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Robust stabilization of cascade switched nonlinear systems subject to input saturation under asynchronous switching

Juan Wang
Tieling Normal College,
Tieling, Liaoning, 112000, China
e-mail: wangjuanneu@163.com

Yanan Xu
Tieling Normal College,
Tieling, Liaoning, 112000, China
e-mail: xuyanan1982@163.com

***Abstract*—This paper addresses the robust stabilization problem for a class of cascade nonlinear switched systems subject to input saturation under asynchronous switching. Convex hull skills are utilized in order to deal with input saturation. A state feedback controller is proposed under the dwell time such that the resulting closed-loop system is exponentially robust stable.**

***Keywords*—asynchronous switching; switched systems; input saturation; dwell time;**

Ⅰ. INTRODUCTION

Switched systems have drawn more attention during the past two decades [1-11]. Robust stabilization is a basic system property in the study of switched systems. Switched systems contain a finite number of subsystems and a logical switching rule, which rules the switching among the subsystems. Therefore, the study of robust stabilization of switched systems is very complicate. A number of techniques have been developed to deal with switched systems, such that the common Lyapunov function[4], the multiple Lyapunov function [5] and the average dwell time [6] and so on.

On the other hand, an important issue is the presence of input saturation, which is inevitable in most practical systems. Input saturation may degrade the performance of practical control systems, what is worse, and even cause instability of the systems. As a result, the robust stabilization of switched systems subject to input saturation has gain wide concern [7-10]. The major approaches of dealing with actuator saturation in the switched systems have anti-windup compensator [7] and convex hull skills [8-10].

Many results on linear switched systems with input saturation have been proposed in the literature. The stabilization and L_2 gain analysis for the linear switched systems with input saturation was addressed in [8]. [9] investigated the linear switched system with input saturation, using convex hull skills and minimal dwell-time. [10] studied the linear switched system subject to actuator saturation with stabilizable and unstabilizable subsystem. So far, few results on the topic for nonlinear switched systems in the presence of actuator saturation have been reported. It is well known that the above results were in terms of an ideal assumption that the switching of the controller is. synchronous with that of the subsystem. As a matter of fact, because it certainly takes some time to recognition the active subsystem and applies the matched controller, the switching instants of controllers can fall behind those of the systems. Hence, asynchronous switching between the subsystems and the controllers arises. Therefore, the study of asynchronous switching is of significance in practice. Many results have been obtained [12-14].

In this paper, the robust stabilization problem for cascade nonlinear switched systems subject to input saturation is addressed under asynchronous switching. Convex hull skills are utilized in order to cope with input saturation. Sufficient conditions are proposed such that the cascade nonlinear switched systems are exponentially robust stable.

Ⅱ. PROBLEM FORMULATION

Consider the following switched systems with input saturation:

$$\begin{aligned}&\dot{x}_1(t)=(A_{1\sigma}+\Delta A_{1\sigma})x_1(t)+A_{2\sigma}x_2(t)+(B_\sigma+\Delta B_\sigma)sat(u_\sigma),\\&\dot{x}_2(t)=f_{2\sigma}(x_2(t)),\end{aligned}\tag{1}$$

where, $x_1(t)\in R^{n-d}, x_2(t)\in R^d$ are the state vector, $\sigma:[0,\infty)\to I_N=\{1,\dots,N\}$ is the switching signal, which is a right continuous piecewise constant function. The switching signal $\sigma(t)$ will be determined later. According to the switching signal, we have the switching sequence $\sum=\{(x_1^T(t_0),x_2^T(t_0))^T;(i_0,t_0),(i_1,t_1),\dots(i_k,t_k),\dots|i_k\in I_N, k=0,1,\dots\}$, which means that $\sigma(t)=\sigma(t_k)=i\in I_N$, i.e., the ith subsystem of switched system is active when $t\in[t_k,t_{k+1})$. Moreover, we suppose that the state of the switched system does not jump at every switching instant. $u_i(t)\in R^m$ are the control input, sat: $R^m\to R^m$ is the vector valued standard saturation function expressed as

$$sat(u_i)=\left[sat(u_{i1})\quad sat(u_{i2})\cdots sat(u_{im})\right]^T,\ i\in I_N,$$
$$sat(u_{ij})=sign(u_{ij})\min\left\{\left|u_{ij}\right|,1\right\},$$

$j\in Q_m=\{1,\cdots,m\}$. A_{1i}, A_{2i}, B_i are constant matrices of suitable dimensions. $f_{2i}(x_2(t))$ are the smooth functions with $f_{2i}(0)=0$. ΔA_{1i}, ΔB_i are uncertain time-varying matrices with the following form, $[\Delta A_{1i}, \Delta B_i] = E_i \Gamma(t)[F_{1i}\ F_{2i}], i \in I_N$, where E_i, F_{1i} and F_{2i} are known constant matrices with proper dimensions, F_{2i} are of full column rank, $\Gamma(t)^T \Gamma(t) \le I$.

The control input is the following form

$$u_\sigma(t) = K_{\sigma(t-\tau)} x_1(t),$$

where, the constant τ is the switching delay, satisfying $\tau \le t_{k+1} - t_k$. On the other hand, the open-loop switched system is activated in the interval $[t_0, t_0+\tau]$.

For a positive definite matrix $P \in R^{n \times n}$, an ellipsoid $\varepsilon(P,\rho)$ is defined as

$$\varepsilon(P,\rho) = \left\{ x \in R^n : x^T P x \le \rho, \rho > 0 \right\}.$$

We define the symmetric polyhedron

$$L(F) = \left\{ x \in R^n : \left| f_l x \right| \le 1, \forall l \in \{1,\cdots,m\} \right\}.$$

Here, f_l are the row of the matrix $F \in R^{m \times n}$.

Lemma 1.*(*[8]*)* Given matrices $K, H \in R^{m\times n}$, $\forall x \in R^n$, if $x \in L(H)$, then

$$sat(Kx) \in co\{D_s K x + D_s^- H x, s \in Q\},$$
$$Q = \{1,2,...,2^m\}$$

where, co denotes the convex hull. Here, D_s are $m \times m$ diagonal matrices with elements either 1 or 0 and $D_s^- = I - D_s$. There are 2^m possible such matrices. Therefore,

$$sat(Kx) = \sum_{s=1}^{2^m} \eta_s (D_s K + D_s^- H)x,$$

where $\sum_{s=1}^{2^m} \eta_s = 1, 0 \le \eta_s \le 1$.

We note that the η_s are functions of the state.

Definition1. For any switching signal $\sigma(t)$ and any $t > t_0 > 0$, let $N_\sigma(t_0,t)$, denote the number of switching of $\sigma(t)$ on the interval (t_0,t). If $N_\sigma(t_0,t) \le \frac{t-t_0}{\tau_a}$ holds for $\tau_a > 0$. The constant τ_a is called the dwell time.

III. ROBUST STABILIZATION

In this section, we derive conditions the robust stabilization of the closed-loop system under the dwell time.

Theorem. If the switched system (1) satisfies the following conditions:

(a) There exist matrices $P_i > 0$, matrices H_i with proper dimensions and a set of scalars $\varepsilon_i > 0, \lambda_0 > 0, \lambda_u > 0, \mu \ge 1$, such that the following inequalities hold.

$$\begin{aligned}&[A_{1i} + B_i(D_s K_i + D_s^- H_i)]^T P_i + P_i[A_{1i} + \\ &B_i(D_s K_i + D_s^- H_i)] + \varepsilon_i P_i E_i E_i^T P_i \\ &+\varepsilon_i^{-1}[F_{1i} + F_{2i}(D_s K_i + D_s^- H_i)]^T [F_{1i} + \\ &F_{2i}(D_s K_i + D_s^- H_i)] + \lambda_0 P_i + I < 0. \end{aligned} \tag{2}$$

$$\begin{aligned}&[A_{1i} + B_i(D_s K_j + D_s^- H_j)]^T P_i + P_i[A_{1i} + \\ &B_i(D_s K_j + D_s^- H_j)] + \varepsilon_i P_i E_i E_i^T P_i \\ &+\varepsilon_i^{-1}[F_{1i} + F_{2i}(D_s K_j + D_s^- H_j)]^T [F_{1i} + \\ &F_{2i}(D_s K_j + D_s^- H_j)] - \lambda_u P_i + I < 0. \end{aligned} \tag{3}$$

$$\begin{aligned}&A_{1i}^T P_i + P_i A_{1i} + \varepsilon_i P_i E_i E_i^T P_i + \varepsilon_i^{-1} F_{1i}^T F_{1i} \\ &-\lambda_u P_i + I < 0. \end{aligned} \tag{4}$$

$$P_i \le \mu P_j, \tag{5}$$

$$\varepsilon(P_i,1) \subset L(H_i) \text{ i.e. } \begin{bmatrix} 1 & H_i^r \\ * & P_i \end{bmatrix} \ge 0, \tag{6}$$

where, $\forall i, j \in I_N, i \ne j$, H_i^r denote the *r*-th row of H_i, $s \in Q = [1, 2^m]$.

(b) There exist functions $W_i(x_2)$ and a set of scalars $\beta_i > 0, \alpha_{i1} > 0, \alpha_{i2} > 0$, $\forall i \in I_N$, such that the following inequalities hold.

$$\alpha_{i1} \left\| x_2 \right\|^2 \le W_i(x_2) \le \alpha_{i2} \left\| x_2 \right\|^2, \tag{7}$$

$$\frac{dW_i(x_2)}{dx_2} f_{2i}(x_2) \le -\beta_i \left\| x_2 \right\|^2, \tag{8}$$

Then the closed-loop switched system is exponentially stable under asynchronous switching for any initial condition $x_0 \in \cap_{i=1}^N \varepsilon(P_i, 1/c), c = \hat{\mu} e^{\beta\tau}$, for arbitrary switching signal satisfying the dwell time

$$\tau_a \ge \tau_a^* = \frac{\ln \hat{\mu} + \tau(\lambda_u + \hat{\lambda}_0)}{\hat{\lambda}_0}, \tag{9}$$

where, $\hat{\lambda}_0 = \min\{\lambda_0, \frac{\beta_i}{\alpha_{i2}} \big| i \in I^+\}$,

$$\hat{\mu}=\max\{\mu,\frac{a_{i2}}{a_{j1}}\Big|i,j\in I_N\}.$$

Proof. By lemma, for every $x\in\varepsilon(P_i,1)\subset L(H_i)$, $sat(u_i)=sat(K_ix_1)\in co\{D_sK_ix_1+D_s^-H_ix_1, s\in Q\}$, choose multiple Lyapunov function

$$V(t)=V_\sigma(t)=x_1^TP_\sigma x_1+kW_\sigma(x_2).$$

The employed controller is

$$u_\sigma(t)=K_{\sigma(t-\tau)}x_1(t).$$

When the *i*th subsystem is active, the matched controller lag behind for τ time.

When $t\in[t_k+\tau,t_{k+1})$,

$$\begin{aligned}\dot V_i(t)&=\dot x_1^TP_ix_1+x_1^TP_i\dot x_1+k\frac{dW_i(x_2)}{dx_2}\\ &\le\max_{s\in Q}2x_1^T\{[A_{1i}+B_i(D_sK_i+D_s^-H_i)]^TP_i+P_iE_i\Gamma(t)[F_{1i}\\ &\quad+F_{2i}(D_sK_i+D_s^-H_i)]\}x_1+2x_2^TA_{2i}^TP_ix_1-k\beta_i\|x_2\|^2,\\ &\le-\lambda_0x_1^TP_ix_1-x_1^Tx_1+2m\|x_1\|\|x_2\|-k\beta_i\|x_2\|^2\\ &\le-\hat{\lambda}_0V_i(t)-(k\beta_i-kl\alpha_{i2}-m^2)\|x_2\|^2.\end{aligned}$$

where, the constant *m* satisfying

$$2x_2^TA_{2i}^TP_ix_1\le2m\|x_1\|\|x_2\|.\ l=\min\{\frac{\beta_i}{\alpha_{i2}}|\ i\in I_N\},$$

$$\hat{\lambda}_0=\min\{\lambda_0,l\}=\min\{\lambda_0,\frac{\beta_i}{\alpha_{i2}}|\ i\in I_N\},$$

$$k\ge\max\{\frac{m^2}{\beta_i-l\alpha_{i2}}|\ i\in I_N\}.$$

Therefore, we have

$$\dot V_i(t)\le-\hat{\lambda}_0V_i(t),$$

which yields that

$$V_i(t)\le e^{-\hat{\lambda}_0(t-t_k-\tau)}V_i(t_{k+\tau}),\ \forall k=0,1,...$$

Similarly, when $t\in[t_k,t_k+\tau)$,

$$\dot V_i(t)\le\lambda_uV_i(t),$$

which yields that

$$V_i(t)\le e^{\lambda_u(t-t_k)}V_i(t_k),\forall k=0,1,...$$

In view of (5), we have

$$V_i(t)\le\hat{\mu}V_j(t),$$

where $\hat{\mu}=\max\{\mu,\frac{\alpha_{i2}}{\alpha_{j1}}\Big|i,j\in I_N\}$.

When $t\in[t_k+\tau,t_{k+1})$.

$$\begin{aligned}V(t)&\le e^{-\hat{\lambda}_0(t-t_k-\tau)}V_i(t_{k+\tau})\\ &\le\cdots\\ &\le\hat{\mu}^{N_\sigma}e^{\lambda_u(N_\sigma+1)\tau}e^{-\hat{\lambda}_0[t-t_0-(N_\sigma+1)\tau]}V(t_0)\\ &\le ce^{-\lambda(t-t_0)}V(t_0),\end{aligned}$$

where, $c=e^{\tau(\lambda_u+\hat{\lambda}_0)}$,

$$\lambda=\hat{\lambda}_0-\frac{\ln\hat{\mu}+\tau(\lambda_u+\hat{\lambda}_0)}{\tau_a}>0.$$

Similarly, when $t\in[t_k,t_k+\tau)$, we have

$$V(t)\le c_0e^{-\lambda(t-t_0)}V(t_0).$$

In view of (7), we easily find constants $c_1>0,c_2>0$,

$$c_1(\|x_1\|^2+\|x_2\|^2)\le V(t)\le c_2(\|x_1\|^2+\|x_2\|^2),$$

$$\|x\|\le\sqrt{\frac{c_2c_0}{c_1}}e^{\frac{-\lambda(t-t_0)}{2}}\|x_0\|.$$

Thus, the closed-loop system is exponential robust stability under asynchronous switching for any initial condition $x_0\in\cap_{i=1}^N\varepsilon(P_i,1/c),c=\hat{\mu}e^{\beta\tau}$.

Remark 1. The gain matrices K_i are obtained easily though solving LMI.

Ⅳ. CONCLUSIONS

This paper has considered the robust stabilization problem for the cascade nonlinear switched systems with input saturation by the dwell time approach. Convex hull skills are employed in order to deal with input saturation. Sufficient conditions are proposed for the cascade switched systems to be exponentially robust stable.

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