2017 11th Asian Control Conference (ASCC)
Gold Coast Convention Centre, Australia
December 17-20, 2017

Barrier coverage by heterogeneous sensor network with input saturation

Thien-Minh Nguyen, Xiuxian Li, Lihua Xie

Abstract— **In this paper we present a decentralized control law for a mobile sensor network with different sensing radii to form a barrier that spans over a corridor with optimal coverage. Instead of using a gradient-descent approach which may only settle to a local minimum, we show that there exists a cost function whose global minimum is achieved when the sensor network reaches the optimal barrier configuration via consensus. Taking into account the heterogeneous sensing capability and input saturation, the proposed control law not only can drive the agents to the desired optimal configuration but also avoid collision and preserve the order of the sensor placement. Analysis and simulation are carried out to verify the result of the algorithm.**

I. INTRODUCTION

One of the most important works on coverage control is [1] which pioneers the treatment of coverage as an optimization problem. In this work, the authors have shown that the gradient of a cost function is negative when each sensor tries to reach the centroid of its Voroinoi partition, which guarantees the convergence to a local minimum of the cost function. Following the same approach are the works of [2], [3] with more complexity when the model of the sensor and mobile platform is taken into account. Also under the same approach of optimization are works in [4], [5] and [6] which provide insights into the cases with nonuniform information value, noisy measurements or intermittent communication in one-dimensional case. In [7], optimization of the probability distribution of some targets in a certain area is studied where agents cooperatively survey and exchange their information of the area. A similar problem can be found in [8], [9] where the awareness level of important points is persistently maintained by optimizing the path and frequency of visits to these points by mobile agents.

Different from the commonly used approach based on gradient descent which may only settle on a local optimum, a series of work [10], [11], [12] is notable when the authors take a different approach to reach the global optimal coverage via consensus of the multi-agent system on a closed loop. Through their works a series of decentralized coverage control laws have been developed with increasing complication. Their analyses successfully demonstrated the convergence of a homogeneous multi-agent system to a globally optimal configuration. The authors then show that the same convergence to the global optimum persists in the case of input saturation.

The authors are with School of Electrical and Electronic Engineering, Nanyang Technological University, Singapore 639798, 50 Nanyang Avenue.
The research was partially supported by the ST Engineering - NTU Corporate Lab through the NRF corporate lab@university scheme.
Emails: elhxie@e.ntu.edu.sg, e150040@e.ntu.edu.sg

However the use of consensus is still not limited to optimizing a cost function as in the aforementioned works. In many applications such as mine-sweeping, surveillance and surface inspection where the area to be covered is mostly two dimensional and there is no relative importance between the locations in the mission area, one may be interested in making the agents form a barrier between some predefined boundaries with some *efficient* spacing between them and sweep over the area to ensure every location is visited at least once. Such strategies are addressed in the literature as barrier and coverage control. Notable studies of barrier coverage for a swarm of agents with nonholonomic dynamics were investigated in [13], [14], [15]. Under a set of assumptions, the algorithm presented in [14] attempts to make a system of homogeneous nonholonomic mobile sensor network reach consensus on the direction of the corridor and arrange themselves evenly into a barrier to sweep over this corridor. The authors also addressed the problem of kinematic constraints by having a predetermined maximum velocity, which is tunable by the time step and communication radius. Since the maximum velocity is predetermined, collision avoidance is also achieved by other assumptions on the communication and obstacle detection radii. However the convergence of the algorithm is based on the assumption that the network is jointly connected (Definition 2 in [16]) as the robots can be detected within some finite radius. This assumption may only work when the number of agents and their communication radii are sufficiently large. When the number of agents and their communication radius are only satisfied marginally, this assumption may no longer hold. Thus it does not guarantee that isolated clusters will not form. If this happens there may not be enough agents to form a connected barrier. One possibility to guarantee this assumption is to augment the control law with a connectivity preservation logic [17]. Whether this augmentation would affect the convergence of the system remains uncertain. However, a similar recent work in [18] has considered the feasibility of maintaining connectivity while still ensuring connectivity.

In this paper we seek to develop an algorithm applicable in barrier coverage and sweep coverage scenarios using consensus. The control law in this paper is inspired by the aforementioned series of work in [10], [11] in terms of treatment for heterogeneous system and and input constraints. However, our work is significantly different since the same theoretical result on consensus of multi-agent system [19] can not be reused in the presence of external inputs. Thus, the contribution of our work can be stated as extending the goal of driving a heterogeneous mobile sensor network to a desired configuration first investigate in [10], [11], [12] to a new scenario as follows::

- We provide a more realistic interpretation of the coverage function the cost function as *"maximum chance an event is overlooked by the sensor network when it crosses the corridor"*.
- We show that the control law can maintain the order of sensor placement and guarantees collision avoidance in the presence of boundaries and input constraint.
- We show that the global optimal coverage function can be achieved in the non-looping one dimension case using a leader-follower consensus.

The rest of the paper is organized as follows. In Section II we introduce the dynamics and communication topology of our system as well as cost function. We then propose the control law in Section III-A and analyse its features in subsections III-B and III-C. Section IV provides numerical simulation to verify the results on the optimization of the cost function. Finally Section V concludes our work.

II. Problem Formulation

A. Dynamics and Topology

We denote the set of n agents as $\mathscr{I} = \{1,2,...,n\}$. We assume that the direction of the barrier is known beforehand by all agents, thus the coordinate of agent i over the barrier direction is a scalar $x_i(k)$ at time step k. The dynamics of each agent over the barrier dimension is described as:

$$x_i(k+1) = x_i(k) + \varepsilon u_i(k), \tag{1}$$

where $u_i(k)$ is the displacement generated in the k-th time step by a control law. It should be noted that $\varepsilon > 0$ acts as both the sample time and a proportional gain.

We assume that via a channelized communication network, agent i can have the information on position and parameters of its two immediate neighbours $x_{i-1}(k)$ and $x_{i+1}(k)$. In case of agent 1 and agent n we treat the two boundaries as neighbours with positions $x_0(k) \equiv \underline{x}$ and $x_{n+1}(k) \equiv \bar{x}$ respectively.

For convenience of notation when the boundaries are involved we make the following definitions:

$$\mathscr{I}^0 = \mathscr{I} \cup \{0\}, \tag{2}$$

$$d_i(k) = x_{i+1}(k) - x_i(k), i \in \mathscr{I}^0. \tag{3}$$

B. Globally minimisable cost function.

In this scenario, n agents are tasked with forming a barrier across a corridor to cover all points between two boundaries. We define such a corridor as $\mathbb{X} = [\underline{x},\ \bar{x}]$ with two boundaries $\underline{x} < \bar{x}$, such that $\underline{x} \le x_i(k) \le \bar{x}$. An agent i is equipped with a sensor that can detect an event crossing the barrier. As the event passes by the agents, each sensor i can detect the presence of this event with a chance of being overlooked as Q_i. The farther the event is from sensor i the more likely it is overlooked by i. Thus we model the coverage cost for a location x inspected by agent i as:

$$Q_i(x) = |x - x_i| / r_i, \tag{4}$$

where $r_i > 0$ is a coefficient to indicate the rate that overlooking will increase over the distance. As r_i is different for each sensor, we say that the system is heterogeneous with respect to their sensing capability. A physical interpretation for the function $Q_i(x)$ is given in Fig. 1.

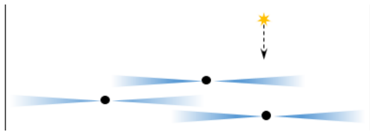

Fig. 1: An illustration of the sensor network. The agents are indicated by black dots, the event is the yellow star and the two boundaries are the lines. Each agent is equipped with some proximity sensors that can detect an event crossing the barrier. The farther the event is from the sensor the more likely it is overlooked, thus we model the sensor performance function as in (4).

Our objective now is to find a control law to minimize the *maximum chance an event is overlooked by the sensor network when it crosses the corridor*. Thus, the following objective function is proposed for this purpose:

$$Q(x_1, x_2, ..., x_n) = \max_{x\in\mathbb{X}} \min_{i\in\mathscr{I}} Q_i(x) = \max_{x\in\mathbb{X}} \min_{i\in\mathscr{I}} |x - x_i| / r_i \tag{5}$$

The cost function (5) in our scenario is similar to [12], as such the proof for its global optimum is also similar, however there is still different in the case where the boundaries are involved. Moreover, as can be seen in later parts our control law as well as analysis on convergence and collision avoidance are substantially different.

We define the upper bound of the r_i as $r_M \ge \max_{i\in\mathscr{I}} r_i$ and the lower bound as $0 < r_m \le \min_{i\in\mathscr{I}} r_i$. These two values are assumed to be known beforehand by all agents in the network. We also denote $r_0 := 0$ and $r_{n+1} := 0$.

We now state the existence of a global minimum of the cost function (5) in the following theorem.

Theorem 1. Given the sensor network with the sensor model (4), the coverage cost function (5) has a global minimum

$$Q \ge X / (2 \sum_{i=1}^{n} r_i),$$

where $X = \bar{x} - \underline{x}$ and the equality occurs if the distances d_i defined in (3) satisfy

$$d_{i-1} / (r_{i-1} + r_i) = d_i / (r_i + r_{i+1}),\ \forall i \in \mathscr{I}. \tag{6}$$

Proof: The proof for this theorem can be similarly obtained by following the steps in **Theorem 2** in [12].

C. Optimal sweep coverage configuration.

Other than optimizing the cost function by moving along the barrier only, one can be interested in a sweep coverage problem illustrated in Fig. 2 where the corridor has width of X. If we assume that each UAV has a camera footprint of radius r_i and the sum of their footprint radii are larger

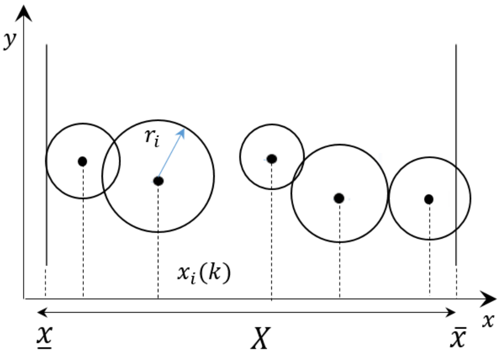

Fig. 2: Illustration of sweep coverage using multiple UAVs equipped with cameras of different fields of view.

than the width of the corridor. Then to cover the corridor, our strategy should be to have the UAVs arrange themselves along the barrier so that their fields of view would touch each other, or even overlap, as long as no location is missed out when the UAVs sweep through an area. For this purpose we create a similar definition to Definition 2.1 in [14] as follows:

Definition II.1. Given n mobile sensors and with the dynamics and communication topology in Section II-A and $2\sum_{j=1}^{n} r_j \geq X$. A decentralized control law is said to be an optimal barrier coverage control law if for almost all initial sensor positions, one has:

$$\lim_{k\to\infty} d_i(k) = X(r_i + r_{i+1})/(2\sum_{j=1}^{n} r_j) \leq r_i + r_{i+1} \tag{7}$$

Thus one may be interested in a formation with a certain configuration $d = [d_0 \ d_1 \ ...d_n]^\top$, where $d_i \leq r_i + r_{i+1}$, $\forall i \in \mathscr{I}^0$. Since $X/(2\sum_{j=1}^{n} r_j) \leq 1$, we can see that $d_i = X(r_i + r_{i+1})/(2\sum_{j=1}^{n} r_j) \leq r_i + r_{i+1}$, which satisfies (6), is such a configuration.

III. Control and Analysis

A. Control law

The following control law is proposed to drive the agents to the optimal configuration subject to the cost function (5):

$$u_i(k) = \text{sign}(\bar{u}_i(k)) \min\{1, |\bar{u}_i(k)|\} \tag{8}$$

where $\bar{u}_i(k)$ is the following non-saturated control law:

$$\bar{u}_i(k) = \lambda_i d_i(k) - \lambda_{i-1} d_{i-1}(k), \tag{9}$$

for $i \in \mathscr{I}$ and λ_i are defined as:

$$\lambda_i = r_M/(r_i + r_{i+1}) \tag{10}$$

where $i \in \mathscr{I}^0$, $x_0(k) = \underline{x}$, $x_{n+1}(k) = \bar{x}$ and $r_0 = r_{n+1} = 0$.

Remark. In general different agents have different speed limits, as such the control input should be defined as $u_i(k+1) = v_i \text{sign}(\bar{u}_i(k)) \min\{1, |\bar{u}_i(k)|\}$, where v_i is the maximum speed of each agents. However since our targeted optimal configuration does not concern the agents' speed as intended in [12], we can reduce the saturated speed of all agents to the minimum value, say v_m, and redo the same analysis in this paper with only slight change in the assumption of ε. More specifically one can change the assumption on the time step to $\varepsilon < r_m/(2v_m r_M)$ instead of $\varepsilon < r_m/(2r_M)$. The same conclusions on collision avoidance and convergence obtained in the later parts will still hold. Without loss of generality we have purposely assumed $v_m = 1$ for convenience of analysis and glossed over a parametrization for it in our control law.

B. Collision avoidance

From the definition of the saturation function we have the following inequalities:

$$\begin{cases} u_i(k) \leq \bar{u}_i(k), & \text{if } u_i(k) \geq 0 \\ u_i(k) \geq \bar{u}_i(k), & \text{if } u_i(k) < 0. \end{cases} \tag{11}$$

From (9) and (3) we have the evolution of $d_i(k)$ as follows:

$$d_i(k+1) = d_i(k) + \varepsilon(u_{i+1}(k) - u_i(k)) \tag{12}$$

where $i = 0, 1...n$; $u_0(k) \equiv 0$ and $u_{n+1}(k) \equiv 0$.

From the definition of λ_i we have:

$$\lambda_i = r_M/(r_i + r_{i+1}) \leq r_M/r_m \tag{13}$$

where the equality occurs when $i = 0$ and $r_1 = r_m$, or $i = n$ and $r_n = r_m$.

Theorem 2. If $\varepsilon < r_m/(2r_M)$ and $x_{i-1}(0) < x_i(0) < x_{i+1}(0)$, $\forall i \in \mathscr{I}$, then under the control law (8), $x_{i-1}(k) < x_i(k) < x_{i+1}(k)$, $i \in \mathscr{I}$, $\forall k > 0$.

Proof: Clearly that $x_{i-1}(k) < x_i(k) < x_{i+1}(k)$ is equivalent with $d_i(k) > 0$, $\forall d_i \in \mathscr{I}^0$. We will show by induction that if $d_i(k) > 0$ holds then $d_i(k+1) > 0$ also holds given $\varepsilon < r_m/(2r_M)$ as follows:

If $u_{i+1}(k) \geq 0$ and $u_i(k) \leq 0$ then from (12) the following holds:

$$d_i(k+1) \geq d_i(k) > 0$$

If $u_{i+1}(k) \geq 0$ and $u_i(k) > 0$ then using (13), the first case in (11) and the assumptions $\varepsilon < r_m/(2r_M)$, we have the following chain:

$$\begin{aligned} d_i(k+1) &\geq d_i(k) - \varepsilon u_i(k) \geq d_i(k) - \varepsilon \bar{u}_i(k) > d_i(k) - \varepsilon \lambda_i d_i(k) \\ &> d_i(k) - \frac{r_m}{2r_M} \frac{r_M}{r_i + r_{i+1}} d_i(k) > d_i(k) - \frac{1}{2} d_i(k) > 0 \end{aligned}$$

If $u_{i+1}(k) < 0$ and $u_i(k) \leq 0$ then the following inequalities hold:

$$\begin{aligned} d_i(k+1) &\geq d_i(k) + \varepsilon \bar{u}_{i+1}(k) > d_i(k) - \varepsilon \lambda_i d_i(k) \\ &> d_i(k) - \frac{r_m}{2r_M} \frac{r_M}{(r_{i+1} + r_i)} d_i(k) > d_i(k) - \frac{1}{2} d_i(k) > 0 \end{aligned}$$

Finally if $u_{i+1}(k) < 0$ and $u_i(k) > 0$ we have:

$$\begin{aligned} d_i(k+1) &\geq d_i(k) + \varepsilon(\bar{u}_{i+1}(k) - \bar{u}_i(k)) > d_i(k) - \varepsilon 2\lambda_i d_i(k) \\ &> d_i(k) - \frac{r_m}{r_M} \frac{r_M}{(r_{i+1} + r_i)} d_i(k) > 0 \end{aligned}$$

Hence after having examined all of the possible motions of the systems we can conclude that no collision can occur and the order of the agents on the barrier is preserved under the control law (8).

C. Convergence analysis

Before proving the convergence of the system, let us recite the following result on leader-following consensus. Assume that we have a leader agent 0 and n follower agents. We define the matrices characterizing the interaction between the agents as $M = L + D$. Here L is the weighted Laplacian of the network of n followers and $D := \mathrm{diag}\{\mathbf{d}_i, \mathbf{d}_2, ..., \mathbf{d}_n\}$, where $\mathbf{d}_i > 0$ if node i can know the leader's state and $\mathbf{d}_i = 0$ otherwise. We make the following assumption on the connectivity of the communication graph:

Assumption 1. *For the graph with a leader, there is at least one directed path from the leader to each follower.*

Lemma 1 ([20])**.** If Assumption 1 holds, then:

1) M is non-singular;
2) all the eigenvalues of M have positive real parts;
3) M^{-1} exists and is non-negative.

For our sensor network, the matrices L and D can be defined as follows:

$$L = \begin{bmatrix} \lambda_1 & -\lambda_1 & 0 & \dots & 0 & 0 \\ -\lambda_1 & \lambda_1+\lambda_2 & -\lambda_2 & \dots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \dots & -\lambda_{n-1} & \lambda_{n-1} \end{bmatrix} \tag{14}$$

$$D = \begin{bmatrix} \lambda_0 & 0 & \dots & 0 \\ 0 & 0 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & 0 \\ 0 & 0 & \dots & \lambda_n \end{bmatrix} \tag{15}$$

We can see that L and D satisfy Assumption 1. Hence the matrix $M = L + D$ has all of the features listed in **Lemma 1**.

Lemma 2. For $\varepsilon < r_m/(2r_M)$ and the matrix $M = L + D$, where L and D are defined in (14) and (15), all eigenvalues of the matrix $A = I - \varepsilon M$ are in the open interval $(0, 1)$.

Proof: It can be seen that all the eigenvalues of A are real since A is symmetric. Moreover, from **Lemma 1** we can see that all of the eigenvalues of M are positive. Hence, combined with the Gershgorin circle theorem, we can see that all eigenvalues of M are in the union of n half-closed intervals: $(0, 2\lambda_i + 2\lambda_{i+1}]$ for $i \neq 1, n$ and $(\lambda_0, \lambda_0 + 2\lambda_1]$, $(\lambda_n, \lambda_n + 2\lambda_{n-1}]$ for $i = 0$ and $i = n$, respectively.

Due to (13), we can see that the union of all of these Gershgorin intervals is a subset of $(0, 2r_M/r_m]$. Since $\varepsilon < r_m/(2r_M)$, we can see that εM will have all of its eigenvalues in the interval $(0, 1)$. Finally, it can be easily seen that $A = I - \varepsilon M$ will have all of its eigenvalues in the open interval $(0, 1)$.

Now we can proceed to prove the convergence of the system.

Theorem 3. If $\varepsilon < r_m/(2r_M)$ and $x_{i-1}(0) < x_i(0) < x_{i+1}(0)$, $i \in \mathscr{I}$ the control law (8) will drive the mobile sensor network to the configuration (6) while preserving the order of the sensor along the barrier without collision

Proof: We will first show that the control inputs under the law (8) converge to 0 when k goes to infinity. Define the following aggregated vectors to describe the evolution of the system in a compact form for analysis: $u(k) = [u_1(k) \ \dots \ u_n(k)]^\top$, $\bar{u}(k) = [\bar{u}_1(k) \ \dots \ \bar{u}_n(k)]^\top$ and $x(k) = [x_1(k) \ \dots \ x_n(k)]^\top$. Hence from (9), the compact form for the non-saturated control law can be put as:

$$\bar{u}(k) = -\varepsilon M x(k) + D \begin{bmatrix} \underline{x} & 0 & \dots & 0 & \bar{x} \end{bmatrix}^\top \tag{16}$$

Taking the difference between two time steps yields:

$$\bar{u}(k+1) - \bar{u}(k) = -\varepsilon M[x(k+1) - x(k)] = -\varepsilon M u(k).$$

We obtain the equation for the evolution of all non-saturated control inputs as:

$$\bar{u}(k+1) = \bar{u}(k) - \varepsilon M u(k+1). \tag{17}$$

Since (17) is nonlinear, we cannot analyze the convergence of $\bar{u}(k)$ directly. However we can show that $\bar{u}(k)$ will enter an invariant set in finite time and once this happens, (17) will become a linear system. We define the invariant set as follows:

$$\Omega = \{\bar{u} \in \mathbb{R}^n \mid -1 \le \bar{u}_i \le 1\}.$$

We can show that $\bar{u}_i(k) \ge -1$ then $\bar{u}_i(k+1) \ge -1$. From (17), we can work out row by row the following equation:

$$\begin{aligned} \bar{u}_i(k+1) = &\bar{u}_i(k) + \varepsilon\lambda_{i-1}u_{i-1}(k) \\ &- \varepsilon(\lambda_{i-1} + \lambda_i)u_i(k) + \varepsilon\lambda_i u_{i+1}(k). \end{aligned} \tag{18}$$

whereas $u_0(k) \equiv 0$ and $u_{n+1}(k) \equiv 0$.

Now let's assume that $\bar{u}(k) \ge -1$, then by substituting $u_{i-1}(k) \ge -1$, $u_{i+1}(k) \ge -1$ and $u_i(k) \le \bar{u}_i(k)$ we have the following inequality:

$$\bar{u}_i(k+1) \ge (1 - \varepsilon(\lambda_{i-1} + \lambda_i))\bar{u}_i(k) - \varepsilon(\lambda_{i-1} + \lambda_i). \tag{19}$$

For $i = 1$ and $i = n$ we have:

$$\lambda_{i-1} + \lambda_i = \frac{r_M}{r_{i-1} + r_i} + \frac{r_M}{r_i} \le \frac{3}{2}\frac{r_M}{r_m}. \tag{20}$$

On the other hand for $i \neq 1, n$ we have,

$$\lambda_{i-1} + \lambda_i = \frac{r_M}{r_{i-1} + r_i} + \frac{r_M}{r_i + r_{i+1}} \le \frac{r_M}{r_m}. \tag{21}$$

Therefore, if $\varepsilon < r_m/(2r_M)$, $\varepsilon(\lambda_{i-1} + \lambda_i) < 3/4 < 1 \ \forall i \in \mathscr{I}$ and thus $1 - \varepsilon(\lambda_{i-1} + \lambda_i) > 0$. This inequality combined with the assumption $\bar{u}_i(k) \ge -1$ leads to:

$$(1 - \varepsilon(\lambda_{i-1} + \lambda_i))\bar{u}_i(k) \ge -1 + \varepsilon(\lambda_{i-1} + \lambda_i). \tag{22}$$

Substituting (22) to (19) one has:

$$u_i(k+1) \ge -1 + \varepsilon(\lambda_{i-1} + \lambda_i) - \varepsilon(\lambda_{i-1} + \lambda_i) = -1. \tag{23}$$

Thus we can conclude that if $\bar{u}_i(k) \ge -1$ then $\bar{u}_i(k+1) \ge -1$. Following the same logic we can conclude that if $\bar{u}_i(k) \le 1$ then $\bar{u}_i(k+1) \le 1$.

If $\bar{u}_i(k) > 1 \ \forall k$, then $u_i(k) = 1$ and infinite time due to (9) $x_i(k)$ will exceed $\bar{x}$. This violates **Theorem 2**, therefore

$\bar{u}_i(k)$ will have to be smaller or equal than 1 at some k. Say, at time k and $k+1$ we have $\bar{u}_i(k) > 1$ and $\bar{u}_i(k+1) \leq 1$, since $\bar{u}_i(k) > -1$, $\bar{u}_i(k+1) > -1$. This means $\bar{u}_i$ has entered Ω at $k+1$. We can also make the same conclusion in the case of $\bar{u}_i(k) < -1$. Hence, we assert that $\exists k_\Omega < +\infty$ such that $\bar{u}(k) \in \Omega\ \forall k > k_\Omega$.

Therefore from $k_\Omega + 1$ onward we can drop the saturation operation and the system (17) becomes:

$$\bar{u}(k+1) = (I - \varepsilon M)\bar{u}(k) = A\bar{u}(k). \tag{24}$$

We have shown in **Lemma 2** that the matrix A has only real eigenvalues in the open interval $(0,1)$, thus A is stable and all control inputs will asymptotically converge to 0.

It can be seen from (9) and (10) that $u(k) = 0$ if and only if $d_i(k)/(r_{i+1}+r_i) = d_{i-1}(k)/(r_i + r_{i-1})$. Since $\lim_{k=0}^{\infty} u_i(k) \to 0$, $\forall i \in \mathscr{I}$ under the control law (8), we can conclude that the control law (8) will drive the agents to the optimal configuration to minimize the cost function (5) as time goes to infinity. Moreover, **Theorem 2** has shown that under the control law and assumption on the order of the sensors at $k=0$, no collision will happen at any later time $k > 0$, therefore the initial order will be preserved at all times.

IV. Simulation

A simulation is done to verify our results for $n = 4$ agents with the parameters taking the following values:

$$x(0) = [5.9264,\ 9.0185,\ 10.9402,\ 14.8939]^\top$$
$$\underline{x} = 0,\ \bar{x} = 20$$
$$r = [1.4558,\ 5.2913,\ 1.4139,\ 2.8390]^\top$$
$$\varepsilon = 0.1309$$

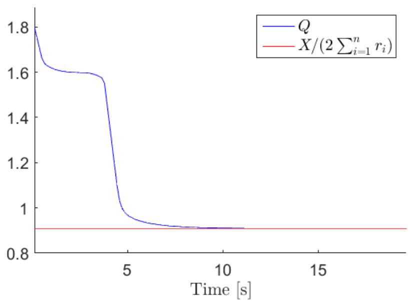

Fig. 3: Time evolution of the cost functions Q.

We can clearly see in Fig. 3 that the cost function Q converges to the value $X/(2\sum_{i=1}^{n} r_i)$ as stated in **Theorem 1**. From Fig. 4a-4b we can observe the convergence of the agents to the optimal configuration where $d_{i-1}/(r_{i-1}+r_i) = d_i/(r_i + r_{i+1})$, $\forall i \in \mathscr{I}$. At each point $x \in \mathbb{X}$, we calculate the cost $\min_{i \in \mathscr{I}} Q_i(x)$. The color of each segment of $\min_{i \in \mathscr{I}} Q_i(x)$ shows which sensor can detect the event crossing x with the least cost. It can be seen that if i is the sensor that minimizes the the cost for x, then this cost will keep increasing for the points farther away from x_i than x until that at some point, the most efficient sensor to cover that point is no longer i. Notice that this change is not necessary just a change from i

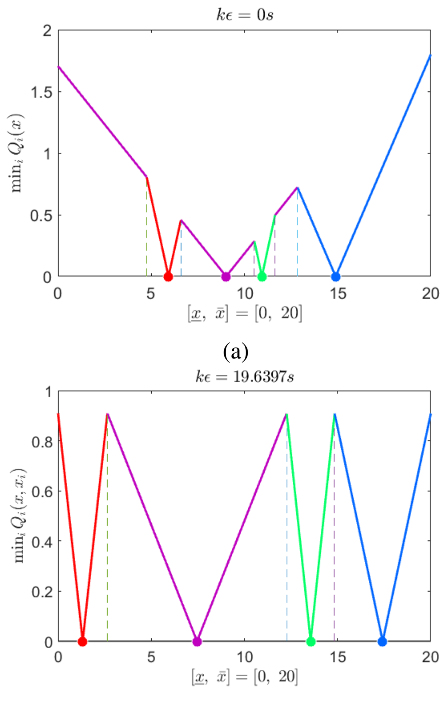

Fig. 4: Time evolution of the agents' position x_i (color-filled circles) and the cost function $\min_{i \in \mathscr{I}} Q_i(x, x_i), \forall x \in \mathbb{X}$ at two different times $k = 0$ and $k = 150$.

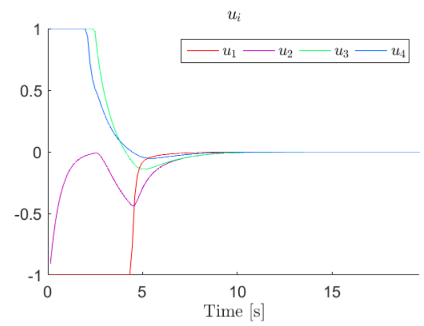

Fig. 5: Time evolution of the control inputs u_i.

to $i+1$ or $i-1$. We see that at $k=0$, the points on the two boundaries are the least covered. The value of the cost at the rightmost boundary is the one that yields the value of the system's cost $\max_{x \in \mathbb{X}} \min_{i \in \mathscr{I}} Q_i(x)$. At $k = 150$, the system's cost approaches the minimum value.

In Fig. 5, we can clearly see that several control inputs did saturate in the beginning, however as the system evolves, all of the control inputs will enter the invariant set and then converge to zero as the proof for **Theorem 3** has pointed out. Finally, in Fig. 6 we demonstrate a sweep coverage scheme of the UAV group over a corridor of width X. Each UAV travels along the corridor at a constant speed $+1$m/s and has camera footprint of radius r_i. We can see that in the beginning while their footprints overlap each other, some regions are not covered at all. As they sweep along the corridor, the control law drives their positions along

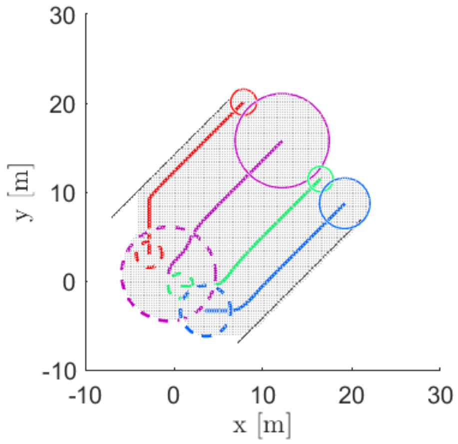

Fig. 6: Sweep coverage of the UAV team using the barrier formed by control law (8).

the barrier to the optimal configuration so that the camera footprints are neatly arranged to cover the whole corridor. Notice also that there will be no collision between the UAVs since we have guaranteed that their position along the barrier will never be equal as has been in Section III-B.

V. Conclusion

In this work we have proposed a control law capable of driving the agents to a desired global optimal configuration with input constraint and collision avoidance taken into account. We also show that the specific cost functions can achieve their globally optimal configuration under the proposed control law and demonstrate the feasibility of the algorithm in an actual implementation.

Further research can be done by extending the results in this paper for mobile agents with high-order dynamics or analyze the system with the more realistic assumption of switching topology. Studies into the case where the nonlinear boundaries would also be an interesting problem.

References

[1] J. Cortes et al. "Coverage control for mobile sensing networks". In: *IEEE Transactions on Robotics and Automation* 20.2 (2004), pp. 243–255.

[2] J. Xu et al. "Distributed coverage control under generalized locational optimization framework". In: *Control Conference (CCC), 2012 31st Chinese*. IEEE. 2012, pp. 6015–6020.

[3] J. Hu and Z. Xu. "Brief paper-distributed cooperative control for deployment and task allocation of unmanned aerial vehicle networks". In: *IET Control Theory & Applications* 7.11 (2013), pp. 1574–1582.

[4] P. Davison et al. "Nonuniform line coverage from noisy scalar measurements". In: *IEEE Transactions on Automatic Control* 60.7 (2015), pp. 1975–1980.

[5] N. E. Leonard and A. Olshevsky. "Nonuniform coverage control on the line". In: *IEEE Transactions on Automatic Control* 58.11 (2013), pp. 2743–2755.

[6] S. Miah et al. "Nonuniform coverage control with stochastic intermittent communication". In: *IEEE Transactions on Automatic Control* 60.7 (2015), pp. 1981–1986.

[7] J. Hu et al. "Multi-agent cooperative target search". In: *Sensors* 14.6 (2014), pp. 9408–9428.

[8] C. Song et al. "Persistent awareness coverage with maximum coverage frequency for mobile sensor networks". In: *Cyber Technology in Automation, Control and Intelligent Systems (CYBER), 2013 IEEE 3rd Annual International Conference on*. IEEE. 2013, pp. 235–240.

[9] C. Song et al. "Persistent awareness coverage control for mobile sensor networks". In: *Automatica* 49.6 (2013), pp. 1867–1873.

[10] C. Song and G. Feng. "Coverage control for mobile sensor networks on a circle". In: *Unmanned Systems* 2.3 (2014), pp. 243–248.

[11] C. Song, L. Liu, and G. Feng. "Coverage Control for Mobile Sensor Networks with Input Saturation". In: *Unmanned Systems* 4.1 (2016), pp. 15–21.

[12] C. Song et al. "Coverage control for heterogeneous mobile sensor networks on a circle". In: *Automatica* 63 (2016), pp. 349–358.

[13] T. M. Cheng and A. V. Savkin. "A distributed self-deployment algorithm for the coverage of mobile wireless sensor networks". In: *IEEE Communications Letters* 13.11 (2009).

[14] T. M. Cheng and A. V. Savkin. "Decentralized control for mobile robotic sensor network self-deployment: Barrier and sweep coverage problems". In: *Robotica* 29.2 (2011), pp. 283–294.

[15] T. M. Cheng and A. V. Savkin. "Self-deployment of mobile robotic sensor networks for multilevel barrier coverage". In: *Robotica* 30.4 (2012), pp. 661–669.

[16] W. Ni and D. Cheng. "Leader-following consensus of multi-agent systems under fixed and switching topologies". In: *Systems & Control Letters* 59.3 (2010), pp. 209–217.

[17] T. Razafindralambo and D. Simplot-Ryl. "Connectivity preservation and coverage schemes for wireless sensor networks". In: *IEEE Transactions on Automatic Control* 56.10 (2011), pp. 2418–2428.

[18] X. Sun and C. G. Cassandras. "Optimal dynamic formation control of multi-agent systems in constrained environments". In: *Automatica* 73 (2016), pp. 169–179.

[19] R. Olfati-Saber, J. A. Fax, and R. M. Murray. "Consensus and cooperation in networked multi-agent systems". In: *Proceedings of the IEEE* 95.1 (2007), pp. 215–233.

[20] X. Li et al. "Distributed Bounds on the Algebraic Connectivity of Graphs With Application to Agent Networks". In: *IEEE Transactions on Cybernetics* (2016).