In the name of God, the Merciful, the Compassionate

Artificial Intelligence Final Exam

Spring 2020 – Group 1

Time: 5 hours

Note: There are 20 bonus points in the exam.

1) **[25 pts.]** Answer each of these questions briefly.
   a. We are using the maximum likelihood estimation to find the conditional probability tables of a large Bayes' net from limited data samples. Explain in details what the possible problem with this idea is.
   b. Suppose that a Markov model always converges to a particular distribution irrespective of the initialization. How can you find this distribution?
   c. What is the role of resampling in the particle filtering?
   d. Explain why naïve Bayes' is not an ideal model for the classification of handwritten digits.
   e. Explain why we are guaranteed to find the solution with the least possible loss function when using logistic regression.
2) **[35 pts.]** We want to predict the future trend of a stock price based on its past values. More specifically, let P_t be the stock price at time t, and $R_t = P_t – P_{t-1}$ be the change in the stock price at time t. We assume that the stock has three states at any given time (denoted as X_t): "growing", "steady", "decreasing". These states each lead to a different distribution of change in the stock price that is given below:

r	$P(R_t = r \mid X_t = \text{growing})$
-5	0.1
+5	0.9

r	$P(R_t = r \mid X_t = \text{steady})$
-5	0.5
+5	0.5

r	$P(R_t = r \mid X_t = \text{decreasing})$
-5	0.9
+5	0.1

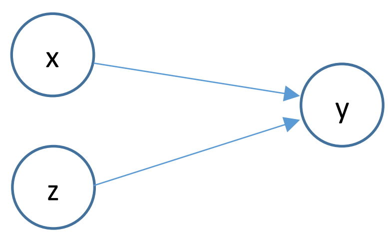
The states also change over time according to the following Markov model:

$P(X_{t+1} \mid X_t)$	X_{t+1} = growing	X_{t+1} = steady	X_{t+1} = decreasing
X_t = growing	0.7	0.2	0.1
X_t = steady	0.05	0.9	0.05
X_t = decreasing	0.05	0.15	0.8

Suppose that we observed following sequence of R_t for t = 1 to t = 6: (5, 5, 5, 5, -5, -5).

a) **[25 pts.]** Find the probability the stock is "growing" at time t=7 based on this observation.
b) **[10 pts.]** Find the probability the stock is "growing" at time t=8 based on this observation.

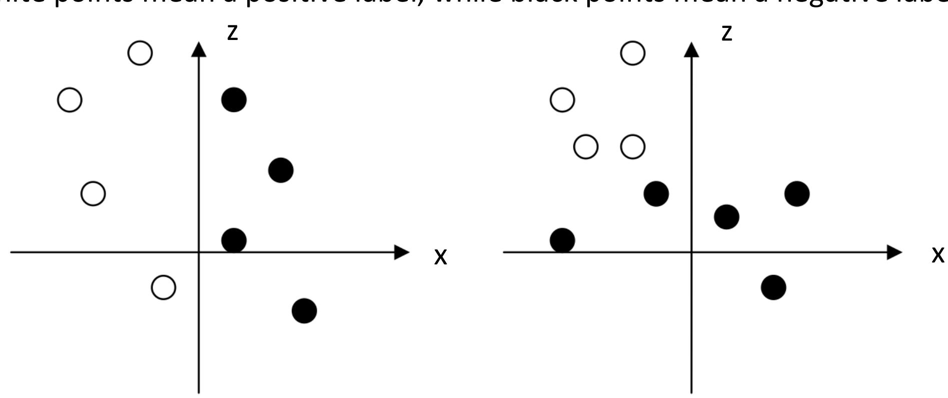
3) **[25 pts.]** Lets' assume that we want to use a perceptron with the following architecture to solve a classification problem in the two-dimensional feature space (x, z).



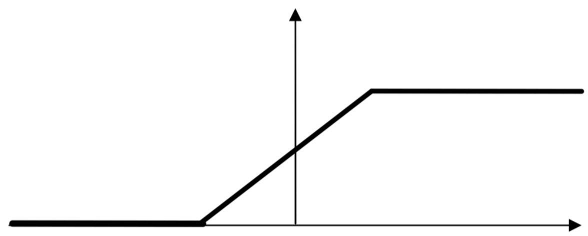
Note that this perceptron does not use a bias and the activation function is

$$f(x) = \begin{cases} -1 & x \leq 0 \\ 1 & x > 0 \end{cases}$$

a) **[10 pts.]** Which one of the following problems can be solved by this network and why? (white points mean a positive label, while black points mean a negative label).



b) **[15 pts.]** Now suppose that we change the activation function to the sigmoid ($f(u) = \frac{1}{1+\exp(-u)}$) and train the network using the backpropagation algorithm. To make the computation at both training and test time simpler, one suggests to use the following activation function as an approximation to the sigmoid function:



Explain the issues with this suggestion at the training time.

4) **[35 pts.]** Suppose that you are playing the following game. At each time a natural number between 1 to 4 is generated uniformly randomly. The numbers are also independent across the time. At each time, you can either stop the game and receive a reward equal to the generated number or you can continue the game with an immediate reward of -1. Let the discount factor be 0.9.
   a) **[5 pts.]** Show that this game can be modeled by an MDP and specify different components of the MDP for this problem.
   b) **[15 pts.]** Use the value iteration algorithm (only up to 3 actions; or 3 iterations) to find the optimal policy.
   c) **[15 pts.]** Now suppose that you don't know the probabilities that are used to generate the numbers, and you also don't know the rewards before playing the game. What would be the final Q-table if you observe the following sequence of (current state, action, next state, reward)?
   (1, cont., 1, -1), (1, cont., 2, -1), (2, cont., 3, -1), (3, stop, 3, 3)
   Assume that the Q-table is initialized to zero, and let the learning rate be 0.1.