Coverage control for homogeneous mobile sensors with input constraints on a unit circle

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***Abstract*—This paper addresses the coverage control problem for a network of homogeneous mobile sensors with first-order discrete-time dynamics, where the goal is to minimize a coverage cost function which is defined as the largest distance from any point on a unit circle to its nearest sensor. Due to the existence of a maximum velocity for each sensor in practice, input saturation for the sensors needs to be taken into consideration. Distributed coverage control laws with input constraints are developed to drive the sensors to the final configuration such that the coverage cost function is minimized. It is also shown that the spatial order of the mobile sensors is preserved throughout the network's evolution. As a result, collision avoidance between mobile sensors is always guaranteed.**

I. INTRODUCTION

This paper considers the coverage problem of a circle using a network of mobile sensors with identical maximum velocity. The goal is to deploy the sensors on the circle such that the largest arrival time from the mobile sensor network to any point on the circle is minimized. This problem is motivated by the facts that there always exists an upper bound on a sensor's moving speed in practice and events taking place in the mission domain only last for a finite time period [1]–[3]. When the sensing range of mobile sensors is negligible with respect to the length of a circle, reduction of the largest arrival time from a sensor network to the points on the circle will increase the possibility of capturing the events taking place on the circle before they fade away.

In the past decade, much effort has been devoted to the coverage control problem for mobile sensor networks, where the goal is to drive the sensors to the optimal locations such that the overall sensing performance of the sensor network is maximized [4]–[6]. The coverage problem for mobile sensors in a one-dimensional mission space has received increasing attention in recent years due to its wide potential applications. A benchmark problem of one-dimensional coverage is the uniform coverage problem. It has been shown that the sensing performance of a homogeneous sensor network is maximized when the sensors are uniformly deployed on a line or circle provided that all points on the lie or circle are identical [7], [8].

Problems that are closely related to the coverage control problem on a circle include circular formation and multi-agent consensus. In the circular formation problems, a team of mobile agents is required to form a formation on a circle and the desired distance between neighboring agents is generally prescribed *a priori* [9]. In contrast, in coverage control problems the desired distance between sensors is unknown beforehand and depends on the coverage cost function to be optimized. The multi-agent consensus problem has been studied extensively in literature in which all agents reach an agreement on a variable of interest [10]. Recently, the consensus problem for multi-agent systems with input saturation has attracted much interest due to the fact that input saturation is ubiquitous in real-world applications. For multi-agent systems with integrator dynamics and fixed topology, it is shown in [11] that if the network topology contains a spanning tree, a linear consensus protocol widely used in literature still guarantees the achievement of consensus when there are saturation constraints on the agents' control inputs.

In this paper, a distributed coverage control scheme is developed for homogeneous mobile sensor networks on a circle to minimize the coverage cost function while preserving the mobile sensors' order on the circle. Input constraints are imposed on the mobile sensors' control laws due to the existence of mobile sensors' maximum velocities. Firstly, it is shown that the mobile sensors' order is preserved throughout the network's evolution under the proposed coverage control scheme and thus collision between the sensors is avoided during the coverage task. The idea is to first prove that the spatial order of the sensors is preserved under the coverage control laws without input constraints. Then, we show that the order preservation property is not affected by the introduction of saturation constraints on the sensors' control inputs. Secondly, it has been proved in [8] that the coverage cost function is minimized when all sensors are uniformly deployed on the circle. It is noted that uniform deployment is achieved when all sensors' control inputs converge to zero provided that the sensors' order is preserved. Using tools from graph theory and matrix analysis, we show that under the distributed coverage control laws the sensors' control inputs reach a consensus as time goes to infinity and the consensus value is equal to zero.

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The rest of the paper is organized as follows. In Section II, preliminaries and notations are presented. The coverage control problem for homogeneous mobile sensor networks on a unit circle is formulated in Section III. Distributed coverage control laws which preserve the order of the mobile sensors are proposed in Section IV and convergence analysis of the coverage control laws is given in Section V. In Section VI, simulation results are provided to illustrate the main results. Finally, Section VII concludes the paper.

II. Preliminaries and Notations

Let $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ be a directed graph of order n with the set of nodes $\mathcal{V}$ and the set of edges $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$. An edge of $\mathcal{G}$ is denoted by (i, j) which indicates node j can receive information from node i. The adjacency matrix of $\mathcal{G}$ is defined as $\mathcal{A} = [a_{ij}] \in \mathbb{R}^{n\times n}$ with $a_{ij} > 0$ if $(j, i) \in \mathcal{E}$ and $a_{ij} = 0$ otherwise. The Laplacian matrix of $\mathcal{G}$ is given by $\mathcal{L} = [l_{ij}] \in \mathbb{R}^{n\times n}$ with

$$l_{ij} = \begin{cases} \sum_{k=1,k\neq i}^{n} a_{ik}, & j = i, \\ -a_{ij}, & j \neq i. \end{cases}$$

A directed graph is strongly connected if there exists a directed path from every node to every other node. A directed graph is said to have a spanning tree if there exists one node which has a directed path to every other node.

Throughout this paper, $\mathbb{Z}$ represents the set of integers. $\mathbf{1}_n$ is a $n \times 1$ column vector of ones. A standard saturation function $\mathrm{sat}(\cdot) : \mathbb{R} \to \mathbb{R}$ is defined by

$$\mathrm{sat}(x) = \begin{cases} 1, & \text{if } x > 1, \\ x, & \text{if } -1 \leq x \leq 1, \\ -1, & \text{if } x < -1. \end{cases}$$

For a vector $x = (x_1, x_2, ..., x_n)^T \in \mathbb{R}^n$, $\mathrm{sat}(x) = [\mathrm{sat}(x_1), \mathrm{sat}(x_2), ..., \mathrm{sat}(x_n)]^T$.

III. Problem Formulation

Consider a network of homogeneous mobile sensors $i,\ i \in \mathcal{I}_n = \{1, ..., n\}$ initially located on a unit circle $\mathbb{S}^1$. The position of an arbitrary point q on the circle is denoted by the angle measured counterclockwise from the positive horizontal axis. The counterclockwise distance from sensor i to point $q \in \mathbb{S}^1$ can be defined as $\bar{d}(q_i, q) = (q - q_i) \bmod 2\pi$, where q_i is the position of sensor i. Then, the clockwise distance from sensor i to point q is $2\pi - \bar{d}(q_i, q)$. The distance between sensor i and point q is defined by

$$d(q_i, q) = \min\{\bar{d}(q_i, q), 2\pi - \bar{d}(q_i, q)\}. \quad (1)$$

In this paper, q is not required to be constrained in $[0, 2\pi]$ and points q and $q + 2k\pi,\ k \in \mathbb{Z}$ refer to the same point on the circle.

Without loss of generality, sensors are labeled counterclockwise in accordance with their initial locations on the circle, that is,

$$0 \leq q_1(0) < \cdots < q_i(0) < q_{i+1}(0) < \cdots < q_n(0) < 2\pi. \quad (2)$$

A mobile sensor network's order is preserved if the following inequalities

$$q_1(k) < \cdots < q_i(k) < q_{i+1}(k) < \cdots < q_n(k) < 2\pi + q_1(k)$$

always hold. In this work, each sensor can only communicate with its right neighbor and left neighbor on the circle. Then, the network topology is fixed and strongly connected if the sensors' order is preserved throughout the entire evolution.

Assume that networked mobile sensors evolve in discrete-time steps according to the following dynamics

$$q_i(k + 1) = q_i(k) + \epsilon u_i(k), \quad (3)$$

where $q_i(k)$ and $u_i(k)$ are the position and control input of sensor i at step k, respectively and $\epsilon > 0$ is the step-size. Note that $u_i(k)$ also denotes the velocity of sensor i. There exists an identical maximum velocity λ for each sensor in the homogeneous sensor network, which is assumed to be known *a priori* by the sensors. Due to the existence of sensors' maximum velocities, there exist saturation constraints on the sensors' control inputs, that is, $-\lambda \leq u_i(k) \leq \lambda,\ \forall k \geq 0,\ \forall i \in \mathcal{I}_n$. For convenience, let $q_{n+1} = q_1$ throughout this paper.

Given these definitions, this paper aims at deploying a network of homogeneous mobile sensors to minimize the following coverage cost function

$$T(q_1, ..., q_n) = \max_{q\in\mathbb{S}^1} \min_{i\in\mathcal{I}_n} \frac{d(q_i, q)}{\lambda}, \quad (4)$$

where $d(q_i, q)/\lambda$ is the shortest arrival time from sensor i to point q on the circle. Define the arrival time from a mobile sensor network to an arbitrary point on $\mathbb{S}^1$ as the minimum of the shortest arrival time from the sensors to the point. Then, T is the largest arrival time from a mobile sensor network to any point on the circle. When the mobile agents' sensing range is negligible with respect to the length of the circle, a smaller T increases the possibility of capturing each event taking place on the circle before they disappear. The goal of this paper is to develop distributed coverage control laws to drive a network of homogeneous mobile sensors to the optimal configuration such that the coverage cost function $T(q_1, ..., q_n)$ is minimized while preserving the mobile sensors' order during the coverage task.

IV. Distributed coverage control laws

Consider the following coverage control laws

$$u_i(k) = \lambda \mathrm{sat}(\bar{u}_i(k)),\ i = 1, ..., n, \quad (5)$$

where $\lambda > 0$ is the sensors' maximum velocities and

$$\begin{aligned} \bar{u}_1(k) &= -2q_1 + q_2 + q_n - 2\pi, \\ \bar{u}_i(k) &= q_{i-1} - 2q_i + q_{i+1},\ i = 2, ..., n-1, \\ \bar{u}_n(k) &= q_1 + q_{n-1} - 2q_n + 2\pi. \end{aligned} \quad (6)$$

The control laws can be rewritten in a compact form

$$u(k) = \lambda \mathrm{sat}(\bar{u}(k)), \quad (7)$$

where $u(k) = [u_1(k), u_2(k), ..., u_n(k)]^T$ and $\bar{u}(k) = [\bar{u}_1(k), \bar{u}_2(k), ..., \bar{u}_n(k)]^T$ is given by

$$\bar{u}(k) = -Lq(k) + B \tag{8}$$

with $q(k) = [q_1(k), q_2(k), ..., q_n(k)]^T$, $B = [-2\pi, 0, ..., 0, 2\pi]^T$ and

$$L = \begin{bmatrix} 2 & -1 & 0 & ... & -1 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ ... & -1 & 2 & -1 & ... \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ -1 & 0 & ... & -1 & 2 \end{bmatrix}. \tag{9}$$

Before we proceed, a property of the saturation function is explored.

Lemma 1. $\text{sat}(x) - \text{sat}(y) \geq x - y$ if $x \leq y$.

Proof. From the definition of the saturation function, one has

$$\text{sat}(x)-\text{sat}(y) = \begin{cases} 0, & \text{if } x > 1,\ y > 1, \\ x-1, & \text{if } -1 \leq x \leq 1,\ y > 1, \\ x-y, & \text{if } -1 \leq x \leq 1,\ -1 \leq y \leq 1, \\ -2, & \text{if } x < -1,\ y > 1, \\ -1-y, & \text{if } x < -1,\ -1 \leq y \leq 1, \\ 0, & \text{if } x < -1,\ y < -1. \end{cases}$$

As a result, it can be verified that $\text{sat}(x) - \text{sat}(y) \geq x - y$ holds when $x \leq y$. ∎

Next, we show that the mobile sensors' order is always preserved by the proposed coverage control laws.

Theorem 1. Given the initial condition (2) and step-size ϵ with $0 < \epsilon < 1/2\lambda$,

$$q_1(k) < \cdots < q_i(k) < q_{i+1}(k) < \cdots < q_n(k) < 2\pi + q_1(k),$$

holds for all $k \geq 0$ under the coverage control laws (7).

Proof. We first prove that the mobile sensors' order is always preserved under the following coverage control laws without input saturation

$$\tilde{u}_i(k) = \lambda \bar{u}_i(k),\ i = 1, ..., n, \tag{10}$$

where $\bar{u}_i(k)$ is given by equation (6). In this case, the dynamics of each sensor can be described by $q_i(k+1) = q_i(k) + \epsilon\lambda\bar{u}_i(k)$. Define

$$\begin{aligned} d_i(k) &= q_{i+1}(k) - q_i(k),\ i = 1, ..., n-1, \\ d_n(k) &= q_1(k) + 2\pi - q_n(k). \end{aligned}$$

The control laws $\bar{u}_i,\ i = 1, ..., n$ can be rewritten as

$$\begin{aligned} \bar{u}_1(k) &= d_1(k) - d_n(k), \\ \bar{u}_i(k) &= d_i(k) - d_{i-1}(k),\ i = 2, ..., n. \end{aligned}$$

Therefore, under the control laws (10), it can be verified

$$\begin{aligned} d_1(k+1) - d_1(k) &= \epsilon\lambda(\bar{u}_2(k) - \bar{u}_1(k)) \\ &= \epsilon\lambda(d_2(k) + d_n(k) - 2d_1(k)) \end{aligned}$$

$$\begin{aligned} d_n(k+1) - d_n(k) &= \epsilon\lambda(\bar{u}_1(k) - \bar{u}_n(k)) \\ &= \epsilon\lambda(d_1(k) + d_{n-1}(k) - 2d_n(k)) \end{aligned}$$

and

$$\begin{aligned} d_i(k+1) - d_i(k) &= \epsilon\lambda(\bar{u}_{i+1}(k) - \bar{u}_i(k)) \\ &= \epsilon\lambda(d_{i-1}(k) + d_{i+1}(k) - 2d_i(k)), \end{aligned}$$

where $i = 2, ..., n-1$. The above equations can be rewritten in a compact form $d(k+1) = (I + \epsilon\lambda H)d(k)$, where $d(k) = [d_1(k), d_2(k), ..., d_n(k)]^T$ and the matrix H is given by

$$H = \begin{bmatrix} -2 & 1 & 0 & ... & 1 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ ... & 1 & -2 & 1 & ... \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 1 & 0 & ... & 1 & -2 \end{bmatrix}.$$

When $0 < \epsilon < 1/2\lambda$, the matrix $I + \epsilon\lambda H$ is nonnegative and at least two elements in each row of $I + \epsilon\lambda H$ are larger than zero. Consequently, $d(k+1) > 0$ if $d(k) > 0$, which implies that the mobile sensors' order is preserved under the coverage control laws (10).

Next, we show that mobile sensors' order is preserved under the coverage control laws (7) with input constraints. Let $\bar{u}_{n+1}(k) = \bar{u}_1(k),\ \forall k \geq 0$. Under the control laws (7), one has

$$d_i(k+1) - d_i(k) = \epsilon\lambda(\text{sat}(\bar{u}_{i+1}(k)) - \text{sat}(\bar{u}_i(k))),\ i \in \mathcal{I}_n.$$

When $\bar{u}_{i+1}(k) \geq \bar{u}_i(k)$, the right hand side of the above equation is larger than zero. Consequently, $d_i(k+1) > 0$ if $d_i(k) > 0$. Next, consider the case when $\bar{u}_{i+1}(k) < \bar{u}_i(k)$. Note that the mobile sensors' order is preserved under the control laws (10), which implies $d_i(k+1) = d_i(k) + \epsilon\lambda(\bar{u}_{i+1}(k) - \bar{u}_i(k)) > 0$ if $d_i(k) > 0$. Then, it follows from Lemma 1 that if $d_i(k) > 0$ and $\bar{u}_{i+1}(k) < \bar{u}_i(k)$, one has $d_i(k) + \epsilon\lambda(\text{sat}(\bar{u}_{i+1}(k)) - \text{sat}(\bar{u}_i(k))) \geq d_i(k) + \epsilon\lambda(\bar{u}_{i+1}(k) - \bar{u}_i(k)) > 0$. Recall that $d_i(k+1) = d_i(k) + \epsilon\lambda(\text{sat}(\bar{u}_{i+1}(k)) - \text{sat}(\bar{u}_i(k)))$ under the control laws (7). Consequently, $d_i(k+1) > 0$ when $d_i(k) > 0$ and $\bar{u}_{i+1}(k) < \bar{u}_i(k)$. Therefore, $d_i(k+1) > 0$ if $d_i(k) > 0$ under the coverage control laws (7). When the initial condition (2) is satisfied and $0 < \epsilon < 1/2\lambda$, $q_1(k) < \cdots < q_i(k) < q_{i+1}(k) < \cdots < q_n(k) < 2\pi + q_1(k)$ always holds under the proposed coverage control laws with input constraints. ∎

Remark 1. It has been shown in Theorem 1 that $q_1(k) < \cdots < q_i(k) < q_{i+1}(k) < \cdots < q_n(k) < 2\pi + q_1(k),\ \forall k \geq 0$ under the coverage control laws (7). As a result, $0 < q_{i+1}(k) - q_i(k) < 2\pi,\ i = 1, ..., n-1$ and $0 < q_1(k) + 2\pi - q_n(k) < 2\pi$ always hold. Recalling the definition of the counterclockwise distance from a sensor to a point on the circle, one has $\bar{d}(q_i(k), q_{i+1}(k)) = q_{i+1}(k) - q_i(k),\ i = 1, ..., n-1$ and $\bar{d}(q_n(k), q_1(k)) = q_1(k) + 2\pi - q_n(k)$. Let $q_{n+1} = q_1$ and $q_0 = q_n$. The coverage control laws (7) can be rewritten as

$$u_i(k) = \lambda \text{sat}(\bar{d}(q_i(k), q_{i+1}(k)) - \bar{d}(q_{i-1}(k), q_i(k))),\ i \in \mathcal{I}_n,$$

which implies that each sensor only requires the information of its right neighbor and left neighbor on the circle to compute its control law. Therefore, the coverage control laws (7) are distributed and can be implemented without the knowledge of the sensors' labels.

V. Convergence analysis

In this section, we will show that a network of homogeneous mobile sensors will be driven to the final configuration under the distributed coverage control laws (7) such that the coverage cost function $T(q_1, ..., q_n)$ is minimized. Before we proceed, some useful lemmas are introduced.

Lemma 2 [8]. The coverage cost function $T(q_1, ..., q_n)$ is minimized if and only if the counterclockwise distance between neighboring sensors $\bar{d}(q_i, q_{i+1})$ is identical for all $i \in \mathcal{I}_n$.

Lemma 3 [10]. Consider a network of n agents with topology G and the following discrete-time consensus protocol

$$x(k+1) = (I - \varepsilon \mathcal{L}) x(k),$$

where $x(k) = [x_1(k), x_2(k), ..., x_n(k)]^T$ is the agents' states at step k, $\mathcal{L} = [l_{ij}] \in \mathbb{R}^{n \times n}$ is the Laplacian matrix of G, and $0 < \varepsilon < 1/\max_{i \in \mathcal{I}_n} l_{ii}$. A consensus is asymptotically reached if G is a strongly connected digraph.

The following lemma has been shown in references [12] and [13].

Lemma 4. The Laplacian matrix $\mathcal{L}$ has the algebraic multiplicity of its eigenvalue 0 equal to one if and only if the graph associated with the matrix L has a spanning tree.

Given Lemmas 3 and 4, following a similar procedure of the proof of theorem 1 in [11] one can prove the convergence of $z(k)$ to zero, which is formally stated in the following lemma.

Lemma 5. Let $z(k) = \bar{u}(k)$. When $0 < \epsilon < 1/2\lambda$, $z(k)$ will be driven to zero as k goes to infinity under the control laws (7).

The detailed proof of Lemma 5 is omitted due to the limit of page length.

Now we are ready to present the main results.

Theorem 2. When the step-size ϵ satisfies $0 < \epsilon < 1/2\lambda$, under the distributed coverage control laws (7) a network of mobile sensors will be driven to the optimal configuration that minimizes the coverage cost function $T(q_1, ..., q_n)$ with order preservation.

Proof. It has been proved in Theorem 1 that the mobile sensor' order is always preserved under the distributed coverage control laws (7) when the step-size ϵ satisfies $0 < \epsilon < 1/2\lambda$. Then, the counterclockwise distance from each sensor to its right neighbor at each time step can be rewritten as

$$\bar{d}(q_i(k), q_{i+1}(k)) = q_{i+1}(k) - q_i(k),\ i = 1, ..., n-1,$$
$$\bar{d}(q_n(k), q_1(k)) = q_1(k) + 2\pi - q_n(k).$$

Note that $\bar{d}(q_i, q_{i+1}) = \bar{d}(q_j, q_{j+1}),\ \forall i, j \in \mathcal{I}_n$ when $\bar{u}(k) = -Lq(k) + B = 0$. It follows from Lemmas 2 and 5 that the mobile sensors can be driven to the configuration that minimizes the cost function $T(q_1, ..., q_n)$ under the distributed control laws (7). ∎

VI. Simulations

Consider a network of homogeneous mobile sensors which are randomly deployed on a unit circle initially. The sensors are denoted by solid circles with different colors as shown in Fig. 1 and are labeled counterclockwise according to their initial positions on the unit circle. The maximum velocity of the sensors is $\lambda = 0.8$ and the step-size is chosen as $\epsilon = 0.02$. The mobile sensors' initial locations and final locations are shown in Fig. 1. It can be seen that under the proposed coverage control law the mobile sensors are driven to the final configuration where sensors are uniformly deployed on the unit circle. Time evolution of the mobile sensors' positions and control inputs are shown in Fig. 2. Note that the sensors' order is preserved throughout the network's evolution and their control inputs converge to zero as stated in Lemma 5. Fig. 3 shows the time evolution of the coverage cost function $T(q_1, ..., q_5)$ and the counterclockwise distance $\bar{d}_i = \bar{d}(q_i, q_{i+1}),\ i = 1, ..., 5$. It is noted that the counterclockwise distance between neighboring sensors reaches a consensus and the coverage cost function $T(q_1, ..., q_5)$ is minimized under the coverage control laws (7).

VII. Conclusion

In this paper the coverage control problem for a network of homogeneous mobile sensors arranged on a circle has been addressed. A distributed coverage control scheme with input constraints has been developed to drive the sensors to the optimal locations such that the largest arrival time from the mobile sensor network to any point on the circle is minimized. Order preservation of the mobile sensors is also guaranteed by the proposed coverage control scheme, which not only prevents the sensors from colliding with each other, but also results in a fixed network topology and thus simplifies the convergence analysis of the distributed coverage control scheme.

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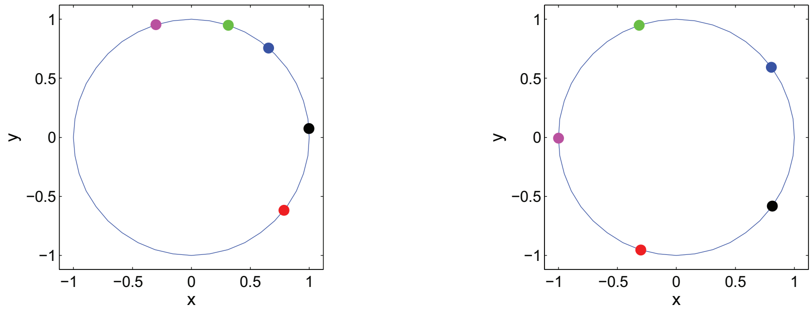

Fig. 1. Homogenous mobile sensors' initial locations and final locations on the circle.

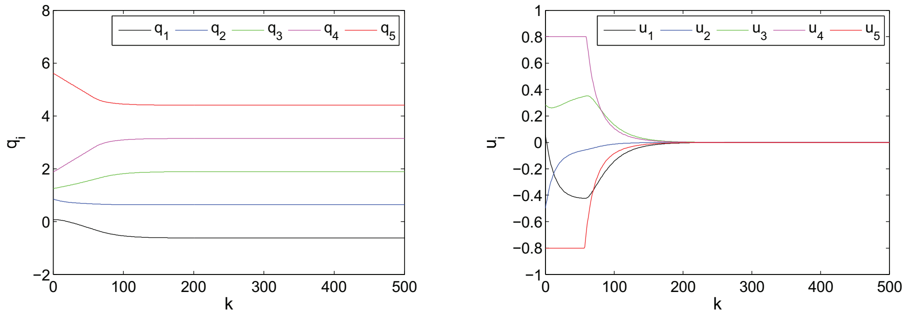

Fig. 2. Position trajectories and control inputs of homogenous mobile sensors.

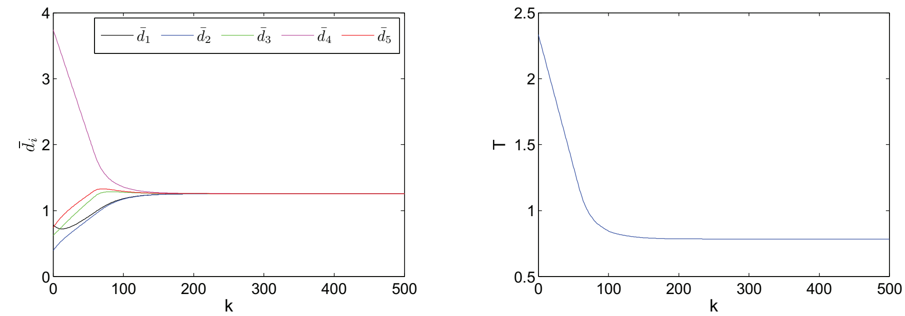

Fig. 3. Time evolution of the functions T_i and T for a homogenous mobile sensor network.

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