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A Design of Adaptive Control Systems with Input Saturation

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Abstract: This paper deals with model reference adaptive control problem for a linear single-input single-output time invariant continuous-time plant with input saturation. We propose an algorithm in which error feedback is effectively used and reference signal conditions are weakened. We have shown that globally asymptotic stability is guaranteed. Using this algorithm, we allow the adaptive system to have higher design flexibility than conventional ones. These results are confirmed by a simulation study.

Keywords: Adaptive control; Input saturation; Globally asymptotic stability

1. INTRODUCTION

Many real systems have input constrains because of the limitation of actuators or protection of systems. When a input that exceeds the limit is required by a controller, the input will be saturated. If a control scheme without considering input saturation would be applied, the performance might not be guaranteed or the system might become unstable, because the controller does not work as we expect. This problem may arise also in an adaptive control scheme as result of excessive adaptation. Accordingly, it is required that the controller design taking input saturation into consideration to realize sufficient control performance.

Many researches have been done on this problem, but there are few adaptive control scheme focused on the globally asymptotic stability. The reason is why it is difficult because of the following facts [1].

(a) Globally asymptotic stability cannot be expected for adaptive systems with plants which are unstable and have input saturation.

(b) For a given plant with input saturation, asymptotic stability cannot be achieved for any reference input or any reference model even if the plant is stable and known.

To this problem, it is proposed that an algorithm which guarantees the globally asymptotic stability for linear single-input single-output (SISO) time-invariant continuous-time plants when some priori information on the plant is available to choose upper limit of the input and a reference input [2].

In this paper, based on [1], [2], we propose an algorithm in which error feedback effectively used and reference signal conditions are weakened compares with conventional ones. We show that globally asymptotic stability is guaranteed when the plant is stable and has relative degree of one.

The paper is organized as follows. Section 2 is for the statement of the problem and several assumptions. An adaptive control system considering input saturation is designed in section 3. Simulation results are given in section 4. Section 5 is for conclusions. Finally stability analysis to prove the global asymptotic stability is shown in appendix.

2. PROBLEM STATEMENT AND ASSUMPTIONS

The SISO time-invariant plant to be considered is described by

$$y(t) = G(s)\sigma(u) \tag{1}$$

$$G(s) = \frac{b_m B(s)}{A(s)} \tag{2}$$

$$A(s) = s^n + \sum_{i=0}^{n-1} a_i s^i, \; B(s) = s^m + \sum_{i=0}^{m-1} b_i s^i \tag{3}$$

where $y(t) \in \Re$, $\sigma(u) \in \Re$ are the plant output and input, respectively, and $s \equiv d/dt$. $\sigma(u)$ is a saturation function to control input $u(t) \in \Re$ as shown in Fig. 1 and Eq. (4).

$$\sigma(u) = \begin{cases} u_{max} & (u_{max} < u(t)) \\ u(t) & (u_{min} \le u(t) \le u_{max}) \\ u_{min} & (u_{min} > u(t)) \end{cases} \tag{4}$$

In addition, we make the following assumptions.

(A1) $A(s),\ B(s)$ are coprime stable polynomials.

(A2) The coefficients of $A(s),\ B(s)$ and b_m are unknown, but these bounds are known.

(A3) The sign of b_m is known ($b_m > 0$).

(A4) n is known.

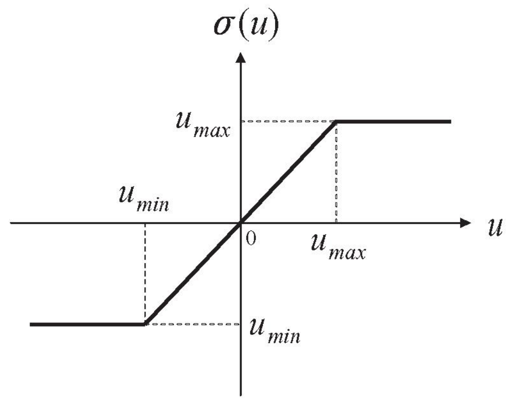


Fig. 1 Input saturation.

(A5) $n - m = 1$.
(A6) Limits of input u_{min}, u_{max} are known.
(A7) The reference signal $y_M(t)$ is uniformly bounded and differentiable.

We shall construct an adaptive control system which satisfies the above conditions.

3. ADAPTIVE CONTROL DESIGN WITH INPUT SATURATION

We obtain a nonminimal realization of Eq. (1)

$$\dot{y}(t) = -\lambda y(t) + \boldsymbol{\theta}^{\mathsf{T}}\boldsymbol{\omega}(t) + b_m\sigma(u) + \varepsilon_1(t) \tag{5}$$

$$\boldsymbol{\omega}(t) = \left[\boldsymbol{v}_{f1}(t)^{\mathsf{T}},\ \boldsymbol{v}_{f2}(t)^{\mathsf{T}},\ y(t)\right]^{\mathsf{T}} \tag{6}$$

$$\dot{\boldsymbol{v}}_{f1}(t) = \boldsymbol{F}\boldsymbol{v}_{f1}(t) + \boldsymbol{g}y(t) \tag{7}$$

$$\dot{\boldsymbol{v}}_{f2}(t) = \boldsymbol{F}\boldsymbol{v}_{f2}(t) + \boldsymbol{g}\sigma(u) \tag{8}$$

$$\dot{\boldsymbol{\varepsilon}}(t) = \boldsymbol{F}_0\boldsymbol{\varepsilon}(t) \tag{9}$$

$$\varepsilon_1(t) = \boldsymbol{g}_0^{\mathsf{T}}\boldsymbol{\varepsilon}(t),\ \ \boldsymbol{g}_0 = \left[1, 0, \cdots, 0\right]^{\mathsf{T}} \tag{10}$$

where $\boldsymbol{\theta} \in \Re^{2n-1}$, $\boldsymbol{F} \in \Re^{n-1\times n-1}$, $\boldsymbol{g} \in \Re^{n-1}$, $\boldsymbol{F}_0 \in \Re^{n-1\times n-1}$, $\boldsymbol{g}_0 \in \Re^{n-1}$ and $\boldsymbol{F}$, $\boldsymbol{F}_0$ are stable matrices, $(\boldsymbol{F}, \boldsymbol{g})$ and $(\boldsymbol{F}_0^{\mathsf{T}}, \boldsymbol{g}_0)$ are controllable pairs. Furthermore, $\boldsymbol{\theta}$, b_m are unknown parameters determined by the plant, $\varepsilon_1(t)$ is an exponential damping term dependant of initial conditions and λ is a positive design parameter. We let $e(t) \equiv y(t) - y_M(t)$ be a tracking error and $p \equiv 1/b_m$, tracking error system is represented as follows

$$\dot{e}(t) = -\lambda e(t) + b_m\left\{\sigma(u) - \Gamma(t)\right\} + \varepsilon_1(t) \tag{11}$$

$$Y_M(t) = \dot{y}_M(t) + \lambda y_M(t) \tag{12}$$

$$\Gamma(t) \equiv -p\{\boldsymbol{\theta}^{\mathsf{T}}\boldsymbol{\omega}(t) - Y_M(t)\}. \tag{13}$$

From Eq. (11), to achieve the condition of $e(t) \to 0$, $\sigma(u)$ must converge to $\Gamma(t)$. In other words, $\Gamma(t)$ should be such that $u_{min} \le \Gamma(t) \le u_{max}$ for allowing $y(t)$ to track $y_M(t)$. To apply a control input $u(t)$, we represent Eq. (11) as follows

$$\begin{aligned}\dot{e}(t) = &-(c_1 + c_2)e(t)\\ &+ b_m\left\{\sigma(u) + p\phi(t)\right\} + \varepsilon_1(t)\end{aligned} \tag{14}$$

$$\phi(t) \equiv (c_1 + c_2 - \lambda)e(t) + \boldsymbol{\theta}^{\mathsf{T}}\boldsymbol{\omega}(t) - Y_M(t) \tag{15}$$

where c_1 and c_2 are design parameters to adjust the transient performanceCand $c_1 > 0$, $\lambda > c2 > 0$. If we assume that the system have no input saturation, we can choose $u(t) = -p\phi(t)$ to achieve asymptotic stability of Eq. (14). Since p, $\boldsymbol{\theta}$ are unknown parameters, we adopt $\hat{p}(t)$ and $\hat{\boldsymbol{\theta}}(t)$ as the estimate values of p and $\boldsymbol{\theta}$, respectively, and then we provide $u(t)$ as follows

$$u(t) = -\hat{p}(t)\hat{\phi}(t) \tag{16}$$

$$\begin{aligned}\hat{\phi}(t) \equiv &(c_1 + c_2 - \lambda)e(t)\\ &+ \hat{\boldsymbol{\theta}}(t)^{\mathsf{T}}\boldsymbol{\omega}(t) - Y_M(t).\end{aligned} \tag{17}$$

When we choose the adaptive laws as follows

$$\dot{\hat{p}}(t) = g_1\hat{\phi}(t)e(t) \tag{18}$$

$$\dot{\hat{\boldsymbol{\theta}}}(t) = \boldsymbol{G}_2\boldsymbol{\omega}(t)e(t) \tag{19}$$

$$g_1 > 0,\ \boldsymbol{G}_2 = \boldsymbol{G}_2^{\mathsf{T}} > 0, \tag{20}$$

asymptotic stability of the system is guaranteed in case of no input saturation. When occurs input saturation, the stability of the system will not be guaranteed. Therefoere we employ novel adaptive laws for guaranteeing the stability of the system. Firstly we let $\phi(u) \equiv u(t) - \sigma(u)$ be a function which represents a degree of input saturation, and add the state as given by

$$\begin{aligned}\dot{x}(t) =&\{c_1 + c_2 - (k_1 + k_2)\}e(t)\\ &- (k_1 + k_2)x(t) + \hat{b}(t)\psi(u)\end{aligned} \tag{21}$$

where k_1 and k_2 are positive design parameters and $\hat{b}(t)$ is an estimated value of b_m. Moreover, we introduce a new state variable as follows

$$w(t) \equiv x(t) + e(t). \tag{22}$$

Then, we can obtain the system representation given by Eqs. (23) $\sim$ (25).

$$\begin{aligned}\dot{e}(t) = &-(c_1 + c_2)e(t) + b_m\tilde{p}(t)\hat{\phi}(t)\\ &+ \tilde{\boldsymbol{\theta}}(t)^{\mathsf{T}}\boldsymbol{\omega}(t) - b_m\psi(u) + \varepsilon_1(t)\end{aligned} \tag{23}$$

$$\begin{aligned}\dot{w}(t) = &-(k_1 + k_2)w(t) + b_m\tilde{p}(t)\hat{\phi}(t)\\ &+ \tilde{\boldsymbol{\theta}}(t)^{\mathsf{T}}\boldsymbol{\omega}(t) - \tilde{b}(t)\psi(u) + \varepsilon_1(t)\end{aligned} \tag{24}$$

$$\begin{aligned}&\tilde{p}(t) \equiv p - \hat{p}(t)\\ &\tilde{\boldsymbol{\theta}}(t) \equiv \boldsymbol{\theta} - \hat{\boldsymbol{\theta}}(t)\\ &\tilde{b}(t) \equiv b_m - \hat{b}(t)\end{aligned} \tag{25}$$

The adaptive laws of $\dot{\hat{p}}(t)$, $\dot{\hat{\boldsymbol{\theta}}}(t)$, $\dot{\hat{b}}(t)$ are constructed as follows.

$$\dot{\hat{p}}(t) = g_1\hat{\phi}(t)\left\{w(t) + \delta(\psi, e)e(t)\right\} \tag{26}$$

$$\dot{\hat{\boldsymbol{\theta}}}(t) = \boldsymbol{G}_2\boldsymbol{\omega}(t)\left\{w(t) + \delta(\psi, e)e(t)\right\} \tag{27}$$

$$\dot{\hat{b}}(t) = -g_3\psi(u)w(t) \tag{28}$$

$$\delta(\psi, e) = \begin{cases}1 & (\psi(u)e(t) \ge 0)\\ 0 & (\psi(u)e(t) < 0)\end{cases} \tag{29}$$

$$g_1,\ g_3 > 0,\ \ \boldsymbol{G}_2 = \boldsymbol{G}_2^{\mathsf{T}} > 0 \tag{30}$$

Now we will get the following theorem.

Theorem. For a plant represented by Eq. (1) satisfying assumptions (A1) $\sim$ (A7) and

(A8) $u_{min} \le \Gamma(t) \le u_{max}\ \ \left({}^{\forall}t \ge 0\right)$,

the adaptive control system constructed by the control law Eq. (16) and the adaptive laws Eqs. (26) $\sim$ (28) guarantees boundedness of all signals of the system and

$$\lim_{t\to\infty} e(t) = 0 \tag{31}$$

for any bounded initial conditions of the plant and the controller. □

The theorem says that we can construct an adaptive control system with input saturation which guarantees globally asymptotic stability.

Remark. The condition (A8) of $\Gamma(t)$ guarantees globally asymptotic stability. Although p and $\boldsymbol{\theta}$ are unknown, we can estimate the maximum and minimum values because we know the bounds of unknown parameters by (A2). Therefore we can provide a range of $\Gamma(t)$.

4. SIMULATIONS

To confirm a validity of the proposed method, we give a simulation example for a second-order plant with relative degree of one plant shown below

$$y(t) = \frac{1.5(s+0.7)}{s^2+1.2s+0.3}\sigma(u). \tag{32}$$

Here we used the following state space representation of Eq. (32)

$$\begin{aligned}\dot{\boldsymbol{x}}_p(t) &= \begin{bmatrix} 0 & 1 \\ -0.3 & -1.2 \end{bmatrix} \boldsymbol{x}_p(t) + \begin{bmatrix} 0 \\ 1.5 \end{bmatrix} \sigma(u) \\ y(t) &= \begin{bmatrix} 0.7 & 1 \end{bmatrix} \boldsymbol{x}_p(t)\end{aligned} \tag{33}$$

to make an initial value easy to consider. We chose reference signal and design parameters as follows

$$\begin{aligned} &y_M(t) = \sin(0.4\pi t) \\ &\lambda = 2 \\ &c_1 = 4,\ c_2 = k_1 = k_2 = 1 \\ &g_1 = g_3 = 1,\ \boldsymbol{G}_2 = \boldsymbol{I}, \end{aligned} \tag{34}$$

and initial states were

$$\begin{aligned} &\boldsymbol{x}_p(0) = [-30/7,\ 0]^{\mathsf{T}} \\ &v_{f1}(0) = v_{f2}(0) = 0. \end{aligned} \tag{35}$$

We set a input constraint such that $u_{min} = -1$, $u_{max} = 1$.

Fig. 2(a) shows the simulation results using adaptive laws Eqs. (18), (19) which do not compensate the input saturation. Fig. 2(b) shows the results using the proposed method. We can find that in Fig. 2(a), the input intermittently exceeds the limitation , and the control performance remarkably deteriorate with that influence. On the other hand, in Fig. 2(b), asymptotic stabilization is achieved by applying the proposed method.

5. CONCLUSIONS

In this paper, we provided an algorithm of adaptive control systems with input saturation which guaranteed globally asymptotic stability. The effectivity was confirmed by a simulation study. It have higher design flexibility than conventional ones because of error feedback is effectively used and reference signal conditions are weakened.

REFERENCES

[1] S. P. Karason and A. M. Annaswamy, "Adaptive Control in the Presence of Input Constraints," *IEEE Trans. Automat. Cont.*, Vol. 39, No. 11, pp. 2325-2330, 1994.

[2] Y. -S. Zhong, "Globally stable adaptive system design for minimum phase SISO plants with input saturation," *Automatica*, Vol. 41, Issue 9, pp. 1539-1547, 2005.

[3] A. Feuer and A. S. Morse, "Adaptive Control of Single-Input, Single-Output Linear Systems," *IEEE Trans. Automat. Cont.*, Vol. 23, No. 4, pp. 557-569, 1978.

APPENDIX

Proof of the theorem

Let $V(t)$ be the positive function defined by

$$\begin{aligned} V(t) \equiv{}& \frac{1}{2}e(t)^2 + \frac{1}{2}w(t)^2 + \frac{b_m}{2g_1}\tilde{p}(t)^2 \\ &+ \frac{1}{2}\tilde{\boldsymbol{\theta}}(t)^{\mathsf{T}}\boldsymbol{G}_2^{-1}\tilde{\boldsymbol{\theta}}(t) + \frac{1}{2g_3}\tilde{b}(t)^2 \\ &+ \left(\frac{1}{c_2}+\frac{1}{k_2}\right)\boldsymbol{\varepsilon}(t)^{\mathsf{T}}\boldsymbol{P}_0\boldsymbol{\varepsilon}(t) \end{aligned} \tag{36}$$

$$\boldsymbol{P}_0 = \boldsymbol{P}_0^{\mathsf{T}} > 0,\ \ \boldsymbol{P}_0\boldsymbol{F}_0 + \boldsymbol{F}_0^{\mathsf{T}}\boldsymbol{P}_0 = -\boldsymbol{I}. \tag{37}$$

We evaluate the time derivative $\dot{V}(t)$ for the conditions of $\sigma(u)$ and $e(t)$.

a) $u_{min} \le u(t) \le u_{max}$

Here $\sigma(u) = u(t)$ and become $\psi(u) = 0$, $\psi(u)e(t) = 0$. Hence, form Eqs. (23), (24), $\dot{V}(t)$ is evaluated as follow

$$\begin{aligned} \dot{V}(t) ={}& -(c_1+c_2)e(t)^2 - (k_1+k_2)w(t)^2 \\ &+ \frac{b_m}{g_1}\tilde{p}(t)\left[g_1\hat{\phi}(t)\{w(t)+e(t)\} - \dot{\hat{p}}(t)\right] \\ &+ \tilde{\boldsymbol{\theta}}(t)^{\mathsf{T}}\boldsymbol{G}_2^{-1}\left[\boldsymbol{G}_2\boldsymbol{\omega}(t)\{w(t)+e(t)\} - \dot{\hat{\boldsymbol{\theta}}}(t)\right] \\ &+ \frac{1}{g_3}\tilde{b}(t)\left\{-g_3\psi(u)w(t) - \dot{\hat{b}}(t)\right\} \\ &+ e(t)\varepsilon_1(t) + w(t)\varepsilon_1(t) \\ &- \left(\frac{1}{c_2}+\frac{1}{k_2}\right)\|\boldsymbol{\varepsilon}(t)\|^2. \end{aligned} \tag{38}$$

Then Eqs. (26) $\sim$ (30) and following formulas

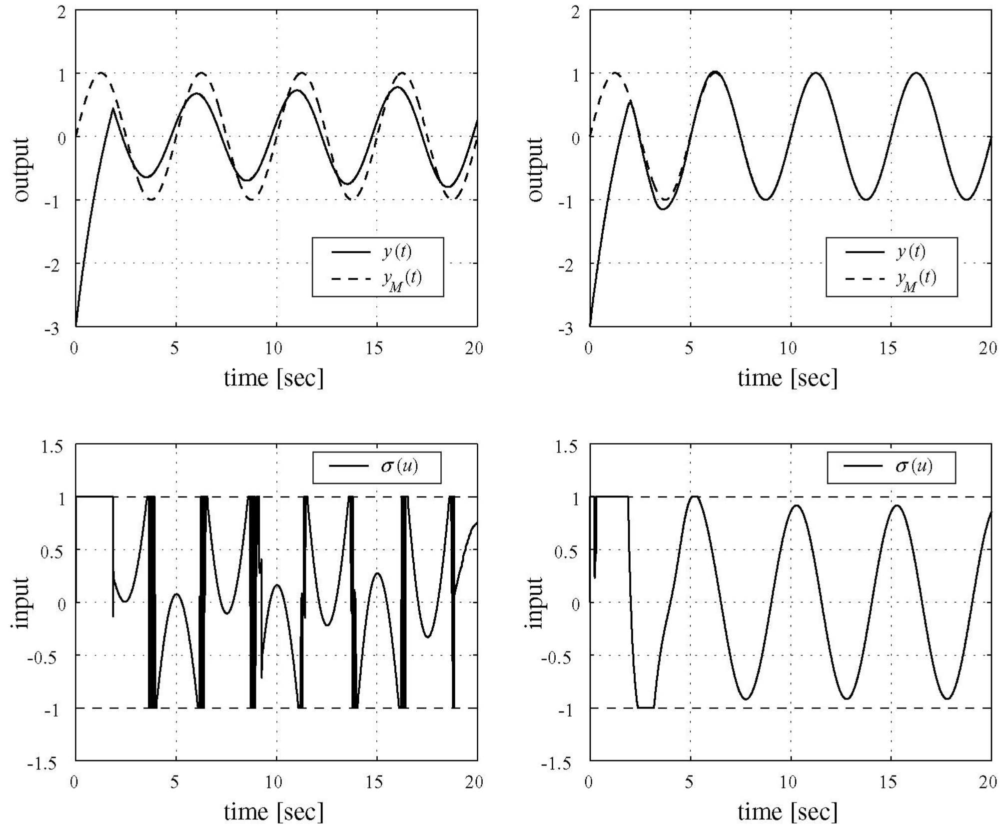
$$e(t)\varepsilon_1(t) \le c_2 e(t)^2 + \frac{1}{4c_2}\varepsilon_1(t)^2 \tag{39}$$

$$w(t)\varepsilon_1(t) \le k_2 w(t)^2 + \frac{1}{4k_2}\varepsilon_1(t)^2, \tag{40}$$

we have

$$\begin{aligned} \dot{V}(t) \le{}& -c_1 e(t)^2 - k_1 w(t)^2 \\ &- \left(\frac{1}{c_2}+\frac{1}{k_2}\right)\Omega(\boldsymbol{\varepsilon}) \end{aligned} \tag{41}$$

$$\Omega(\boldsymbol{\varepsilon}) \equiv \|\boldsymbol{\varepsilon}(t)\|^2 - \frac{1}{4}\varepsilon_1^2(t) > 0. \tag{42}$$



(a) No saturation compensate

(b) Saturation compensate

Fig. 2 Simulation results.

b) $u(t) > u_{max}$

Here $\sigma(u) = u_{max}$. From (A8), we obtain

$$
\begin{aligned}
0 &\leq u_{max} - \Gamma(t) \\
&< u(t) - \Gamma(t) \\
&= -p(c_1 + c_2 - \lambda)e(t) + \tilde{p}(t)\hat{\phi}(t) \\
&\quad + p\tilde{\boldsymbol{\theta}}(t)^{\mathsf{T}}\boldsymbol{\omega}(t).
\end{aligned}
\tag{43}
$$

If $e(t) \geq 0$, $\psi(u)e(t) \geq 0$ and then

$$
\begin{aligned}
&b_m e(t)\{\sigma(u) - \Gamma(t)\} \\
&\leq -(c_1 + c_2 - \lambda)e(t)^2 + b_m\tilde{p}(t)\hat{\phi}(t)e(t) \\
&\quad + \tilde{\boldsymbol{\theta}}(t)^{\mathsf{T}}\boldsymbol{\omega}(t)e(t).
\end{aligned}
\tag{44}
$$

Hence, from Eqs. (11), (24)‘(30), $\dot{V}(t)$ is evaluated as well as Eq. (41).

Also if $e(t) < 0$, $\psi(u)e(t) < 0$ and then

$$
b_m e(t)\{\sigma(u) - \Gamma(t)\} < 0, \tag{45}
$$

from Eqs. (11), (24) $\sim$ (30), (39) and (40), $\dot{V}(t)$ is evaluated as follow

$$
\begin{aligned}
\dot{V}(t) < &-(\lambda - c_2)e(t)^2 - k_1 w(t)^2 \\
&- \left(\frac{1}{c_2} + \frac{1}{k_2}\right)\Omega(\varepsilon).
\end{aligned}
\tag{46}
$$

c) $u(t) < u_{min}$

Here $\sigma(u) = u_{min}$. From (A8), we obtain

$$
\begin{aligned}
0 &\geq u_{min} - \Gamma(t) \\
&> -p(c_1 + c_2 - \lambda)e(t) + \tilde{p}(t)\hat{\phi}(t) \\
&\quad + p\tilde{\boldsymbol{\theta}}(t)^{\mathsf{T}}\boldsymbol{\omega}(t).
\end{aligned}
\tag{47}
$$

Hence, if $e(t) > 0$, $\psi(u)e(t) < 0$ and then Eq. (45) holds, $\dot{V}(t)$ is evaluated as Eq. (46). Further if $e(t) \leq 0$, $\psi(u)e(t) \geq 0$ and then Eq. (44) holds, $\dot{V}(t)$ is evaluated as Eq. (41).

From above discussion, we obtain

$$
\dot{V}(t) \leq 0 \tag{48}
$$

for all $t \geq 0$. Therfore boundedness $V(t)$ is guaranteed, and $e(t)$, $w(t)$, $\hat{p}(t)$, $\hat{\boldsymbol{\theta}}(t)$, $\hat{b}(t) \in \mathcal{L}^\infty$ are implied by Eq. (36). Hence Eqs. (41), (46) lead

$$
\int_0^\infty e(t)^2 dt < \infty, \quad \int_0^\infty w(t)^2 dt < \infty, \tag{49}
$$

$e(t), w(t) \in \mathcal{L}^2$ holds.

Finally we find that the boundedness of all signals are guaranteed analytically, and

$$
\lim_{t\to\infty} e(t) = 0 \tag{50}
$$

holds since $e(t) \in \mathcal{L}^\infty \cap \mathcal{L}^2$ and $\dot{e}(t) \in \mathcal{L}^\infty$.□