Appendix 1: Dictionary of cities

```
In [1]:
# Here is the list of cities
cities list = ["Seoul", "Hyderabad", "Ho Chi Minh", "Kuala Lumpur",
              "Singapore", "Bangkok", "Tokyo", "Hongkong", "Mumbai"]
# Transform the list to dictionary so we can access id number of each city later
cities = {key: value for value, key in enumerate(cities list)}
cities
Out[1]:
{'Seoul': 0,
 'Hyderabad': 1,
 'Ho Chi Minh': 2,
 'Kuala Lumpur': 3,
 'Singapore': 4,
 'Bangkok': 5,
 'Tokyo': 6,
 'Hongkong': 7,
 'Mumbai': 8}
In [2]:
# Create an inversed dictionary to translate the id number to city name later
cities reverse = {value: key for key, value in cities.items()}
cities_reverse
Out[2]:
{0: 'Seoul',
1: 'Hyderabad',
 2: 'Ho Chi Minh',
 3: 'Kuala Lumpur',
 4: 'Singapore',
 5: 'Bangkok',
 6: 'Tokyo',
 7: 'Hongkong',
 8: 'Mumbai'}
```

Appendix 2: Test cases

```
In [3]:
```

Appendix 3: 3 graph algorithms

Bellman-Ford algorithm

```
In [4]:
```

```
# Declare global variables
INF = float("inf")
MAX = 9 # MAX is 9 because we have 9 cities in the list of cities
# Class Edge to store the information of each edge in the edge list
class Edge:
   def init (self, source, target, weight):
        self.source = source
        self.target = target
        self.weight = weight
# Function to create one-dimensional graph from the edge list
def graph 1D(dictionary, edge list):
   graph = []
    # Loop through each edge in the edge list
    for i in range(len(edge_list)):
        # u, v are the 2 cities
        \# w is the price flying from city u to city v
       u, v, w = edge_list[i]
        # Get the id number of each city through the dictionary
        u = dictionary[u]
       v = dictionary[v]
        # Append it to the one-dimensional graph
        graph.append(Edge(u, v, w))
    return graph
# Function Bellman-Ford
def BellmanFord(start, target, nodes, dictionary, dictionary reverse, edge list):
    Inputs:
    start: Starting node
    target: Target node
    nodes: Number of nodes in the graph
    dictionary: The dictionary to translate the names of the node to their ID numbers
    dictionary_reverse: The dictionary to translate the ID numbers of the nodes to their names
    edge list: The edge list representing the flying routes between cities
    dist[dictionary[target]]: The shortest distance from the start node to the target node
    # Prepare the data structures
    graph = graph 1D(dictionary, edge list) # To store the nodes in the graph
    dist = [INF for i in range(MAX)] # To store the distance from one node to another
    global pathBF # To store the path from one node to another
    pathBF = [-1 for i in range(MAX)]
    edges = len(edge list) # Number of edges in the graph
   dist[dictionary[start]] = 0 # Initialize the function by setting the distance from the
starting node to itself as 0
    # Loop through all the nodes (n-1) times
    for i in range(1, nodes):
        # Loop through all the edges
        for j in range(edges):
            u = graph[j].source
            v = graph[j].target
            w = graph[j].weight
            # If there exists a route from source u to target v with lower cost than the current or
            # then we update the distance of target v
            if (dist[u] != INF) and (dist[u] + w < dist[v]):</pre>
                dist[v] = dist[u] + w
                \# We also update the path to node v through u
                pathBF[v] = u
    # Return the shortest distance from start node to target node
    return dist[dictionary[target]]
def printPath(start, target, dictionary, dictionary reverse):
    Inputs:
    start: Starting node
```

```
target: larget noue
    dictionary: The dictionary to translate the names of the node to their ID numbers
    dictionary_reverse: The dictionary to translate the ID numbers of the nodes to their names
    Output:
    The function directly prints out the path from the starting node to the target node
    # Take the ID number of the start and target node
    s, t = dictionary[start], dictionary[target]
    # The list to store the sequence of the nodes
    # Base case: If the target node is the same as the start node, then we just simply print it ou
    if t == s:
        print(dictionary reverse[s])
        return
    \# Base case: There is no path from the start node to the target node, noted by "-1"
    # So we print "No path"
    elif pathBF[t] == -1:
       print("No path")
        return
    while True:
        # Tracking the path from the start node to the target node in the reversed order
        \ensuremath{\text{\#}} We are back tracking from the target node back to the start node
        lst.append(t)
        t = pathBF[t]
        # When we reach the starting node, break the while loop
        if t == s:
            lst.append(s)
            break
    # Reverse the sequence to print it out
    lst.reverse()
    # Print out the order from start node to target node
    for i in range(len(lst)-1):
       print(dictionary reverse[lst[i]], end = " => ")
    print(dictionary reverse[lst[-1]])
                                                                                                  I
In [5]:
ans = BellmanFord("Seoul", "Hyderabad", 5, cities, cities reverse, test1)
print("Minimum cost from Seoul to Hyderabad is ${}".format(ans))
# Print the path
printPath("Seoul", "Hyderabad", cities, cities reverse)
Minimum cost from Seoul to Hyderabad is $350
Seoul => Kuala Lumpur => Hyderabad
In [6]:
ans = BellmanFord("Seoul", "Hyderabad", 6, cities, cities_reverse, test2)
print("Minimum cost from Seoul to Hyderabad is ${}".format(ans))
# Print the path
printPath("Seoul", "Hyderabad", cities, cities reverse)
Minimum cost from Seoul to Hyderabad is $350
Seoul => Kuala Lumpur => Hyderabad
In [7]:
ans = BellmanFord("Tokyo", "Hyderabad", 6, cities, cities_reverse, test3)
print("Minimum cost from Tokyo to Hyderabad is ${}".format(ans))
# Print the path
printPath("Tokyo", "Hyderabad", cities, cities reverse)
```

Dijkstra's algorithm

```
In [8]:
```

```
# Import library
import queue
# Declare global variables
INF = float("inf")
MAX = 9
# Class Node to store the information of each node in the graph: its ID number and distance
class Node:
   def __init__(self, ID, distance):
       self.ID = ID
        self.distance = distance
    # To organize the node from the shortest distance to longest
    def lt (self, other):
        return self.distance <= other.distance</pre>
# Function to create two-dimensional graph from the edge list
def graph_2D(dictionary, edge_list):
    graph = [[] for i in range(MAX)]
    for i in range(len(edge list)):
       city1 = dictionary[edge_list[i][0]]
        city2 = dictionary[edge_list[i][1]]
        distance = edge list[i][2]
        graph[city1].append(Node(city2, distance))
    return graph
# Function Dijkstra
def Dijkstra(start, target, nodes, dictionary, dictionary_reverse, edge_list):
    11 11 11
    Inputs:
    start: Starting node
    target: Target node
    nodes: Number of nodes in the graph
    dictionary: The dictionary to translate the names of the node to their ID numbers
    dictionary reverse: The dictionary to translate the ID numbers of the nodes to their names
    edge list: The edge list representing the flying routes between cities
    Output:
    dist[dictionary[target]]: The shortest distance from the start node to the target node
    # Prepare the data structures
    graph = graph 2D(dictionary, edge list) # To store the nodes in the graph
    dist = [INF for i in range(MAX)] # To store the distance from one node to another
    global pathD # To store the path from one node to another
    pathD = [-1 for i in range(MAX)]
    # Priority queue to get the nodes with shortest distance
    pq = queue.PriorityQueue()
    \# Initialize the function by putting the start node to the queue, distance 0
    pq.put(Node(dictionary[start], 0))
    dist[dictionary[start]] = 0
    # Run the while loop when we still have nodes in the priority queue
    while not pq.empty():
        # Get the top node in the queue
        top = pq.get()
       u = top.ID
        w = top.distance
        # For all neighbor nodes in graph of u
        for neighbor in graph[u]:
            # If there exists a route from node u to neighbor with lower cost than the current one
            # then we update the distance of neighbor node
            if neighbor.distance + w < dist[neighbor.ID]:</pre>
               dist[neighbor.ID] = w + neighbor.distance
```

```
# We put this node and its updated distance to the priority queue
                pq.put(Node(neighbor.ID, dist[neighbor.ID]))
                # We also update the path to node neighbor through u
                pathD[neighbor.ID] = u
    # Return the shortest distance from start node to target node
    return dist[dictionary[target]]
def printPath(start, target, dictionary, dictionary reverse):
    Inputs:
    start: Starting node
    target: Target node
    dictionary: The dictionary to translate the names of the node to their ID numbers
    dictionary reverse: The dictionary to translate the ID numbers of the nodes to their names
    Output:
    The function directly prints out the path from the starting node to the target node
    # Take the ID number of the start and target node
    s, t = dictionary[start], dictionary[target]
    # The list to store the sequence of the nodes
    lst = []
    # Base case: If the target node is the same as the start node, then we just simply print it ou
    if t == s:
        print(dictionary reverse[s])
        return
    # Base case: There is no path from the start node to the target node, noted by "-1"
    # So we print "No path"
    elif pathD[t] == -1:
       print("No path")
        return
    while True:
        # Tracking the path from the start node to the target node in the reversed order
        # We are back tracking from the target node back to the start node
        lst.append(t)
        t = pathD[t]
        # When we reach the starting node, break the while loop
        if t == s:
            lst.append(s)
            break
    # Reverse the sequence to print it out
    lst.reverse()
    # Print out the order from start node to target node
    for i in range(len(lst)-1):
       print(dictionary reverse[lst[i]], end = " => ")
    print(dictionary_reverse[lst[-1]])
In [9]:
# Test cases
ans = Dijkstra("Seoul", "Hyderabad", 5, cities, cities_reverse, test1)
print("Minimum cost from Seoul to Hyderabad is ${}".format(ans))
# Print the path
printPath("Seoul", "Hyderabad", cities, cities reverse)
Minimum cost from Seoul to Hyderabad is $350
Seoul => Kuala Lumpur => Hyderabad
```

ans = Dijkstra("Seoul", "Hyderabad", 6, cities, cities_reverse, test2)
print("Minimum cost from Seoul to Hyderabad is \${}".format(ans))

printPath("Seoul", "Hyderabad", cities, cities reverse)

In [10]:

Print the path

```
Seoul => Kuala Lumpur => Hyderabad

In [11]:
ans = Dijkstra("Tokyo", "Hyderabad", 6, cities, cities_reverse, test3)
print("Minimum cost from Tokyo to Hyderabad is ${}".format(ans))
# Print the path
```

Minimum cost from Tokyo to Hyderabad is \$400 Tokyo => Bangkok => Mumbai => Hyderabad

printPath("Tokyo", "Hyderabad", cities, cities_reverse)

Minimum cost from Seoul to Hyderabad is \$350

Floyd-Warshall algorithm

In [12]:

```
# Declare global variables
INF = float("inf")
MAX = 9
# Function Floyd-Warshall algorithm
def FloydWarshall(start, target, nodes, dictionary, dictionary_reverse, edge_list):
    Inputs:
    start: Starting node
    target: Target node
    nodes: Number of nodes in the graph
    dictionary: The dictionary to translate the names of the node to their ID numbers
    dictionary_reverse: The dictionary to translate the ID numbers of the nodes to their names
    edge list: The edge list representing the flying routes between cities
   dist[dictionary[start]][dictionary[target]]: The shortest distance from the start node to the
target node
    # Initialize the data structures
    graph = [[INF for i in range(MAX)] for j in range(MAX)] # To store the nodes in the graph
    \texttt{dist} = \texttt{[[INF for i in range(MAX)] for j in range(MAX)]} \ \textit{\# To store the distance from one node to}
another
    global pathFW
    pathFW = [[-1 for i in range(MAX)] for j in range(MAX)] # To store the path from one node to an
    # If the starting and ending nodes are the same, then there is no route between them
    for i in range(MAX):
        for j in range(MAX):
            if i == j:
                graph[i][j] = 0
    # Update the information in graph, dist, path according to the edge list
    for i in range(len(edge list)):
       city1 = dictionary[edge_list[i][0]]
        city2 = dictionary[edge list[i][1]]
        distance = edge_list[i][2]
        graph[city1][city2] = distance
        dist[city1][city2] = distance
        pathFW[city1][city2] = city1
    # Run 3 for loops to update the distance
    for k in range(MAX):
        for i in range(MAX):
            for j in range(MAX):
            # If there exists a node k such that going from i to j through k is shorter than going
directly from i to j
            # then we update the distance from i to j
                if dist[i][j] > dist[i][k] + dist[k][j]:
                    dist[i][j] = dist[i][k] + dist[k][j]
                    # We also undate the nath from node i to node i accordingly
```

```
pathFW[i][j] = pathFW[k][j]
    # Return the shortest distance from start node to target node
    return dist[dictionary[start]][dictionary[target]]
def printPath(start, target, dictionary, dictionary reverse):
    Inputs:
    start: Starting node
    target: Target node
    dictionary: The dictionary to translate the names of the node to their ID numbers
    dictionary reverse: The dictionary to translate the ID numbers of the nodes to their names
    Output:
    The function directly prints out the path from the starting node to the target node
    # Take the ID number of the start and target node
    s, t = dictionary[start], dictionary[target]
    # The list to store the sequence of the nodes
    1st = [1]
    # Run the while loop until we reach the start node
    while s != t:
        # Tracking the path from the start node to the target node in the reversed order
        \# We are back tracking from the target node back to the start node
        lst.append(t)
        t = pathFW[s][t]
    # Finally, add the start node to the list
    lst.append(s)
    # Reverse the sequence to print it out
    lst.reverse()
    # Print out the order from start node to target node
    for i in range(len(lst)-1):
       print(dictionary reverse[lst[i]], end = " => ")
    print(dictionary_reverse[lst[-1]])
In [13]:
# Test cases
ans = FloydWarshall("Seoul", "Hyderabad", 5, cities, cities reverse, test1)
print("Minimum cost from Seoul to Hyderabad is ${}".format(ans))
# Print the path
printPath("Seoul", "Hyderabad", cities, cities_reverse)
Minimum cost from Seoul to Hyderabad is $350
Seoul => Kuala Lumpur => Hyderabad
In [14]:
ans = FloydWarshall("Seoul", "Hyderabad", 6, cities, cities reverse, test2)
print("Minimum cost from Seoul to Hyderabad is ${}".format(ans))
# Print the path
printPath("Seoul", "Hyderabad", cities, cities_reverse)
Minimum cost from Seoul to Hyderabad is $350
Seoul => Kuala Lumpur => Hyderabad
In [15]:
ans = FloydWarshall("Tokyo", "Hyderabad", 6, cities, cities reverse, test3)
print("Minimum cost from Tokyo to Hyderabad is ${}".format(ans))
# Print the path
printPath("Tokyo", "Hyderabad", cities, cities reverse)
```

Appendix 4: Running time examination

I'm writing modified code for each algorithm because the input from the randomly generated graph function does not require a dictionary to translate from string to integer.

Bellman-Ford algorithm to test running time

In [16]:

```
# Declare global variables
INF = float("inf")
# Class Edge to store the information of each edge in the edge list
class Edge:
    def init (self, source, target, weight):
        self.source = source
       self.target = target
       self.weight = weight
# Function to create one-dimensional graph from the edge list
def graph 1D(edge list):
   graph = []
    # Loop through each edge in the edge list
    for i in range(len(edge_list)):
       # u, v are the 2 cities
        # w is the price flying from city u to city v
        u, v, w = edge_list[i]
       # Append it to the one-dimensional graph
       graph.append(Edge(u, v, w))
    return graph
# Function Bellman-Ford
def BellmanFord General(start, target, nodes, edge list):
   Inputs: s, n, m
    start: Starting node
    target: Target node
    nodes: Number of nodes in the graph
    edge list: The edge list representing the flying routes between cities
    Output:
    True/False: whether we can find the shortest distance
    # Prepare the data structures
   MAX = nodes + 1
    graph = graph 1D(edge list) # To store the nodes in the graph
    dist = [INF for i in range(MAX)] # To store the distance from one node to another
    \# Initialize the function by setting the distance from the starting node to itself as 0
    dist[start] = 0
    # Loop through all the nodes (n-1) times
    for i in range(1, nodes):
        # Loop through all the edges
        for j in range(len(edge list)):
            u = graph[j].source
            v = graph[j].target
            w = graph[j].weight
            \# If there exists a route from source u to target v with lower cost than the current or
е
            # then we update the distance of target v
            if (dist[u] != INF) and (dist[u] + w < dist[v]):</pre>
               dist[v] = dist[u] + w
    # Return the shortest distance from start node to target node
    return dist[target]
                                                                                                 Þ
```

```
In [17]:
# Import library
import queue
# Declare global variables
INF = float("inf")
# Class Node to store the information of each node in the graph: its ID number and distance
class Node:
   def init (self, ID, distance):
        self.ID = ID
        self.distance = distance
    # To organize the node from the shortest distance to longest
    def lt (self, other):
        return self.distance <= other.distance</pre>
# Function to create two-dimensional graph from the edge list
def graph 2D(edge_list):
    graph = [[] for i in range(len(edge list)+1)]
    for i in range(len(edge_list)):
       city1 = edge list[i][0]
       city2 = edge list[i][1]
       distance = edge_list[i][2]
        graph[city1].append(Node(city2, distance))
    return graph
def Dijkstra_General(start, target, nodes, edge_list):
    Inputs:
    start: Starting node
    target: Target node
    nodes: Number of nodes in the graph
    dictionary: The dictionary to translate the names of the node to their ID numbers
    dictionary_reverse: The dictionary to translate the ID numbers of the nodes to their names
    edge_list: The edge list representing the flying routes between cities
    dist[dictionary[target]]: The shortest distance from the start node to the target node
    # Prepare the data structures
    MAX = nodes + 1
    graph = graph_2D(edge_list)
    dist = [INF for i in range(MAX)]
    # Priority queue to get the nodes with shortest distance
    pq = queue.PriorityQueue()
    # Initialize the function by putting the start node to the queue, distance 0
    pq.put(Node(start, 0))
    dist[start] = 0
    # Run the while loop when we still have nodes in the priority queue
    while not pq.empty():
        # Get the top node in the queue
       top = pq.get()
        u = top.ID
        w = top.distance
        # For all neighbor nodes in graph of u
        for neighbor in graph[u]:
            # If there exists a route from node u to neighbor with lower cost than the current one
            # then we update the distance of neighbor node
            if neighbor.distance + w < dist[neighbor.ID]:</pre>
                dist[neighbor.ID] = w + neighbor.distance
                \# We put this node and its updated distance to the priority queue
```

return dist[target]

pq.put(Node(neighbor.ID, dist[neighbor.ID]))

Return the shortest distance from start node to target node

```
In [18]:
```

```
# Declare global variables
INF = float("inf")
# Function Floyd-Warshall
def FloydWarshall General(start, target, nodes, edge list):
   Inputs:
   start: Starting node
   target: Target node
   nodes: Number of nodes in the graph
    dictionary: The dictionary to translate the names of the node to their ID numbers
    dictionary reverse: The dictionary to translate the ID numbers of the nodes to their names
    edge_list: The edge list representing the flying routes between cities
   Output:
   dist[dictionary[start]][dictionary[target]]: The shortest distance from the start node to the
target node
    11 11 11
    # Initialize the data structures
   MAX = nodes + 1
    graph = [[INF for i in range(MAX)] for j in range(MAX)] # To store the nodes in the graph
   dist = [[INF for i in range(MAX)] for j in range(MAX)] # To store the distance from one node t
o another
    # If the starting and ending nodes are the same, then there is no route between them
    for i in range(MAX):
        for j in range (MAX):
            if i == j:
                graph[i][j] = 0
    # Update the information in graph, dist, path according to the edge list
    for i in range(len(edge list)):
       city1 = edge list[i][0]
       city2 = edge list[i][1]
       distance = edge list[i][2]
        graph[city1][city2] = distance
        dist[city1][city2] = distance
    # Run 3 for loops to update the distance
    for k in range(MAX):
        for i in range(MAX):
            for j in range(MAX):
            # If there exists a node k such that going from i to j through k is shorter than going
directly from i to j
            # then we update the distance from i to j
                if dist[i][j] > dist[i][k] + dist[k][j]:
                    dist[i][j] = dist[i][k] + dist[k][j]
    # Return the shortest distance from start node to target node
    return dist[start][target]
```

Generate data to push into the algorithms

In [19]:

```
# Import library
import random

def random_graph_generator(n):
    """
    Function to generate random graphs to push to the 3 graph algorithms

    Input: n: Number of nodes in the graph

    Output: graph: An edge list with each edge comprised of a start node, end node, and cost from start to end node
    """
```

```
# Declare the number of edges in the graph
# Actually the number of edges can be any as long as it is less than n*(n-1)/2
# But for the sake of calculating the running time, we predefine the number of edges here
num_edges = n*(n-1)//2

# Initialize the edge list
edge_list = []

# Run through all edges in the edge list
for i in range(num_edges):
    a = random.randint(0, n-1) # Start node
    b = random.randint(0, n-1) # End node
    w = random.randint(0, 1000) if a != b else 0 # Random traveling cost from node a to b
    edge_list.append((a, b, w)) # Append the edge to the edge list

# Return the edge list
return edge_list
```

Plot the results

In [21]:

```
# Import plotting and time libraries
import matplotlib.pyplot as plt
import time
# Initialize the storage array to store the running time of 3 algorithms
bf_time = [] # Bellman-Ford function
d time = [] # Dijkstra function
fw time = [] # Floyd-Warshall function
x = [i \text{ for } i \text{ in } range(3, 101)] \# This is the x-axis: Input size from 3 to 100 (inclusize)
# Running time of each algorithm with respect to the growth in input size
# Input ranges from 3 to 100 (inclusive)
for n in range (3, 101, 1):
    # Create temporary arrays to store the time
    \# For each length we would run the function 50 times, then average the results
    bf time temp = []
    d time temp = []
    fw_time_temp = []
    # Run each function 50 times for each input size
    for i in range (50):
        # Create the edge list with nodes
        data = random_graph_generator(n)
        # Timing the running time of Bellman-Ford algorithm
        start BF = time.time()
        BellmanFord General(0, n-1, n, data)
        end BF = time.time()
        bf time temp.append(end BF-start BF)
        # Timing the running time of Dijkstra's algorithm
        start_D = time.time()
        Dijkstra General(0, n-1, n, data)
        end_D = time.time()
        d time temp.append(end D-start D)
        # Timing the running time of Floyd-Warshall algorithm
        start FW = time.time()
        FloydWarshall General(0, n-1, n, data)
        end FW = time.time()
        fw time temp.append(end FW-start FW)
    # Average the result of 50 running times. Then append it to the storage array
    bf time.append(sum(bf time temp)/50)
    d time.append(sum(d time temp)/50)
    fw_time.append(sum(fw_time_temp)/50)
\# Plot the running time of each function versus its growth in input size
plt.plot(x, bf_time, color = "orange", linestyle = "dashed", marker='o', label = "Bellman-Ford")
plt.plot(x, d_time, color = "green", linestyle = "dashed", marker='o', label = "Dijkstra's")
plt.plot(x, fw time, color = "blue", linestyle = "dashed", marker='o', label = "Floyd-Warshall")
plt.xlabel("Number of nodes")
plt.ylabel("Running time")
```

