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Conference Paper *in* Proceedings of the IEEE Conference on Decision and Control · January 2007

DOI: 10.1109/CDC.2006.376796 · Source: IEEE Xplore

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Proceedings of the 45th IEEE Conference on Decision & Control
Manchester Grand Hyatt Hotel
San Diego, CA, USA, December 13-15, 2006

Discrete-time sliding mode path-tracking control for a wheeled mobile robot

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Abstract— **In this work, it is presented a discrete time control strategy for the solution of the path-tracking problem for a wheeled mobile robot of the type (2,0). It is assumed that the mobile robot is remotely controlled over a communication network that induces a transport delay. The exact discrete-time model of the mobile robot including the induced delay is developed. It is presented a discrete-time strategy control based on a sliding mode approach that allows to solve the path-tracking problem. The closed loop stability of the overall system is clearly stated. The proposed control strategy is evaluated by mean of computer simulation.**

I. INTRODUCTION

A mobile robot is a class of electromechanical system capable of autonomous motion. It can be classified into several mobility configurations as wheels number and type, single- or multi-body structure, so on. Also, they can be classified according to the number of degrees of mobility and steeribility, [2]. In this work, a wheeled mobile robot of the type (2,0) (two degrees of mobility and zero degrees of steeribility) is considered.

In this work, it will be considered remote path-tracking control of a mobile robot of the type (2,0) subject to transport delay. The control strategy will be based on the exact discrete time model of the robot. In practice, the practical implementation of a remote control strategy for a mobile robot, involves the consideration of transport delays due to the communication network linking the sensors and actuators placed on the robot and the controller placed on a remote location. A schematic representation of the communication network considered in this work is depicted in Fig. 1. A remotely controlled mobile robot is an example of a time delay system where the effect of the transport delay on the stability of the overall system has to be considered.

The use of discrete-time models and discrete-time controllers has been motivated by the necessity to consider the action of the transport delay and to analyze its effects on the overall control scheme, [10]. The complexity of the discretization of nonlinear systems precludes, in most of the cases, the use of discrete-time controllers. Nevertheless, a discrete-time control scheme based on an exact discrete-time model of the plant presents significant advantages over the restriction on the sampling period encountered when considering a digitally implemented continuous-time controller.

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The control of wheeled mobile robots is not a trivial task due to the nonlinearities present on their model and due to the non integrable kinematics constraints that cannot be eliminated from the model. Nevertheless, there are several works in the literature [12], [15], [5] where the authors proposed several continuous sliding mode control schemes to solve the path tracking. Similarly in [9] a navigation function is proposed to generate a set of desired trajectories, in order to ensure that the continuous sliding mode control scheme guarantee the asymptotic convergence of the tracking errors to zero.

In general, controllers for non-linear systems are designed by considering continuous-time models, obtaining consequently, continuous-time controllers. However, in most cases, continuous time controllers are digitally implemented and therefore operate in discrete-time. In addition, a disadvantage of digitally operated continuous-time controllers is that they require, for its implementation, small sampling periods. For these reasons, some authors proposed discrete-time control schemes based on approximate discrete models. In [3], the stabilization problem for a wheeled mobile robot was addressed by considering the presence of disturbances violating the nonholonomic constraint. Starting from the results obtained in [3]; in [4] a discrete-time sliding mode control technique is proposed for trajectory tracking of a wheeled mobile robot.

In [11] the exact discrete-time model of a mobile robot (2,0) is obtained by direct integration and a discrete-time nonlinear control scheme is developed. In [6] the authors present the transforming of the original kinematic system in chained form through an appropriate feedback transfor-

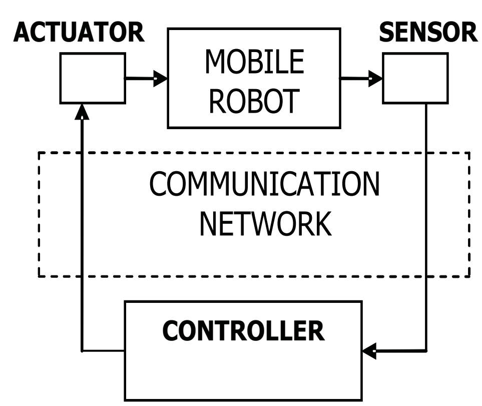

Fig. 1. Communication network for a remote control system.

mation. The obtained equations are closed-form integrable, thereby yielding a linear discrete-time model which provides exact odometric prediction and associated covariance.

In general, transport delay problems in mobile robots have not been yet extensively discussed. In [13], [14], the authors consider an approximate discrete-time model of a mobile robot that is controlled remotely introducing the effects of communication network delay. Specifically, in [13] it is obtained a control law that stabilizes the mobile robot in the presence of a time delay less than the sampling period of the system.

Additionally, the problem addressed in this work, a discrete-time sliding mode control strategy to solve the path-tracking problem of a mobile robot of the type (2,0) considering the effects of the communication network based on the exact discrete-time model of the system has not been considered in the literature.

The paper is organized as follows: In Section II several models for the mobile robot of the type (2,0) are presented including continuous and discrete time representations. In Section III a discrete-time sliding mode control scheme to solve the path following problem is presented based on the exact discrete time model of the robot. Computational experiments of the proposed discrete time strategy are presented in Section IV where it is shown its performance. Finally the overall conclusions are presented in Section V.

II. Discrete-time models

The continuous-time kinematic model of a mobile robot of the type (2,0), shown in Fig. 2, is given by,

$$\begin{aligned}\dot{x}_1 &= u_1 \cos x_3 \\ \dot{x}_2 &= u_1 \sin x_3 \\ \dot{x}_3 &= u_2,\end{aligned} \tag{1}$$

where x_1 and x_2 represent the coordinates of the center of the axis of the actuated wheels on the plane X_1-X_2 and x_3 is the angle formed by the longitudinal axis of the robot and the X_1 axis. u_1 represents the linear velocity while u_2 represents the angular velocity.

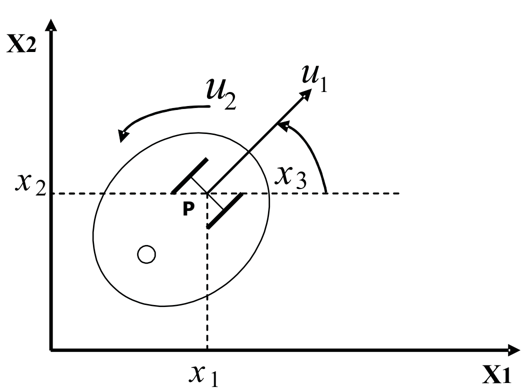

Fig. 2. Mobile Robot (2,0).

Considering the communication network and the induced transport delay effects, system (1) can be rewritten as,

$$\dot{x}_1(t) = u_1(t-\tau)\cos x_3(t) \tag{2a}$$
$$\dot{x}_2(t) = u_1(t-\tau)\sin x_3(t) \tag{2b}$$
$$\dot{x}_3(t) = u_2(t-\tau) \tag{2c}$$

where τ is the period of time between the generation of the signal in the controller and its effective application to the remote system. Note that the transmission time delay could be in fact negligible, as a matter of fact, the time delay is mainly due to the period of time required to process the information from the sensors of the robot, multiplexing and demultiplexing signals and so on. . As usual, it is assumed that the time delay τ is greater than the time required to overcome these operations.

A. Approximate discrete-time model with delay

An approximate discrete-time model for system (2) can be obtained from a Taylor's series expansion, [13]. In this case, the approximate model is given by,

$$\begin{aligned}x_1(kT+T) &= x_1(kT) \\ &\quad + (T-\tau)u_1(kT)\cos x_3(kT+\tau) \\ &\quad + \tau u_1(kT-T)\cos x_3(kT) \\ x_2(kT+T) &= x_2(kT) \\ &\quad + (T-\tau)u_1(kT)\sin x_3(kT+\tau) \\ &\quad + \tau u_1(kT-T)\sin x_3(kT) \\ x_3(kT+T) &= x_3(kT) + (T-\tau)u_2(kT) \\ &\quad + \tau u_2(kT-T).\end{aligned} \tag{3}$$

This model is valid only when the sampling period T is sufficiently small [13], [14]. When T can not be reduced to a sufficiently small value, the possible control strategy may show appreciable degradation on the general system performance.

B. Exact discrete-time model

The exact discrete-time model of a mobile robot of the type (1) was obtained in [11] by direct integration of the continuous-time model under the standard assumption that control inputs remain constant between sampling instants, producing for a sampling period T,

$$\begin{aligned}x_1(kT+T) &= x_1(kT) \\ &\quad + 2u_1(kT)\psi\left(u_2(kT)\right) \\ &\quad \cos\left(x_3(kT) + \frac{T}{2}u_2(kT)\right) \\ x_2(kT+T) &= x_2(kT) \\ &\quad + 2u_1(kT)\psi\left(u_2(kT)\right) \\ &\quad \sin\left(x_3(kT) + \frac{T}{2}u_2(kT)\right) \\ x_3(kT+T) &= x_3(kT) + Tu_2(kT)\end{aligned} \tag{4}$$

where,

$$\psi\left(u_2(kT)\right) = \begin{cases} \dfrac{\sin\left(\frac{T}{2}u_2(kT)\right)}{u_2(kT)}, & \text{if } u_2(kT) \neq 0 \\ \dfrac{T}{2}, & \text{if } u_2(kT) = 0. \end{cases}$$

Notice that the function $\psi\left(u_2(kT)\right)$ satisfies,

$$\lim_{u_2(kT)\to 0} \psi\left(u_2(kT)\right) = \frac{T}{2} = \psi\left(0\right).$$

C. Exact discrete-time model with time-delay

The effects of the communication network (as shown in Fig. 1) on the discrete time model will be considered in this subsection. As in the case of systems (1), the exact discretization of system (2) will be obtained by direct integration of the continuous-time equations under the assumption that the time delay τ is less than the sampling period T.

First consider the solution $x_3(t)$ of equation (2c), for the time interval $[t_0, t]$. Assuming that $\tau < T$, it follows that,

$$x_3\left(t\right) = x_3\left(t_0\right) + \int_{t_0}^{t} u_2(s-\tau)ds \tag{5}$$

Due to the fact that $\tau < T$, the value of $u(t)$ changes from $u(kT-T)$ to $u(kT)$ at $t_0+\tau$, as shown on the Fig. 3 (See [1] for the linear case).

Assuming that τ has a known constant value, (5) can be rewritten according to Fig. 3 as,

$$\begin{aligned} x_3\left(t\right) = x_3\left(t_0\right) &+ \int_{t_0}^{t_0+\tau} u_2(kT-T)ds \\ &+ \int_{t_0+\tau}^{t} u_2(kT)ds \end{aligned} \tag{6}$$

where $u_2(kT)$ represents the current value of $u_2\left(t\right)$ and $u_2(kT-T)$ represents the value of $u_2\left(t\right)$ during the precedent sampling interval.

Notice that when $x_3\left(t\right)$ from (6) is evaluated on the time interval $t_0 \leq t < t_0+\tau$ it produces,

$$x_3\left(t\right) = x_3\left(t_0\right) + \left(t-t_0\right)u_2(kT-T) \tag{7}$$

but when (6) is evaluated for $t_0 \leq t$, it is obtained,

$$\begin{aligned} x_3\left(t\right) = x_3\left(t_0\right) &+ \tau u_2(kT-T) \\ &+ \left(t-t_0-\tau\right)u_2(kT). \end{aligned} \tag{8}$$

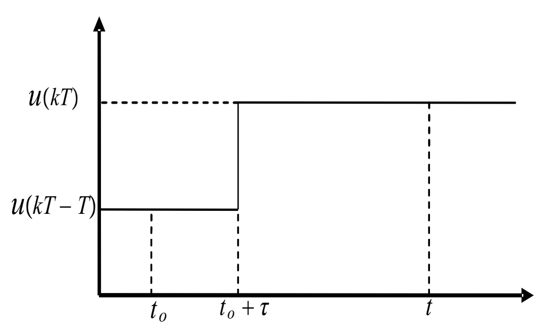

Fig. 3. Control signal. $u(t-\tau)$

Following the same arguments used for (2c) and considering (7) and (8), the equation (2a) can be integrated as,

$$\begin{aligned} x_1\left(t\right) \;=\; & x_1\left(t_0\right) \\ & + u_1(kT-T)\int_{t_0}^{t_0+\tau} \sin\left(x_3\left(t_0\right)\right. \\ & \left.+\left(s-t_0\right)u_2(kT-T)\right)ds \\ & + u_1(kT)\int_{t_0+\tau}^{t} \cos\left(x_3\left(t_0\right)+\tau u_2(kT-T)\right. \\ & \left.+\left(s-t_0-\tau\right)u_2(kT)\right)ds. \end{aligned} \tag{9}$$

The same procedure is apply for the solution $x_2\left(t\right)$ of equation (2b).

For the sake of conciseness, in the rest of the paper the following notation will be adopted: Let $\eta\left(t\right)$ be a real valued continuous function of time. Then define

$$\eta\left(kT \pm mT\right) = \eta^{\pm m} \quad with \quad m = 0, 1, 2, \ldots$$

To finally obtain the exact discrete-time model, it suffices to consider $t_0 = kT$ and evaluate (8), (9) and the one obtained for $x_2(t)$ at the end of the interval $[kT, kT+T]$, that is, $t = kT+T$, producing

$$\begin{aligned} x_1^+ &= x_1 + 2u_1^-\psi_1\left(u_2^-\right)\cos\beta + 2u_1\psi_2\left(u_2\right)\cos\theta \\ x_2^+ &= x_2 + 2u_1^-\psi_1\left(u_2^-\right)\sin\beta + 2u_1\psi_2\left(u_2\right)\sin\theta \\ x_3^+ &= x_3 + \tau u_2^- + \left(T-\tau\right)u_2 \end{aligned} \tag{10}$$

with

$$\theta = x_3 + \tau u_2^- + \frac{T-\tau}{2}u_2,$$
$$\beta = x_3 + \frac{\tau}{2}u_2^-$$

and

$$\psi_1\left(u_2^-\right) = \begin{cases} \dfrac{\sin\left(\dfrac{\tau}{2}u_2^-\right)}{u_2^-} & \text{if} \quad u_2^- \neq 0 \\ \dfrac{\tau}{2} & \text{if} \quad u_2^- = 0 \end{cases}$$

$$\psi_2\left(u_2\right) = \begin{cases} \dfrac{\sin\left(\dfrac{T-\tau}{2}u_2\right)}{u_2} & \text{if} \quad u_2 \neq 0 \\ \dfrac{T-\tau}{2} & \text{if} \quad u_2 = 0. \end{cases} \tag{11}$$

Notice that in this case, $\psi_1(u_2^-)$ and $\psi_2(u_2)$ satisfies,

$$\lim_{u_2^-\to 0} \psi_1(u_2^-) = \frac{\tau}{2} = \psi_1(0)$$
$$\lim_{u_2\to 0} \psi_2(u_2) = \frac{T-\tau}{2} = \psi_2(0)$$

III. PATH TRACKING CONTROL STRATEGY

The discrete-time sliding mode strategies are in practical implementations, due to the simplicity and robustness of the obtained controllers, [4], [7] and [8].

In [7], it is presented a stable discrete time sliding mode control insensitive to the choice of sampling period and free of chattering. The controller is designed on the basis of a discrete-time Lyapunov function. The solution of the path-tracking problem proposed in this work will be based on the results presented in [7].

Contrary to the continuous case, in the discrete-time sliding mode control present in [7] and used in [4], the considered switching surface is different from the sliding surface and, in a neighborhood along the sliding mode, there is a region where a linear control is used.

A. *Path tracking error dynamics*

Consider the path tracking errors,

$$\begin{aligned} e_{x_1} &= x_1 - x_{1d} \\ e_{x_2} &= x_2 - x_{2d} \\ e_{x_3} &= x_3 - x_{3d}. \end{aligned}$$

The error dynamics are given by,

$$\begin{aligned} e^+_{x_1} &= e_{x_1} + 2\psi_1 u_1^- \cos\beta + 2\psi_2 u_1 \cos\theta - \Delta x_{1d} \\ e^+_{x2} &= e_{x_2} + 2u_1^- \psi_1 \sin\beta + 2u_1\psi_2 \sin\theta - \Delta x_{2d} \\ e^+_{x_3} &= e_{x_3} + \tau u_2^- + (T-\tau)\, u_2 - \Delta x_{3d} \end{aligned} \tag{12}$$

with

$$\begin{aligned} \theta &= e_{x_3} + x_{3d} + \tau u_2^- + \frac{T-\tau}{2} u_2, \\ \beta &= e_{x_3} + x_{3d} + \frac{\tau}{2} u_2^- \end{aligned}$$

and ψ_1 and ψ_2 defined as in (11).

B. *Discrete-Time sliding mode control*

Consider the representation given in (12) and define the input *virtual* signals,

$$w_{1x} = u_1 \psi_2 \cos\theta \tag{13}$$
$$w_{1y} = u_1 \psi_2 \sin\theta, \tag{14}$$

and also, the auxiliary signals,

$$\begin{aligned} \upsilon_{1x} &= u_1^- \psi_1 \cos\beta \\ \upsilon_{1y} &= u_1^- \psi_1 \sin\beta. \end{aligned}$$

Now, the system (12) can be rewritten in the form,

$$\begin{aligned} e^+_{x_1} &= e_{x_1} + 2\upsilon_{1x} + 2w_{1x} - \Delta x_{1d} \\ e^+_{x_2} &= e_{x_2} + 2\upsilon_{1y} + 2w_{1y} - \Delta x_{2d} \\ e^+_{x_3} &= e_{x_3} + \tau u_2^- + (T-\tau)\, u_2 - \Delta x_{3d} \end{aligned} \tag{15}$$

For this new representation, and for design purpose only, it will be assumed that w_{1x}, w_{1y}, u_2 are the input signals of the system. Under these conditions, consider now the sliding surfaces $\sigma_x = 0$, $\sigma_y = 0$ and $\sigma_t = 0$, defined as,

$$\sigma_x = e^+_{x_1} - e_{x_1} + \gamma_x e^-_{x_1} \tag{16a}$$
$$\sigma_y = e^+_{x_2} - e_{x_2} + \gamma_y e^-_{x_2} \tag{16b}$$
$$\sigma_t = e^+_{x_3} - e_{x_3} + \gamma_t e^-_{x_3}, \tag{16c}$$

where the positive real parameters γ_x, γ_y, γ_t will be determined later.

Considering the function σ_x given in (16a),

$$\begin{aligned} \sigma_x &= e^+_{x_1} - e_{x_1} + \gamma_x e^-_{x_1} \\ \sigma_x &= (e_{x_1} + 2\upsilon_{1x} + 2w_{1x} - \Delta x_{1d}) - e_{x_1} + \gamma_x e^-_{x_1} \\ \sigma_x &= 2\upsilon_{1x} + 2w_{1x} - \Delta x_{1d} + \gamma_x e^-_{x_1} \end{aligned}$$

it is possible to define the feedback,

$$w_{1x} = \frac{1}{2}\left[\Delta x_{1d} - \gamma_x e^-_{x_1} - 2\upsilon_{1x} + \alpha\sigma_x^-\right] \tag{17}$$

producing in the closed-loop (16a)-(17),

$$\sigma_x = \alpha\sigma_x^-.$$

In the same way, the virtual control w_{1y} is obtained from the consideration of the surface (16b), this is,

$$w_{1y} = \frac{1}{2}\left[\Delta x_{2d} - \gamma_y e^-_{x_2} - 2\upsilon_{1y} + \beta\sigma_y^-\right] \tag{18}$$

getting in closed loop (16b)-(18),

$$\sigma_y = \beta\sigma_y^-.$$

Notice that the real control signal u_1 can be determined from the *virtual* signals w_{1x} and w_{1y}, since,

$$w_{1x}^2 + w_{1y}^2 = u_1^2\psi_2^2,$$

therefore

$$u_1 = \frac{\sqrt[2]{w_{1x}^2 + w_{1y}^2}}{\psi_2}. \tag{19}$$

Notice also that u_1 can be obtained from (13),(14) as

$$u_1 = \frac{w_{1x}}{\psi_2\cos\theta} \text{ or } u_1 = \frac{w_{1y}}{\psi_2\sin\theta} \tag{20}$$

and that w_{1x} and w_{1y} satisfy

$$\frac{w_{1y}}{w_{1x}} = \tan\theta.$$

Finally, consider the surface σ_t shown in (16c),

$$\begin{aligned} \sigma_t &= e^+_{x3} - e_{x3} + \gamma_t e^-_{x3} \\ \sigma_t &= \left(e_{x_3} + \tau u_2^- + (T-\tau)\, u_2 - \Delta x_{3d}\right) - e_{x3} + \gamma_t e^-_{x3} \\ \sigma_t &= \tau u_2 + (T-\tau)\, u_2 - \Delta x_{3d} + \gamma_t e^-_{x3}, \end{aligned}$$

defining the feedback,

$$u_2 = \frac{1}{(T-\tau)}\left[\Delta x_{3d} - \gamma_t e^-_{x_3} - \tau u_2^- + \delta\sigma_t^-\right], \tag{21}$$

the closed loop (16c)-(21) produces,

$$\sigma_t = \delta\sigma_t^-.$$

C. *Stability analysis*

Due to the control strategy proposed in the previous subsection, w_{1x} and w_{1y}, defined in (17) and (18) can be used separately to impose a sliding mode on surfaces $\sigma_x = 0$ and $\sigma_y = 0$ respectively. Similarly, u_2 defined on (21) can be used to impose a sliding mode on surface $\sigma_t = 0$.

As shown previously, the dynamics of the sliding surfaces are given by

$$\begin{aligned} \sigma_x &= \alpha\sigma_x^- \\ \sigma_y &= \beta\sigma_y^- \\ \sigma_t &= \delta\sigma_t^-. \end{aligned} \tag{22}$$

where a suitable election of the α, β and δ provides the desired convergence.

Consider the feedback (17), in closed-loop with system (12). Since u_1 can be rewritten of the form (20), the dynamics of e_{x_1} is given by,

$$e_{x_1}^+ = e_{x_1} + 2v_{1x} + 2\psi_2 \cos\theta \left[\frac{w_{1x}}{\psi_2 \cos\theta}\right] - \Delta x_{1d}$$

replacing w_{1x} defined in (17) it produces,

$$\begin{aligned} e_{x_1}^+ &= e_{x_1} + 2v_{1x} + \left[\Delta x_{1d} - \gamma_x e_{x_1}^- - 2v_{1x} + \alpha\sigma_x^-\right] \\ &\quad - \Delta x_{1d} \\ e_{x_1}^+ &= e_{x_1} - \gamma_x e_{x_1}^- + \alpha\sigma_x^- \end{aligned}$$

considering σ_x^- from (16a) it is possible to obtain,

$$\begin{aligned} e_{x_1}^+ - e_{x_1} + \gamma_x e_{x_1}^- - \alpha\left[e_{x_1} - e_{x_1}^- + \gamma_x e_{x_1}^{-2}\right] &= 0 \\ e_{x_1}^+ - (1+\alpha)\, e_{x_1} + (\gamma_x + \alpha)\, e_{x_1}^- - \alpha\gamma_x e_{x_1}^{-2} &= 0 \\ e_{x_1}^{+3} - (1+\alpha)\, e_{x_1}^{+2} + (\gamma_x + \alpha)\, e_{x_1} - \alpha\gamma_x e_{x_1} &= 0 \end{aligned}$$

where the suitable election of the parameters α and γ_x allows to establish the desired error convergence.

Now consider that u_1 defined as in (20), the dynamics of e_{x_2} is given by

$$e_{x_2}^+ = e_{x_2} + 2v_{1y} + 2\psi_2 \sin\theta \left[\frac{w_{1y}}{\psi_2 \sin\theta}\right] - \Delta x_{1d},$$

replacing w_{1y} from (18), it is possible to obtain

$$\begin{aligned} e_{x_2}^+ &= e_{x_2} + 2v_{1y} + \left[\Delta x_{2d} - \gamma_y e_{x_2}^- - 2v_{1y} + \beta\sigma_y^-\right] \\ &\quad - \Delta x_{2d} \\ e_{x_2}^+ &= e_{x_2} - \gamma_y e_{x_2}^- + \beta\sigma_y^- \end{aligned}$$

or equivalently from (16b),

$$\begin{aligned} e_{x_2}^+ - e_{x_2} + \gamma_y e_{x_2}^- - \beta\sigma_y^- &= 0 \\ e_{x_2}^+ - e_{x_2} + \gamma_y e_{x_2}^- - \beta\left[e_{x_2} - e_{x_2}^- + \gamma_y e_{x_2}^{-2}\right] &= 0 \\ e_{x_2}^+ - (1+\beta)\, e_{x_2} + (\gamma_y + \beta)\, e_{x_2}^- - \beta\gamma_y e_{x_2}^{-2} &= 0 \\ e_{x_2}^{+3} - (1+\beta)\, e_{x_2}^{+2} + (\gamma_y + \beta)\, e_{x_2} - \beta\gamma_y e_{x_2} &= 0. \end{aligned}$$

where again the election of the parameters β and γ_y allows to ensure the desired convergence. Finally, the dynamics of e_{x_3} in closed loop with (21) is given by,

$$e_{x_3}^+ = (1+\delta)\, e_{x_3} - (\gamma_t + \delta)\, e_{x_3}^- - \delta\gamma_t e_{x_3}^{-2}$$

therefore,

$$e_{x_3}^{+3} - (1+\delta)\, e_{x_3}^{+2} + (\gamma_t + \delta)\, e_{x_3}^+ + \delta\gamma_t e_{x_3} = 0,$$

that can be also stabilized.

IV. Simulation results

In order to show the performance of the proposed control strategy, the results of a simulation experiment are presented. In the experiments the sampling period was set to $T = 0.1\ sec$. The time delay τ induced by the communication network was considered equal to $0.025s$. It is desired that the system follows a circular path of radius $r = 3$, with the initial conditions,

$$\begin{aligned} &x_1 = 0.2,\ x_2 = 2.5,\ x_3 = 0.02 \\ &u_2 = 0, \quad u_1 = 0. \end{aligned}$$

The control parameters for (16) and (22) were given by,

$$\begin{aligned} \gamma_x = \gamma_y = \gamma_t = 0.24 \\ \alpha = 0.4,\ \beta = 0.4,\ \delta = 0. \end{aligned}$$

The desired and the real trajectory of the mobile robot, on the plane $X_1 - X_2$, are shown in Fig. 4 where it is possible to see that in spite of the initial conditions of the mobile robot, the desired convergence is obtained.

The evolution of path tracking errors for the states x_1, x_2 and x_3 with respect to time is depicted in Fig. 5, showing the asymptotic convergence of the errors.

In Fig. 6, it is possible to see that the obtained control signals u_1 and u_2 are free of chattering. Fig. 7 shows the evolution of the virtual control signals w_{1x} and w_{1y}. Finally, in Fig. 8 is possible to see that the evolution with respect to time of the considered sliding surfaces.

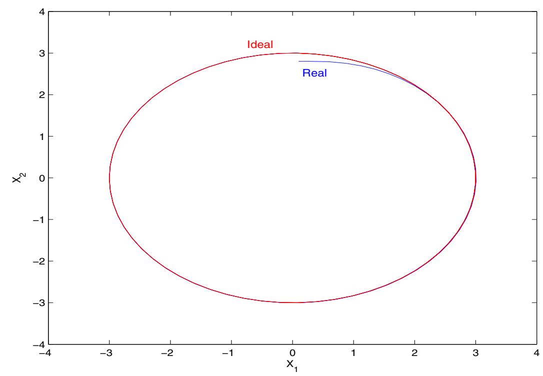

Fig. 4. Ideal trajectory and real trajectory of mobile robot in the plane $X_1 - X_2$.

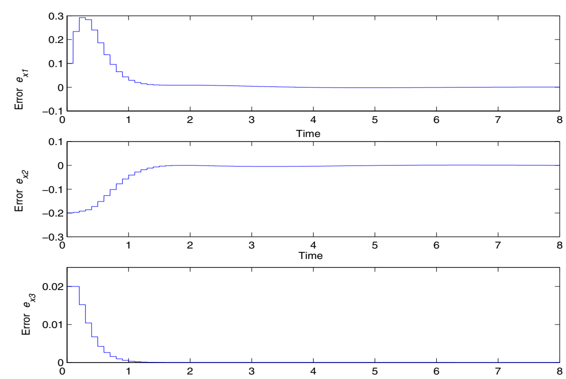

Fig. 5. Path tracking error dynamics.

V. Conclusions

In this work it is considered the remote control of a mobile robot of the type (2,0) assuming that there exits a communication network between the sensors and actuators on the mobile robot and the remote control system. It is proposed a discrete-time control strategy for the mobile robot

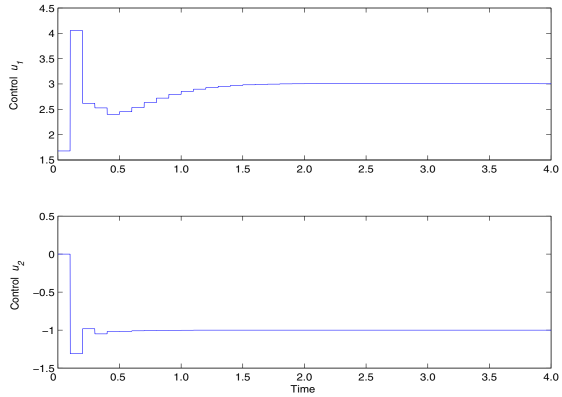

Fig. 6. Control signals. u_1 and u_2.

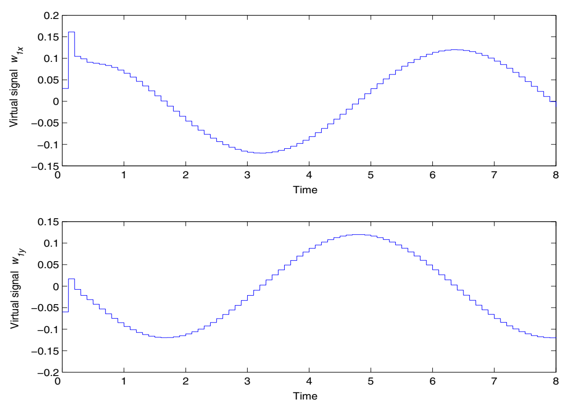

Fig. 7. Virtual control signals. w_{1x} y w_{1y}

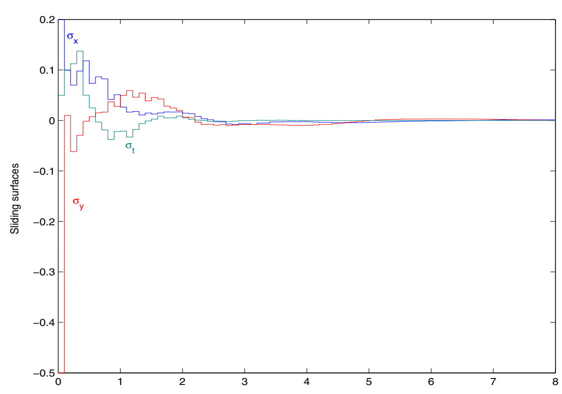

Fig. 8. Sliding surfaces σ_x, σ_y and σ_t.

subject to transport delay induced by the communication network. In order to develop the discrete time controller it is obtained the exact discrete time model for the mobile robot including the effects of the time-delay. A solution for the path tracking problem is obtained based on a discrete-time sliding mode approach that allows a simple implementation of the controller. The closed loop stability of the proposed strategy was stated by considering the convergence of the tracking errors. The performance of obtained model is better compared to the approximate model because it does not need a sampling period T sufficiently small. The proposed discrete-time control strategy can be extended for tracking mobile robots with greater degree of mobility and steeribility in the case that their exact discrete time model is available.

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