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Coverage Control of Unicycle Agents under Constant Speed Constraints ⋆

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Abstract: This paper studies the problem of optimal coverage control over a convex bounded region by a group of unicycle-type mobile agents with constant speeds. We further assume that the constant speeds for different agents may be non-identical. Thus, a unicycle is controlled solely via steering of its orientation in which a virtual centre is defined for each unicycle. A basic controller is proposed which asymptotically drive the virtual centres to a Centroidal Voronoi configuration, thus achieving the optimal coverage objective. At the Centroidal Voronoi configuration, each unicycle executes steady-state circular orbit about its virtual centre. To overcome a drawback of the basic controller that requires a careful selection of initial conditions, a modified switching controller is proposed which achieves a Centroidal Voronoi configuration for arbitrarily chosen initial conditions. Simulations for both the basic and modified controllers are presented for a five-unicycle group.

Keywords: Coverage control, unicycle agent, non-identical speed

1. INTRODUCTION

In the last two decades, a significant amount of interest has been generated in the field of multi-agent systems, see (Cao et al., 2013; Knorn et al., 2015) for a recent overview. In particular, focus has been given to coordinating multiple agents so that they cooperatively complete tasks with higher efficiency, better reliability and robustness against the failure of individual agents or communication loss.

Coverage control is a particular problem of importance (Cortes et al., 2004; Martinez et al., 2007; Schwager et al., 2009; Gusrialdi et al., 2008; Breitenmoser et al., 2010; Gusrialdi and Yu, 2014). A coordinated group of mobile agents (e.g. mobile robots with sensors in the context of coverage control) can provide coverage over a large area better than a single complex agent. Applications of coverage control involving multiple robots include search and rescue operations, surveillance, environmental monitoring and exploration of hazardous/inaccessible regions. The coverage control problem involves solving a locational optimization problem to deploy a group of mobile robots in a manner which covers a specified region. This optimality is obtained by optimizing a coverage performance function which might be related to the quality of sensing or the cumulative probability of the sensor network for detecting an event within the region.

The dynamic model of the agents must be considered when designing the control algorithms for solving the coverage control problem. The pioneering work (Cortes et al., 2004) considered mobile agents modelled by fully actuated single integrators. In (Jiang et al., 2015), multiple single-integrator agents covering the same partition of the region is considered. Unicycle models better represent modern mobile robots such as Unmanned Aerial Vehicles (UAVs) or ground vehicles (Sepulchre et al., 2008; Seyboth et al., 2014; Sun et al., 2015). However, unicycle models have complex dynamics; control algorithms for single-integrator models cannot be applied.

This paper considers the coverage control problem where each agent has dynamics described by the unicycle model. Furthermore, we consider the case that each agent has a fixed cruising speed. A typical example of such agent model is fixed-wing UAVs which have a nominal airspeed for increasing fuel efficiency and airborne duration. A novel problem considered in this paper is that the agents may have fixed speeds which are not necessarily identical, allowing greater flexibility. For example, two different UAVs may have different sensory payloads or flight capabilities

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and thus fly at different speeds. Although non-identical fixed speed is considered in (Seyboth et al., 2014), their considered problem focuses on formation control.

We note that the problem considered in this paper has a similar formulation with that in paper (Kwok and Martìnez, 2010) which discussed the case that all agents have identical unit speeds. In comparison, the control algorithm namely *basic controller* (see Subsection 3.1) proposed in this paper provides another point of view and is intuitively straightforward to understand. The proposed algorithm ensures that the *virtual centre* (the exact definition of the virtual centre is given in the sequel) of each agent asymptotically converge to the set of Centroidal Voronoi configurations if the initial values are carefully selected, thus solving the optimal coverage control problem. When the virtual centres are approaching a desired configuration, the agents trajectories will converge to a steady-state circular orbit about the virtual centres. Moreover, this paper presents a key novel contribution in proposing a *modified controller* (see Subsection 3.2) to ensure that a virtual centre associated with an individual unicycle agent remains inside the coverage region while approaching coverage objective. The design of the modified controller is inspired to remove the drawback of basic controller where the virtual centres can leave the coverage region if their initial values are not properly selected. In fact, the virtual centre is undefined outside the coverage region, and therefore the basic controller for that specific robot becomes undefined and the coverage problem cannot be solved. To the author's knowledge, this paper is the first to design a controller which allows us to explicitly conclude that the virtual centre cannot leave the coverage region and is therefore well defined, for all time.

The rest of this paper is structured as follows. Section 2 introduces concepts in locational optimization, unicycle dynamics and provides a formal definition of the problem. Section 3 presents the two controllers with detailed theoretical analysis. Simulations are provided in Section 4 and the paper is concluded in Section 5.

2. PRELIMINARIES

2.1 Notations

The notations used throughout this paper are fairly standard. The set of real numbers and the set of non-negative real numbers are denoted by $\mathbb{R}$ and $\mathbb{R}_+$, respectively. The set of complex numbers is denoted by $\mathbb{C}$. The space of a unit circle is denoted by $\mathbb{S}^1$ and an angle θ is a point in $\mathbb{S}^1$. The n-torus is the Cartesian product $\mathbb{T}^n = \mathbb{S}^1 \times \cdots \times \mathbb{S}^1$. The imaginary unit is $i := \sqrt{-1}$. A complex number $z \in \mathbb{C}$ is denoted as $\mathbf{Re}(z) + i\mathbf{Im}(z)$, where $\mathbf{Re}(z)$ and $\mathbf{Im}(z)$ are its real and imaginary part, respectively. The complex conjugate of z is denoted by $\bar{z}$. For $z_1, z_2 \in \mathbb{C}$, the scalar product is defined by $\langle z_1, z_2\rangle = \mathbf{Re}(\bar{z}_1 z_2)$. The norm of $z \in \mathbb{C}$ is defined as $\|p\| = \langle p, p\rangle^{\frac{1}{2}}$. In this paper, we shall refer to a vector in the complex plane simply as a "vector" for convenience.

2.2 Background: locational optimization

Let Q be a convex polygon in $\mathbb{R}^2$ to be covered by n mobile agents. An arbitrary point in Q is denoted as q. We call a map $\Phi : Q \to \mathbb{R}_+$ a distribution density function if it represents a measure of information or probability that some event takes place over Q. The location of n agents is represented by $P = (p_1, \ldots, p_n)$, each moving in the space Q. A coverage performance function is defined as

$$H(P) = \int_Q \min_{k\in\{1,\cdots,n\}} \|q - p_k\|^2 \Phi(q) dq \qquad (1)$$

where $\Phi(q)$ is a distribution density function known to all agents and it is assumed that $\Phi(q)$ is at least two times continuously differentiable. One can consider $\|q - p_k\|^2$ as a quantitative measure of how poorly a sensor positioned at p_k senses the point q.

The minimum operation inside the integral of the above performance function (1) induces a so-called Voronoi partition $V(P) = (V_1, \ldots, V_n)$ of polygon Q. The Voronoi partition is defined as follows:

$$V_k = \{q \in Q | \ \|q - p_k\| \le \|q - p_j\|, \forall j \neq k\} \qquad (2)$$

The set of regions $V_1, \ldots, V_n$ is called the Voronoi diagram for the generators $p_1, \ldots, p_n$. Note that each Voronoi cell is convex. When two Voronoi regions V_k and V_j are adjacent (i.e., they share an edge), p_k is called a neighbour of p_j (and vice versa). The set of indexes of the Voronoi neighbours of p_k is denoted by N_k. According to the definition of the Voronoi partition, we have $\min_{k\in\{1,\cdots,n\}} \|q-p_k\|^2 = \|q - p_j\|^2$ for any q inside V_j. Therefore, the performance function (1) can be rewritten as

$$H(P) = \sum_{k=1}^{n} \int_{V_k} \|q - p_k\|^2 \Phi(q) dq \qquad (3)$$

The locational optimization aims to move p_k to minimize the coverage performance function (3).

The following lemma shows an important fact about the performance function (3).

Lemma 1. (Lemma 2.1, Schwager (2009)). The gradient of $H(P)$ is given by

$$\frac{\partial H}{\partial p_k} = \int_{V_k} \frac{\partial}{\partial p_k} \|q - p_k\|^2 \Phi(q) dq \qquad (4)$$

□

From Lemma 1, it is simple to obtain that

$$\frac{\partial H}{\partial p_k} = 2p_k \int_{V_k} \Phi(q)dq - 2\int_{V_k} q\Phi(q)dq$$

By defining the (generalized) mass, centroid (or centre of mass) as follows:

$$M_{V_k} = \int_{V_k} \Phi(q)dq, \quad C_{V_k} = \frac{1}{M_{V_k}} \int_{V_k} q\Phi(q)dq$$

we finally obtain

$$\frac{\partial H}{\partial p_k} = 2M_{V_k}(p_k - C_{V_k}) \qquad (5)$$

Critical points of H are those in which every agent is at the centroid of its Voronoi cell. The resulting Voronoi configuration is called Centroidal Voronoi configuration.

2.3 Background: unicycle models with constant speed

Unicycles are nonholonomic systems which can be used to describe fixed-wing UAVs and ground based vehicles, among other systems. The unicycle model for the k^{th} agent is described by

$$\begin{aligned}\dot{x}_k &= v_k \cos(\theta_k)\\ \dot{y}_k &= v_k \sin(\theta_k)\\ \dot{\theta}_k &= u_k\end{aligned}$$

where $x_k \in \mathbb{R}$, $y_k \in \mathbb{R}$ are the coordinates of agent k in the real plane and θ_k is the heading angle. The fixed forward velocity (cruising speed) $v_k \in \mathbb{R}_+$ is by definition strictly positive, and is not required to be identical between different agents. The control input u_k is to be designed for steering the orientation of agent k.

In complex notation, let $r_k = x_k + iy_k := \|r_k\|e^{i\varphi_k} \in \mathbb{C}$ denote the position of agent k in the complex plane. Then the above dynamics can be reformulated as

$$\begin{aligned}\dot{r}_k &= v_k e^{i\theta_k}\\ \dot{\theta}_k &= u_k\end{aligned} \tag{6}$$

With the complex notation, a virtual centre, z_k, for each agent k is defined as

$$z_k = r_k + \frac{v_k}{\omega_0} i e^{i\theta_k} \tag{7}$$

and the dynamics of the virtual centre can be written as:

$$\dot{z}_k = v_k e^{i\theta_k} - \frac{v_k}{\omega_0} e^{i\theta_k} u_k, \quad k = 1, \cdots, n \tag{8}$$

The following observations are made which will be used throughout the rest of the paper

Observation 1. Observe that

- O1 If the control input u_k is zero, then each agent travels in a straight line.
- O2 If the control input $u_k = \omega_0$ where ω_0 is a non-zero constant, then each agent moves in a circle of radius $v_k/|\omega_0|$. The rotation direction is determined by the sign of ω_0. If $\omega_0 > 0$, each agent rotates in a counter-clockwise direction with respect to the centre of the circular motion, otherwise clockwise.
- O3 If $u_k = \omega_0$ then the centre of circular orbit is equal to the virtual centre, $c_k = z_k$. Moreover, $\dot{z}_k = 0$.

2.4 Problem statement

Suppose there are n unicycle agents with fixed, not necessarily identical forward speeds. The objective is to steer the virtual centre, defined in (7), of each unicycle agent to a desired Voronoi centroid so that the polygon Q can be covered optimally. Once the virtual centre has arrived at the centroid, the agent will orbit about the centroid with fixed forward speed and angular speed, i.e. steady-state circular orbit. Note that under these conditions, there holds $z_k = c_k$, as in Observation O2.

The control input, u_k, is to be designed to drive z_k to a desired Voronoi centroid C_{V_k} and thus the following performance function

$$H_V(Z) = \sum_{k=1}^{n} \int_{V_k} \|q - z_k\|^2 \Phi(q) dq \tag{9}$$

is to be minimized. Here, Z is the stacked column vector of all z_k, i.e. $Z = (z_1, \cdots, z_n)$.

3. CONTROLLER DESIGN AND PERFORMANCE ANALYSIS

In this section, we introduce two controllers to achieve optimal coverage control. We call the first one the *basic controller* and the second the *modified controller*.

3.1 Basic controller

The aim of the controller design in this subsection is to drive each virtual centre z_k to its corresponding Voronoi cell centroid C_{V_k} so that each unicycle agent will orbit around C_{V_k} with an angular speed $|\omega_0|$ and radius $v_k/|\omega_0|$. To achieve this, the controller for each agent is designed as

$$u_k = \omega_0 + \gamma\omega_0 \langle z_k - C_{V_k}, v_k e^{i\theta_k} \rangle \tag{10}$$

where $\gamma > 0$ is a positive control gain. Recall that the Voronoi partitions are defined in (2) with $p_k = z_k$. Then the virtual centre of agent k is not defined outside of Q. In other words, the controller (10) is undefined for agent k. When z_k is undefined outside of Q, we shall say that "z_k is outside of Q" or that "z_k leaves Q" if z_k is undefined. This is to make explanation for certain parts of future proofs easier. We introduce the following assumption for the basic controller, which will be removed for the modified controller design.

Assumption 1. The initial conditions of the virtual centres $z_k(0)$ are selected such that $z_k(t), \forall\, k \in \{1, ..., n\}$, is defined for all $t \geq 0$.

We now present the main result of the basic controller.

Theorem 2. Consider a group of n unicycle type agents with constant, non-identical speeds modelled by (6), and driven by controller (10). If the initial positions of the virtual centres $z_1, \cdots, z_n$ satisfy Assumption 1, then they converge asymptotically to the set of centroidal Voronoi configurations on Q.

Proof. By using Lemma 1 and (5), the time derivative of the performance function (9) along the trajectory of system (8), is

$$\begin{aligned}\dot{H}_V(Z) &= \sum_{k=1}^{n} 2M_{V_k} \langle z_k - C_{V_k}, \dot{z}_k \rangle\\ &= \sum_{k=1}^{n} 2M_{V_k} \langle z_k - C_{V_k}, \frac{v_k}{\omega_0} e^{i\theta_k}(\omega_0 - u_k) \rangle\end{aligned}$$

By substituting control input (10) into the above equation, we obtain

$$\begin{aligned}\dot{H}_V(Z) &= -\sum_{k=1}^{n} 2M_{V_k} \langle z_k - C_{V_k}, v_k e^{i\theta_k} \gamma \langle z_k - C_{V_k}, v_k e^{i\theta_k} \rangle \rangle\\ &= -\sum_{k=1}^{n} 2\gamma M_{V_k} \langle z_k - C_{V_k}, v_k e^{i\theta_k} \rangle^2 \leq 0\end{aligned}$$

Since $\dot{H}_V(Z)$ is non-positive because γ and $M_{V_k}, \forall\, k$ is strictly positive. Next, define ϱ as the set of Z, θ for which $\dot{H}_V(Z) = 0$. Here, θ is the stacked column vector of all θ_k, i.e. $\theta = (\theta_1, \ldots, \theta_n)$. The set ϱ is expressed as

$$\varrho = \{(Z, \theta) : \langle z_k - C_{V_k}, v_k e^{i\theta_k} \rangle = 0, \; k = 1, \cdots, n\} \tag{11}$$

Firstly, observe that, given u_k in (10), the equality $\langle z_k - C_{V_k}, v_k e^{i\theta_k}\rangle = 0$ implies $\dot{z}_k = 0\,\forall\, k \in \{1,\dots,n\}$. It follows that z_k is constant for all k, which implies that C_{V_k} is constant for all k (C_{V_k} is a function of Z alone). Next, observe that for all k and for all t, $v_k(t)e^{i\theta_k(t)} \neq 0 + 0i$ because $v_k > 0$ is fixed. It is then straightforward to verify that $\langle z_k - C_{V_k}, v_k e^{i\theta_k}\rangle = 0$ if and only if 1) $z_k = C_{V_k}$ or 2) with $z_k - C_{V_k} \neq 0$, there holds $z_k - C_{V_k} \perp v_k e^{i\theta_k}$.

Consider firstly case 2), for any k. Observe that if $z_k - C_{V_k} \perp v_k e^{i\theta_k}$ then from (10) it is concluded that $u_k = w_0$. Observations O2 and O3 will allow us to conclude that agent k is conducting circular orbit about z_k. In other words, the vector $v_k e^{i\theta_k}$ changes direction with angular frequency w_0. Because $v_k e^{i\theta_k}$ is non-constant but $z_k - C_{V_k}$ is constant, it is straightforward to verify that the set $\{(Z,\theta) : z_k - C_{V_k} \perp v_k e^{i\theta_k}, z_k - C_{V_k} \neq 0, k = 1,\dots n\}$ is *not* an invariant set of ϱ.

Next consider case 1). It is straightforward to verify that $z_k = C_{V_k}$ implies $u_k = w_0$ which in turn implies that agent k is undergoing a steady-state circular orbit about C_{V_k} and $\dot{z}_k = 0$. This allows us to conclude that the set

$$S = \{(Z,\theta) : z_k = C_{V_k}, \forall\, k \in \{1,\dots,n\}\} \qquad (12)$$

is the largest invariant set in ϱ. LaSalle's Invariance Principle (Khalil, 2002) is then used to conclude that, with $\dot{H}_V(Z) \leq 0$, we have $\lim_{t\to\infty} z_k = C_{V_k}$. In other words, the trajectory of $H_V(Z)$ converges to S along which $\partial H/\partial z_k = 0\,\forall\, k \in \{1,\dots,n\}$. It follows that the virtual centres of all unicycles asymptotically converge to the set of centroidal Voronoi configurations on Q. □

Note that controller (10) cannot ensure that each virtual centre z_k remains in Q if $z_k(0)$ does not satisfy Assumption 1.[1] We will present some further analysis for this phenomenon in the remaining part of this subsection.

Nagumo's Theorem [page 26, (Aubin, 1991)] will be firstly used to show that for arbitrary initial conditions $z_k(0) \in Q$, z_k can leave Q if agent k is driven by controller (10). This then provides the motivation for the *modified* controller which will be introduced in the following subsection. The new controller will, with the use of Nagumo's Theorem, show that z_k remains in Q for all time. Because z_k is continuous, if z_k is to leave Q then it must first be on the boundaries of Q (i.e. z_k cannot jump from the interior of Q to the outside). Without loss of generality, assume that $z_k(T_1) = p_Q$ for some time $T_1 > 0$. Here, p_Q is a point of the joint boundaries denoted by $\partial Q \cap \partial V_k$ of Q and V_k. Let δ_Q be the normal vector of $\partial Q \cap \partial V_k$ pointing outside of Q, from p_Q. Nagumo's Theorem is used to conclude that $z_k(t) \in Q$ for all t if and only if $\langle \delta_Q, \dot{z}_k\rangle \leq 0$. In other words, if $\langle \delta_Q, \dot{z}_k\rangle > 0$ then z_k may leave Q and thus become undefined.

In order to aid in the intuitive explanation of Nagumo's Theorem as applied to controller (10), Fig. 1 is provided, showing the system at $t = T_1$. The black lines denote the boundaries of a Voronoi cell V_k. At $t = T_1$, $z_k(T_1) = p_Q$ is on the joint boundaries of Q and V_k, which is denoted by $\partial Q \cap \partial V_k$. The red dot denotes the virtual centre,

[1] We provide a video to show a failure of coverage by the use of controller (10) with improperly chosen initial conditions, which is available at `https://www.youtube.com/watch?v=LdVJ-_c07N8`

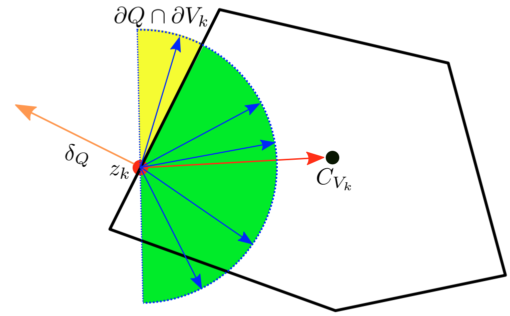

Fig. 1. Diagram used to explain Nagumo's Theorem as applied to the basic controller (10).

z_k. The black dot denotes the cell centroid C_{V_k}. The red arrow denotes the vector $C_{V_k} - z_k$. The orange arrow denotes the normal vector δ_Q, at point p_Q. The blue arrows denote the possible directions of the vector $\dot{z}_k$. As will be shown below, $\dot{z}_k$ can point in any radial direction pointing outwards within the blue semicircle. We shall explain the yellow and green regions shortly.

The idea behind Nagumo's theorem is that one should check whether the directions of velocity vector fields are pointing inside or outside of the convex polygon if the positions are located on the boundaries of the convex polygon. In our case, we study the vector field $\dot{z}_k$. By substituting (10) into (8), we have

$$\dot{z}_k = \gamma v_k e^{i\theta_k}\langle C_{V_k} - z_k, v_k e^{i\theta_k}\rangle \qquad (13)$$

Note that $\langle C_{V_k} - z_k, v_k e^{i\theta_k}\rangle$ is a scalar and $\gamma > 0$. If $C_{V_k} - z_k$ and $v_k e^{i\theta_k}$ are perpendicular then $\dot{z}_k = 0$. If $C_{V_k} - z_k$ and $v_k e^{i\theta_k}$ are not perpendicular then either $\dot{z}_k$ and $v_k e^{i\theta_k}$ point in the same direction (i.e. $\langle C_{V_k} - z_k, v_k e^{i\theta_k}\rangle > 0$) or $\dot{z}_k$ and $v_k e^{i\theta_k}$ point in the opposite direction (i.e. $\langle C_{V_k} - z_k, v_k e^{i\theta_k}\rangle < 0$).

By letting β_k denote the angle between $C_{V_k} - z_k$ and $\dot{z}_k$, it follows that

$$\cos\beta_k = \frac{\langle C_{V_k} - z_k, \dot{z}_k\rangle}{\|C_{V_k} - z_k\|\|\dot{z}_k\|} = \frac{\langle C_{V_k} - z_k, v_k e^{i\theta_k}\rangle^2}{\|C_{V_k} - z_k\|\|\dot{z}_k\|} \geq 0$$

The fact that $\cos\beta_k$ is non-negative reveals an interesting relationship between the directions of $C_{V_k} - z_k$ and $\dot{z}_k$ at $t = T_1$. This relationship is clearly shown in Fig. 1. Observe that the vector $\dot{z}_k$ at $t = T_1$ can be pointing in any radially outwards direction within the blue semicircle since $\cos\beta_k \geq 0$. Note that C_{V_k} is by definition always in the interior of Q (else it is undefined), which implies that $C_{V_k} - z_k$ points inside of Q. However, it is not guaranteed that $\dot{z}_k$ is pointing towards the interior of Q. This is because the boundary $\partial Q \cap \partial V_k$ is not perpendicular to the vector $C_{V_k} - z_k$ in general. As shown in Fig. 1, we use a yellow region to represent the possible directions of $\dot{z}_k$ that point away from the interior of Q. In the green region of Fig. 1 there holds $\langle \delta_Q, \dot{z}_k\rangle \leq 0$. However, in the yellow region, there holds $\langle \delta_Q, \dot{z}_k\rangle > 0$. Using Nagumo's Theorem, we conclude that controller (10) cannot ensure the virtual centre z_k will remain in Q for all time $t > T_1$ if $z_k(0)$ does not satisfy Assumption 1.

3.2 Modified controller

In the above subsection, we have analysed the performance of the basic controller (10) and showed that the initial values of each virtual centre z_k must be carefully selected to satisfy Assumption 1 if the coverage objective was to be successfully achieved. Unfortunately, we cannot provide quantitative results for selecting initial values such that Assumption 1 holds. This is a significant weakness of the *basic* controller and limits application of the proposed basic controller in real world cooperative UAV scenarios.

To remove such a drawback, a modified controller is designed in this subsection. By following the analysis of the previous subsection, a switching controller is proposed and analysed, which ensures that z_k does not leave Q and the optimal coverage control objective is achieved.

Let $Q \setminus \partial Q$ represent the interior of Q. The quantity δ_Q is a normal vector defined in the last subsection, γ is strictly positive and $\omega_0 \neq 0$. The implementation of the algorithm is given in the following algorithm block.

Algorithm 1: Control input determination

Data: boundary information of polygon Q, forward speed v_k and global position r_k of agent k.
Result: determination of the control input u_k.
for $k = 1$ **to** n **do**
  if $z_k \in Q \setminus \partial Q$ **then**
    $u_k = \omega_0 + \gamma\omega_0 \langle z_k - C_{V_k}, v_k e^{i\theta_k} \rangle$
  else
    if $\langle \delta_Q, v_k e^{i\theta_k} \langle C_{V_k} - z_k, v_k e^{i\theta_k} \rangle \rangle \leq 0$ **then**
      $u_k = \omega_0 + \gamma\omega_0 \langle z_k - C_{V_k}, v_k e^{i\theta_k} \rangle$
    else
      $u_k = \omega_0$
    end
  end
end
return $u_k, k = 1, \cdots, n$

Now we state our main results for implementing Algorithm 1 on system (6).

Theorem 3. Consider a group of n unicycle type agents modelled by (6), and driven by Algorithm 1. If the virtual centre $z_k(t)$ of each agent satisfies $z_k(t_0) \in Q$ at time t_0, then $z_k(t) \in Q$ for all $t > t_0$. Moreover, $z_1, \cdots, z_n$ will converge asymptotically to the set of centroidal Voronoi configurations on Q.

Proof. The proof is divided into two parts. In the first part, we will show that no agent k using Algorithm 1 will leave Q if $z_k(t_0)$ is chosen inside or on the boundaries of Q. In the second part, we will prove that the virtual centres converge to the set of Centroidal Voronoi configurations on Q. Due to space limitation, the second part is omitted and will be provided elsewhere.

Part 1: As discussed before, because z_k is continuous then if z_k is to leave Q, it must first be at a point on the boundary. We call this point p_Q as introduced in the previous subsection. Fig. 1 will be used again to help explain the analysis. As will be seen, the proposed algorithm ensures that $\langle \delta_Q, \dot{z}_k \rangle \leq 0$. Assume that z_k is at p_Q at time T_1, where T_1 was defined in the last subsection. Note that function

$$\langle \delta_Q, v_k e^{i\theta_k} \langle C_{V_k} - z_k, v_k e^{i\theta_k} \rangle \rangle \tag{14}$$

is used to decide when to switch the control input when $z_k \notin Q \backslash \partial Q$. Observe that (14) can be any real number at T_1. We now show that both $u_k = \omega_0 + \gamma\omega_0 \langle z_k - C_{V_k}, v_k e^{i\theta_k} \rangle$ and $u_k = w_0$ ensures that z_k does not leave Q.

Consider firstly the case where at $t = T_1$ there holds $\langle \delta_Q, v_k e^{i\theta_k} \langle C_{V_k} - z_k, v_k e^{i\theta_k} \rangle \rangle \leq 0$. In the argument beginning at (13), we pointed out that the direction of vector $v_k e^{i\theta_k} \langle C_{V_k} - z_k, v_k e^{i\theta_k} \rangle$ is restricted to pointing radially outwards from p_Q and only within the blue semicircle shown in Fig. 1. The green region in Fig. 1 corresponds to the situation where (14) is nonpositive, i.e. when the angle between δ_Q and the vector $v_k e^{i\theta_k} \langle C_{V_k} - z_k, v_k e^{i\theta_k} \rangle$ is greater than $\pi/2$. In this case, we have

$$\begin{aligned} \langle \delta_Q, \dot{z}_k \rangle &= \langle \delta_Q, v_k e^{i\theta_k} - \frac{v_k}{\omega_0} e^{i\theta_k} u_k \rangle \\ &= \langle \delta_Q, \gamma v_k e^{i\theta_k} \langle C_{V_k} - z_k, v_k e^{i\theta_k} \rangle \rangle \\ &\leq 0 \end{aligned} \tag{15}$$

with the first equality obtained by substituting in (8), the second equality obtained by substituting in the control input u_k and the inequality obtained by the arguments immediately above.

Next, consider the case where at $t = T_1$, we have $\langle \delta_Q, v_k e^{i\theta_k} \langle C_{V_k} - z_k, v_k e^{i\theta_k} \rangle \rangle > 0$; this is the yellow region in Fig. 1. In this scenario, the control input is set to be $u_k = \omega_0$ according to Algorithm 1. The substitution of $u_k = \omega_0$ to (8) straightforwardly leads to the result that $\dot{z}_k = 0$, as in Observation O3. Consequently, we have that $\langle \delta_Q, \dot{z}_k \rangle = 0$.

We thus conclude that for any value of $\langle \delta_Q, v_k e^{i\theta_k} \langle C_{V_k} - z_k, v_k e^{i\theta_k} \rangle \rangle$ at $t = T_1$ (i.e. for either control sequence u_k when z_k is at p_Q), there holds $\langle \delta_Q, \dot{z}_k \rangle \leq 0$. Nagumo's Theorem can then be used to conclude that z_k will not leave Q if $z_k(t_0) \in Q$. $\square$

4. SIMULATION

In this section, we provide two separate simulation examples to demonstrate the performances of basic sand modified controllers with application to a five-unicycle group. In both examples, we assume that the density function $\Phi(q) = 1$.

The first example shows the performance of the basic controller. Each unicycle is denoted by a yellow triangle and its corresponding virtual centre is denoted by a blue +. The trajectories of different agents are represented by solid lines with different colours. The simulation results of the first example is shown in Fig. 2, from which can see that the optimal coverage objective has been successfully achieved.

The second example illustrates the performance of the modified controller, [2] with simulation results shown in Fig.

[2] We recorded a video to better demonstrate the second example. The video is available on YouTube: `https://www.youtube.com/watch?v=sminHMDIm5Y`. From the video, it can be observed that each virtual cannot leave Q and optimal coverage can be achieved, which verifies the efficiency of our proposed modified controller.

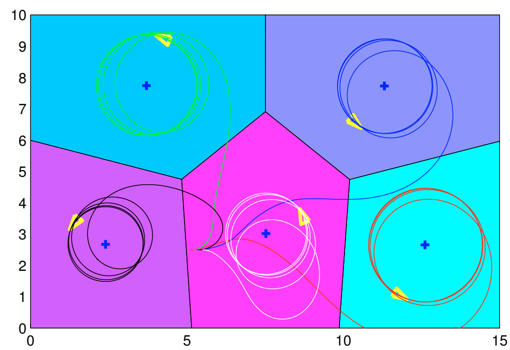

Fig. 2. Unicycle coverage control with basic controller

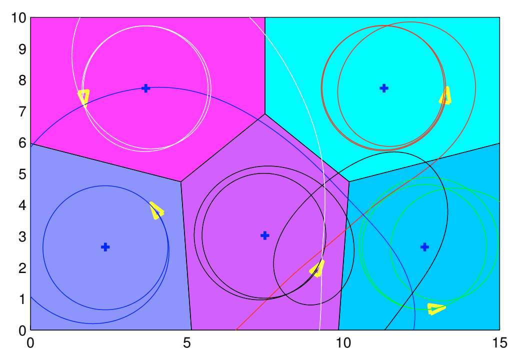

Fig. 3. Unicycle coverage control with modified controller

3. From Fig. 3, one can see that the optimal coverage objective is also asymptotically achieved.

5. CONCLUSION

In this paper, we proposed a basic controller and a modified controller to optimally cover a convex region by using a group of unicycle agents whose forward speeds are fixed, but nonidentical. By defining a virtual centre for each agent, both controllers ensure that the virtual centres asymptotically reach the set of Centroidal Voronoi configurations on the coverage region. When using the basic controller, the initial conditions of the virtual centres must be carefully selected to ensure that the virtual centres remain inside the coverage region for all time. Using Nagumo's Theorem we design a modified switching controller which ensures that the virtual centres remain inside the coverage region for all time, for any initial conditions. Simulations illustrated the theoretical conclusions and showed the effectiveness of both proposed controllers.

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