GRÖBNER BASIS FOR POLYNOMIAL SYSTEMS IN ROBOTICS

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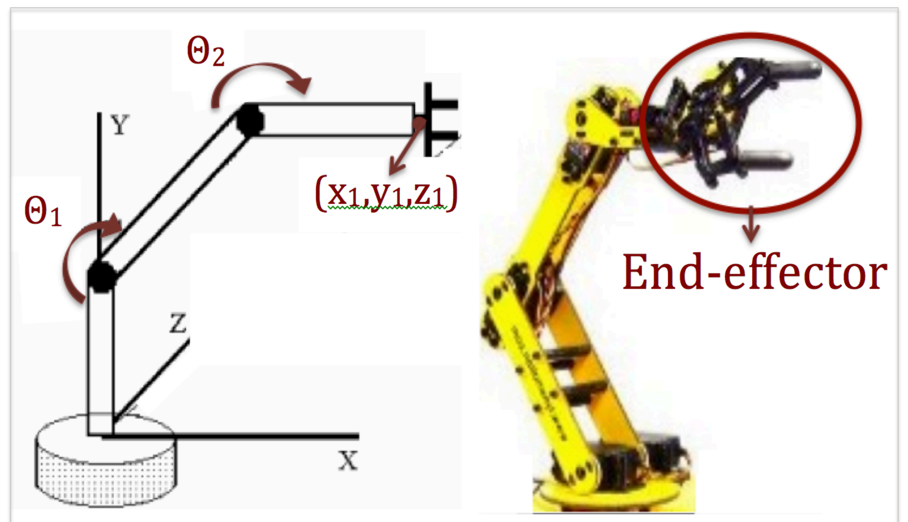
Motivation: *Efficiently* find *exact* solutions to multivariate polynomial equations in robot control and motion planning.

I. ROBOT CONTROL

- Robot control involves solving polynomial equations in inverse kinematics (positions) and dynamics (forces) problems for robots.

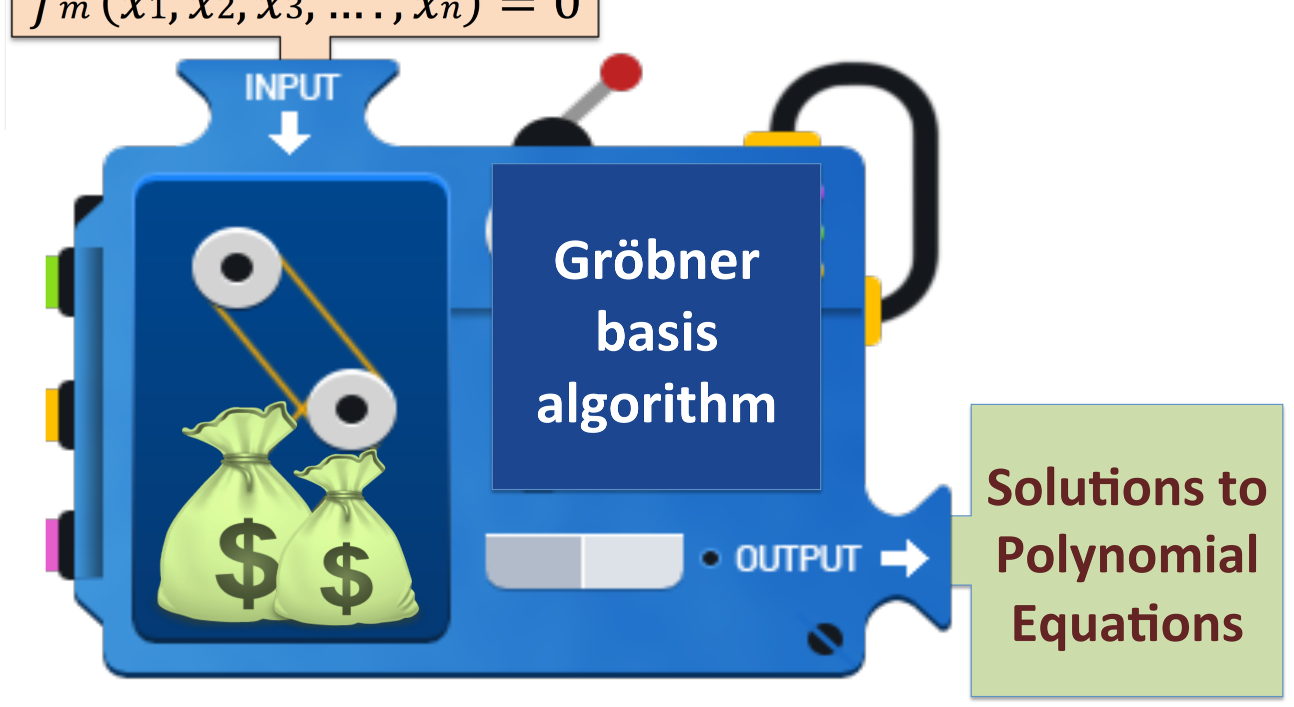


Serial N-link manipulator model (*left*) for a robot arm (*right*)

II. GRÖBNER BASIS

- Numerical root finding methods are widely used, but these methods trade exactness for efficiency.
- ✔ Gröbner basis can be used to find exact solutions to multivariate polynomial equations.
- ✖ Gröbner basis algorithms have double exponential worst-case complexity, so we aim to find efficient Gröbner basis algorithms.

$$
\begin{aligned}
f_1(x_1, x_2, x_3, \ldots, x_n) &= 0\\
f_2(x_1, x_2, x_3, \ldots, x_n) &= 0\\
&\vdots\\
f_m(x_1, x_2, x_3, \ldots, x_n) &= 0
\end{aligned}
$$



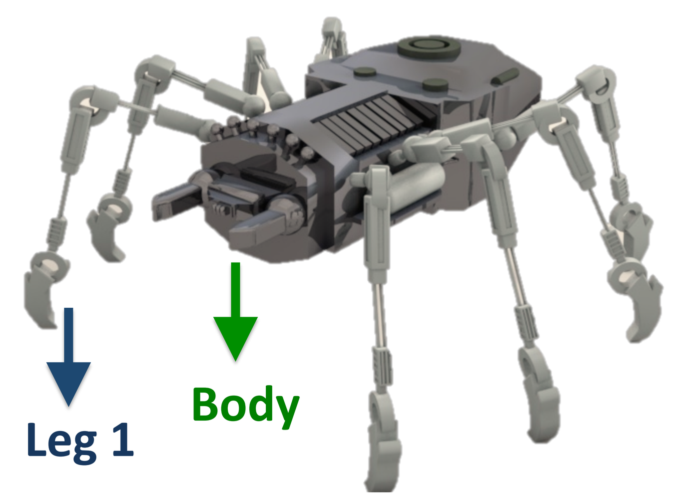
III. ROBOT STRUCTURES

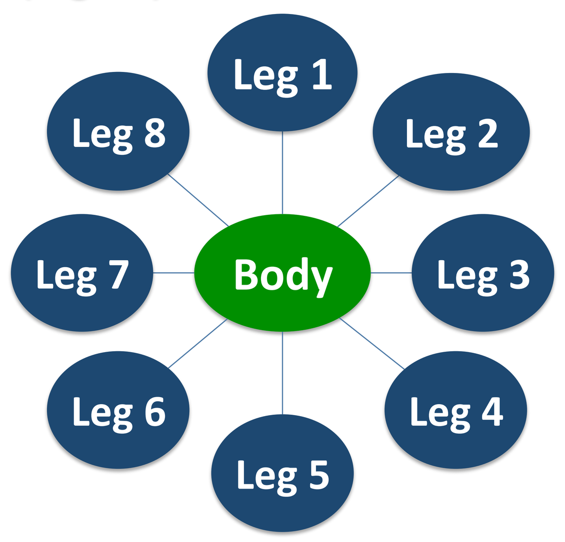
- Sparse matrix computations are simpler than dense matrix computations.

$$
\begin{vmatrix} 1 & 0 & 0 & 0\\ 0 & 2 & 0 & 0\\ 0 & 0 & 3 & 0\\ 0 & 0 & 0 & 4 \end{vmatrix} = 24
$$

Determinant of a sparse diagonal matrix

We created a spider robot (*left*). Each spider leg is connected to the body, but not to other legs, yielding *chordal sparsity* in robot graphs (*right*).





IV. COMPUTATIONS

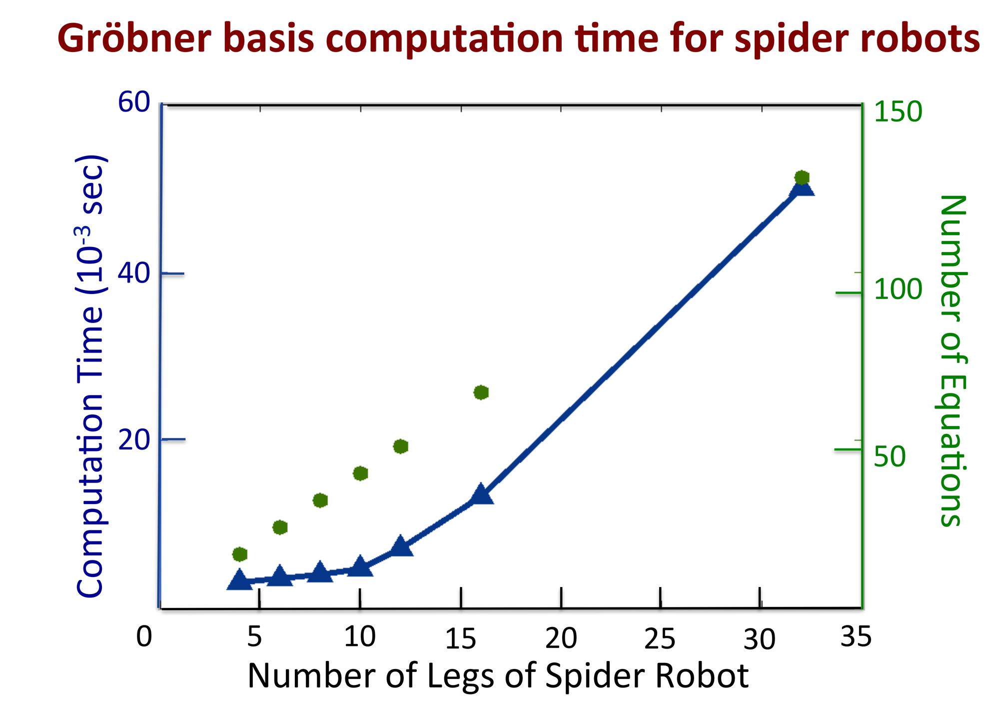
- Gröbner basis algorithms include Buchberger's algorithm, Faugère's F4, F5 and FGLM.

✖ Maple, MATLAB and Macaulay2 *cannot* compute Gröbner basis for a polynomial system representing even one robotic spider leg.

- Sage is Python-based math software that uses C++ libraries in Singular and Magma to compute Gröbner bases.
- ✔ Sage allows fast Gröbner basis computations.

V. CONCLUSIONS

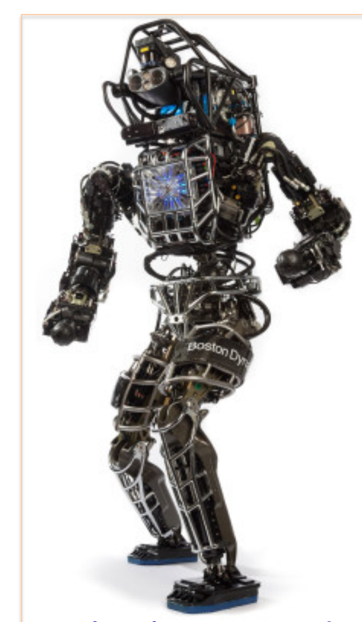
- Sage can efficiently compute Gröbner basis using a 1.3 GHz processor. We achieved a *50 milliseconds* computation time for Gröbner basis for a system of *128 polynomials in 67 variables* for a 32-leg spider robot.



Computation time for Gröbner basis using Sage. Our polynomial systems for the spider robot had 16 to 128 equations.

VI. FUTURE WORK

- Integrate Sage's Gröbner basis algorithms in our existing polynomial solvers to improve the performance of robots, especially Boston Dynamics "Atlas".
- Investigate chordal graphs to further leverage polynomial sparsity in Gröbner basis algorithms.



Atlas humanoid robot

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Picture Courtesy

ROSARIO, J. M. et al. Proposal of methodology for the modeling and control of manipulators. J. Braz. Soc. Mech. Sci. 2002, vol.24, n.3; http://www.societyofrobots.com/robot_arm_tutorial.shtml; http://www.mathwarehouse.com/algebra/relation/evaluating-function.php; http://physicssum.deviantart.com/art/3D-Robot-Spider-280498884; Logos: MATLAB, Maple, Macaulay2, Sage, Python, Magma, Singular, C/C++ (http://www.techhui.com/group/candcppdevelopers); http://www.fairfaxunderground.com/forum/file.php?40,file=113550,filename=Atlas-p2_nt.jpg