

ADAPTIVE CONTROL IN PRESENCE OF INPUT SATURATION CONSTRAINT AN INPUT-OUTPUT APPROACH

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Abstract. The problem of controlling time-invariant linear systems subject to parametric uncertainty and input saturation constraint is considered. A cautious combination of a pole placement control design and a least squares parameter adaptation is used to this end. The global stability and convergence analysis of the involved control system is carried out by investigating the l_2 -stability of an adequate *nonlinear feedback* system. The stability condition turns out to be positive-realness of a transfer function involving the (estimated) model and the specified closed-loop poles. Such a stability requirement is shown to be satisfied using a suitable modification of the indirect pole placement adaptive control, namely

- i) a suitable self-excitation capability ensuring the asymptotic admissibility of the estimated plant model, e.g. the asymptotic stability.
- ii) an appropriate adaptation of the regulator parameters as well as the desired closed-loop poles.

Besides the closed-loop global stability, the proposed control algorithm guarantees asymptotic tracking of constant reference sequences and enforces the input control to stop saturating asymptotically.

Key Words. Pole placement, Adaptive control, Input constraints, Stability, Convergence.

1. INTRODUCTION

Input saturation constraints may arise from physical limitations as those imposed to avoid damage to actuators of the feedback systems. It is widely recognized that the performances of the resulting regulators depend, partly but heavily, upon the way the input constraints are accounted for in the regulator design, see for instance [1] to [7].

The problem of (null) controllability and global stability of linear systems subject to input saturation constraint has received much interest over the last years as pointed out in [8] to [12]. It was shown that global stability, or the possibility to drive the output of a constrained system to zero from any initial conditions, can be achieved when the system is not strictly unstable, i.e. has no pole strictly outside the stability region. This was generally established using discontinuous or nonlinear control laws. Global stabilisation of a constrained linear system using standard linear feedbacks has been particularly investigated in [13] to [16].

A parallel research activity has focused on the problem of adaptively stabilizing linear systems which are subject to parameter uncertainty and control input constraint, see for instance [17] to [24]. In [17], [18] and [20], it was shown that a direct adaptive one-step ahead control law converges to the underlying linear control law corresponding to the true plant parameters, when applied to an asymptotically stable or a type-1 linear system. The question that was not

answered is whether the involved linear control law actually stabilizes the constrained system. In [21] and [24], it was shown that a pole placement indirect adaptive control law yields a closed-loop system with bounded sequences, when applied to type-k systems. However, the involved adaptive controller has not shown to have those fundamental performances as output asymptotic tracking of a reference sequence or the possibility of the plant input to stop saturating. Such an issue has been solved using direct and indirect adaptive pole placement controllers in [22] and [23]. It was shown that the main design feature, from the performance viewpoint, is the choice of the closed-loop poles. More specifically, in the case of a constant reference sequence, the control input to stop saturating and the tracking error vanish asymptotically, provided that the closed-loop poles are not faster than those of the plant.

In this paper, the importance of the closed-loop poles assignment in dealing with the constraint issue is further emphasized following an *input-output* approach. More precisely, we consider an indirect adaptive pole placement controller and show that investigating global stability and convergence of the resulting closed-loop system amounts to analyzing l_2 -stability of a *non-linear feedback* system. Then, applying the well known *circle criterion* [25], the stability condition turns out to be the positive-realness of a transfer function involving the poles of the (estimated) plant model and those specified for the closed-loop system. Accordingly, the former should be stable while the latter should belong to a specified neighbourhood of the former. Of particular interest, the closed-loop poles are allowed to be *faster* than those of the estimated model. In order to satisfy these stability requirements, two major modifications in the standard adaptive pole placement controller are introduced, namely

- i) a self-excitation capability which enforces the estimated model to be asymptotically stable (and controllable),
- ii) an appropriate tuning of the pole placement control objective, i.e. the desired closed-loop poles, taking into account the estimated plant model variations. Such a tuning ensure a positive-realness condition that should be satisfied all time.

The resulting adaptive controller guarantees global boundedness of all the closed-loop sequences. Furthermore, in the case of a constant reference sequence, it ensures global convergence of the plant output to the reference value, while enforcing the control input to stop saturating asymptotically.

The paper is organized as follows. In section 2, we state The control problem is stated in section 2. The systems to be controlled and the desired control objectives are described, and the existence of a solution is investigated. The adaptive

control design is presented in section 3, it includes successively a parameter identification algorithm, an self exciting adaptive control law and a regulator parameter adaptation law. In section 4, the proposed adaptive controller is shown to meet its objectives. A conclusion end the paper.

2. FORMULATION OF THE CONTROL PROBLEM

2.1. Plant model description

We are intrested in controlling SISO linear systems whose input-output behavior is described by the following difference equation

$$\begin{aligned} A(q^{-1}) y(t) &= B(q^{-1}) u(t) \\ u(t) &= \text{sat}(v(t)) := \text{sign}(v(t)) \cdot \min\{u_M, |v(t)|\} \end{aligned} \quad (2.1)$$

with

$$\begin{aligned} A(q^{-1}) &= 1 + a_1 q^{-1} + \dots + a_n q^{-n} \\ B(q^{-1}) &= b_1 q^{-1} + \dots + b_n q^{-n} \end{aligned}$$

where t denotes sampling time instant, $v(t)$, $u(t)$ and $y(t)$ are respectively the computed input, the applied input and the output, u_M denotes the maximal value allowed for the control magnitude and q^{-1} is the backward-shift operator .

Thus, the system is composed of a linear part (plant dynamics) and a static nonlinearity reduced in the present case to a simple saturation. Many practical situations can be described with an admissible accuracy by such a representation with different types of nonlinearities, namely hysteresis and dead-zones. The following assumptions complete the system description

- A1.** n is known.
- A2.** $A(q^{-1})$ is Hurwitz
- A3.** $A(q^{-1})$ and $B(q^{-1})$ are coprime.
- A4.** $B(q^{-1})$ and $D(q^{-1}) := 1 - q^{-1}$ are coprime.

Assumptions A1 is standard, it stipulates that a high bound on the plant model is a priori known. From A2-A3, it follows that the system linear part is exponentially stable and controllable. The plant stability condition A2 is quite standard in the present context as it is attempting to get a system output regulation in presence of input constraint using a regulator with an integral action. Finally, A3 implies that the plant static gain $B(1)/A(1)$ is nonzero, this will make it possible for the output to track a nonzero reference. Apart from these assumptions the linear part of the system could be inversely unstable phase, some zeros of $B(q^{-1})$ could be outside the unit disc, the parameters (a_i , b_i) are not a priori known, and the system delay is not exactly known ($b_1 = \dots = b_d = 0$ for some $d < 0$).

The presence of the saturation implies some usual properties of the system to be controlled as pointed out by the following result

Proposition 2.1.

- a) $|u(t)| \leq |v(t)|$ and $|u(t)| \leq u_M$ for all t .
- b) $\{y(t)\} \in l_\infty$
- c) $\text{sign}(u(t)) = \text{sign}(v(t))$ for all t and hence $|v(t)| - |u(t)| = |v(t) - u(t)|$ for all t .
- d) If $|v(t) - u(t)| \neq 0$ for all t , then $|u(t)| = u_M$.
- e) i) For any reals v_1 and v_2 one has
 $|\text{sat}(v_1) - \text{sat}(v_2)| \leq |v_1 - v_2|$
 $\text{sign}[\text{sat}(v_1) - \text{sat}(v_2)] = \text{sign}[v_1 - v_2]$
- ii) For all t one has
 $|u(t) - u(t-1)| \leq |v(t) - v(t-1)|$
 $\text{sign}[u(t) - u(t-1)] = \text{sign}[v(t) - v(t-1)]$

An outline of the proof is given in the full version of the paper .

2.2. Control objective statement

Given the considered system and a reference sequence $\{y^*(t)\}$ satisfying

$$\sup_{0 < t < \infty} |y^*(t)| < \frac{|B(1)|}{|A(1)|} u_M \quad (2.2)$$

one aim at designing a controller that should be able to achieve the following objectives, in spite of the parameter uncertainty.

- a) All the closed-loop system signals are bounded
- b) if $\{y^*(t) - y^*(t-1)\} \in l_2$ then $\{y(t) - y^*(t)\}$ converges to zero

Notice that inequality (2.2) ensures the compatibility of the tracking objective with the constraint imposed on the control magnitude.

2.3 Existence of a linear regulator

In this section, it will show that the abovesly stated control problem has a solution in the case of known plant parameters. More precisely, there exists a linear controller meeting the desired objective despite the control constraint as pointed out by the result given below.

Theorem 2.1. Consider the system (2.1) subject to assumptions A1-A4, and the linear pole placement controller defined by

$$\begin{aligned} v(t) &= [1 - R(q^{-1})D(q^{-1})]u(t) - S(q^{-1})e_y(t) \\ \text{with} \quad e_y(t) &= y(t) - y^*(t) \end{aligned} \quad (2.3)$$

$$A(q^{-1})D(q^{-1})R(q^{-1}) + B(q^{-1})S(q^{-1}) = C(q^{-1})$$

where $C(q^{-1})$ is an Hurwitz polynomial of degree smaller or equal to $2n-1$. Then one has, for any initial conditions

- a) All the closed-loop sequences remain bounded
- b) Furthermore if

$$\{y^*(t) - y^*(t-1)\} \in l_2$$

and

$$\inf_{\omega \in [0, 2\pi[} \text{Re} \left[\frac{C(e^{-j\omega})}{A(e^{-j\omega})} \right] > 0$$

then one has

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- i) $\{D(q^{-1})v(t)\} \in l_2$, $\{D(q^{-1})u(t)\} \in l_2$ and $\{D(q^{-1})y(t)\} \in l_2$
- ii) $\{v(t) - u(t)\}$ and $\{y(t) - y^*(t)\}$ converge to zero.

The proof is given in the full paper. Part b-ii) shows that the control law asymptotically reduces to the standard pole placement control law [26]. The *positive-real condition* clearly defines a neighbourhood of $A(q^{-1})$ in which $C(q^{-1})$ has to be chosen. Accordingly, the poles of the closed-loop system are allowed to be relatively faster than those of the system to be controlled unlike in [23]. The particular choice $C(q^{-1}) = A(q^{-1})$ has been made popular by the internal model control approach [27]. It was also recommended for robustness considerations in practical situations [28].

3. ADAPTIVE REGULATOR DESIGN

3.1. System identification

The system to be controlled can be given the following regression form

$$\begin{aligned} y(t) &= \phi(t)^T \theta^* \\ \text{with} \quad \theta^* &= [a_1 \dots a_n \quad b_1 \dots b_n]^T \\ \phi(t-1) &= [-y(t-1) \dots -y(t-n) \quad u(t-1) \dots u(t-n)]^T \end{aligned} \quad (3.1)$$

where the parameters could be estimated using the following least squares algorithm

$$\begin{aligned} \theta(t) &= \theta(t-1) + \frac{F(t-1)\phi(t-1)e(t)}{1 + \phi(t-1)^T F(t-1)\phi(t-1)} \\ e(t) &= y(t) - \phi(t-1)^T \theta(t-1) \\ F(t) &= F(t-1) + \frac{F(t-1)\phi(t-1)\phi(t-1)^T F(t-1)}{1 + \phi(t-1)^T F(t-1)\phi(t-1)} \end{aligned} \quad (3.2)$$

where $\theta(0)$ is any vector such that the induced model satisfies Assumptions A1-A4 and inequality (2.2), and $P(0)$ is any positive definite matrix.

Let

$$\begin{aligned} A(t, q^{-1}) &= 1 + a_1(t) q^{-1} + \dots + a_n(t) q^{-n} \\ B(t, q^{-1}) &= b_1(t) q^{-1} + \dots + b_n(t) q^{-n} \\ e_p(t) &= y(t) - \phi(t-1)^T \theta(t) \end{aligned}$$

the system can be rewritten as follows

$$A(t, q^{-1}) y(t) = B(t, q^{-1}) u(t) + e_p(t)$$

and one has the following well known result ([26], [29] to [31]).

Proposition 3.1. The parameter adaptation algorithm (3.2), when applied to the system (3.1), satisfies the following properties

- a) $\{\theta(t)\}$ converges to some θ_∞
- b) $e_p(t) \in l_2$ and $\|\theta(t) - \theta(t-1)\| \in l_2$

The above proposition shows that the involved parameter adaptation algorithm least is quite satisfactory from an input-output point of view. Indeed, the estimated model asymptotically restores the input-output behaviour of the system, independently of the nature of the input and output sequences. More specifically, it is not required that such sequences be persistently exciting or even bounded. However, it is not shown that the estimated model satisfies in turn Assumptions A2-A4. More particularly, the compatibility condition between the reference sequence and the input constraint is not likely to be preserved for the estimated model when. In the general case, such a type of problem is referred to as *admissibility problem* of the estimated model, (see [29] and [32] for more details). In the next subsection, we will present a control law including an ad-hoc excitation capability that guarantees the involved model admissibility.

3.2. Control law design

One aims at designing an adaptive controller which is able to monitor its prime objective from plant *stabilization* to plant *identification*. Switching from stabilisation to identification should occur whenever the estimated model is not likely to be not asymptotically admissible. The evaluation of estimated model admissibility at a given time-instant t is made through the following functions

$$M_C(\theta(t)) = |\det S(A_t, B_t)|$$

$$M_S(\theta(t)) = 1 - \rho_{\max}(t)$$

$$M_r(\theta(t), y^*(t)) = \frac{|B(t, 1)|}{|A(t, 1)|} u_M - \sup_{0 < \tau < t} |y^*(\tau)|$$

where $S(A_t, B_t)$ denotes the Sylvester matrix of $A(t, q^{-1})$ and $B(t, q^{-1})$ and $\rho_{\max}(t)$ is the maximal spectral radius of the polynomial $A(t, q^{-1})$.

Notice that $M_C(\theta(t))$ and $M_S(\theta(t))$ are respectively measures of the estimated model *controllability* degree and *stability* margin, while $M_r(\theta(t), y^*(t))$ is a measure of the *compatibility* degree between the reference sequence and the input constraint when the estimated model is substituted to the system. For the estimated model to be asymptotically *admissible*, the above measures should not vanish asymptotically. Since $M_C(\theta^*)$, $M_S(\theta^*)$ and $M_r(\theta^*, y^*(t))$ are not known, one cannot say whether or not the measures $M_C(\theta(t))$, $M_S(\theta(t))$ and $M_r(\theta(t), y^*(t))$ are sufficiently large. Then, a comparison should be made with a moving threshold, i.e. any decreasing positive sequence $\{\alpha(t)\}$ that converges to zero. More precisely, an admissibility indicator I_{adm} would be defined as follows where $i_0 = 0$.

Admissibility Test Algorithm

If $M_C(\theta(t))$, $M_S(\theta(t))$ and $M_r(\theta(t), y^*(t))$ are greater than α_i ,
then $I_{\text{adm}}(t) = 1$ and $i_{t+1} = i_t$,
else $I_{\text{adm}}(t) = 0$ and $i_{t+1} = i_t + 1$.

The model admissibility is considered to be sufficient whenever the indicator I_{adm} is equal to 1. Then, the comparison threshold is kept unchanged. When $I_{\text{adm}} = 0$, then the admissibility degree is relatively small and hence the threshold is decreased because it could be too large. The admissibility indicator being now available, let us define a time sequence $\{t_j\}$ including time instants at which should occur the switching from stabilization to identification process

t_0 denotes the first time instant such that
 t_j ($j \geq 1$) denotes the first time instant such

The control input is now computed according to the following law

$$v(t) = f(u(t), y(t), y^*(t), \theta(t)) \text{ for } t \in [0, t_0] \quad (3.3)$$

and for all $j \geq 1$ and $t \in [t_{j-1}, t_j[$

$$\begin{aligned} &0 \text{ if } t \in [t_{j-1}, t_{j-1} + 2n - 2] \\ &u_M \text{ if } t = t_{j-1} + 2n - 1 \\ v(t) = &0 \text{ if } t \in [t_{j-1} + 2n, t_{j-1} + 4n - 1] \\ &f(u(t), y(t), y^*(t), \theta(t)) \text{ if } t \in [t_{j-1} + 4n, t_j[\end{aligned} \quad (3.4)$$

where $f(\cdot)$ is any stabilizing feedback control law, namely

$$\begin{aligned}
& f(u(t), y(t), y^*(t), \theta(t)) = \\
& \quad [1 - R(t, q^{-1})D(q^{-1})]u(t) - S_t(t, q^{-1})e_y(t) \\
& \text{with} \\
& R(t, q^{-1}) = 1 + r_1(t)q^{-1} + \dots + r_{n-1}(t)q^{-n+1} \\
& S(t, q^{-1}) = s_0(t) + s_1(t)q^{-1} + \dots + s_{n-1}(t)q^{-n+1}
\end{aligned} \tag{3.5}$$

The relation between the regulator parameters, $r_i(t)$ and $s_i(t)$, and the estimated model parameter vector $\theta(t)$ will be precised later in subsection 3.3. Now, let us focus on the excitation-capability of the proposed control law.

From (3.5), it follows that the control sequence looks like a Dirac impulse within the time-intervals $[t_j, t_{j+4n-1}]$. This makes the sequence $\{\phi(t)\}$ persistently exciting "at the time-instants t_j ", which in turn enforces the estimated model to be admissible after a finite time. These results are formalized in the following proposition.

Proposition 3.2. Consider the system (2.1) subject to assumptions A1-A3, the parameter adaptation algorithm (3.2) and the control law (3.5) to (3.5). Whatever is the stabilizing feedback control law $f(u(t), y(t), y^*(t), \theta(t))$, the sequences $\{\phi(t)\}$ and $\{\theta(t)\}$ satisfy the following properties.

a) There exists a positive real β such that

$$\sum_{k=1}^{4n} \phi(t_j + k) \phi^T(t_j + k) > \beta I \text{ for all } t$$

b) There exist a positive real constant ε_a and a finite integer T_a such that for all $t \geq T_a$ one has

$$\begin{aligned}
M_C(\theta(t)) &\geq \varepsilon_a \text{ and } M_S(\theta(t)) \geq \varepsilon_a \\
\text{and } M_{f^*}(\theta(t), y^*(t)) &\geq \varepsilon_a
\end{aligned}$$

and consequently for all $t \geq T_a$ one has

$$v(t) = f(u(t), y(t), y^*(t), \theta(t))$$

The proof is given in the full paper. The main arguments have been already used in [22] and [31] to [33].

3.3. Controller parameter adaptation

The control law design is now completed by incorporating an adaptation algorithm for the controller parameters from the pole placement design technique (2.3) where the key point is to make an adequate choice of the desired closed loop poles, i.e. the polynomial $C(q^{-1})$. Such a choice should be made taking particularly into account the input constraint. When the system parameters are known, this amounts to satisfy positivity condition of theorem 2.1. In the case of unknown parameters, the positivity condition has to be satisfied, at least asymptotically, when the time-varying

estimated model is substituted to the system. This suggests that the desired closed loop poles should be continuously monitored to account for the estimated model time evolution. Let us now propose an example of a pole assignment procedure.

The polynomial $A(t, q^{-1})$ may factorized as follows

$$A(t, q^{-1}) = \prod_{i=1}^n (1 - \alpha_i(t)q^{-1})$$

where $\alpha_i(t)$ denotes the i -th estimated model pole which can be given the following complex form

$$\alpha_i(t) = \rho_i(t)e^{j\varphi_i(t)} \text{ where } \rho_i(t) \geq 0 \text{ and } \varphi_i(t) \in [-\pi, +\pi[$$

To each open loop pole $\alpha_i(t)$ one will associate a closed loop pole $\gamma_i(t)$ of the form

$$\gamma_i(t) = \rho_i^*(t)e^{j\varphi_i(t)} \tag{3.6}$$

to build up a polynomial $C(t, q^{-1})$ as follows

$$C(t, q^{-1}) = \prod_{i=1}^n (1 - \gamma_i(t)q^{-1}) \tag{3.7}$$

The modules $\rho_i^*(t)$'s should be chosen so that the polynomial pair $(A(t, q^{-1}), C(t, q^{-1}))$ uniformly satisfies the positivity condition.

Proposition 3.3. Let $\rho_i^*(t)$ be chosen as follows

$$\rho_i^*(t) = \max \left(0, \frac{\rho_i(t) - \lambda_i(t) \sqrt{\lambda_i(t)^2 - \rho_i(t)^2 + 1}}{1 + \lambda_i(t)^2} \right) \tag{3.8}$$

with

$$\lambda_i(t) = \text{tg} \left(\varepsilon \frac{\rho_i(t)}{\sum_{i=1}^n \rho_i(t)} \frac{\pi}{2} \right) \sqrt{1 - \rho_i(t)^2}$$

where $\varepsilon \in]0, 1[$ is an arbitrary real constant. Then one has the following properties

- $\{A(t, q^{-1})\}, \{\rho_i(t)\}, \{\varphi_i(t)\}, \{C(t, q^{-1})\}$ and $\{\rho_i^*(t)\}$ converge.
- $0 \leq \rho_i^*(\infty) \leq \rho_i(\infty)$ for $i = 1$ to n
-

$$\inf_{\omega \in]0, 2\pi[} \text{Re} \left[\frac{C(\infty, e^{-j\omega})}{A(\infty, e^{-j\omega})} \right] > 0$$

The proof is given in the full paper. Notice that $\{C(t, q^{-1})\}$ asymptotically has all the required properties, namely the limiting polynomial exists, is Hurwitz and satisfies the required positivity condition. This will be shown to be sufficient to meet the control objective.

Before going back to the controller parameter adaptation, let us first introduce the following notations

$$\theta_r(t) = [1 \ r_1(t) \ \dots \ r_{n-1}(t) \ s_0(t) \ \dots \ s_{n-1}(t)]^T$$

$$\theta_c(t) = [1 \ c_1(t) \ \dots \ c_{2n-1}(t)]^T$$

where the $c_i(t)$'s denote the coefficients of $C(t, q^{-1})$ which identically zero for $i \geq n+1$.

The controller parameters $\theta_r(t)$ are updated according to the following adaptation law

$$\begin{aligned} \text{If } \det[S(A_t D, B_t)] &\neq 0 \\ \text{then } \theta_r(t) &= [S(A_t D, B_t)]^{-1} \theta_c(t) \\ \text{else } \theta_r(t) &= \theta_r(t-1) \end{aligned} \quad (3.9)$$

where $S(A_t D, B_t)$ denotes the Sylvester matrix associated to the pair $\{A(t, q^{-1})D(q^{-1}), B(t, q^{-1})\}$.

The adaptive controller, we are concerned with, consists of the parameter adaptation algorithm (3.1)-(3.2), the control law (3.3)-(3.5), the closed-loop pole tuning procedure (3.6)-(3.8) and the regulator parameter updating procedure (3.9). The performances of such a controller are investigated in the next section.

4. CONTROL SYSTEM ANALYSIS

The following results provide the key properties of the adaptive controller and show that the considered control objective has been achieved.

Proposition 4.1. The regulator parameter updating procedure (3.9) has the following properties.

- a) There exist a real $\varepsilon_r > 0$ and a finite integer $T_r \geq T_a$ such that

$$|\det[S(A_t D, B_t)]| > \varepsilon_r \text{ for all } t \geq T_r$$

and hence, one has

$$\theta_r(t) = [S(A_t D, B_t)]^{-1} \theta_c(t) \text{ for all } t \geq T_r$$

or equivalently

$$A(t, q^{-1})D(q^{-1})R(t, q^{-1}) + B(t, q^{-1})S(t, q^{-1}) = C(t, q^{-1})$$

$$\text{b) } \{\theta_r(t)\} \in \ell_\infty \text{ and } \|\theta_r(t) - \theta_r(t-1)\| \in \ell_2$$

The proof is given in the full version of the paper.

Theorem 4.1. The adaptive control system under consideration satisfies the following properties for any initial conditions.

- a) All the closed-loop sequences are bounded.

- b) If $\{y^*(t) - y^*(t-1)\} \in \ell_2$ then one has

$$\text{i) } \{D(q^{-1})v(t)\} \in \ell_2, \{D(q^{-1})u(t)\} \in \ell_2 \text{ and } \{D(q^{-1})y(t)\} \in \ell_2$$

$$\text{ii) } \limsup_{t \rightarrow \infty} |v(t)| \leq u_M$$

$$\text{iii) } \limsup_{t \rightarrow \infty} |u(t) - v(t)| = 0$$

$$\text{iv) } \limsup_{t \rightarrow \infty} |y^*(t) - y^*(t-1)| = 0$$

The proof is given in the full version of the paper. It is worth noticing that the positive-realness property is crucial for achieving the tracking objective in presence of the considered class of reference sequences.

5. CONCLUSION

An adaptive controller has been proposed to deal with the those ubiquitous input constraints using an input-output approach. Two design features are worth to be mentionned. Firstly, an adequate exciting procedure has been used to achieve the estimated model admissibility, namely the asymptotic stability and controllability, as well as the compatibility condition between the input constraints and the reference sequence. Secondly, an adequate updating procedure of the closed loop poles has been used to ensure the required positive-realness condition

REFERENCES.

For the sake of place limitation, the references as well as the involed proofs have been omitted. Please, ask for the full version of the paper if needed.

- F.Z. Chaoui, F. Giri and M. M'Saad (1997). Adaptive control in presence of input saturation constraint : an input-output approach. Internal report 1997.