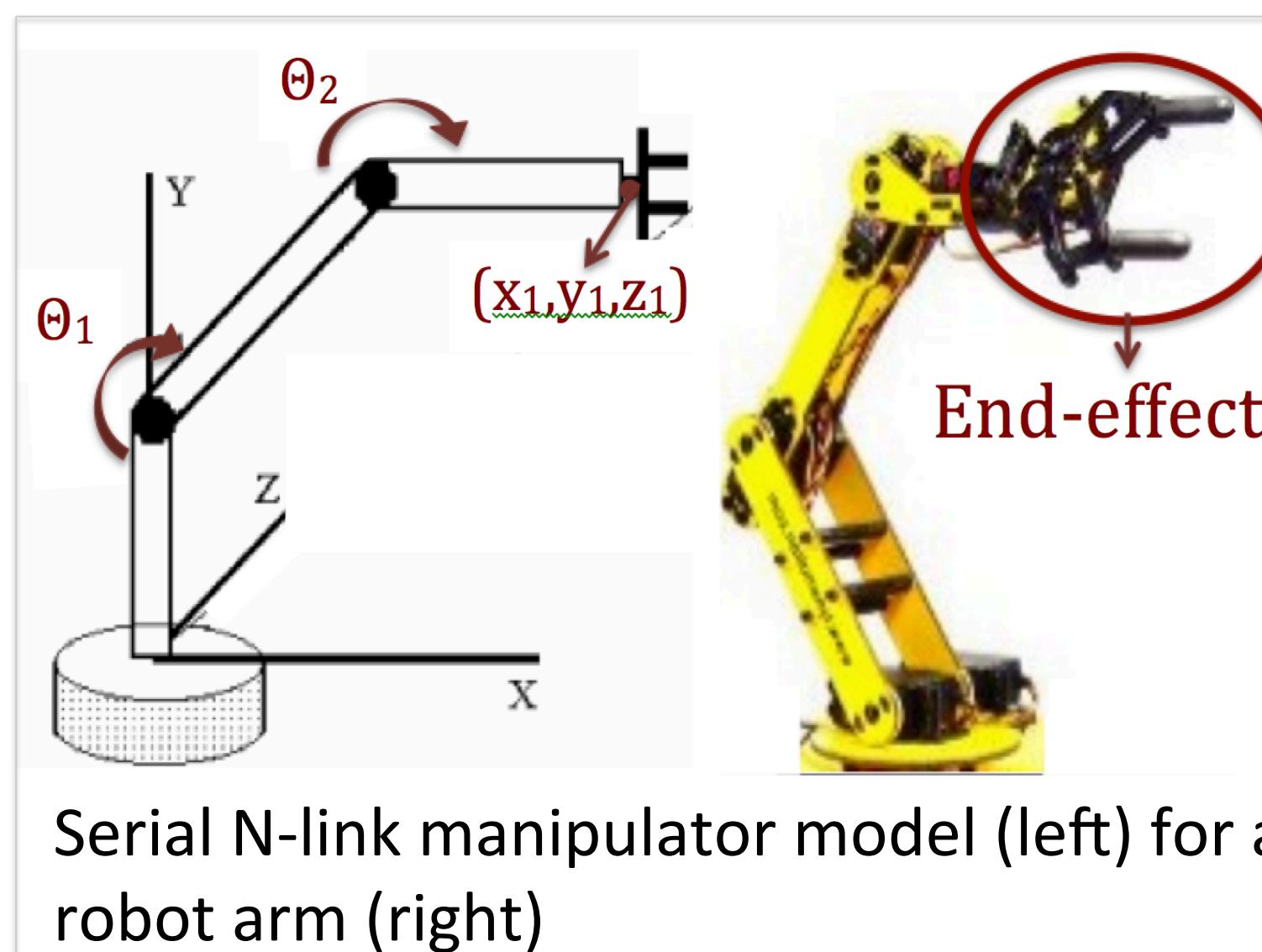


**Motivation:** Efficiently find exact solutions to multivariate polynomial equations in robot control and motion planning.

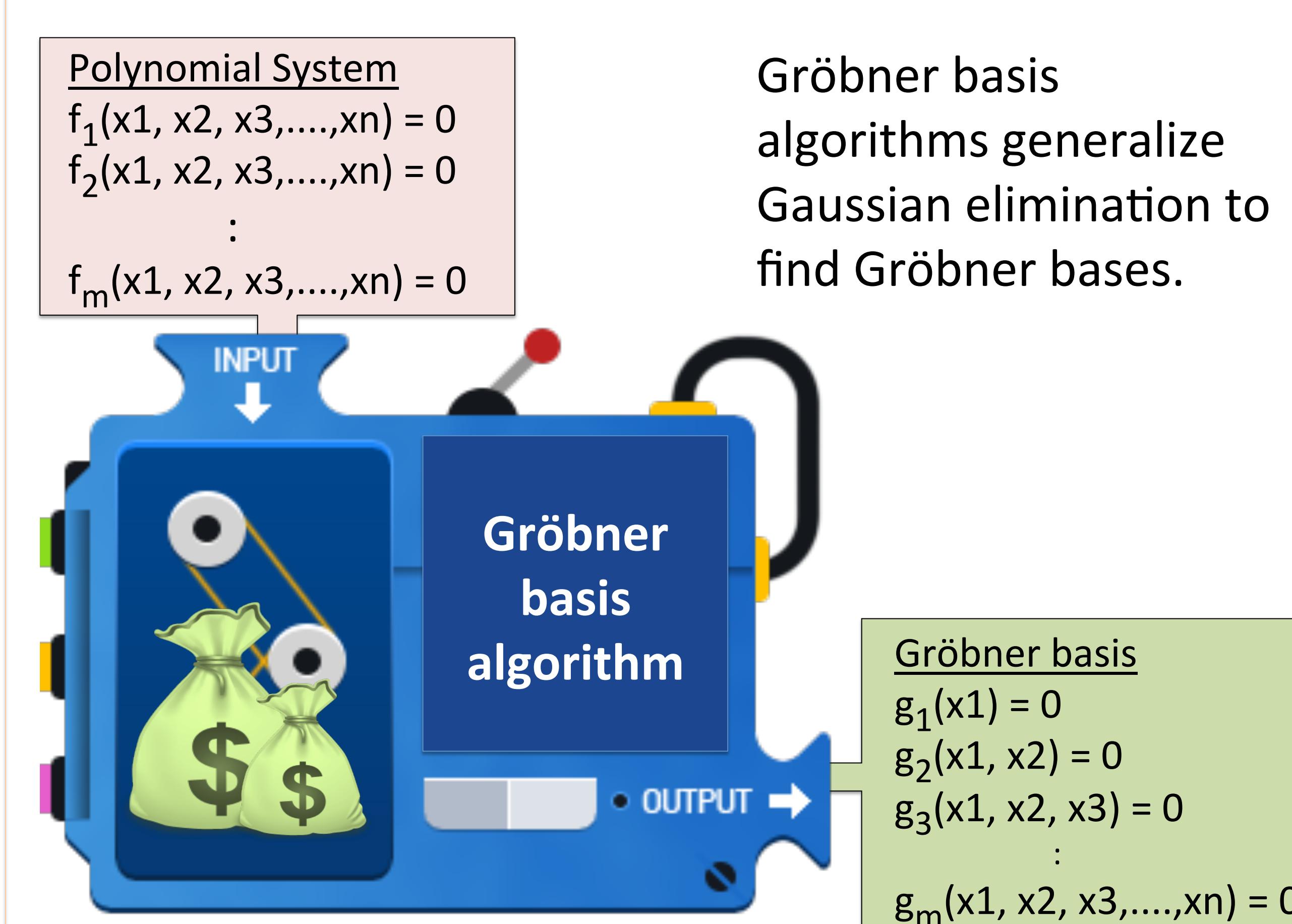
## I. ROBOT CONTROL

- Robot control involves solving polynomial equations in inverse kinematics (positions) and dynamics (forces) problems for robots.



## II. GRÖBNER BASES

- ✓ Gröbner bases can be used to find solutions to systems of multivariate polynomial equations.
- ✗ Gröbner basis algorithms have double exponential worst-case complexity, so we aim to find efficient Gröbner basis algorithms.



## III. ROBOT STRUCTURES

- Sparse matrix computations are simpler than dense matrix computations.

$$\begin{vmatrix} 1 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 4 \end{vmatrix} = 24$$

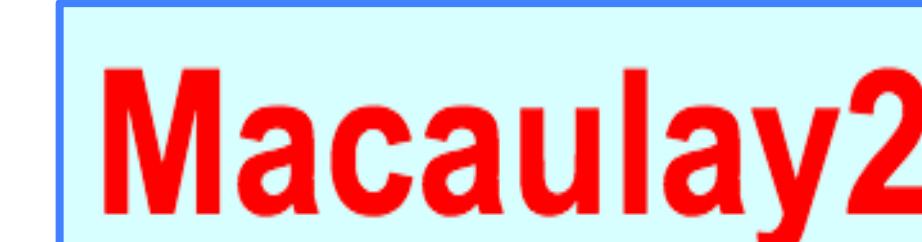
Determinant of a sparse diagonal matrix

We created a spider robot (left). Each spider leg is connected to the body, but not to other legs, yielding chordal sparsity in robot graphs (right).



## IV. COMPUTATIONS

- Gröbner basis algorithms include Buchberger's algorithm, Faugère's F4, F5 and FGFM.
- ✗ Maple, MATLAB and Macaulay2 *cannot* compute Gröbner basis for a polynomial system representing even one robotic spider leg.

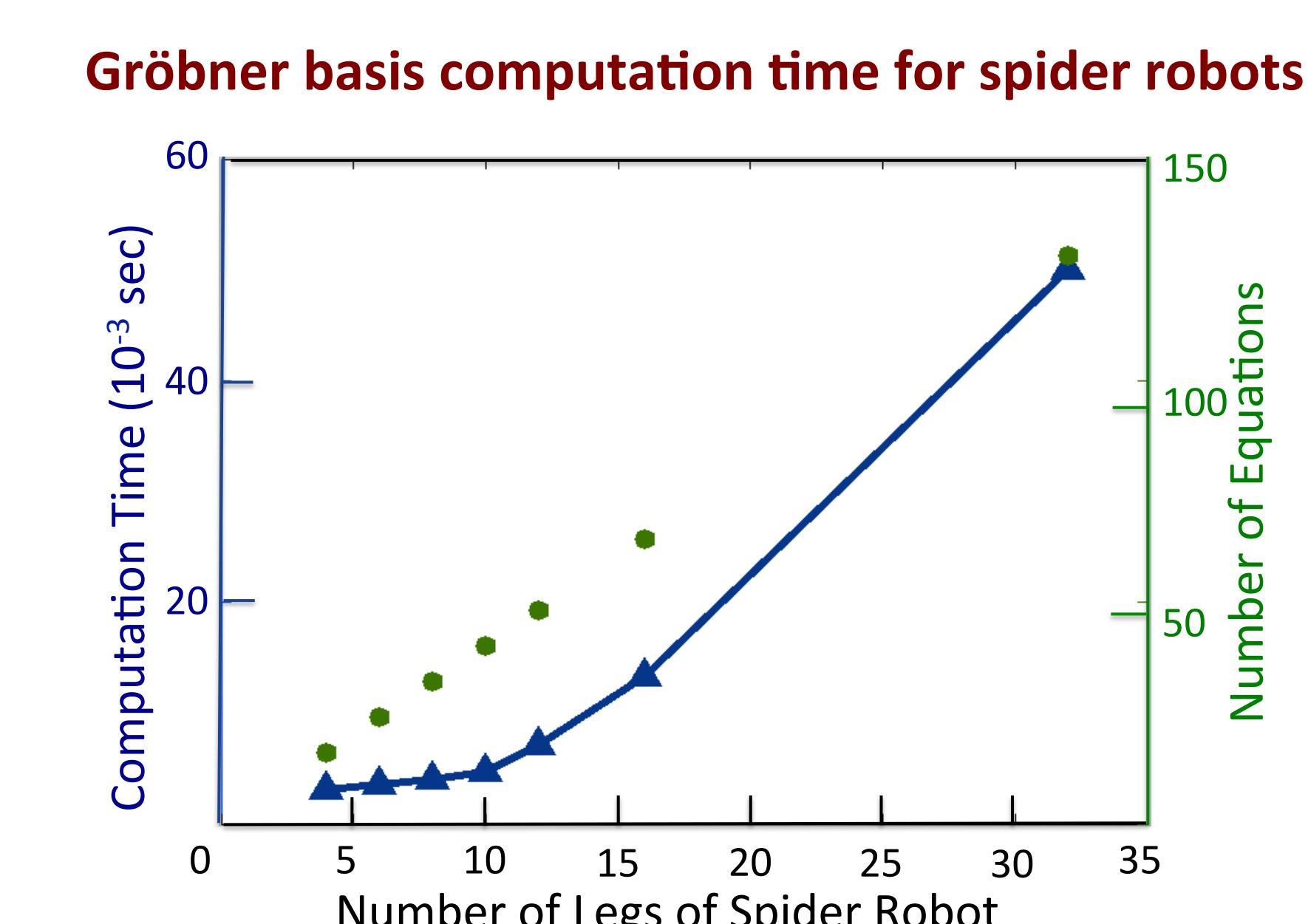


- Sage is Python-based math software that uses C++ libraries in Singular and Magma to compute Gröbner bases.
- ✓ Sage allows fast Gröbner basis computations.



## V. CONCLUSIONS

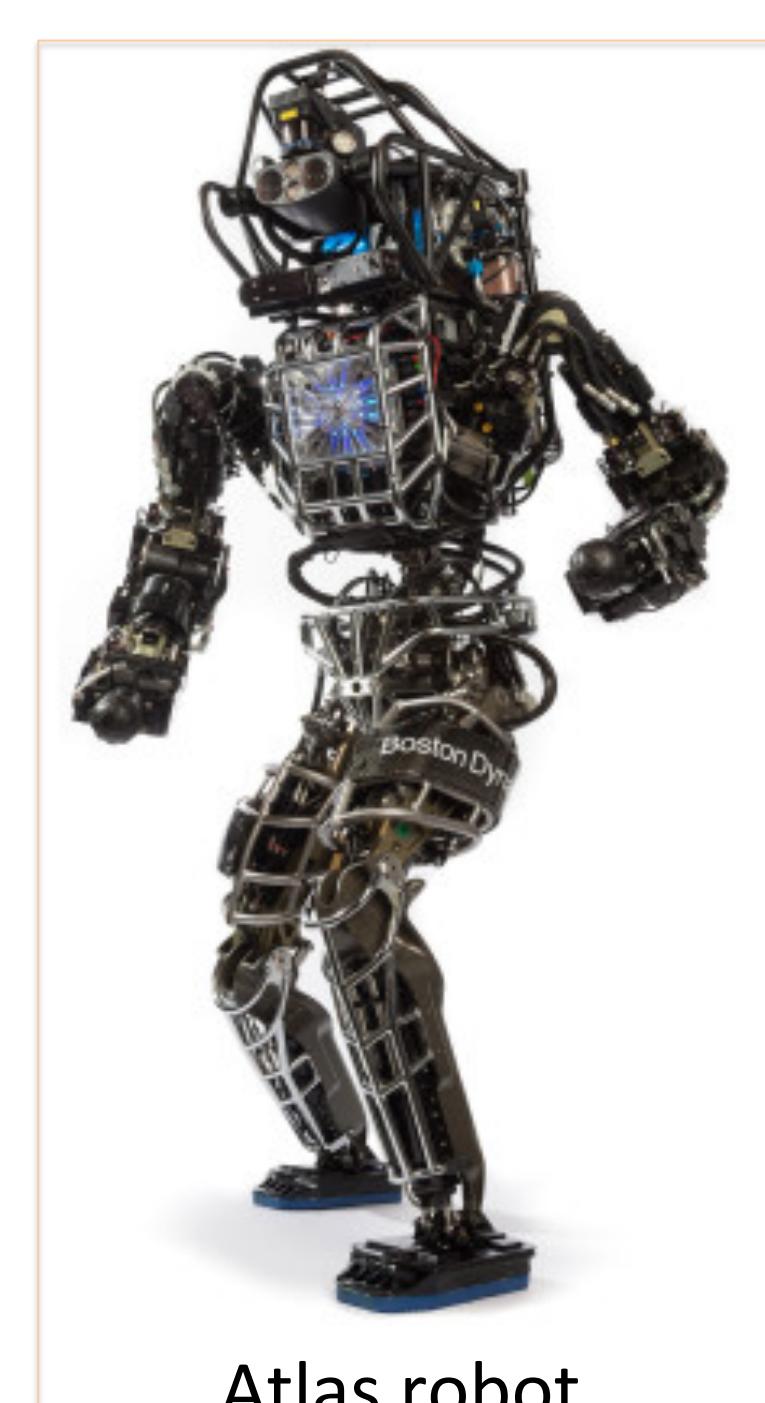
- Sage efficiently computes Gröbner bases using a 1.3 GHz Intel Core i5 processor. We achieved a 50 milliseconds computation time for Gröbner basis for a system of 128 polynomials in 67 variables for a 32-leg spider robot.



Computation time for Gröbner bases using Sage. Our polynomial systems for the spider robot had 16 to 128 equations.

## VI. FUTURE WORK

- Integrate Sage's Gröbner basis algorithms in our existing polynomial solvers to improve the performance of robots, especially Boston Dynamics "Atlas".
- Investigate chordal graphs to further leverage polynomial sparsity in Gröbner basis algorithms. Our existing algorithm using chordal graphs is slower than Sage's algorithm.



## Acknowledgements

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## Picture Courtesy

ROSARIO, J. M. et al. Proposal of methodology for the modeling and control of manipulators. J. Braz. Soc. Mech. Sci. 2002, vol.24, n.3; [http://www.societyofrobots.com/robot\\_arm\\_tutorial.shtml](http://www.societyofrobots.com/robot_arm_tutorial.shtml); <http://www.mathwarehouse.com/algebra/relation/evaluating-function.php>; <http://physicssum.deviantart.com/art/3D-Robot-Spider-28049884>; Logos: MATLAB, Maple, Macaulay2, Sage, Python, Magma, Singular, C/C++ (<http://www.techui.com/group/candcppdevelopers>); [http://www.fairfaxunderground.com/forum/file.php?40,file=113550,filename=Atlas-p2\\_nt.jpg](http://www.fairfaxunderground.com/forum/file.php?40,file=113550,filename=Atlas-p2_nt.jpg)