

Lamiah Khan
 → Signals PS-9:

2) ARMA and AR Models:

$$1) x[n] = v[n] + 0.4v[n-1] + 0.2v[n-2] + 1.6x[n-1] - 0.81x[n-2]$$

$$\text{ARMA}(N, M) \Rightarrow v[n] = \frac{1}{b_0} x[n] + \dots + \frac{a_m}{b_0} x[n-M] - \frac{b_1}{b_0} v[n-1] \dots - \frac{b_N}{b_0} v[n-N]$$

$$\rightarrow \text{so } \boxed{\text{ARMA}(2, 2)}, p=2, q=2$$

2) both current and past vals
 ⇒ innovations filter

$$3) v[n] = x[n] - 0.4v[n-1] - 0.2v[n-2] - 1.6x[n-1] + 0.81x[n-2]$$

$$\hat{v}[n] = x[n] - 0.4\hat{v}[n-1] - 0.2\hat{v}[n-2] - 1.6x[n-1] + 0.81x[n-2]$$

$$4) x(z) = v(z) + 0.4z^{-1}v(z) + 0.2z^{-2}v(z) + 1.6z^{-1}x(z) - 0.81z^{-2}x(z)$$

$$= v(z)(1 + 0.4z^{-1} + 0.2z^{-2}) + x(z)(1.6z^{-1} - 0.81z^{-2})$$

$$H(z) = \frac{x(z)}{v(z)}$$

$$\frac{x(z)(1 - 1.6z^{-1} + 0.81z^{-2})}{\cancel{v(z)}(1 - 1.6z^{-1} + 0.81z^{-2})} = \frac{\cancel{v(z)}(1 + 0.4z^{-1} + 0.2z^{-2})}{\cancel{v(z)}(1 - 1.6z^{-1} + 0.81z^{-2})}$$

$$H(z) = \frac{1 + 0.4z^{-1} + 0.2z^{-2}}{1 - 1.6z^{-1} + 0.81z^{-2}}$$

$$5) H(z) = \frac{1 + 0.4z^{-1} + 0.2z^{-2}}{1 - 1.6z^{-1} + 0.81z^{-2}} \quad \left\{ \begin{array}{l} S_x(\omega) = |H(e^{j\omega})|^2 \cdot S_v(\omega) \\ \sigma_v^2 = 2, S_v(\omega) = 2 \end{array} \right.$$

$$|H(e^{j\omega})|^2 = |CH(z)|^2|_{z=e^{j\omega}}$$

$$S_x(\omega) = |CH(z)|^2|_{z=e^{j\omega}} \cdot 2$$

$$= 2 |H(e^{j\omega})|^2, \text{ where } H(z)$$