Optimal Coverage of Unicycle Robots with Constant Speed and Input-Saturation Constraints

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***Abstract*—The abstract goes here.**

***Index Terms*—IEEE, IEEEtran, journal, LATEX, paper, template.**

I. Introduction

COVERAGE control targets at deploying a set of mobile agents in a finite domain such that a certain coverage metric is optimized. ...

Corresponds to a voronoi tenssellation

The coverage control of fully actuated agents have been widely investigated...... The classical solution for optimal converge control is based on a gradient-descending paradigm. In general, the optimal coverage does not necessarily render a convex programming since the metric function is usually non-convex. This usually indicates that only a local optimal solution can be solved, although it is sufficient to solve most problems in practice. ....

In this paper, we consider the coverage control of a set of underactuated unicycle robots......., which renders a more chanllenging problem than the fully actuated agents. The unicycle robots have a fundamental basis to depict the behavior of ...... like Unmanned Aerial Vehicles (UAV) or ground vehicles......... that navigate a cruise with a cruise in a certain task. Compared to drones............ the UAVs are advantagous to the drones.... in terms of reliability or consumption....... In this paper, we consider that the UAVs navigate a cruise within the confined domain with their virtual centers achieve an optimal deployment. In this scenario, the cruise velocity are set as constants..... and the motion of the agents are dominated by the steering angles, which renders a optimal deployment with under-actuation....(to be added: Why should we model like this? What is the advantage to keep a constant spped) A fomulation as such renders an underactuated optimization......... In general, it is not possible to achieve the gradient-descending paradigm.......to ....In .....(cite), a ... basic controller is proposed...... ... This leads to that the virtual centers may exceed the boundary of the finite domain..... Therefore, it is not desired....... The Voroi.... tenssellation is not defined ..... In this paper, without relaxing the constant speed, we involve a barrier function to confine the ....motion of the agents. When the agents.....go beyond the domain, the ...... can be corrected to .... to resolve the overlarge control input of the ..... close to the boundary of the domain, we .... a modified controller of .... A ... is used to prove that for any initial conditions, the ..... are confined within the boundary and asymptotically converge to the local optimal deployment....

The main contribution of this paper is ...... The paper is organized as following

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II. Preliminaries

III. Main Results

A. Proposed Barrier Lyapunov Function

For a given $A = [a_1, a_2, ..., a_m] \in ?, b = [b_1, b_2, ...b_m] \in ?$, a convex region is defined as $\Omega = \Omega^o \cup \partial\Omega = \{q \in ?| \langle q, a_j \rangle - b_j \leq 0, \forall j\}$. We define the barrier Lyapunov function $V(Z)$

$$\begin{aligned} V(Z) &= \sum_{k=1}^{n} V_k(Z) \\ V(Z) &= \sum_{k=1}^{n}\sum_{j=1}^{m} \frac{1}{2} \frac{\langle z_k - C_k(Z), z_k - C_k(Z) \rangle}{\langle z_k, a_j \rangle - b_j} \end{aligned} \tag{1}$$

where

$$\begin{aligned} V_k(Z) &= \sum_{j=1}^{m} \frac{1}{2} \frac{\langle z_k - C_k(Z), z_k - C_k(Z) \rangle}{\langle z_k, a_j \rangle - b_j} \\ C_k(Z) &= \frac{\int_{\Omega_k(Z)} q\rho(q)dq}{\int_{\Omega_k(Z)} \rho(q)dq} \end{aligned} \tag{2}$$

The proposed BLF in (1) has the following properties:

1) $V(Z)$ is positive definite
Proof.

$$\begin{aligned} V_k(Z) \geq 0 &\iff z_k \in \Omega, \ \forall k \in \{1, ..., n\} \\ V_k(Z) = 0 &\iff z_k \to C_k(Z), \ \forall k \in \{1, ..., n\} \end{aligned} \tag{3}$$

2) $V(Z)$ grows to infinity if and only if at least one agent crosses the boundary of the coverage region
Proof.

$$\begin{aligned} V(Z) \to \infty &\iff \exists k : V_k(Z) \to \infty \\ &\iff \exists k, j : \langle z_k, a_j \rangle - b_j \to 0^+ \\ &\iff \exists k : z_k \to \partial\Omega \end{aligned} \tag{4}$$

It is shown that $V(Z)$ is a feasible candidate for a Barrier Lyapunov function.

B. Control Law for State feasibility

The following control law ensures the state and input feasibility of this coverage problem.

$$u_k = \omega_{k_0} + \frac{\mu_k(\psi_k)\text{sign}(\omega_{k_0})}{\left\| \left\langle \sum_{i\in\tilde{K}} \frac{\partial V_i(Z)}{\partial z_k}, e^{i\theta_k} \right\rangle \right\|} \left\langle \sum_{i\in\tilde{K}} \frac{\partial V_i(Z)}{\partial z_k}, e^{i\theta_k} \right\rangle \tag{5}$$

where

$$\begin{aligned} \omega_{k_0} &: \text{Desired orbiting velocity} \\ \tilde{K} &: \{j \in 1,...,n|\Omega_k \cup \Omega_j \neq \emptyset\}\text{: Set of adjacent agents} \\ \mu(\psi_k) &: \text{Positive control gain} \\ \psi_k &: \text{Angle ...} \end{aligned} \tag{6}$$

By using the proposed control law in (5), the dynamic of each agent's virtual mass is described as

$$\begin{aligned} \dot{z}_k &= v_k e^{i\theta_k} - \frac{v_k}{w_0} e^{i\theta_k} u_k \\ &= -\frac{\mu_k(\psi_k)v_k}{\|\omega_{k_0}\|} \frac{\left\langle \sum_{i\in\tilde{K}} \frac{\partial V_i(Z)}{\partial z_k}, e^{i\theta_k} \right\rangle}{\left\| \left\langle \sum_{i\in\tilde{K}} \frac{\partial V_i(Z)}{\partial z_k}, e^{i\theta_k} \right\rangle \right\|} e^{i\theta_k} \end{aligned} \tag{7}$$

In order to study the time derivative of the proposed BLF $V(Z)$, we introduce the partial derivative of the Voronoi Centroidal and each sub BLF $V_i(Z)$. Appendix ... introduced the previous work from Du [5], which determines the partial derivative of the Voronoi Centroidal C_i of the region Ω_i. The region Ω_i is monitored by agent i and is defined by virtual mass z_i and the virtual masses of the adjacent agents z_k. Therefore, each Voronoi Centroidal $C_i(Z)$ is defined from $Z = [z_1, ..., z_i, ...z_n]$ and there exists a partial derivative $\frac{\partial C_i(Z)}{\partial z_k}$. Since the term adjacent is relative, agent k belongs to the adjacent set of agent i also implies that agent i belongs to the adjacent set of agent k. From the appendix ..., we have

$$\frac{\partial C_i}{\partial z_k} = \begin{cases} \frac{\partial C_k}{\partial z_k} & i = k \\ \frac{\partial C_i}{\partial z_k} & i \in \tilde{K} \text{ or } k \in \tilde{I} \\ 0 & i \notin \tilde{K} \text{ or } k \notin \tilde{I} \end{cases}$$

Furthermore, the partial derivative of $V_i(Z)$ is determined from appendix ... as

- $i = k$

$$\begin{aligned} \frac{\partial V_k(Z)}{\partial z_k} = &\left\langle 1 - \frac{\partial C_k(Z)}{\partial z_k}, z_k - C_k(Z) \right\rangle \sum_{j=1}^{m} \frac{1}{\langle z_k, a_j \rangle - b_j} \\ &- \frac{\langle z_k - C_k(Z), z_k - C_k(Z) \rangle}{2} \sum_{j=1}^{m} \frac{a_j}{\left(\langle z_k, a_j \rangle - b_k\right)^2} \end{aligned}$$

- $i \in \tilde{K}$

$$\frac{\partial V_i(Z)}{\partial z_k} = \left\langle -\frac{\partial C_i(Z)}{\partial z_k}, z_i - C_i(Z) \right\rangle \sum_{j=1}^{m} \frac{1}{\langle z_i, a_j \rangle - b_j}$$

- $i \notin \tilde{K}$

$$\frac{\partial V_i(Z)}{\partial z_k} = 0$$

The time derivative of the introduced BLF $V(Z)$ from (1) is shown to be non-positive

$$\begin{aligned} \dot{V}(Z) &= \sum_{k=1}^{n} \dot{V}_k(Z) \\ &= \sum_{k=1}^{n}\sum_{i=1}^{n} \left\langle \frac{\partial V_k(Z)}{\partial z_i}, \dot{z}_i \right\rangle \\ &= \sum_{k=1}^{n}\sum_{i=1}^{n} \left\langle \frac{\partial V_i(Z)}{\partial z_k}, \dot{z}_k \right\rangle \\ &= \sum_{k=1}^{n} \left\langle \sum_{i=1}^{n} \frac{\partial V_i(Z)}{\partial z_k}, \dot{z}_k \right\rangle \\ &= \sum_{k=1}^{n} \left\langle \sum_{i\in\tilde{K}} \frac{\partial V_i(Z)}{\partial z_k}, \dot{z}_k \right\rangle \end{aligned} \tag{8}$$

Substitute $\dot{z}_k$ from (7), we have

$$\dot{V}(Z) = -\sum_{k=1}^{n} \frac{\mu_k(\psi_k)v_k}{\|\omega_{k_0}\|} \frac{\left\langle \sum_{i\in\tilde{K}} \frac{\partial V_i(Z)}{\partial z_k}, e^{i\theta_k} \right\rangle^2}{\left\| \left\langle \sum_{i\in\tilde{K}} \frac{\partial V_i(Z)}{\partial z_k}, e^{i\theta_k} \right\rangle \right\|} \leq 0 \tag{9}$$

TO DO.
- Independence of C and V in relation to the "neighbors of neighbors" in appendix
- Scale factor μ_k for feasible control input
- Numerical solution of term dV/dz -¿ set of K tilde

C. Designing proper scaling factor for input constraint

By designing the proper positive scaling factor $\mu_k(\psi_k)$ as

$$\mu_k(\psi_k) = \begin{cases} k_1 \in \mathbb{R},\ 0 < k_1 \leq U_{up} - \|\omega_{k_0}\| & ,\ \psi_k \in [\frac{\pi}{2}\ \frac{3\pi}{2}] \\ k_2 \in \mathbb{R},\ 0 < k_2 \leq U_{low} + \|\omega_{k_0}\| & ,\text{else} \end{cases} \tag{10}$$

IV. Simulation

V. Experimental Validation

VI. Conclusion

The conclusion goes here.

Appendix A
Barrier Lyapunov Function for coverage control

Appendix B
Partial derivative of BLF

FIRST MULTIPLICAND

$$\begin{aligned} &\frac{\partial}{\partial z_k} \langle z_i - C_i(Z), z_i - C_i(Z) \rangle \\ &= 2 \left\langle \frac{\partial}{\partial z_k}(z_i - C_i(Z)), z_i - C_i(Z) \right\rangle \\ &= \begin{cases} 2\left\langle 1 - \frac{\partial C_i(Z)}{\partial z_k}, z_i - C_i(Z) \right\rangle & i = k \\ 2\left\langle -\frac{\partial C_i(Z)}{\partial z_k}, z_i - C_i(Z) \right\rangle & i \in \tilde{K} \\ 0 & i \notin \tilde{K} \end{cases} \end{aligned} \tag{11}$$

SECOND MULTIPLICAND

$$\begin{aligned}&\frac{\partial}{\partial z_k}\left(\sum_{j=1}^{m}\frac{1}{\left\langle z_i,a_j\right\rangle - b_j}\right)\\&=\begin{cases}-\sum_{j=1}^{m}\frac{a_j}{\left(\left\langle z_i,a_j\right\rangle - b_j\right)^2} & i=k\\ 0 & i\neq k\end{cases}\end{aligned}\tag{12}$$

Together

$$\begin{aligned}&\frac{\partial V_i(Z)}{\partial z_k}\\&=\frac{\partial}{\partial z_k}\left(\sum_{j=1}^{m}\frac{1}{2}\frac{\left\langle z_i-C_i(Z),z_i-C_i(Z)\right\rangle}{\left\langle z_i,a_j\right\rangle - b_j}\right)\\&=\frac{\partial}{\partial z_k}\left(\frac{1}{2}\left\langle z_i-C_i(Z),z_i-C_i(Z)\right\rangle\sum_{j=1}^{m}\frac{1}{\left\langle z_i,a_j\right\rangle - b_j}\right)\\&=\left\langle\frac{\partial z_i}{\partial z_k}-\frac{\partial C_i(Z)}{\partial z_k},z_i-C_i(Z)\right\rangle\sum_{j=1}^{m}\frac{1}{\left\langle z_i,a_j\right\rangle - b_j}\\&-\ldots\end{aligned}\tag{13}$$

APPENDIX C
PARTIAL DERIVATIVE OF VORONOI CENTROIDAL ALGORITHM IMPLEMENTATION

NOTE. CHANGE TO COMPLEX PLANE

In [5], Lee formulated the term $\frac{\partial C_i}{\partial z_k}$ as

$$\begin{aligned}\frac{\partial C_i^{(a)}}{\partial z_k^{(b)}}=&\frac{\int_{\partial\Omega_{i,k}}\rho(q)q^{(a)}\frac{q^{(b)}-z_k^{(b)}}{\|z_k-z_i\|}dq}{m_i}\\&-\frac{(\int_{\partial\Omega_{i,k}}\rho(q)\frac{q^{(b)}-z_k{}^{(b)}}{\|z_k-z_i\|}dq)(\int_{\Omega_i(Z)}\rho(q)q^{(a)}dq)}{m_i^2}\end{aligned}\tag{14}$$

with

$$\begin{aligned}&a,b\in\{x,y\}\\&m_k=something\end{aligned}\tag{15}$$

APPENDIX D

Appendix two text goes here.

It is known that

$$\frac{\partial\left\langle z,a\right\rangle}{\partial z}=a,\tag{16}$$

$$\frac{\partial\left\langle C_v(z),a\right\rangle}{\partial z}=\tag{17}$$

$$\dot{V}(z)=\left\langle\frac{\partial V}{\partial z},\dot{z}\right\rangle,\tag{18}$$

where

$$\begin{aligned}\frac{\partial V}{\partial z}=&\sum_{k=1}^{n}\sum_{j=1}^{m}\frac{\left\langle z-C_v(z),a\right\rangle}{\left\langle z,a\right\rangle - b}\left(\frac{1}{\left\langle z,a\right\rangle - b}\frac{\partial\left\langle z-Cv,a\right\rangle}{\partial z}\right.\\&\left.-\frac{\left\langle z-C_v,a\right\rangle}{\left(\left\langle z,a\right\rangle - b\right)^2}\frac{\partial\left\langle z,a\right\rangle}{\partial z}\right)\end{aligned}\tag{19}$$

Therefore

$$\frac{\partial V}{\partial z}=\sum_{k=1}^{n}\sum_{j=1}^{m}\frac{\left\langle z-C_v(z),a\right\rangle}{\left(\left\langle z,a\right\rangle - b\right)^3}\tag{20}$$

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REFERENCES

[1] H. Kopka and P. W. Daly, *A Guide to LATEX*, 3rd ed. Harlow, England: Addison-Wesley, 1999.

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Michael Shell Biography text here.

John Doe Biography text here.

Jane Doe Biography text here.