O.2) Let (x_1, x_2, x_3) be a solution of the above system satisfying $x_1 = x_2$.

   Find x_3.

   (A) $x_3 = -9/11$ (B) $x_3 = -7/13$ (C) $x_3 = -9$ (D) None of the others (E) $x_3 = -4/7$

9. (L.O.1,L.O.2) In $\mathbb{R}_3$, let $x = (x_1, x_2, x_3)$ be a solution of the above system (a vector in $\mathbb{R}_3$), given that x is a linear combination of $M = \{(1,1,0), (1,5,4)\}$. Find x_3.

   (A) None of the others (B) $x_3 = -7/13$ (C) $x_3 = -8/13$ (D) $x_3 = -3/4$ (E) $x_3 = -9$

Questions 10 through 11

In the vector space $\mathbb{R}_3$, given that $E = \{(1,2,-1), (2,5,0), (-1,0,m)\}$

and $F = \{(1,0,0), (1,1,0), (1,1,1)\}$ are two bases.

10. (L.O.1,L.O.2) Determine all values $m \in \mathbb{R}$ to make sure that E is a basis of $\mathbb{R}_3$.

    (A) $m \neq 4$ (B) $m \neq -2$ (C) None of the others (D) $m \neq -6$ (E) $m \neq 5$

11. (L.O.1,L.O.2) Let $m = 0$. Find the coordinate vector of a vector u with respect to the basis F, given that the coordinate vector of u with respect to the basis E is $[u]_E = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$.

    (A) $\begin{bmatrix} 0 \\ 1 \\ -13 \end{bmatrix}$ (B) None of the others (C) $\begin{bmatrix} -10 \\ 23 \\ -14 \end{bmatrix}$ (D) $\begin{bmatrix} 2 \\ 10 \\ -13 \end{bmatrix}$ (E) $\begin{bmatrix} -13 \\ 14 \\ -6 \end{bmatrix}$

Questions 12 through 13

In the vector space $M_{3\times 2}(R)$ (*the set of all real matrices size of* 3×2),

let $V = \{X \in M_{3\times 2}(R) | XA = 0\}$ be a subspace of $M_{3\times 2}(R)$ where A$=\begin{bmatrix} 1 & 2 \\ 3 & 6 \end{bmatrix}$.

12. (L.O.1,L.O.2) Find all values $m \in \mathbb{R}$ such that $\begin{bmatrix} -12 & 4 \\ m & 4 \\ -m-12 & 0 \end{bmatrix} \in V$.

   Ⓐ None of the others Ⓑ $m = -6$ Ⓒ $m = 7$ Ⓓ $m = -10$ Ⓔ $m = -12$

13. (L.O.1,L.O.2) Find the dimension of V.

   Ⓐ 1 Ⓑ 3 Ⓒ None of the others Ⓓ 6 Ⓔ 5

Questions 14 through 17

Assume that the input-output matrix of a economic system with 3 sectors: Industry, Agriculture, Service is given as below: $A = \begin{bmatrix} 0.2 & 0.2 & 0.2 \\ 0.1 & 0.15 & 0.1 \\ 0.15 & 0.2 & m \end{bmatrix}$, $m \in \mathbb{R}$. Let $B = 20A$. Given that the output values (in the order: Industry, Agriculture, Service) of each sector is 60, 40, 50 (billion USD).

14. (L.O.1,L.O.2) Find all values $m \in \mathbb{R}$ such that A is invertible.

   Ⓐ $m \neq 3/20$ Ⓑ $m \neq 1/3$ Ⓒ $m \neq 21/80$ Ⓓ $m \neq 1/5$ Ⓔ $m \neq 1/4$

15. (L.O.1,L.O.2) What is the total value of products supplied by the industry to the agricultural sector in billions of USD?

   Ⓐ None of the others Ⓑ 12.5 Ⓒ 12 Ⓓ 8 Ⓔ 9.5

16. (L.O.1,L.O.2) Find the total input values (billion USD) of the industry sector (*the total value that all sectors supply to the industry*)?

   Ⓐ 27.0 Ⓑ None of the others Ⓒ 18.0 Ⓓ 23.2 Ⓔ 18.5

17. (L.O.1,L.O.2) For a sector, let out := the total output value and in := the total input value. Then the profit margin is defined by: $\boxed{\text{roe} = \frac{\text{out -in}}{\text{in}} \cdot 100\%}$. Compute the profit margin of the industry sector.

   Ⓐ 178.22% Ⓑ 99.22% Ⓒ 122.22% Ⓓ 33.22% Ⓔ None of the others

Questions 18 through 20

Given the life span of a population of species is 9 months (after 9 months it will be sold). This population is divided into 3 classes: $0 < \text{age} \leq 3$ (I), $3 \leq \text{age} < 6$ (II), $6 \leq \text{age} < 9$ (III). The average numbers of offsprings produced in 3 months by the age class I, II, III are: 1.4, 8, 4, respectively. The probabilities of survival after 3 months of the age classes I and II are 80% and 90%, respectively. Suppose that at the initial moment, one has 100 individuals in age class I while there is not any individual in classes II and III.

18. (L.O.1,L.O.2) Find the Leslie matrix L.

   Ⓐ $\begin{bmatrix} 1.4 & 8.0 & 4.0 \\ 0.8 & 0.0 & 0.2 \\ 0.0 & 0.9 & 0.1 \end{bmatrix}$ Ⓑ $\begin{bmatrix} 0.0 & 4.0 & 4.0 \\ 0.8 & 0.0 & 0.0 \\ 0.0 & 0.9 & 0.0 \end{bmatrix}$ Ⓒ $\begin{bmatrix} 0.0 & 8.0 & 4.0 \\ 0.8 & 0.0 & 0.0 \\ 0.0 & 0.9 & 1.0 \end{bmatrix}$ Ⓓ None of the others Ⓔ $\begin{bmatrix} 1.4 & 8.0 & 4.0 \\ 0.8 & 0.0 & 0.0 \\ 0.0 & 0.9 & 0.0 \end{bmatrix}$

19. (L.O.1,L.O.2) After 1 year, how many individuals are there in the age class III? (Round the result to the nearest integer).

   Ⓐ 602.0 Ⓑ None of the others Ⓒ 615.0 Ⓓ 421.0 Ⓔ 722.0

20. (L.O.1,L.O.2) Each newborn individual will get a vaccination dose. After one year, what is the total number of doses will have been used? (Round the result to the neareast integer).

Ⓐ 1367 Ⓑ None of the others Ⓒ 12380.0 Ⓓ 15906.0 Ⓔ 11557.0

Answers Sheet

Question sheet code 3131:
1 B. 2 E. 3 C. 4 C. 5 B. 6 A. 7 D. 8 B. 9 D. 10 C. 11 B.
12 A. 13 D. 14 A. 15 A. 16 E. 17 C. 18 A. 19 A. 20 D.

Question sheet code 3132:
1 D. 2 E. 3 D. 4 E. 5 A. 6 E. 7 A. 8 B. 9 D. 10 C. 11 A.
12 B. 13 B. 14 A. 15 D. 16 E. 17 E. 18 D. 19 D. 20 B.

Question sheet code 3133:
1 E. 2 D. 3 B. 4 A. 5 D. 6 D. 7 E. 8 E. 9 D. 10 A. 11 B.
12 E. 13 C. 14 D. 15 E. 16 D. 17 D. 18 D. 19 B. 20 A.

Question sheet code 3134:
1 B. 2 C. 3 C. 4 A. 5 C. 6 E. 7 D. 8 E. 9 B. 10 A. 11 B.
12 D. 13 C. 14 B. 15 E. 16 A. 17 E. 18 D. 19 E. 20 C.

Question sheet code 3135:
1 C. 2 B. 3 C. 4 B. 5 C. 6 D. 7 A. 8 B. 9 C. 10 E. 11 D.
12 D. 13 E. 14 C. 15 C. 16 D. 17 B. 18 A. 19 E. 20 E.

Question sheet code 3136:
1 E. 2 B. 3 B. 4 A. 5 D. 6 B. 7 B. 8 E. 9 A. 10 D. 11 B.
12 E. 13 B. 14 E. 15 D. 16 A. 17 E. 18 B. 19 D. 20 E.

Question sheet code 3137:
1 A. 2 C. 3 B. 4 B. 5 A. 6 C. 7 D. 8 B. 9 B. 10 D. 11 C.
12 A. 13 D. 14 B. 15 A. 16 B. 17 B. 18 E. 19 B. 20 B.

Question sheet code 3138:
1 D. 2 A. 3 C. 4 A. 5 B. 6 A. 7 E. 8 E. 9 C. 10 E. 11 D.
12 E. 13 B. 14 A. 15 D. 16 A. 17 C. 18 E. 19 A. 20 C.

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