ROBUST STABILIZATION OF SWITCHED SYSTEMS WITH INPUT SATURATION

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Abstract:

This paper deals with the robust stabilization of switched systems with input saturation under arbitrarily switching sequences in the discrete-time domain. The proposed approach in this note detects the existence of a common Lyapunov function to check the asymptotic stability of the switched systems with input saturation under arbitrarily switching sequences. According to these conditions, the state feedback control law is given to ensure the robust stabilization of switched systems with input saturation. At last, a numerical example illustrates the feasibility and availability of theory.

Keywords:

Switched system; robust stabilization; input saturation

1. Introduction

In recent years, the investigation of switched systems has received a growing attention. Switched system is a class of hybrid dynamical systems consisting of a family of continuous-time (or discrete-time) subsystems, and a rule that orchestrates the switching sequences between them [1]-[4]. A survey of basic problems on the stability and design of switched systems has been proposed recently in [5]. Some simple stable conditions based on the linear matrix inequalities have been already developed in [6]. Among the large varieties of problems encountered in practice, they can be divided into three parts that are the existence of a switching rule which can stabilize switched systems, stable conditions under arbitrarily switching sequences assuming that switching sequence is not known a priori, and the stability of switched systems under some useful classes of switching sequences.

Switched systems have many applications in the reality. Therefore, there have been many researches about the stable analysis and design of switched systems without uncertainties [7]-[11]. The quadratic stability and stabilization by state-based feedback for both continuous-time and discrete-time switched linear systems is studied in [12]. [13] and [14] consider the robust stabilization of switched systems. But for the input saturation, there are a few researches. The problem of input saturation is a practical problem which often happened in the applications, and is very serious so that the system may be unstable. Thus, in this paper, we consider the problem into the designation of state feedback control law such that the system is asymptotic stable and have not input saturation.

2. Problem formulation

Consider the discrete-time switched linear systems with input saturation under arbitrary switching sequences described by

$$\begin{cases} x(k+1) = \widetilde{A}_i x(k) + \widetilde{B}_i Sat(u(k)) \\ y(k) = \widetilde{C}_i x(k) \\ \begin{bmatrix} \widetilde{A}_i & \widetilde{B}_i \end{bmatrix} = \begin{bmatrix} A_i + \Delta A_i & B_i + \Delta B_i \end{bmatrix} \\ \begin{bmatrix} \Delta A_i & \Delta B_i \end{bmatrix} = D_i F(k) \begin{bmatrix} M_i & N_i \end{bmatrix} \end{cases} \tag{2.1}$$

where $x(k) \in R^n$ is the state vectors, $u(k) \in R^m$ is the control input vectors, $y(k) \in R^p$ is the measurement output vectors, $F(t) \in R^{s\times s}$ is the uncertainty with $F^T(t)F(t) < I$. $D_i \in R^{n\times s}$, $M_i \in R^{s\times n}$, $N_i \in R^{s\times m}$, $A_i \in R^{n\times n}$, $B_i \in R^{n\times m}$, $i \in \widetilde{N} = \{1,2,....,N\}$ are known matrices, and

$$Sat(u(k)) = \begin{cases} u(k), & |u(k)| \le \rho \\ \rho, & |u(k)| > \rho \end{cases} \tag{2.2}$$

When the following state feedback control law

$$Sat(u(k)) = Sat(K_i x(k)) \\ i \in \widetilde{N},\ K_i \in R^{m\times n} \tag{2.3}$$

is applied to the system (2.1), the closed-loop system is obtained in the following form

$$\begin{cases} x(k+1) = \widetilde{A}_i x(k) + \widetilde{B}_i Sat(K_i x(k)) \\ y(k) = \widetilde{C}_i x(k) \end{cases} \tag{2.4}$$

Problem: Choose some suitable feedback gain matrices to stabilize the closed-loop system (2.4).

Before describing the main results, we give the following two lemmas which are the significant theoretical basis of the proof of conclusions.

Lemma 1: Let Y, H, E to be given matrices of appropriate dimensions, then for any $F(t)$ satisfying $F^T(t)F(t) < I$, there holds $Y + HFE + (HFE)^T < 0$ if and only if there exists a constant value $\varepsilon > 0$ such that $Y + \varepsilon HH^T + \varepsilon^{-1}E^T E < 0$.

For a positive definite matrix $P \in R^{n\times n}$ and a positive value $\rho \in R$, we define an ellipsoid as

$$\varepsilon(P,\rho) = \left\{ x \in R^n \mid x^T P x \le \rho \right\} \tag{2.5}$$

For a feedback gain matrix $K \in R^{m\times n}$, define the set of the state for which saturation does not occur as

$$L(K) = \left\{ x \in R^n \mid \left| K^i x \right| \le \rho, i = 1, 2, \cdots, m \right\} \tag{2.6}$$

where K^i is the ith row of K.

Let H be the set of $m \times m$ diagonal matrices whose diagonal elements are either 1 or 0. There are 2^m elements in H and we denote its elements as H_s, $s = 1, 2, 3, \cdots, 2^m$. Denote $H_s^- = 1 - H_s$. It is easy to see that $H_s^- \in H$. Then we can have the following lemma.

Lemma 2 [15]:

Let $u, v \in R^m$ with $u = \{u_1 \ u_2 \ \cdots \ u_m\}^T$ and $v = \{v_1 \ v_2 \ \cdots \ v_m\}^T$. Suppose that $|v_i| \le \rho$, for all $i = 1, 2, \cdots, m$. Then

$$Sat(u) \in co\left\{ H_i u + H_i^- v : i = 1, 2, \cdots, m \right\} \tag{2.7}$$

where co denotes the convex hull.

3. Main results

In this section, we will use the common Lyapunov function to check the stability of the system (2.4). Assuming that the switched system (2.4) has a common Lyapunov function, we define it as

$$V(k, x(k)) = x^T(k) P\, x(k) \tag{3.1}$$

where $P \in R^{n\times n}$ are symmetric positive-definite matrices.

For clearly describing our result, at first, we suppose that there are not the uncertainties in (2.4). Based on the Lyapunov approach, the following theorem can be given.

Theorem 1: If there exist some positive definite matrices $S \in R^{n\times n}$, matrices $\widetilde{Q}_i \in R^{n\times m}$ and $\widetilde{K}_i \in R^{n\times m}$ satisfying

$$\begin{bmatrix} -S & (A_i S + B_i(H_s \widetilde{K}_i + H_s^- \widetilde{Q}_i))^T \\ A_i S + B_i(H_s \widetilde{K}_i + H_s^- \widetilde{Q}_i) & -S \end{bmatrix} < 0$$

$$\begin{bmatrix} -\rho I & \widetilde{Q}_i^T \\ \widetilde{Q}_i & -S \end{bmatrix} < 0$$

$$s = 1, 2, 3, \cdots, 2^m, \quad \forall i \in \widetilde{N} \tag{3.2}$$

Then, the system (2.4) without uncertainties is asymptotically stable, and the feedback gains are given by

$$K_i = \widetilde{K}_i S^{-1}$$

Proof: Based on the Lyapunov function (3.1), we have

$$\begin{aligned} \Delta V &= V(k+1, x(k)) - V(k, x(k)) \\ &= (A_i x(k) + B_i Sat(K_i x(k)))^T P \\ &\quad (A_i x(k) + B_i Sat(K_i x(k))) - x(k) P\, x(k) \end{aligned} \tag{3.3}$$

Let $\varepsilon(P, \rho)$ be an ellipsoid and $Q_i \in R^{n\times n}$ such that $\varepsilon(P, \rho) \subset L(Q_i)$. According to the lemma 2, we give the following inequality and $x(k) \in \varepsilon(P, \rho)$.

$$\begin{aligned} &(A_i x(k) + B_i Sat(K_i x(k)))^T P\, (A_i x(k) + B_i Sat(K_i x(k))) - x(k) P\, x(k) \\ &\le \max_{s \in [1 \ 2^m]} (A_i x(k) + B_i(H_s K_i + H_s^- Q_i) x(k))^T \\ &\quad P(A_i x(k) + B_i(H_s K_i + H_s^- Q_i) x(k)) - x(k) P x(k) \end{aligned} \tag{3.4}$$

From (3.4), we can see that if the following inequality is satisfied, then $\Delta V(k, x(k)) < 0$.

$$\begin{aligned} &x^T(k)(A_i + B_i(H_s K_i + H_s^- Q_i))^T P \\ &(A_i + B_i(H_s K_i + H_s^- Q_i)) x(k) - x^T(k) P x(k) < 0 \end{aligned} \tag{3.5}$$

which is equivalent to

$$\begin{bmatrix} -P^{-1} & P^{-1}(A_i + B_i(H_s K_i + H_s^- Q_i))^T \\ A_i P^{-1} + B_i(H_s K_i + H_s^- Q_i) P^{-1} & -P^{-1} \end{bmatrix} < 0 \tag{3.6}$$

Let $S = P^{-1}, \widetilde{K}_i = K_i P^{-1}$ and $\widetilde{Q}_i = Q_i P^{-1}$. Then we can conclude (3.2). Integrating both sides of $\Delta V(k, x(k))$ from 0 to t, we give in $V(x(k)) < V(x(0))$. This shows that if there is $V(x(0)) < \rho$, $x(0) \in \varepsilon(P, \rho)$, then $V(x(k)) < \rho$ and $x(k) < \varepsilon(P, \rho)$ which means that the trajectories of the closed-loop system starting from $\varepsilon(P,\rho)$ will remain inside $\varepsilon(P,\rho)$.

Based on the lemma 2, we know if the saturation does not occur, then the following inequalities must be satisfied.

$$\varepsilon(S^{-1},\rho) \subset L(\widetilde{Q}_i)$$

which is equivalent to $\widetilde{Q}_i S^{-1} \widetilde{Q}_i^T < \rho I$. Based on the Schur complement formula, we directly conclude

$$\begin{bmatrix} -\rho I & \widetilde{Q}_i^T \\ \widetilde{Q}_i & -S \end{bmatrix} < 0$$

Therefore, the switched system (2.4) without uncertainties is asymptotic stability, and we choose some suitable initial values and feedback gain matrices to stabilize the closed-loop system. The proof is completed.

Based on the result of Theorem 1, when considering the uncertainties, we have the following theorem.

Theorem 2: If there exist some positive definite matrices $S \in R^{n\times n}$, matrices $\widetilde{K}_i \in R^{m\times n}$ and $\widetilde{Q}_i \in R^{m\times n}$, and some positive constant values $\varepsilon_i \in R, \forall i \in \widetilde{N}$ satisfying

$$\begin{bmatrix} -S & (A_i S + B_i(H_s\widetilde{K}_i + H_s^-\widetilde{Q}_i))^T & (M_i S + N_i(H_s\widetilde{K}_i + H_s^-\widetilde{Q}_i))^T \\ A_i S + B_i(H_s\widetilde{K}_i + H_s^-\widetilde{Q}_i) & \varepsilon_i D_i D_i^T - S & 0 \\ M_i S + N_i(H_s\widetilde{K}_i + H_s^-\widetilde{Q}_i) & 0 & -\varepsilon_i I \end{bmatrix} < 0$$

$$\begin{bmatrix} -\rho I & \widetilde{Q}_i^T \\ \widetilde{Q}_i & -S \end{bmatrix} < 0, \quad s = 1, 2, \cdots, 2^m, \quad \forall i \in \widetilde{N} \tag{3.7}$$

Then the system (2.4) is robustly stable, and the feedback gains are given by

$$K_i = \widetilde{K}_i S^{-1} \tag{3.8}$$

Proof : According to (3.2), we have

$$\begin{bmatrix} -S & ((A_i+\Delta A_i)S + (B_i+\Delta B_i)(H_s\widetilde{K}_i + H_s^-\widetilde{Q}_i))^T \\ (A_i+\Delta A_i)S + (B_i+\Delta B_i)(H_s\widetilde{K}_i + H_s^-\widetilde{Q}_i) & -S \end{bmatrix} < 0 \tag{3.9}$$

which is equivalent to

$$\begin{bmatrix} -S & (A_i S + B_i(H_s\widetilde{K}_i + H_s^-\widetilde{Q}_i))^T \\ A_i S + B_i(H_s\widetilde{K}_i + H_s^-\widetilde{Q}_i) & -S \end{bmatrix} + \begin{bmatrix} 0 & (\Delta A_i S + \Delta B_i(H_s\widetilde{K}_i + H_s^-\widetilde{Q}_i))^T \\ \Delta A_i S + \Delta B_i(H_s\widetilde{K}_i + H_s^-\widetilde{Q}_i) & 0 \end{bmatrix} < 0 \tag{3.9}$$

Through some transformations, we can conclude the following result

$$\begin{bmatrix} -S & (A_i S + B_i(H_s\widetilde{K}_i + H_s^-\widetilde{Q}_i))^T \\ A_i S + B_i(H_s\widetilde{K}_i + H_s^-\widetilde{Q}_i) & -S \end{bmatrix} + \begin{bmatrix} 0 \\ D_i \end{bmatrix} F(k)\begin{bmatrix} M_i S + N_i(H_s\widetilde{K}_i + H_s^-\widetilde{Q}_i) & 0 \end{bmatrix} + \left\{\begin{bmatrix} 0 \\ D_i \end{bmatrix} F(k)\begin{bmatrix} M_i S + N_i(H_s\widetilde{K}_i + H_s^-\widetilde{Q}_i) & 0 \end{bmatrix}\right\}^T < 0 \tag{3.10}$$

According to lemma 1, we directly conclude (3.7). The proof is completed.

4. Numerical evaluations

In this section, we consider the velocity control of VTOL aircraft under velocity variation. The dynamical description is

$$\begin{cases} x(k+1) = A(\alpha)x(k) + B(\alpha)u(k) \\ y(k) = C(\alpha)x(k) \end{cases} \tag{4.1}$$

where $A(\alpha), C(\alpha) \in R^{4\times 4}$, $B(\alpha) \in R^{4\times 2}$, $x^1(k)$ is the horizontal velocity, $x^2(k)$ is the vertical velocity, $x^3(k)$ is the pitch rotational velocity, $x^4(k)$ is the pitch angle, $u^1(k)$ is the vertical control, $u^2(k)$ is the horizontal control and α is the speed of aircraft. The sampling time t is 0.05 second. The maximum of $u^1(k)$ is 2 knots. The maximum of $u^2(k)$ is 300 knots.

From the dynamical description of the VTOL aircraft, we find that system matrices are changing with α. In this example, we consider the control of velocity which is ranging from 135 knots to 170 knots. In these two situations, we linearize the mathematic model (4.1) and receive a linear switched system as following:

Subsystem 1:

$$A_1 = \begin{bmatrix} 0.9987 & 0.001325 & 0 & -0.023 \\ 0.0023398 & -0.95 & -0.00488 & -0.1962 \\ 0.005 & 0.017646 & 0.96697 & 0.067945 \\ 0 & 0 & 0.04915 & 1.0017 \end{bmatrix}$$

$$B_1 = \begin{bmatrix} 0.0221 & 0 \\ 0.17331 & -0.37 \\ -0.2697 & 0.21733 \\ -0.0068 & 0.0055 \end{bmatrix}, \quad C_1 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$D_1 = \begin{bmatrix} 0 \\ 0.1 \\ 0.1 \\ 0 \end{bmatrix}, \quad M_1 = [0 \quad 0.1 \quad 0 \quad 0.1], \quad N_1 = [0.1 \quad 0]$$

Subsystem 2:

$$A_2 = \begin{bmatrix} 0.99817 & 0.001327 & 0 & -0.0228 \\ 0.00234 & 0.95071 & -0.00488 & -0.19628 \\ 0.00495 & 0.024189 & 0.9683 & 0.1217 \\ 0 & 0 & 0.04918 & 1.0031 \end{bmatrix}$$

$$B_2 = \begin{bmatrix} 0.022185 & 0.0086 \\ 0.24973 & -0.37054 \\ -0.26827 & 0.21618 \\ -0.006769 & 0.0054721 \end{bmatrix}, \quad C_2 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$D_2 = \begin{bmatrix} 0 \\ 0.1 \\ 0.1 \\ 0 \end{bmatrix}, \quad M_2 = [0 \quad 0.1 \quad 0 \quad 0.1], \quad N_2 = [0.1 \quad 0]$$

Based on theorem 2, we can have the following results:

$$K_1 = \begin{bmatrix} 27.328 & 6.411 & 10.0284 & 11.2524 \\ 41.447 & 8.9397 & 7.8277 & -51.5057 \end{bmatrix}$$

$$K_2 = \begin{bmatrix} -3.7374 & 3.65 & 6.7878 & 16.784 \\ 21.261 & 9.0716 & 9.1797 & -39.279 \end{bmatrix}$$

$$\tilde{Q}_1 = \begin{bmatrix} 32.343 & 7.1921 & 11.157 & 12.054 \\ 41.453 & 8.9663 & 7.8468 & -51.8 \end{bmatrix}$$

$$\tilde{Q}_2 = \begin{bmatrix} -44.833 & -1.4657 & -0.1024 & 18.026 \\ 27.77 & 9.9677 & 10.407 & -39.336 \end{bmatrix}$$

$$S = \begin{bmatrix} 0.00354 & -0.10529 & 0.057602 & -0.00023653 \\ -0.10529 & 9.5702 & -6.3494 & 0.17139 \\ 0.057602 & -6.3494 & 4.2789 & -0.12315 \\ -0.00023653 & 0.17139 & -0.12315 & 0.0067209 \end{bmatrix} \times 10^{-7}$$

$$\varepsilon_1 = 3.149 \times 10^{-8}, \quad \varepsilon_2 = 2.5257 \times 10^{-8}$$

5. Conclusions

In this paper, at first we analyze the stability of the switched systems with input saturation under arbitrarily switching sequences. Based on the Lyapunov stable rule, a sufficient condition which can guarantee the stability of switched systems with input saturation under arbitrarily switching sequences in discrete-time domain is given. Depending on this condition, we synthesize a state feedback controller which robustly stabilizes the switched systems. Finally, a numerical example is employed to illustrate the advantage of theory.

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References

[1] Y. Orlov, "Extended invariance principle for nonautonomous switched systems," IEEE Trans. Automat. Contr., vol. 48, No. 8, August, 2003.

[2] X. Xuping and J. Antsaklis, "Optimal control of switched systems based on parameterization of the switching instants," IEEE Trans. Automat. Contr., vol. 49, no. 1, January, 2004

[3] H. Ye, A. N. Michel and L. Hou, "Stability analysis of switched systems, Conf. Decision Control," Kobe, Japan, 1996.

[4] R. DeCarlo, M. S. Branicky, S. Pettersson, and B. Lennartson, "Perspective and results on the stability and stabilizability of hybrid systems," Proc. IEEE, pp. 1069-1082, July, 2000.

[5] D. Liberzon and A. S. Morse, "Basic problem in stability and design of switched system," IEEE Control System Mag., vol. 19, no. 5, Oct., 1999.

[6] J. Daafouz, P. Riedinger and C. Iung, "Stability analysis and Control synthesis for switched systems: A switched Lyapunov function Approach," IEEE Trans. Automat. Contr., vol. 47, no. 11, November, 2002.

[7] M. Vidyasagar, Nonlinear Systems Analysis. Upper Saddle River, NJ: Prentice-Hall, 1993, ch 5.

[8] G. Michaletzky and L. Gerencser, "BIBO Stability of Linear Swithced Systems", IEEE Trans. Automat. Contr. vol. 47, no. 11, November 2002.

[9] A. A. Agrachev and D. Liberzon, "Lie-algebraic stability criteria for switched systems," SIAMJ. Control Optim., vol. 40, 2001, pp. 253-270.

[10] W.Dayawansa and C.F. Martin, "A converse Lyapunov theorem for a class of dynamical systems which undergo switching," IEEE Trans. Automat. Contr. vol. 44, Apr. 1999, pp. 751-760.

[11] R.Vidal, S. Schaffet, J. Lyageros, and S. Sastry, "Controlled invariance of discrete time systems," in Proc. 3rd Int. Workshop, HSCC'00, Hybrid Systems: Computation Control, vol. 1790, Lecture Notes in Computer Science, N. Lybch and B. H. Krogh, Eds., 2000, pp. 437-450.

[12] G. Zhai , L. Hai, and P. J Antsaklis,., "Quadratic stabilisability of switched linear systems of linear systems with polytopic uncertainties," Int. J. Control, 2003, 76, (7), pp. 747-753.

[13] P. P. Khargonekar, , I. R Petersen., and K. Zhou, "robust stabilization of uncertain linear systems: quadratic stabilization and $H\infty$ control theory," IEEE Trans. Automat. Control., 1990, 35, (3), pp. 356-361.

[14] M.A. Rotea, M. Corless, D. Da, and J. R. Petersen, "systems with structured uncertainty: relation between quadratic and robust stability," IEEE Trans. Automat. Control, 1993, 38 (5),pp.799-803.

[15] F. Haijun, Z. Lin and T. Shu, "Analysis of linear systems in the presence of actuator saturation and L_2 disturbances," Proceedings of the 42nd IEEE Conference on Decision and Control, Manui, Hawaii USA, December 2003.