Exploring the Nature of Environmental Feedback in Daisyworld

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Abstract

Daisyworld is a hypothetical planet with a simple biosphere that demonstrates how a planet *might* self-regulate its climate. This model provides a mathematical basis for the idea that all life on the earth and its physical environment might be part of an interconnected web (Gaia hypothesis) and is used extensively in studying various ecological processes. The planet consists of two species of daisies whose growth rate is dependent upon temperature. However, the death rate of the daisies remains constant. It seems likely that the death rate should also be affected by climatic factors such as temperature. The objective of my research is to investigate the impact of altering the nature of environmental feedback in Daisyworld by making the death rate of daisies functionally dependent on temperature as well. My project questions the fundamental assumptions on which the environmental feedback in Daisyworld is based. I observed that making the death rate dependent on temperature does not alter the basic qualitative features of the model.

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Chapter 1

INTRODUCTION

In the 1960's, the National Aeronautics and Space Administration (NASA) invited atmospheric chemist James Lovelock to the Jet Propulsion Laboratories in Pasadena, California. NASA's mission was to use Lovelock's help in designing instruments that would help them detect life on Mars. He thought deeply about the nature of life, and more importantly, how experiments could be designed to detect it [4].

Lovelock speculated that life on a planet would constantly interact with the atmosphere and modify its composition. Thus, analyzing the chemical composition of the Martian atmosphere, he argued, should provide insight into the problem of detecting life on Mars. In Lovelock's opinion, it was not actually necessary to visit Mars in order to detect life there [4].

Lovelock and his colleague Dian Hitchcock then systematically analyzed the Martian atmosphere and compared it to the Earth's atmosphere. They discovered that the two atmospheres are very different. The Earth's atmosphere contains significant amounts of oxygen and methane, and relatively little carbon dioxide. The Martian atmosphere, on the other hand, contains a significant amount of carbon dioxide, very little oxygen and no methane. Lovelock theorized that this difference arises because Mars is a lifeless planet, and the chemical reactions among its constituent gases had been completed, resulting in an atmosphere that is now in an equilibrium state. The Earth's atmosphere, on the other hand, is not in an equilibrium state [4].

In 1965, while in discussion with Hitchcock, Lovelock hypothesized that it was possible that life on Earth did not just influence its atmosphere, but also regulated it. Additionally, since life first began on Earth four billion years ago, the luminosity of the sun has increased by approximately twenty five per cent, yet the Earth's temperature has remained remarkably constant over this time period [4]. Although the climate of the Earth has changed very little over the past four billion years, its surface properties and atmospheric composition have varied greatly. To Lovelock, the high order of chemical disequilibria in Earth's atmosphere suggested that the atmosphere was a "biological construction" [8], i.e. the conditions required for life to perpetuate on the planet are not an accident, rather that life on the planet is somehow orchestrating these conditions. Lovelock further suggested that the Earth was regulating its planetary conditions, just as living organisms are able to regulate their conditions such as bodily temperature.

Based upon this theory, Lovelock and Hitchkock defined the Gaia[1] hypothesis [8]:

> Gaia is a complex entity involving the Earth's biosphere, atmosphere, oceans, and soil; the totality constituting a feedback or a cybernetic system which seeks an optimal physical and chemical environment for life on this planet.

According to the Gaia hypothesis, the biotic organisms on the Earth have had a more than significant influence on its environment, while the environment has imposed certain constraints on the biota. Thus, Earth's environment and its life comprise two parts of a tightly coupled system that is extremely complex [12]. James Lovelock and Andrew Watson developed the daisyworld model in 1983 to demonstrate how a system capable of self regulation *might* work [12]. Daisyworld thus provides a mathematical basis for studying the Gaia hypothesis [9].

Daisyworld is an imaginary planet, which receives energy from a star that is similar to the Earth's sun and is populated by two species of daisy, black and white, The

[1] Gaia is an ancient Greek name for the Earth goddess

surface of the planet contains seeds for both species of daisy, these seeds sprout when the climatic conditions are favorable. Temperature is the only quantifiable parameter of climate. Daisyworld's sun is assumed to increase in luminosity with respect to the Earth's sun over time [1, 2, 3, 5, 6, 9, 10, 11, 12, 13, 14].

Black daisies, white daisies and bare ground have albedos of 0.25, 0.75, and 0.50 respectively. Albedo is the proportion of the incident sunlight that is reflected back. So, black daisies reflect 25 per cent of the light incident on them. At the start of the Daisyworld simulation, no life exists on the planet. However, as the luminosity of the sun increases, and the average planetary temperature reaches a certain value, daisies begin to sprout. Since the mean planetary temperature is still relatively low, only black daisies, which absorb more heat than white daisies, begin to sprout. Once the black daisies appear, positive feedback takes place between temperature and daisy growth, and a significant portion of the planet gets covered by black daisies [9, 10, 12].

The growth of black daisies reduces the overall albedo of the planet, since black daisies reflect less sunlight than bare ground. The planet becomes warmer, and the growth rate of the black daisies increases. However, as the black daisy population increases, planetary temperature rises high enough that black daisies can no longer survive. White daisies have relatively better capacity for growth in warmer conditions on account of their high reflectivity. Eventually, the planet moves from a state where it is dominated by black daisies, to a state that is dominated by white daisies. The black daisies become extinct first, leaving the white daisies as the only species alive on the planet. Eventually, the planet becomes too hot for the white daisies as well, and they also disappear [9, 10, 12].

As the population of daisies (either black or white) increases, and more of the planet becomes occupied, less and less space is available for new daisies to grow. At the same time, daisies are also naturally dying, because of old age. Thus, the limitations of available space imposes a negative feedback on the system. The counteracting effects of positive and negative feedback lead to regulation of the planet's temperature. Thus, the Daisyworld model demonstrates the planet's capacity for self-regulation [9, 10, 12].

The system eventually collapses when white daisies cannot keep the planet sufficiently cool, due to the increasing solar luminosity. The important point that emerges out of the Daisyworld model is not that it eventually collapses, but that it self regulates so effectively for a wide range of solar luminosity [9, 10, 12]. The creators of this model are not "trying to model the Earth, but rather a fictional world which displays clearly a property which they believe is important for the Earth" [12]. This property is self regulation, and the model effectively demonstrates an explanation for Lovelock's observation that while the luminosity of Earth's sun has increased by 25 per cent, the temperature of the Earth has remained remarkably constant.

Since Daisyworld was first introduced, many sophisticated versions of the model have been studied. Simulations studying planets covered by numerous types of daisy, various herbivores that feed on the daisies, and carnivores that feed on herbivores have been developed. Besides providing a mathematical basis for the Gaia hypothesis, the Daisyworld idea has also been used to study biological diversity [9], relationships between climate and and patterns of biodiversity [5], impacts of habitat fragmentation on ecological systems [11], and investigating environmental feedback [2], to name a few. The Daisyworld model is thus extremely important in studying ecology from a theoretical perspective.

Daisyworld has also been the object of tremendous criticism. Zeng *et. al* criticized Daisyworld by saying that it displays chaotic behavior and sensitive dependence on initial conditions [9, 14]. Lovelock [9] answered this criticism by saying:

> Their [Zeng *et. al*] attempt was not convincing as it involved the expedient of introducing a time lag between the sensing of a change of heat input and the response of the system. Engineers and physiologists familiar with the properties of feedback control systems know that such an act is itself a recipe for instability and chaos in otherwise stable systems.

Feedback is produced on Daisyworld by making the growth rate of daisies and planetary temperature dependent on each other. However, in the original model, the death rate of the daisies remains constant, independent of temperature. It is biologically consistent that under unfavorable planetary conditions, such as extreme temperatures, the daisies would be dying faster, and vice versa. Thus, the death rate of the daisies should also be explicitly linked to temperature. The current work investigates the effect of making death rate functionally dependent on the local temperature and investigating the impact of this variation on the planet's capacity for climate regulation.

Chapter 2

THE ORIGINAL MODEL

In order to study the nature of environmental feedback in the Daisyworld model, it is important to firmly grasp how the model works. With this end in mind, the basic model is reproduced in this chapter.

2.1 Planet Daisyworld

Lovelock and Watson [12] describe Daisyworld as

> a cloudless planet with a negligible atmospheric greenhouse on which the only plants are two species of daisy of different colors. One species is dark – ground covered by it reflects less light than bare ground – while the other is light and reflects more light than the bare ground. To emphasize the contrast, we will refer to them as "black" and "white" though the black daisies need not be perfectly black, nor the white ones completely white.

The growth rates of black and white daisies are based on traditional population ecology models and are expressed as

$$\frac{d\alpha_i}{dt} = \alpha_i(x\beta_i - g) \tag{2.1}$$

where the subscript i denotes either black or white daisies, α_i denotes the fractional surface area of the planet occupied by each species of daisy, β_i denotes the growth rate of daisies per unit time per unit area, g represents the death rate of the daisies per unit time and x denotes the fertile ground not covered by daisies of either species [10, 12]. Note that

$$x = p - \alpha_b - \alpha_w \tag{2.2}$$

where p is the proportional arable surface area of the planet. For the purposes of the model, p is taken to be 1, and α_b and α_w denote the proportional surface areas occupied by black and white daisies respectively [10, 12].

The growth rate β_i of the daisies is

$$\beta_i = 1 - 0.003265(295.5 - T_i)^2 \tag{2.3}$$

where T_i is the local temperature of either species in degrees Kelvin. The growth rate equation is chosen so that it is zero when the local temperature is 278 K and 313 K, and has a maximum at 295.5 K where it equals 1. In the calculations, g is taken to be 0.3. The value of death rate seems to be arbitrarily chosen [10, 12]. Figure 2.1 shows growth rate as a function of local temperature. The growth rate function is defined to be non zero only when the local temperature ranges between 278 K and 313 K, which can be seen in Figure 2.1.

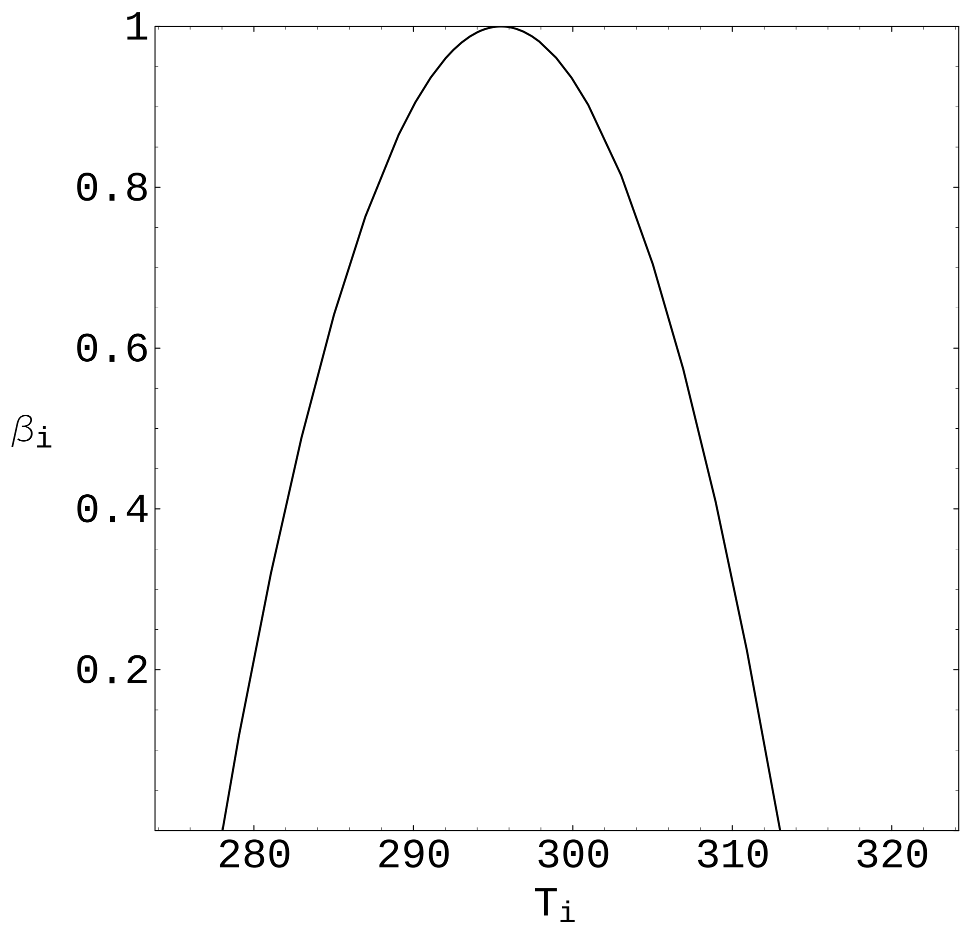

Figure 2.1: Original Daisyworld Model. Growth Rate per unit area per unit time as a function of local temperature; β_i is growth rate and T_i is local temperature.

The radiation emitted by the sun that is incident upon the planet must equal the sum of the radiation absorbed by the planet and the heat radiated back to space so that energy is conserved. Therefore

$$\sigma T_e^4 = SL(1 - A) \tag{2.4}$$

where σ is the Stephan-Boltzmann constant, 5.75×10^{-5}, T_e is the mean planetary temperature in Kelvin, S denotes solar flux and is taken to be 9.17×10^5, L is the luminosity of Daisyworld's sun relative to the earth's sun, and so L is approximately 1 (implying that the luminosity of Daisyworld's sun is chosen to be almost equal to the earth's sun). A is the albedo of the planet and is given by the weighted average

$$A = \alpha_g A_g + \alpha_b A_b + \alpha_w A_w. \tag{2.5}$$

In Equation 2.5, α_g is the proportional surface area of bare ground, and A_g, A_b, A_w are the albedos of bare ground, black daisies and white daisies respectively, where $A_g = 0.50$, $A_w = 0.75$ and $A_b = 0.25$ [10, 12]. Finally, the relationship between local temperature of the daisies and the variables defined above is

$$T_i^4 = q(A - A_i) + T_e^4 \tag{2.6}$$

where T_i denotes the local temperature of either daisy species in Kelvin, and q is a constant that provides a measure of the degree of redistribution of solar energy. If $q > \frac{SL}{\sigma}$, heat flows from a region of low temperature to a region of high temperature, which is not physically possible. If $q = 0$, then the local temperatures of each area (area covered by white daisies, black daisies and bare ground) are equal to the mean planetary temperature [10]. The value of q is chosen to be 2.06×10^9, which is less than $\frac{SL}{\sigma}$ [10, 12]. The condition $T_b > T_g > T_w$ follows from Equation 2.6 [12].

In order to examine the temperature regulation capacity of the planet, Lovelock and Watson generated a series of curves displaying mean planetary temperature and proportional area of the planet covered by daisies as a function of luminosity. They considered four distinct cases: when the planet is barren, when it is occupied either by black or white species exclusively, and when both species coexist. They used the following procedure to generate these curves [12]:

> For a fixed value of L, initial values of α_b, α_w were set to the previous steady state values or 0.01 if these were zero; the equations were integrated forward in time until a steady state value was reached; the value of L was incremented and the procedure was repeated. Thus the curves show the effect of increasing the luminosity slowly, so that the system has time to reach steady state at each value of L.

The remainder of the chapter reproduces the results published by Lovelock and Watson [12] and Saunders [10]. The techniques used are similar to those used by Saunders, and involve solving for steady state solutions directly; these techniques are explained later in this chapter. Lovelock's and Watson's techniques in [12] involve studying time evolution on Daisyworld, and are different from the techniques we have used. Derivations (of equations) needed to reproduce results are shown in Appendix A. All temperature values are in Kelvin, unless otherwise stated. The graphs and results are discussed below, and the concept of stable and unstable states in Daisyworld is explained. The four conditions - lifeless planet, planet covered by black daisies exclusively, planet covered by white daisies exclusively, and the condition when both species coexist, are each considered separately.

2.2 Lifeless Planet

First consider the case where no life can exist on the planet; the planet can be considered infertile. Figure 2.2 illustrates the mean temperature of this planet as a function of the luminosity of its sun. This graph shows that there is no temperature regulation on the planet, since mean planetary temperature increases directly with increasing solar luminosity. If there were temperature regulation, the relationship between temperature and luminosity would not be a simple function relationship as described by Equation 2.7.

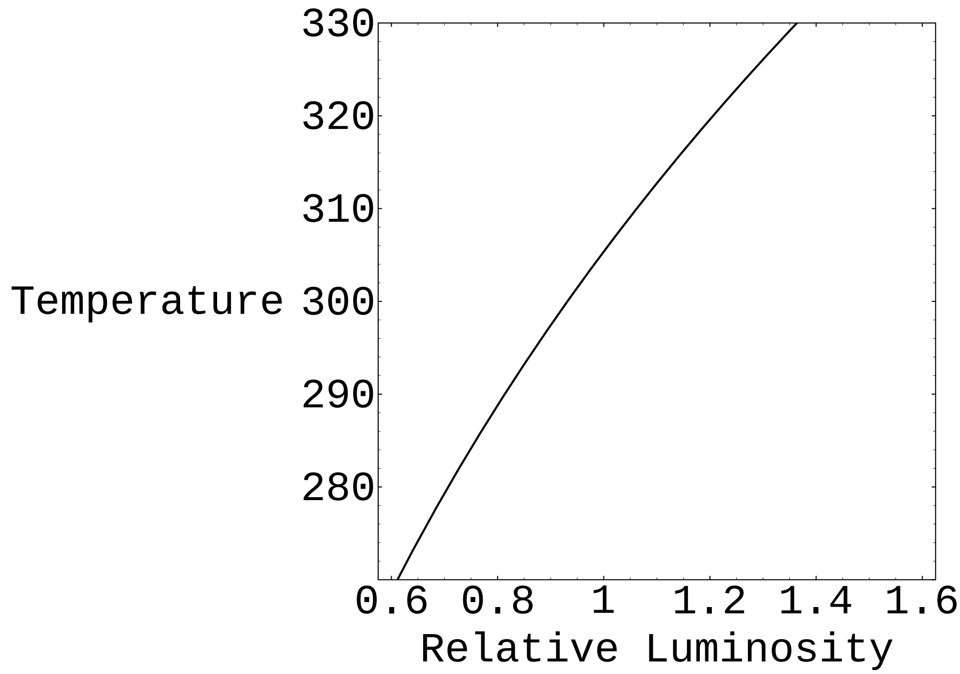

Figure 2.2: Mean Planetary Temperature as a function of relative luminosity for a lifeless planet.

To produce Figure 2.2, we solve Equation 2.7 for T_e as relative solar luminosity L ranges between 0.6 and 1.6, and we obtain

$$T_e^4 = \frac{SL}{2\sigma}. \tag{2.7}$$

Equation 2.7 shows the functional relationship between mean planetary temperature T_e and relative solar luminosity L. Unstable states on Daisyworld are defined as those states where daisy population is not maximized. Since a lifeless planet contains no daisies, there are no unstable states on a lifeless planet. We see from Figure 2.2 that when the planet is barren, it is incapable of regulating its temperature.

2.3 One Species Models

We now consider the condition when seeds for only one species of daisy, either black or white, are present in the ground. We will first explain some general results that apply to both black and white daisies, and then study each particular species separately.

Since the planet is of unit area, and the area covered by daisies can never be negative, $0 \leq \alpha_i \leq 1$. At equilibrium, the proportional area occupied by daisies does not change. Hence, $\frac{d\alpha_i}{dt} = 0$. From Equation 2.1, we know that $\frac{d\alpha_i}{dt} = \alpha_i(x\beta_i - g)$. Therefore, $x = 1 - \alpha_i = \frac{g}{\beta_i}$, which implies

$$\alpha_i = 1 - \frac{0.3}{1 - 0.003265(295.5 - T_i)^2}. \tag{2.8}$$

Thus, max $(\alpha_i) = 1 - \frac{0.3}{1} = 0.7$, implying $0 \leq \alpha_i \leq 0.7$. Therefore, the maximum planetary area occupied by the daisies, when the ground contains seeds for only one type of daisy, is 0.7.

From Equation 2.8 we know the relationship between α_i and T_i. By solving the inequality $0 \leq \alpha_i \leq 0.7$, we calculate the range of acceptable values of T_i for which life exists on the planet. The condition $0 \leq \alpha_i \leq 0.7$ implies that $280.86 \leq T_i \leq 310.14$. Thus daisies can only grow when their local temperature is between 280.86 K and

310.14 K. Daisies first appear when the local temperature is 280.86 K, and become extinct when their local temperature exceeds 310.14 K. For any local temperature that is not in the domain specified above, the planet is lifeless.

Figure 2.3 shows a graph of Equation 2.8. The graph makes it clear that when only one species exists, maximum planetary area is covered when the local temperature of daisies is 295.5 K, and $\alpha_i \geq 0$ only when the local temperature is between 280.86 K and 310.14 K, implying that daisies are alive only when local temperature is in this range. For any other local temperature α_i would be less than 0, implying that the planet is bare.

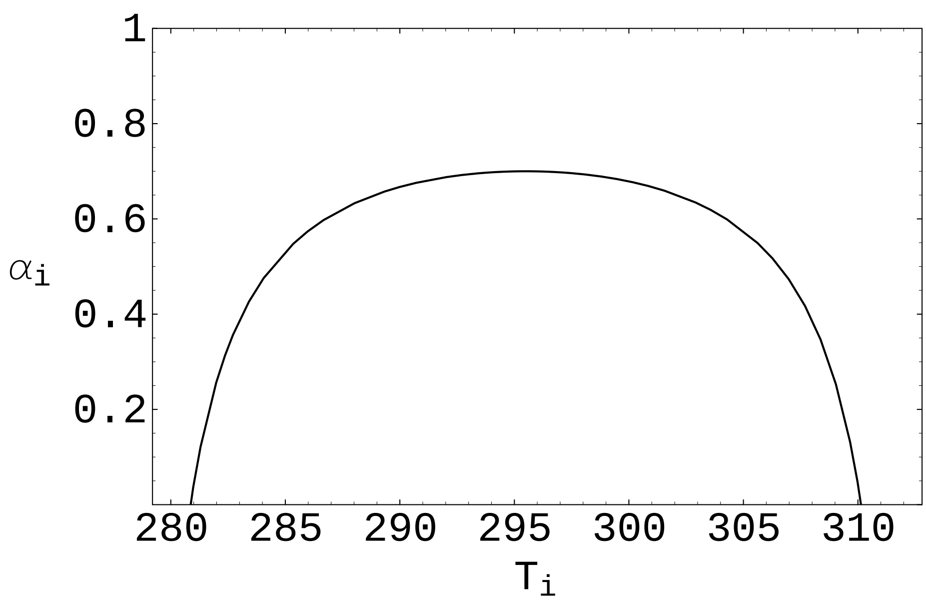

Figure 2.3: Fractional area occupied by daisies in one species model as function of local temperature. Note α_i represents fractional area occupied by daisies, and T_i represents local temperature of daisies. The two vertical asymptotes show T_i values where α_i is not defined.

2.3.1 Black Daisies Exclusively

We now examine the case where the ground contains seeds only for black daisies. Consequently, the planet can only support black daisies, otherwise it remains barren. The equations used to produce the graphs that demonstrate temperature regulation under this condition are given below and the derivation of these equations is shown in Appendix A.

The fractional area occupied by black daisies (α_b) as a function of local temperature (T_b) is given by

$$\alpha_b = \frac{17.5^2(1-g) - (T_b - 295.5)^2}{17.5^2 - (T_b - 295.5)^2}. \tag{2.9}$$

Equation 2.9 is obtained by replacing the subscript i in Equation 2.8 with the subscript b. In other words, the relation for the specific case where the ground contains seeds only for black daisies is derived from the general relationship that holds true in a one species model, and applies to either daisy species. The algebraic relationship between local temperature T_b and relative solar luminosity L is

$$T_b^4 = 0.25q + \frac{0.5SL}{\sigma} + \alpha_b \left(\frac{0.25SL}{\sigma} - 0.25q \right). \tag{2.10}$$

Finally, the algebraic relationship between mean planetary temperature T_e and relative solar luminosity L is

$$T_e^4 = \frac{SL}{\sigma} \left(0.5 + 0.25\alpha_b\right). \tag{2.11}$$

In order to generate the graphs, Equation 2.9 is substituted in Equation 2.10 and Equation 2.11. For a particular luminosity L, the local temperature T_b is then calculated from Equation 2.10. Subsequently, the luminosity L and local temperature T_b are substituted in Equation 2.11 to obtain the mean planetary temperature for those particular values of L and T_b. The luminosity L is then slightly increased, and a new set of values for T_b and T_w are obtained. This process is repeated 1000 times, and several values of local temperature and mean planetary temperature are obtained. Figure 2.4 shows the mean planetary temperature as a function of luminosity under this condition. Figure 2.5 shows the complete picture; both local temperature and mean planetary temperature as functions of relative solar luminosity.

Figure 2.4 demonstrates that the planet is now capable of regulating its temperature. In Figure 2.2, we observed that mean planetary temperature rises monotonically with increasing solar luminosity. When black daisies exist, the rise in mean planetary temperature is different from that in the state when the planet is lifeless. At the temperatures that the daisies are alive, the rise in temperature is much slower than it is when the daisies do not exist. Thus, the daisies regulate the mean temperature of the planet.

Figure 2.5 shows the mean planetary temperature in red, and the local temperature of daisies in blue. It is worth noticing that the local temperature of black daisies is always greater than the mean planetary temperature, which is to be expected since black daisies absorb more heat than bare ground. Notice mean planetary temperature is always lower than local temperature of black daisies. Portion of curve between Points A and X corresponds to stable states (green region in Figure 2.4), and region between Points X and B corresponds to unstable states (red region in Figure 2.4). The portions of the red and blue curves that lie to the right of their respective intersection points with the black curve are physically meaningless because if these portions were meaningful, they would imply greater mean temperature for a bare planet than the mean temperature of a planet covered by black daisies.

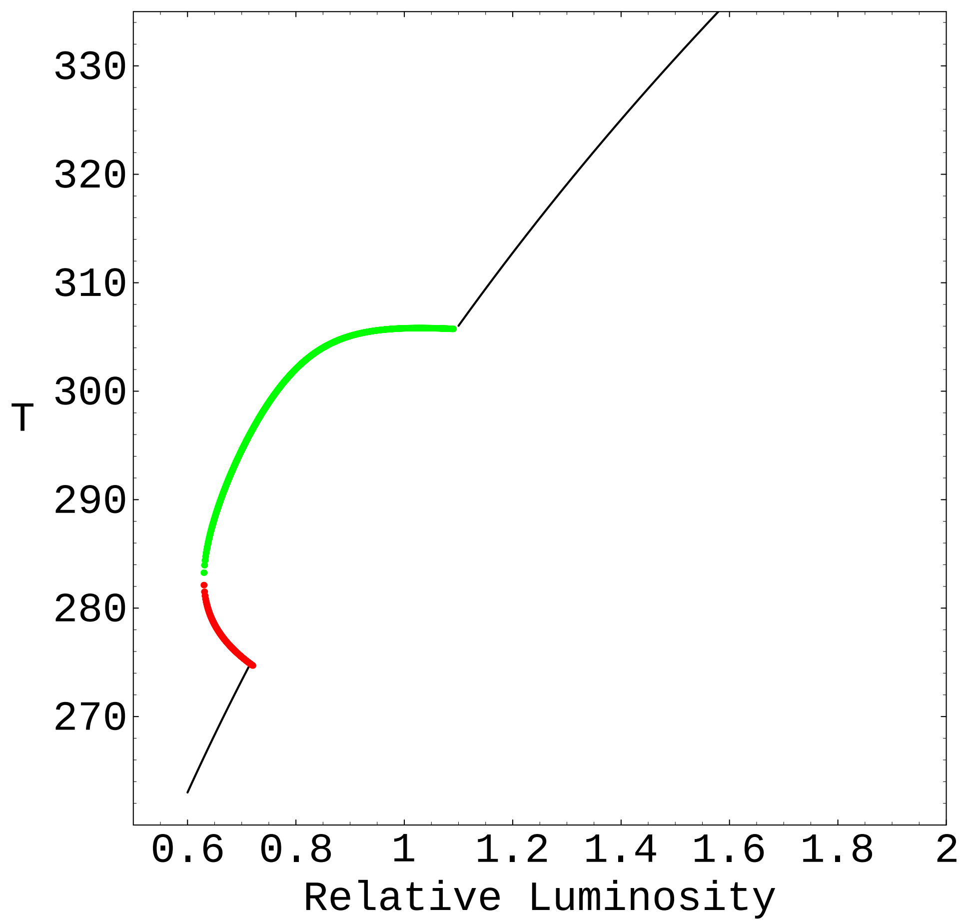

Figure 2.4: Black Daisies Exclusively: Mean Planetary Temperature as a function of relative solar luminosity. Dark portion of the graph corresponds to the state in which no life exists on the planet. The red and green portions of the graph correspond to the state where black daisies are alive on the planet. The green portion corresponds to stable equilibria, while the red portion corresponds to unstable equilibria. On the vertical axis, T represents temperature in Kelvin.

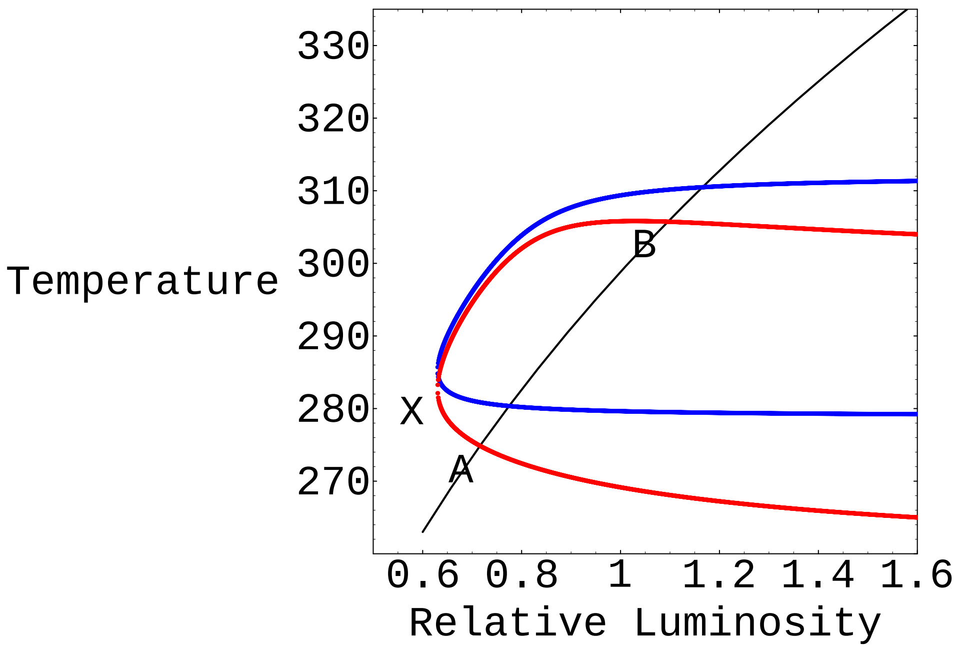

Figure 2.5: Black Daisies Exculsively. Mean planetary temperature and local temperatures as functions of relative solar luminosity. Red curve corresponds to mean planetary temperature, and the blue curve corresponds to local temperature.

Now, let us discuss the concept of stability and instability in this model. In order to define stable and unstable states, it is important to ascertain what ranges of luminosity correspond to each state . Equation 2.10 gives the relationship between T_b and L and the terms may be rearranged to obtain

$$L = \frac{T_b^4 + 0.25q\alpha_b - 0.25q}{\frac{0.5S}{\sigma} + \alpha_b \frac{0.25S}{\sigma}}. \tag{2.12}$$

Figure 2.6 shows the relationship between luminosity and local temperature; this figure is useful because it helps us see the difference in the relationship between relative solar luminosity and local temperature of black daisies, and the relationship between relative solar luminosity and the local temperature of white daisies. We are only interested in the temperature range $280.86 \leq T_b \leq 310.42$ because it is only in this temperature range that daisies exist on the planet. Figure 2.6 clarifies for the ranges of luminosity where Daisyworld is unstable. The stable states are those that correspond to maximum daisy population. Any other state is unstable. In Figure 2.6, points represented by $T_b < 285.25$ correspond to unstable states. These values of luminosity are represented by the red portion of the curve in Figure 2.4.

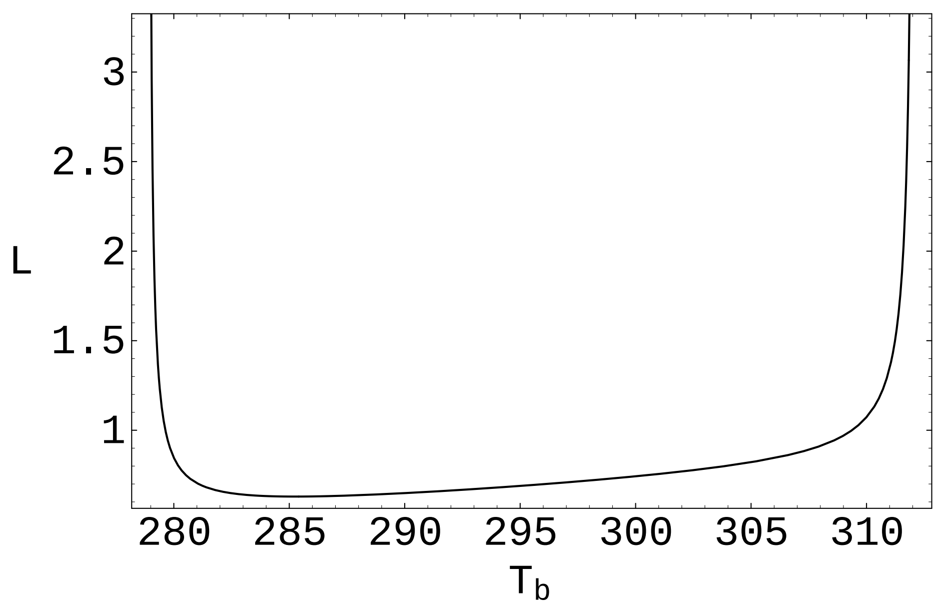

Figure 2.6: The relationship between Relative Solar Luminosity and Local Temperature for planet occupied by only Black Daisies. Relative solar luminosity is represented by L and local temperature of black daisies is represented by T_b.

In [10], Saunders discusses the concept of stability and instability in Daisyworld. He shows unstable and stable regions in graphs that describe relative planetary temperature as a function of relative luminosity. However, he does not clarify how these stable and unstable regions are defined.

In the process of reproducing Saunders' work, it became clear that the unstable regions are those where daisy population is not maximized. Stable regions correspond to those where daisy population is at a maximum. In Figure 2.4, the stable region (green) corresponds to higher mean planetary temperature than the unstable region (red). Since black daisies absorb more heat than bare ground, higher mean temperatures also correspond to higher daisy populations. Thus, the system has a tendency to move towards a state where daisy population is maximum. This tendency may also be explained by the Second Law of Thermodynamics, which says that the entropy of a system always increases (if we consider higher population to correspond to higher entropy).

The unstable region (red in Figure 2.4) shows that with increasing luminosity temperature decreases and vice versa. Hence, solving for the unstable states implies $\frac{dT_e}{dL} < 0$ and $\frac{dT_b}{dL} < 0$, which implies $\frac{dL}{dT_b} < 0$. The relation $\frac{dL}{dT_b}$ can be calculated from Equation 2.12[1]. Solving this inequality provides us with the range of local temperatures for which Daisyworld is in an unstable state. Figure 2.6 shows these unstable states as being those where with luminosity and temperature have an inverse relationship. The graph shows that for $T_b < 285.25$, T_b and L are inversely related. For other values of $T_b > 285.25$, the two parameters share a direct relationship.

When $\frac{dL}{dT_b} < 0$, then either $T_b < -9.43K$, $271.90 < T_b < 278.89$ or $278.89 < T_b < 285.25$. The first solution is physically meaningless because it shows temperatures below absolute zero, and the second solution is meaningless for our purposes because

[1]We picked $\frac{dL}{dT_b}$ instead of $\frac{dL}{dT_e}$, because in Equation 2.12 we obtained a direct functional relationship between L and T_b, which cannot be obtained between L and T_e. The functional relationship between L and T_b is physically meaningless, but it is easier to manipulate mathematically.

daisies only sprout above $T_b = 280.85K$. The third solution is the only one that is meaningful to us, and since the black daisies only survive when their local temperature is at least 280.85 K, the solution is physically meaningful only for $280.85 < T_b < 285.25$. Substituting these constraints on T_b in Equation 2.12 we can obtain the range of luminosity for unstable states in Daisyworld, $L = 0.72$ corresponds to $T_b = 280.85$ and $L = 0.62$ corresponds to $T_b = 285.25$.

The graph in Figure 2.4 can now be understood qualitatively. Initially, the planet is lifeless. At the start of the simulation, the relative luminosity of the sun increases. When the luminosity reaches 0.72, black daisies sprout, but this state is unstable. So, the local temperature of the daisies jumps to a stable value at the same luminosity (the stable region is green in Figure 2.4.) When the luminosity L is 0.72, the local temperature of the daisies is 280.85 K, and the daisies begin to sprout. Hence, the next stable local temperature at $L = 0.72$ can be calculated by solving $L(280.85) = L(x)$, where x is the stable local temperature. Solving this equation for local temperature yields $x = 297.65$ K in the permissible range.

Hence, when the daisies sprout at a relative solar luminosity of 0.72, the local temperature of the daisies jumps from an unstable value of 280.85 K to a stable value of 297.65 K. The mean planetary temperature also changes correspondingly.

Black daisies remain on the planet until their local temperature reaches 310.14 K, which corresponds to a relative solar luminosity of 1.09. At this point, the planet becomes too hot for black daisies to survive, and they become extinct.

It is also important to note the effect of a sun whose luminosity is monotonically decreasing on the planet because there is hysteresis in system. When the sun's luminosity monotonically decreases, daisies appear at $L = 1.09$ and persist until the luminosity reaches 0.62, which is the lowest luminosity that corresponds to a stable state. This luminosity is significantly lower than the luminosity at which daisies would have sprouted ($L = 0.72$) if the sun's luminosity were increasing [10]. Numerical ranges of stability and instability for this condition are summarized in Table

2.1.

Table 2.1: Original Daisyworld Model: Planet Inhabited by Black Daisies only. Stable and Unstable States.

	L	α_b	T_b	T_e
Unstable Region	(0.62, 0.72]	[0.00, 0.54]	[280.86, 285.25)	[274.86, 282.68)
Stable Region	[0.62, 1.09]	[0.00, 0.70]	(285.25, 310.14]	(282.68, 305.73]

2.3.2 White Daisies Exclusively

In this section, the condition where seeds for only white daisies exist in the ground, is discussed. We have used the same numerical and graphical techniques here that were used earlier in the case where only black daisies existed on the planet. Hence, the procedure to obtain equations and results is not described in great detail, but the results are shown and explained.

The equations used to generate graphs that demonstrate temperature regulation in this model are given below and explained in Section A.2.2 of Appendix A. The fractional area occupied by white daisies (α_w) as a function of local temperature (T_w) is given by

$$\alpha_w = \frac{17.5^2(1-g) - (T_w - 295.5)^2}{17.5^2 - (T_w - 295.5)^2}. \tag{2.13}$$

Equation 2.13 is obtained by replacing the subscript i in Equation 2.8 by the subscript w. The algebraic relationship between local temperature (T_w) and relative solar luminosity (L) is given by

$$T_w^4 = -0.25q + \frac{0.5SL}{\sigma} + \alpha_w\left(0.25q + \frac{-0.25SL}{\sigma}\right). \tag{2.14}$$

The algebraic relationship between mean planetary temperature (T_e) and relative solar luminosity (L) is

$$T_e^4 = \frac{SL}{\sigma}(0.5 - 0.25\alpha_w) \tag{2.15}$$

Notice that Equation 2.13 and Equation 2.9 are identical, except for the subscripts w and b. In a one species model, the relationship between area covered by daisies and the local temperature of the species is identical for both black and white daises. Figure 2.7 illustrates mean planetary temperature as a function of relative solar luminosity in this condition where only white daisies exist on the planet. Figure 2.8 shows both local and mean planetary temperatures as functions of relative solar luminosity.

As in Section 2.3.1 when only black daisies existed, the presence of only white daisies also results in temperature regulation on the planet. In Figure 2.7, mean planetary temperature is clearly regulated in the range of luminosity where white daisies are alive.

Figure 2.8 shows mean planetary temperature in red, and local temperature of white daisies in blue. Notice that the local temperature of white daisies is always lower than the mean planetary temperature, which is to be expected since white daisies absorb less heat than bare ground. Notice mean planetary temperature is always greater than local temperature of white daisies. The red curve between points A and X corresponds to stable states (shown in green in Figure 2.7), and the curve between points X and B corresponds to unstable states (shown in red in Figure 2.7). The portions of the red and blue curves that lie to the left of their respective intersection points with the black curve are physically meaningless because they would denote lower mean temperature for a bare planet than the mean temperature of a planet covered by white daisies.

In Figure 2.7, the stable region is shown in green and the unstable region is shown in red. The stable region corresponds to lower mean planetary temperatures than the unstable region, since white daisies absorb less heat than bare ground, lower mean planetary temperatures correspond to higher white daisy populations.

From Figure 2.7, it becomes clear that unstable regions are those where increasing luminosity results in decreasing temperature, and vice versa. Thus, the unstable ranges can be calculated by solving for where $\frac{dL}{dT_e} < 0$, which implies $\frac{dL}{dT_w} < 0$.

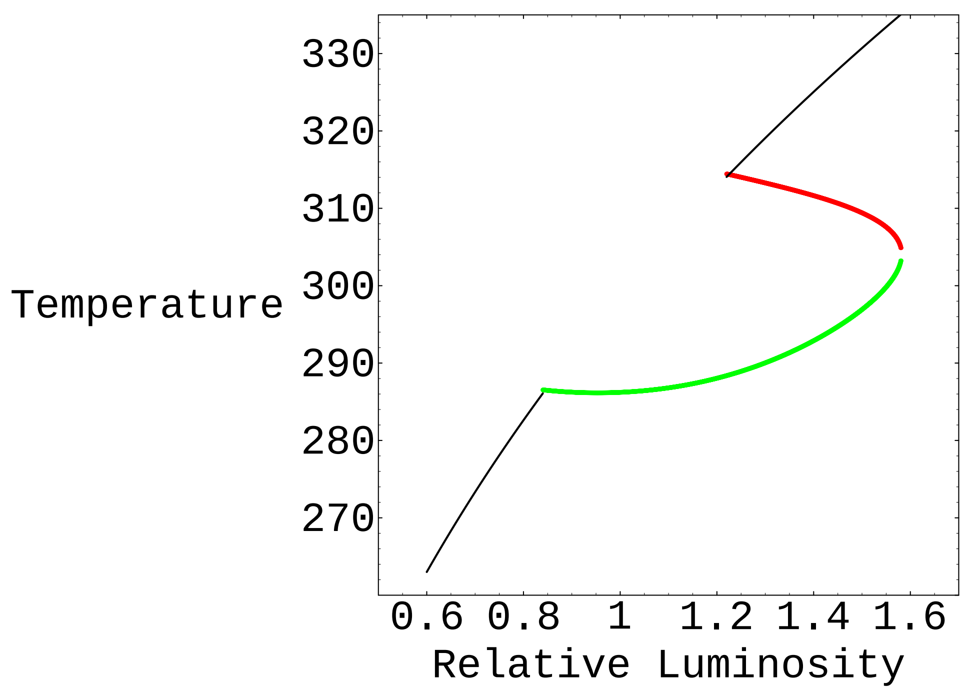

Figure 2.7: White Daisies Exclusively: Mean Planetary Temperature as a function of Relative Solar Luminosity. Dark portion of the graph corresponds to the state where no life exists on the planet. The red and green portions of the graph correspond to the state in which only only white daisies can exist on the planet. The green portion shows stable equilibria, while the red portion corresponds to unstable equilibria.

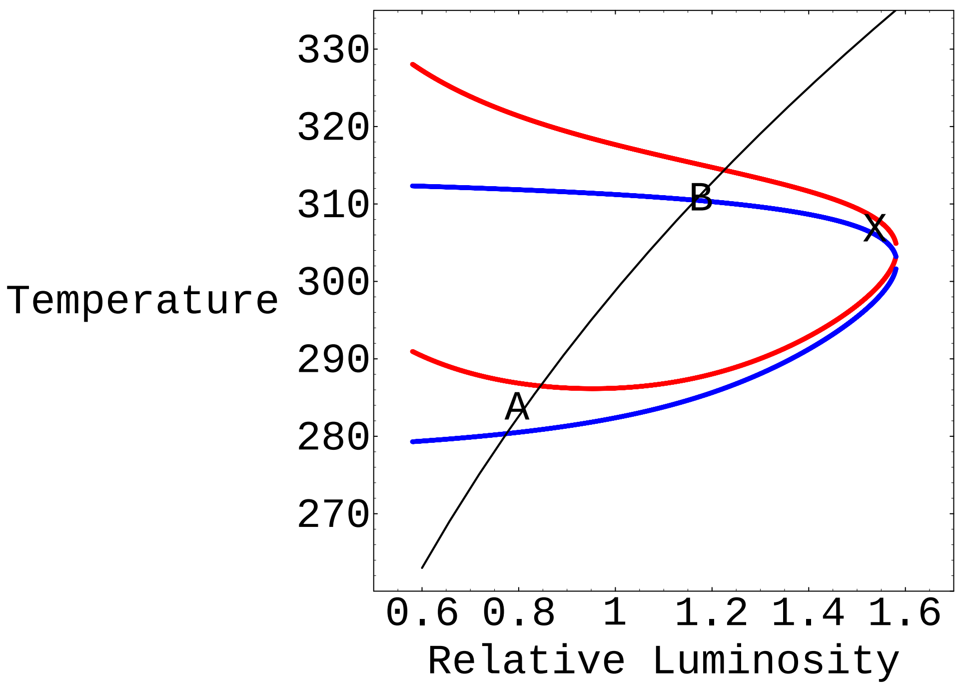

Figure 2.8: White daisies exclusively. Mean planetary temperature and local temperatures as functions of relative solar luminosity. Red curve corresponds to mean planetary temperature and blue curve corresponds to local temperature.

At this point, let us return to the question of instability and stability. We use the same techniques of analysis that were used in the earlier case where only black daisies populated the planet. From Equation 2.14, we obtain

$$L = \frac{T_w^4 - 0.25q\alpha_w + 0.25q}{\frac{0.5S}{\sigma} - \alpha_w \frac{0.25S}{\sigma}}. \tag{2.16}$$

Figure 2.9 shows the relationship between relative solar luminosity and local temperature for a planet occupied by only white daisies. Notice that Figure 2.9 is different from Figure 2.6, that describes the relationship between luminosity and local temperature of black daisies. While the graph in Figure 2.6 was concave upwards, the graph in Figure 2.9 is concave downwards. When the original model is covered only by white daisies, it is unstable for $T_w > 302.43$. The different shapes of the luminosity graphs for the white and black daisies imply that while Daisyworld is in an unstable state when black daisies first sprout and then moves into a stable state, with white daisies in existence, the opposite is true. This result is interesting because clearly the albedo of species plays a role in where stable and unstable states occur.

The stable and unstable states seen in Figure 2.7 are also consistent with the idea that stable states are those where daisy population is maximized. The green (stable) region in Figure 2.4 denotes lower mean planetary temperature than the red (unstable) region. Since white daisies absorb lesser amount of heat than bare ground, the stable region corresponds to greater daisy population than the unstable region.

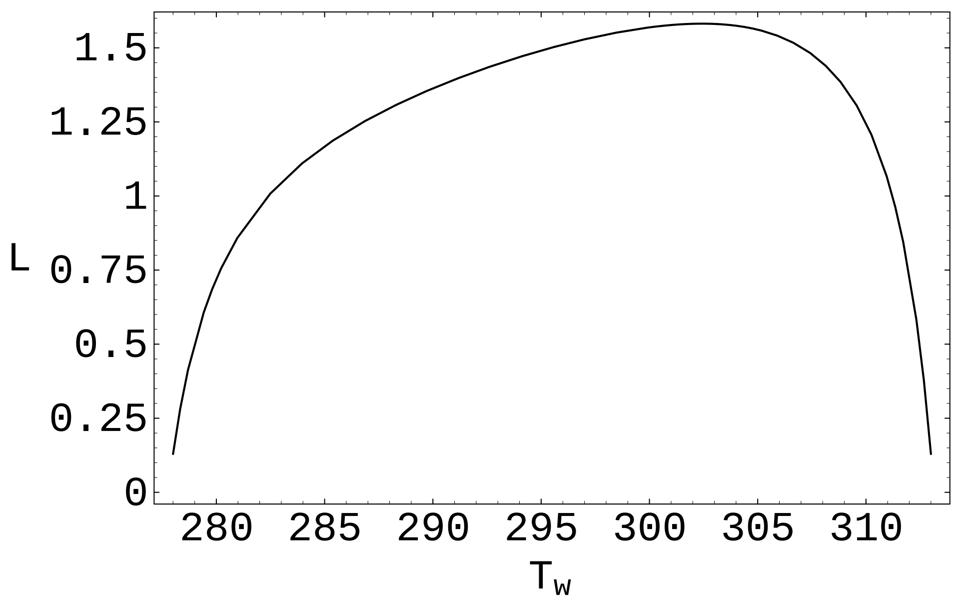

Figure 2.9: The relationship between relative solar luminosity and local temperature for planet inhabited by white daisies only. The axis marked L represents luminosity, and the axis marked T_w represents local temperature of white daisies. Points represented by $T_w > 302.43$ correspond to unstable states. These values of luminosity are represented by the red portion of the curve in Figure 2.4.

Notice the relationship between L and T_b in Figure 2.6 and the relationship between L and T_w depicted in Figure 2.9 are completely different. Applying the instability condition $\frac{dL}{dT_w} < 0$, we find the range of local temperatures where Daisyworld is in an unstable state. If $\frac{dL}{dT_w} < 0$, then $302.43 < T_w < 315.45$; thus Daisyworld is unstable when the local temperature of white daisies is between 302.43K and 315.45K. However, the daisies become extinct when the local temperature reaches 310.14K. Hence the true range of instability is $302.43 < T_w < 310.14$.

From Equation 2.16, the range of luminosity corresponding to unstable states can be obtained, $L = 1.58$ corresponds to $T_w = 302.43$ and $L = 1.22$ corresponds to $T_w = 310.14$. However, when the luminosity is 1.58, the local temperature of white daisies has two mathematically feasible values: 315.45 K and 302.45K. In this model, the range of local temperatures for which life exists is 280.86 K to 310.14 K. Hence, the local temperature value 315.45K is meaningless for our purpose.

The numerical ranges of stability for the condition where the ground contains seeds for only white daisies are summarized in Table 2.2. The stable region is shown in green in Figure 2.7 and the unstable region is in red.

Table 2.2: Original Daisyworld Model: Planet Inhabited by White Daisies Only. Stable and Unstable States for Increasing Solar Lumninosity.

	L	α_w	T_w	T_e
Stable Region	[0.84, 1.58]	(0.00, 0.64]	[280.86, 302.45]	[286.51, 304.08]
Unstable Region	[1.22.1.58)	[0.00, 0.64)	(302.45, 310.14]	(304.08, 314.38]

Thus as the sun is increasing in luminosity, the planet is initially lifeless. Daisies begin to sprout at a relative solar luminosity of 0.84. They exist in a stable state until the luminosity reaches 1.58. However, if the solar luminosity increases further, Daisyworld becomes unstable, and the daisies become extinct. If the solar luminosity were decreasing, the daisies would first appear at a luminosity of 1.22. However, since at this luminosity, the local temperature is in an unstable state (red region in Figure 2.7), it would change to a value in the stable region (green region in Figure 2.7). This value can be found by solving the equation $L(310.42) = L(x)$, where x is a value of local temperature in the stable region. Solving this equation yields $x = 285.19K$. Note also that when the sun is decreasing in luminosity, life appears at a much lower luminosity ($L = 1.22$) than it lasts until when the sun is increasing in luminosity ($L = 1.58$).

2.4 Both Species Coexisting

Now we will discuss the condition where seeds for both black and white daisies are present in the background. The important equations for this condition are shown below. The derivation of these equations is shown in Section A.3 of Appendix A. When both species are coexisting and alive,

$$\alpha_b + \alpha_w = 0.673, \tag{2.17}$$

because the sum of fractional areas occupied by both daisy species ($\alpha_b + \alpha_w$) remains constant. However, individual values of α_b and α_w are variable. Local temperatures in the two species model are constant, as opposed the one species model where they are variable. The reasons are explained in Appendix A. The local temperatures are $T_b = 300.5K$ and $T_w = 290.5K$. When both species coexist, the fractional surface area occupied by daisies is a function of relative solar luminosity, while in a one species model the fractional surface area is a function of local temperature[2]. The

[2] Equation 2.18 is equivalent to the equation derived by Saunders, $\alpha_b = \frac{12.2}{14.7L-1.9} - 0.663$ [10].

relationship between fractional area occupied by black daisies α_b as function of relative solar luminosity L when both species coexist is

$$\alpha_b = \frac{0.829}{L - 0.129} - 0.664. \tag{2.18}$$

Finally, the mean planetary temperature T_e as a function of relative solar α_b is

$$T_e^4 = \frac{SL}{\sigma}(0.332 + \alpha_b). \tag{2.19}$$

Since α_b is a function of L, and T_e is a function of L, T_e is functionally dependent on L. In the one species models, a functional relationship between mean planetary temperature T_e and luminosity L does not exist.

Figure 2.10 shows the mean planetary temperature of the planet as function of relative solar luminosity. This graph provides a means of comparing temperature regulation when both species are present, with the one species models. Notice that when both species coexist (shown in pink in Figure 2.10), the mean planetary temperature actually decreases with increasing luminosity. The temperature regulation when both species coexist occurs over a larger range of luminosity than in the one species cases. Since planetary temperature actually reduces with increasing luminosity, it can also be argued that temperature regulation is also more effective when both species exist.

In Figure 2.10, it might seem contradictory that the region where both species are alive (shown in blue) is defined as stable even though temperature decreases with increasing luminosity. However, recall that the fundamental definition of stable states is that at these states daisy population is maximum. Daisy population in the blue region of the curve (which corresponds to the state where both species are alive) is greater than it is in either of the one species models where only black daisies or only white daisies occupy the planet at the same luminosity. Hence, the two species model population is stable for these luminosities (0.76 to 1.38) and the states predicted by either of the one species models is unstable.

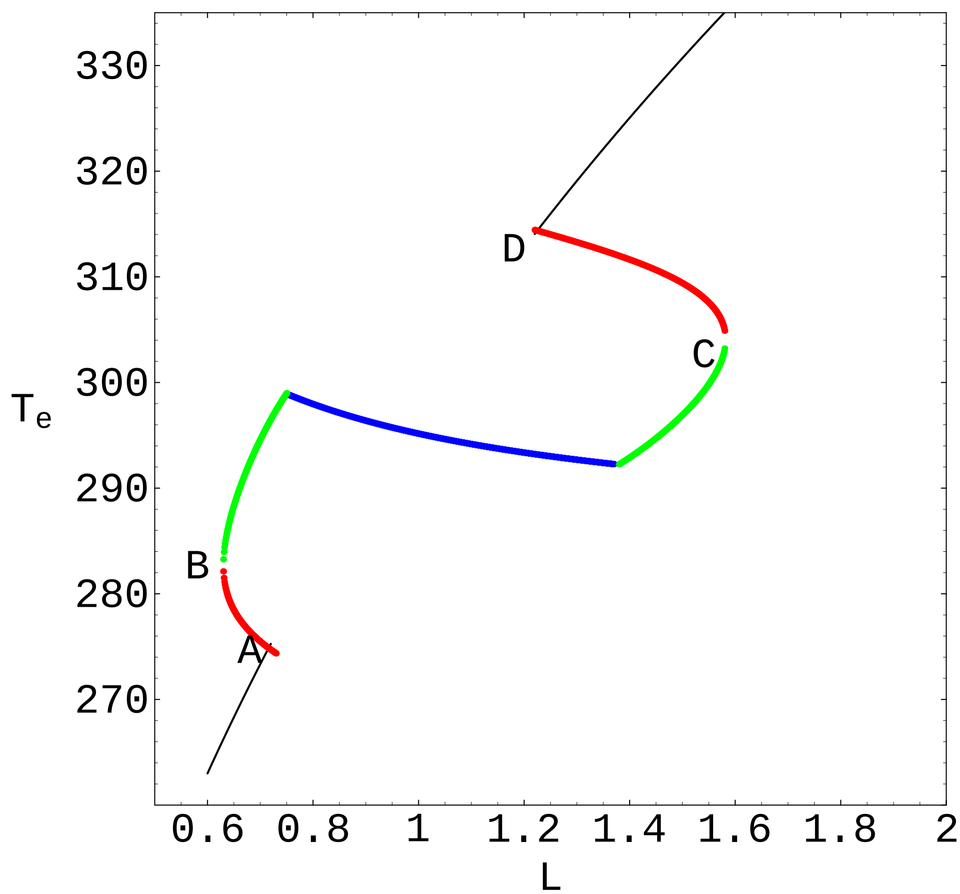

Figure 2.10: Mean Planetary Temperature in a Two Species Model. The vertical axis shows mean planetary temperature denoted by T_e and the horizontal axis shows relative solar luminosity represented by L. The dark portion of the graph corresponds to the state where no life exists on the planet. The red portion shows unstable equilibria and the green portion corresponds to stable equilibria. The red and green portions of the curve connecting to Point B correspond to the state where only black daisies exist on the planet, and those connecting to Point C correspond to the state where only white daisies exist on the planet. The blue portion shows the state where both species coexist.

Let us now numerically analyze the condition where both species coexist. In Equation 2.17 we saw that the sum of the areas occupied by both species when they coexist is 0.673. Hence solving the inequality $0 < \alpha_b \leq 0.673$ gives the range of luminosity for which both species coexist, constraining L to $0.76 < L \leq 1.38$. From Figure 2.10 it is clear that when both species are in existence, the planetary temperature decreases with luminosity, yet this condition is not unstable because at these luminosities daisy population is higher when both species inhabit the planet, than when only one species inhabits it. Stable states are those where daisy population is maximized. Also, consider

$$\frac{dT_e}{dL} = -\frac{9.19}{(L - 0.129)^2(\frac{L}{L-0.129})^{\frac{3}{4}}}. \tag{2.20}$$

Thus $\frac{dT_e}{dL} < 0$ if and only if $(\frac{L}{L-0.129})^{\frac{3}{4}} > 0$. However, the luminosity values we are concerned with are all greater than 0.129. If $\frac{L}{L-0.129} > 0$, then $(\frac{L}{L-0.129})^{\frac{3}{4}} > 0$. Hence $\frac{dT_e}{dL} < 0$. Therefore, when both species coexist, with increasing luminosity, the mean planetary temperature decreases. Hence the blue portion of the curve in Figure 2.10 is decreasing with increasing luminosity.

The numerical ranges for stable states for the two species model are displayed in Table 2.3.

Table 2.3: Original Daisyworld Model: Planet Inhabited by Both Species. Stable States for Increasing Solar Luminosity.

	L	α_b	α_w	T_e
Only Black Daisies Exist	[0.72, 0. 76]	[0.66, 0.69]	[0.00, 0.00]	[296.53, 299.71]
Both Species Coexist	[0.76, 1.38)	[0.00, 0.64]	[0.02, 0.67]	(292.22, 298.92]
Only White Daisies Exist	[1.38, 1.58]	[0.00, 0.00]	[0.64, 0.67]	[292.26, 304.08]

It is clear that when the ground contains seeds for both species of daisy, i.e. when both species can potentially exist, the luminosity range over which each individual species survives is increased. In the one species model, black daisies sprouted at a luminoity of 0.72 and became extinct at a luminosity of 1.09, while in the two species model they existed in the luminosity range 0.72 to 1.38. Similarly, in the one species model, white daisies existed in the luminosity range 0.84 to 1.43, while in the two species model they existed from 0.76 to 1.43. This observation can be used to conjecture that greater biological diversity is better for the health of the ecosystem (i.e. its capacity to respond to perturbation).

We have now developed the necessary numerical and graphical techniques to analyze temperature regulation in Daisyworld. In Chapter 3, we will analyze a slightly modified Daisyworld, where the birth rate of daisies is constant, but the death rate is functionally dependent on temperature.

Chapter 3

CONSTANT GROWTH RATE; VARIABLE DEATH RATE

Now we can begin to investigate the role of death rate in environmental feedback. This investigation will be a two step process. The first step is to investigate Daisyworld's capacity for temperature regulation when the growth rates of daisies is constant but the death rate is functionally dependent on local temperature of the daisies, which is the content of Chapter 3. In Chapter 4. we will make both death and growth rates functionally dependent on temperature, and investigate the impact of this variation on Daisyworld's capacity for self-regulation of climate.

In order to hold the growth rate constant, we took the average value of the growth rate function used in the original Daisyworld model and expressed in Equation 2.3. From this equation, $\beta_i = 1 - 0.003265(295.5 - T_i)^2$. The average value $\overline{\beta_i}$ of this function is

$$\overline{\beta_i} = \frac{1}{T_f - T_i} \int_{T_i}^{T_f} \beta_i(T_i) dT_i \tag{3.1}$$

where T_f is the maximum tolerable local temperature (in Kelvin), T_i is the minimum tolerable local temperature (in Kelvin). Substituting $T_i = 278$ and $T_f = 313$ in Equation 3.1 yields $\overline{\beta_i} = 0.33$.

Since the constant value of death rate in the original function was 0.3, we tried to make the average value of the new death rate function as close to 0.3 as possible. The definition of this new death rate function g_i is

$$g_i = 0.00326204(295.5 - T_i)^2 + 0.001 \tag{3.2}$$

and the average value of the death rate function $\overline{g_i}$ is 0.33. In order to make $\overline{g_i} = 0.30$, the parameters of the equation would have to be changed so that at the optimum temperature (295.5 K), almost no daisies would be dying. Since even at the optimum temperature some daisies should be dying, this model would not be realistic. Hence, we choose the constants in Equation 3.2 so the functional relationship between death rate and local temperature is such that even at the optimum temperature there are daisies dying. Thus, in this new model, the average numerical values of both growth and death rates are nearly equal[1].

Figure 3.1 shows graphs for the two equations of death rate (Equation 3.2 and the definition of g in the original model, i.e. $g = 0.30$) that we have seen so far. This graph illustrates that the new death rate function (dark curve in Figure 3.1) affects daisies depending on their local temperature, white the old death rate function (shown in blue in Figure 3.1) affects daisies independently of temperature. The average values of the two functions are not very different so that any difference in the system in the new model can be attributed to the functional dependence of death rate on local temperature.

[1] Remember that β_i has units of daisy area per unit time per unit area, while g has units of daisy area per unit time.

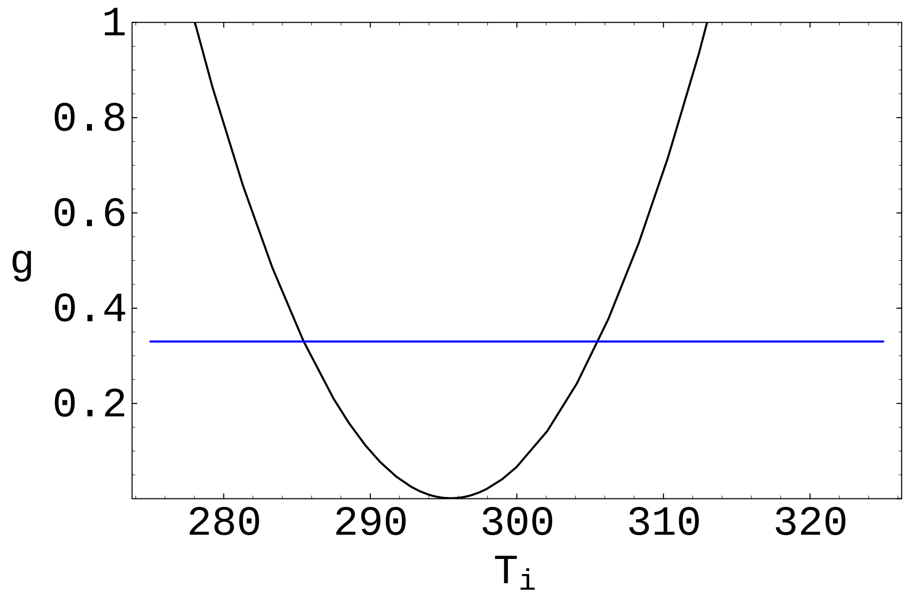

Figure 3.1: The two definitions of death rate. The dark colored graph shows the definition for death rate developed in Chapter 3. The blue colored graph corresponds to the definition of death rate originally developed by Lovelock and Watson that is explained in Chapter 2. The death rate g is in units of fractional area per unit time, and T_i is the local temperature in Kelvin.

3.1 One Species Models

In this section, the two conditions where black daisies and white daisies grow separately are considered. We initially study the general case where where daisies of either species could be growing on the planet, and then analyze the particular condition where black and white daisies occupy the planet exclusively.

In Equation 2.1, $\frac{d\alpha_i}{dt} = \alpha_i(x\beta_i - g)$. At equilibrium, the area occupied by daisies remains constant, hence $\frac{d\alpha_i}{dt} = 0$. Applying the modified growth rates, Equation 2.1 may be rewritten to obtain

$$\frac{d\alpha_i}{dt} = \alpha_i(x\overline{\beta_i} - g_i) = 0 \tag{3.3}$$

the new equilibrium condition. In Equation 3.3, $\overline{\beta_i} = 0.33$ and g_i is the functional relationship between death rate and local temperature defined in Equation 3.2. In Equation 3.3 it is reasonable to assume that $\alpha_i \neq 0$, since we are concerned with the condition where one species of daisy is alive, and if daisies are alive they occupy some area. Thus, $x\overline{\beta_i} - g_i = 0$, so $g_i = x\overline{\beta_i}$. In the one-species model, $x = 1 - \alpha_i$. Substituting for g_i, $\overline{\beta_i}$ and x in $x\overline{\beta_i} - g_i = 0$ and solving for T_i from this equation (since g_i is a function of T_i) yields

$$T_i = 295.5 \pm \sqrt{\frac{0.33(1 - \alpha_i) - 0.001}{0.00326024}}. \tag{3.4}$$

Solving for α_i in terms of T_i yields

$$\alpha_i = 0.999 - \frac{0.0032604}{\overline{\beta_i}}(T_i - 295.5)^2. \tag{3.5}$$

From Equation 3.5, it is clear that the maximum value of α_i is 0.999. Hence, at most 99.9 percent of the planet can be occupie. The reason 100 percent of the planet is not covered is because of the constant 0.001 in Equation 3.2; if this constant were not present, the consequence would be that at optimum temperature 100 per cent of the planet would be covered. This consequence is not realistic because even at the optimum temperature daisies should be dying. By solving the inequality $0 \leq \alpha_i \leq 0.999$, we obtain the range of local temperatures for which daisies are alive in a one species model. The condition $0 \leq \alpha_i \leq 0.999$ implies that $285.45 \leq T_i \leq 305.56$. Thus, in the one species model, either species of daisy is alive only when the local temperature of the plants is in the range 285.45 K to 305.56 K. Recall that in Chapter 2, the range of local temperatures for which daisies were alive in the one species model was 280.85 K to 310.14 K. Thus, the range of permissible local temperatures is reduced in this model.

Figure 3.2 shows a graph of Equation 3.5. This graph shows how planetary area occupied by daisies varies with local temperature. As we saw in Section 2.3, planetary area occupied is maximum at the optimum temperature of 295.5 K, and minimum at the extreme temperatures. The maximum arable area of the planet in the current model is 99.9 per cent, whereas in the original model studied in Chapter 2 maximum fertile area was 70 per cent. In both models, however, the relationship between fractional area occupied and local temperature of daisies is qualitatively very similar.

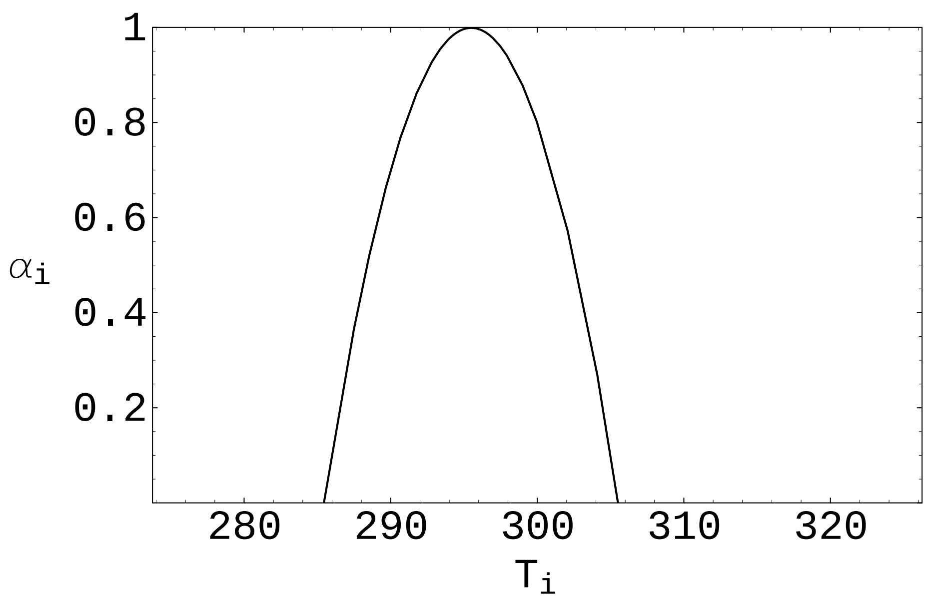

Figure 3.2: Fractional Area Occupied By Daisies in One Species Model when growth rate is constant and death rate is variable. α_i represents fractional area occupied by daisies, and T_i represents local temperature of daisies.

3.1.1 Black Daisies Exclusively

In this case the ground contains seeds for black daisies only. Hence, either the black daisies are alive, or the planet is lifeless. Certain equations were derived to study the mean planetary temperatures when the growth rate of the daisies is constant, their death rate is functionally dependent on local temperature, and the ground contains seeds only for black daisies. These equations are shown below, and their derivations are explained in Appendix B.

The functional relationship between fractional area occupied by daisies (α_b) and local temperature (T_b) is

$$\alpha_b = 0.999 - \frac{0.0032604}{\overline{\beta_b}}(T_b - 295.5)^2. \tag{3.6}$$

Equation 3.6 is obtained by replacing the subscript i with the subscript b in Equation 3.5. The algebraic relationship between local temperature of white daisies T_w and relative solar luminosity L is

$$T_b^4 = 0.25q + \frac{0.5SL}{\sigma} + \alpha_b \left(\frac{0.25SL}{\sigma} - 0.25q \right). \tag{3.7}$$

Finally, the algebraic relationship between mean planetary temperature T_e and relative solar luminosity L is

$$T_e^4 = \frac{SL}{\sigma}(0.5 + 0.25\alpha_b). \tag{3.8}$$

Figures 3.3 and 3.4 show temperature regulation on Daisyworld under the conditions being studied in this case. In Figure 3.4, portion of red curve between Points A and X corresponds to unstable state and portion of the curve between Points B and X corresponds to stable states. The portions of the red and blue curves that lie to the right of their respective intersection points with the black curve are physically meaningless for reasons explained in Section 2.3.1.

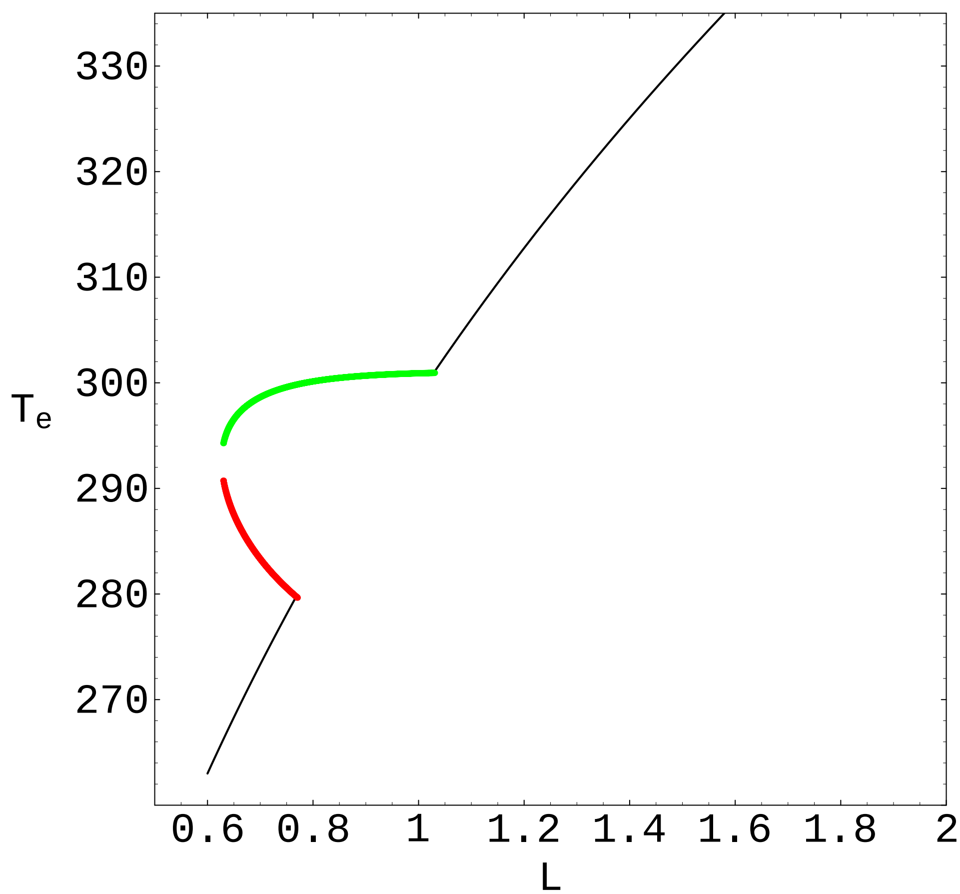

Figure 3.3: Black Daisies Exclusively: Mean Planetary Temperature as a function of Relative Solar Luminosity. Dark portion of the graph corresponds to the state in which no life exists on the planet. The red and green portions of the graph correspond to the state in which black daisies are alive on the planet. The green portion corresponds to stable equilibria, while the red portion corresponds to unstable equilibria.

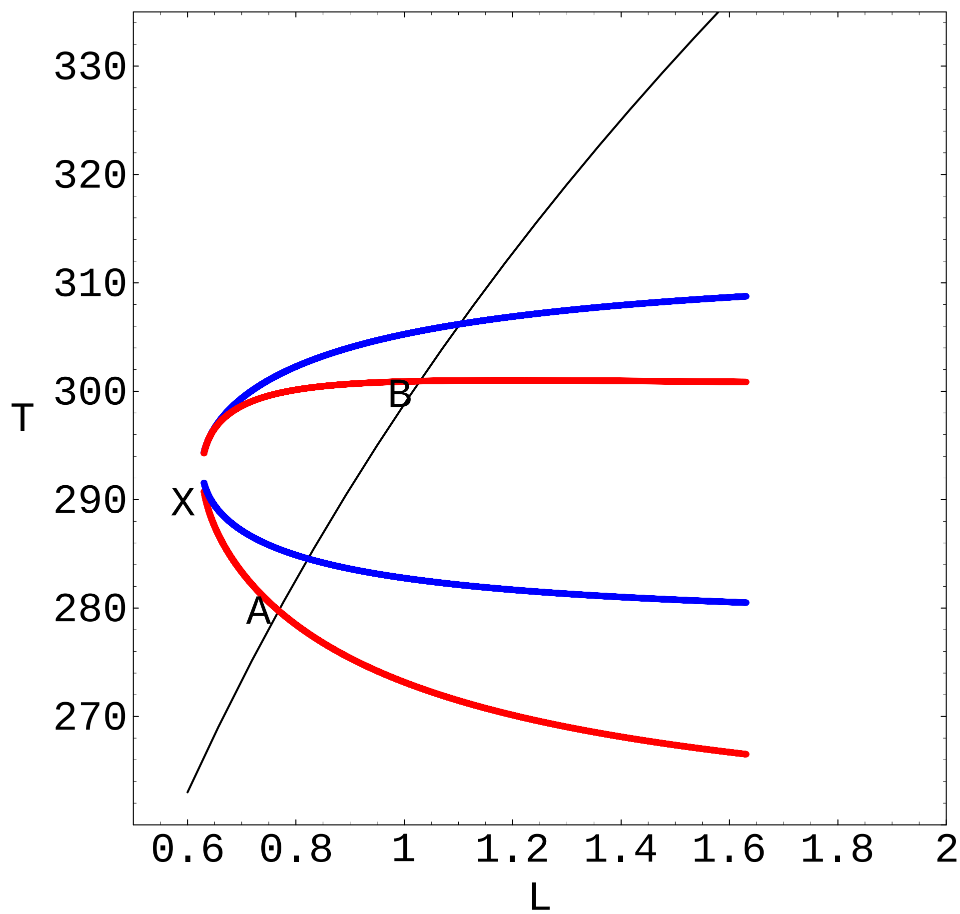

Figure 3.4: Black Daisies Exculsively. Mean Planetary Temperature and Local Temperatures as functions of Relative Solar Luminosity. Red curve corresponds to mean planetary temperature, blue curve corresponds to local temperature.

As in Section 2.3.1, the presence of daisies results in regulation of temperature. The rise in temperature in Figure 3.3 is very similar to that seen in Figure 2.4, which depicts mean planetary temperature on a planet covered by only black daisies in the basic model. The range of luminosity over which daisies survive is slightly different in the two models, but the two models are very similar in their capacity for temperature regulation.

In Figure 3.4, the red curve denotes mean planetary temperature, and the blue curve denotes local temperature. Once again, we see that the local temperature of black daisies is always greater than the mean planetary temperature. However, in the current model, in certain regions the two temperatures are almost equal because in this model almost the entire planet can be covered by black daisies. When that happens, the local temperature of black daisies is almost equal to mean planetary temperature.

We are now in a position to numerically analyze the various states that exist in this model. Rearranging the terms in Equation 3.7 yields

$$L = \frac{T_b^4 + 0.25q\alpha_b - 0.25q}{\frac{0.5S}{\sigma} + \alpha_b \frac{0.25S}{\sigma}}. \tag{3.9}$$

From Equation 3.9, we can calculate the values of luminosity where important transformations occur, i.e. when daisies sprout, for what range of luminosity they survive, and at what luminosity they become extinct[2].

Figure 3.5 shows the relationship between relative solar luminosity and local temperature of back daisies. Now unstable states occur when $T_b < 293.93K$. This value of local temperature is greater than the limit for instability ($T_b = 285.5K$) seen in Section 2.3.1. Thus, the current model has a greater range of instability than the original model.

[2]The relationship between L and T_b obtained in this case might appear identical to the relationship obtained in the first chapter, however, they are not, because the two models have different values for α_b .

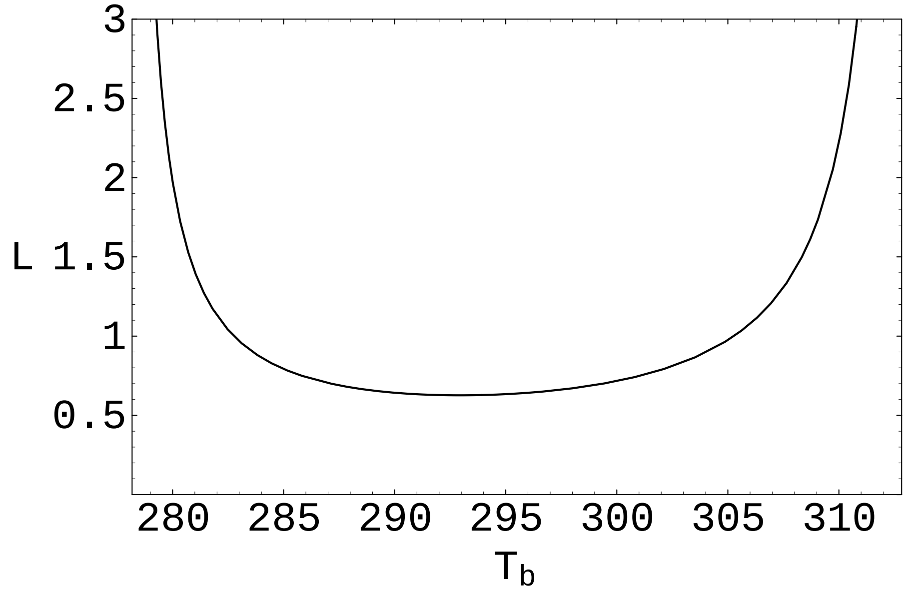

Figure 3.5: The relationship between relative solar luminosity and local temperature for planet occupied by only black daisies, when growth rate is constant and death rate is variable. On the vertical axis, L is luminosity and on T_b on the horizontal axis is local temperature.

In order to find unstable states, we use the same process as was used in Section 2.3.1 of Chapter 2. This process involved calculating the range where $\frac{dL}{dT_b} < 0$ (in order to find unstable states). In the current model, $\frac{dL}{dT_b} < 0$ implies $278.08 < T_b < 292.93$. However, under the conditions we are currently studying, daisies do not sprout until their local temperature reaches 285.44 K; as in Section 2.3.1, the previous inequality actually implies $285.44 < T_b < 292.93$. Thus when the local temperature is between 285.44 K and 292.93 K, this version of Daisyworld is in an unstable state.

Numerical ranges of stability and instability for the condition we are studying are summarized in Table 3.1.

Table 3.1: Constant Growth Rate, Variable Death Rate. Planet Inhabited by Black Daisies Only. Stable and Unstable States.

	L	α_b	T_b	T_e (Kelvin)
Unstable States	(0.63, 0.77]	[0.00, 0.93]	[285.44, 292.93)	[279.73, 292.59)
Stable States	[0.63, 1.03]	[0.00, 0.93]	[292.93, 305.56]	[292.59, 300.94]

Thus, as the solar luminosity is increasing, initially there are no daisies on the planet. Daisies are first able to survive at a luminosity of 0.77. The local temperature and mean planetary temperature are now 285.44 K and 279.93 K respectively. However, since this is an unstable state, the temperature of the daisies jumps to the stable region. This stable temperature value is calculated by solving $L(285.44) = L(x)$ where x is local temperature in stable region and 285.44 K is the value of local temperature in the unstable region. The immediately previous equation implies $x = 301.54K$. Thus the local temperature now becomes 301.54 K, and the mean planetary temperature at this point is 299.83 K. The daisies then survive until relative solar luminosity reaches 1.03, at which point their local temperature is 305.56 K and mean planetary temperature is 300.94 K. At any higher luminosity, the planet remains barren.

If Daisyworld's sun were decreasing in luminosity, daisies would first appear at a luminosity of 1.03 and be alive until the luminosity were reduced to 0.63. Any further decline in luminosity would lead to an unstable temperature, and the planet would become barren. Note again that when the sun's luminosity is reducing, and the planet can grow only black daisies, the daisies survive until a much lower luminosity than at which they appear if the solar luminosity is escalating.

This model is not qualitatively much different from the original Daisyworld model studied in Chapter 2. The only difference in this model is that the ranges of luminosity over which daisies survive are reduced.

3.1.2 White Daisies Exclusively

Now, let us consider the case where the ground contains seeds for white daisies only. The equations used to study temperature regulation under this condition are shown below and derived in Section B.1.2 of Appendix B. Equation 3.10 shows the functional relationship between fractional area occupied by white daisies α_w and local temperature of white daisies T_w that is given by

$$\alpha_w = 0.999 - \frac{0.0032604}{\beta}(T_w - 295.5)^2. \tag{3.10}$$

Equation 3.11 The algebraic relationship between local temperature of white daisies T_w and relative solar luminosity L is

$$T_w^4 = -0.25q + \frac{0.5SL}{\sigma} + \alpha_w\left(0.25q + \frac{-0.25SL}{\sigma}\right). \tag{3.11}$$

The functional relationship between mean planetary temperature T_e and relative solar luminosity L is

$$T_e^4 = \frac{SL}{\sigma}\left(0.5 - 0.25\alpha_w\right). \tag{3.12}$$

Figure 3.6 shows mean planetary temperature as a function of relative solar luminosity and Figure 3.7 shows both local temperature and mean planetary temperature as functions of relative solar luminosity. These graphs are very similar to the graphs for the equivalent case in the original model, seen in Section 2.3.2. In Figure 3.7, portion of the curve between Points A and X corresponds to stable states, and portion of curve between Points B and X corresponds to unstable states. The portions of the red and blue curves that lie to the right of their respective intersection points with the black curve are physically meaningless for reasons explained in Section 2.3.2.

The critical states for this case were calculated in the same manner that was used for a planet covered by only white daisies. The relationship between L and T_w is given by

$$L = \frac{T_w^4 - 0.25q\alpha_w + 0.25q}{\frac{0.5S}{\sigma} - \alpha_w\frac{0.25S}{\sigma}}. \tag{3.13}$$

Figure 4.7 shows a graph of Equation 3.13. This graph has the same shape as the luminosity graph for white daisies seen in the original model.

As in Section 2.3.2 of Chapter 2, the unstable states are obtained by solving the inequality $\frac{dL}{T_w} < 0$. When $\frac{dL}{T_w} < 0$, then $296.24 < T_w < 587.77$. However, since daisies only survive when their local temperatures are in the range 285.44 K to 305.56 K, the meaningful range of instability is $296.24 < T_w < 305.56$. Thus, in the current model, the white daisies have a broader range of stability than white daisies in the original model. Recall that in the current model black daisies have a smaller range of stability than the previous model. This result, however, does not denote any significant qualitative differences between the two models.

Numerical ranges of stability and instability for this model are shown in Table 3.2.

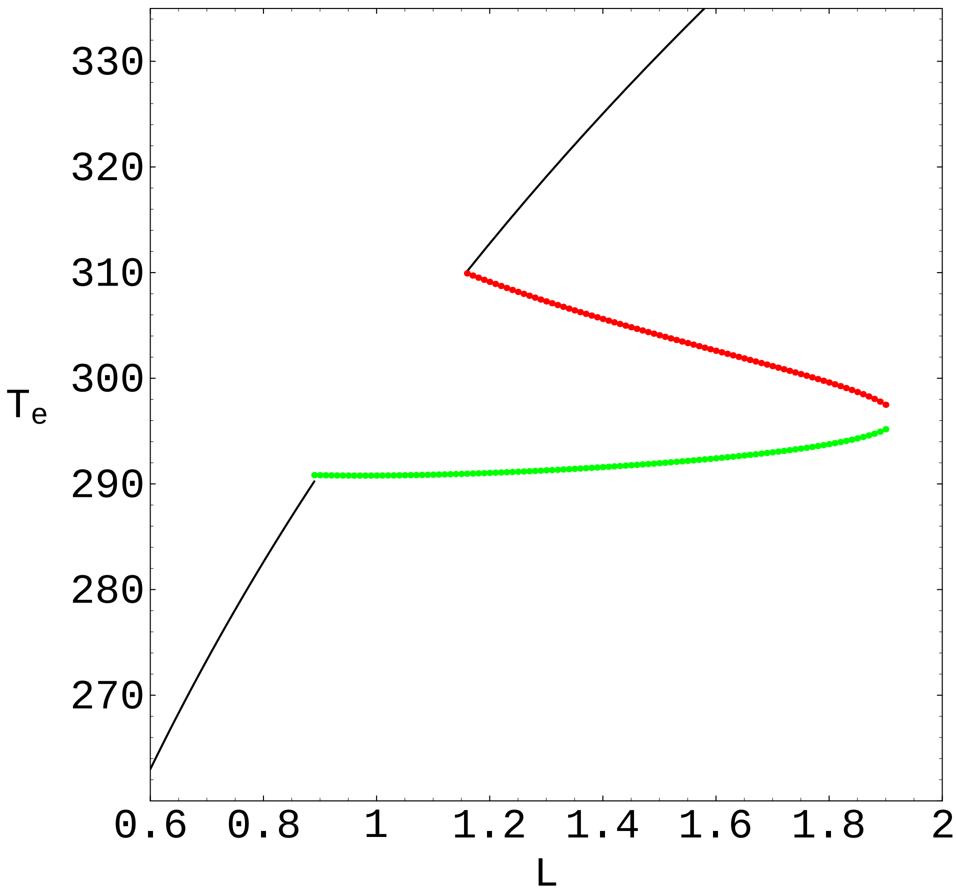

Figure 3.6: White Daisies Exclusively: Mean Planetary Temperature as a function of Relative Solar Luminosity. Daisy growth rate is constant, and death rate is variable. Dark portion of the graph corresponds to the state in which no life exists on the planet. The red and green portions of the graph correspond to the state in which white daisies exist on the planet. The green portion shows stable equilibria, while the red portion corresponds to unstable equilibria.

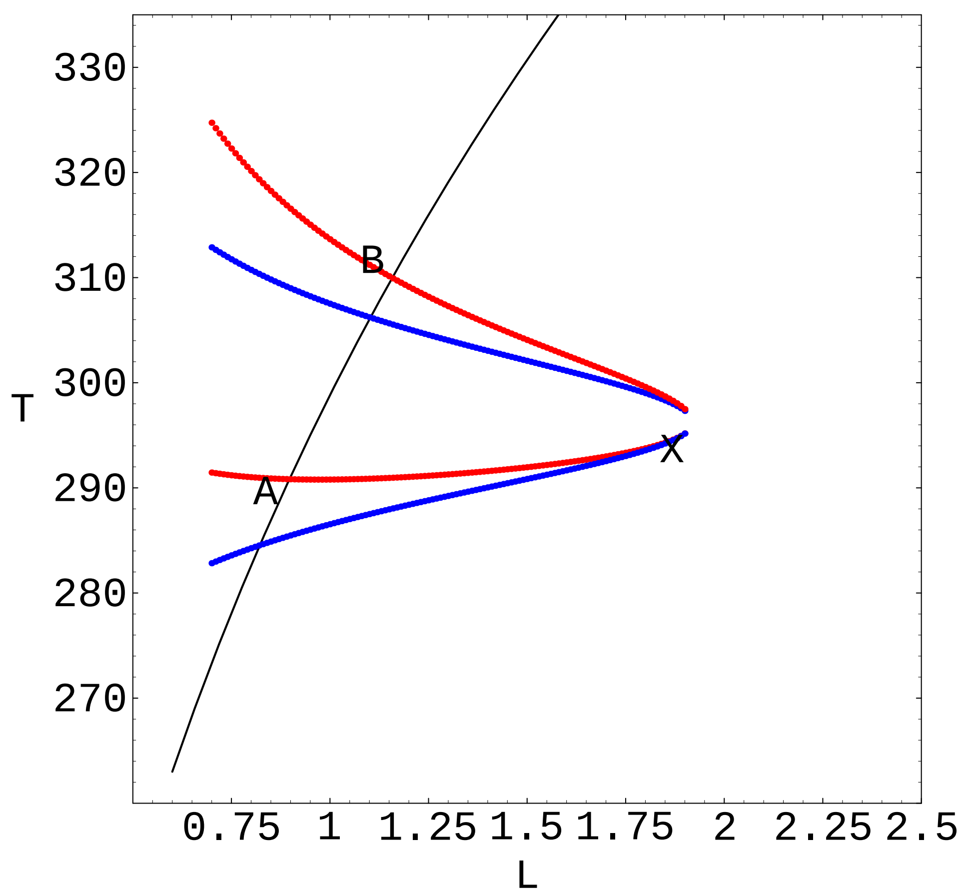

Figure 3.7: White Daisies Exculsively. Mean Planetary Temperature and Local Temperatures as functions of Relative Solar Luminosity. Red curve corresponds to mean planetary temperature, blue curve corresponds to local temperature.

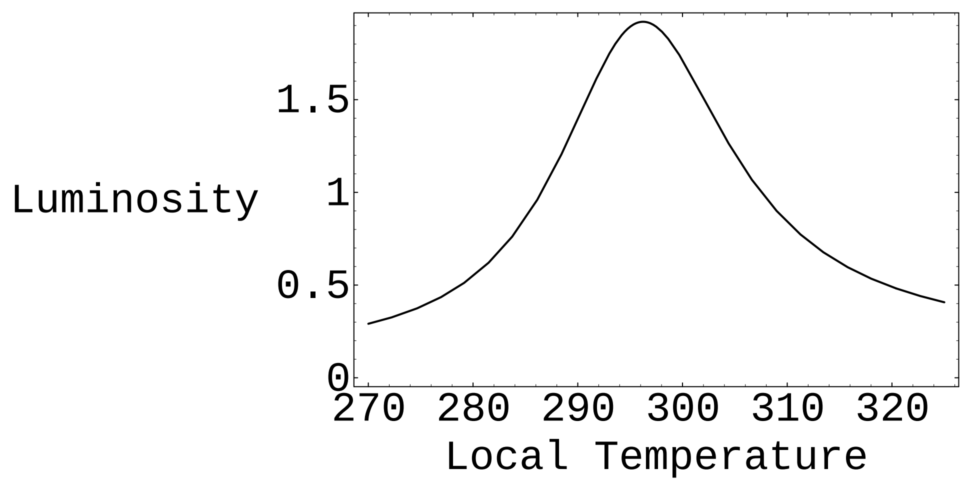

Figure 3.8: The relationship between Relative Solar Luminosity and Local Temperature for planet inhabited by white daisies only. The regions where $\frac{dL}{T_w} < 0$ correspond to unstable states. Thus, luminosities corresponding to $T_w > 296.24$ represent unstable states, shown in red in Figure 3.6.

Table 3.2: Constant Growth Rate, Variable Death Rate: Planet Inhabited by White Daisies Only. Stable and Unstable States.

	L	α_w	T_w	T_e
Stable States	[0.89, 1.92]	(0.00, 0.98]	[285.44, 294.55]	[290.83, 296.27]
Unstable States	[1.16, 1.92)	[0.00, 0.99]	(296.24, 305.56]	(296.27, 309.98]

In the condition that relative solar luminosity is increasing, daisies first appear when luminosity reaches 0.89. At this point the local temperature and planetary temperature are 285.44 K and 290.83 K respectively. Daisies survive until the relative luminosity reaches 1.92 (and consequently the local and planetary temperatures are 296.24 K and 296.27K respectively.). If the luminosity increases further, Daisyworld enters an unstable state.

If the sun's luminosity were reducing, daisies would appear when the luminosity reached 1.15. Since the value of planetary temperature at 1.15 is unstable, the temperature would change to a value which corresponds to a stable population, and they would survive until the luminosity reduced to 0.89. Notice this luminosity is much lower than the maximum luminosity that the daisies would be able to tolerate if the sun's luminosity were increasing.

3.2 Both Species Coexisting

3.2.1 The Two Species Model

We now consider the case where the ground contains seeds for both black and white daisies; thus both species can potentially coexist. In order to study the temperature regulation capacity of the planet, it is once again necessary to study the effect of changing solar luminosity on mean temperature of the planet. The equations required to study this effect are shown below and derived in Appendix B.

The sum of areas occupied by both species ($\alpha_b + \alpha_w$) remains constant and is given by

$$\alpha_b + \alpha_w = 0.750. \tag{3.14}$$

The functional relationship between fractional area occupied by black daisies α_b and relative solar luminosity L is

$$\alpha_b = \frac{0.829}{L - 0.129} - 0.625. \tag{3.15}$$

Finally, the functional relationship between mean planetary temperature T_e and relative solar luminosity L is

$$T_e^4 = \frac{SL}{\sigma}\left(0.313 + 0.5\alpha_b\right). \tag{3.16}$$

From Equation 3.14, we know that when both species coexist, seventy five percent of the planet is covered. As in Section 2.4, by solving the inequality $0 < \alpha_b \leq 0.750$ and using Equation 3.15, we see that the range of luminosity for which both species coexist is $0.73 < L < 1.45$. From Section 3.1.1 we know that in this model black daisies do not sprout until the luminosity reaches 0.77 (and when it does initially Daisyworld is in an unstable state). Hence, the true range of luminosity for which both species coexist in this case is 0.77 to 1.45.

Both daisy species can theoretically exist as the luminosity ranges from 0.73 to 1.45. Hence when the black daisies initially sprout at luminosity 0.77 and Daisyworld is in unstable state, Daisyworld jumps to a stable state where both species coexist. Thus black daisies sprout at Point A in Figure 3.9, which is unstable, and the jump to the stable state takes Daisyworld to the blue region in the figure.

Figure 3.9 shows mean planetary temperature as a function of relative solar luminosity. Once again mean planetary temperature decreases with increasing luminosity when both species coexist. However, as in Section 2.4 of Chapter 2, this region is stable because in this state daisy population is maximum.

Numerical ranges of stability for the two species model are described in Table 3.3.

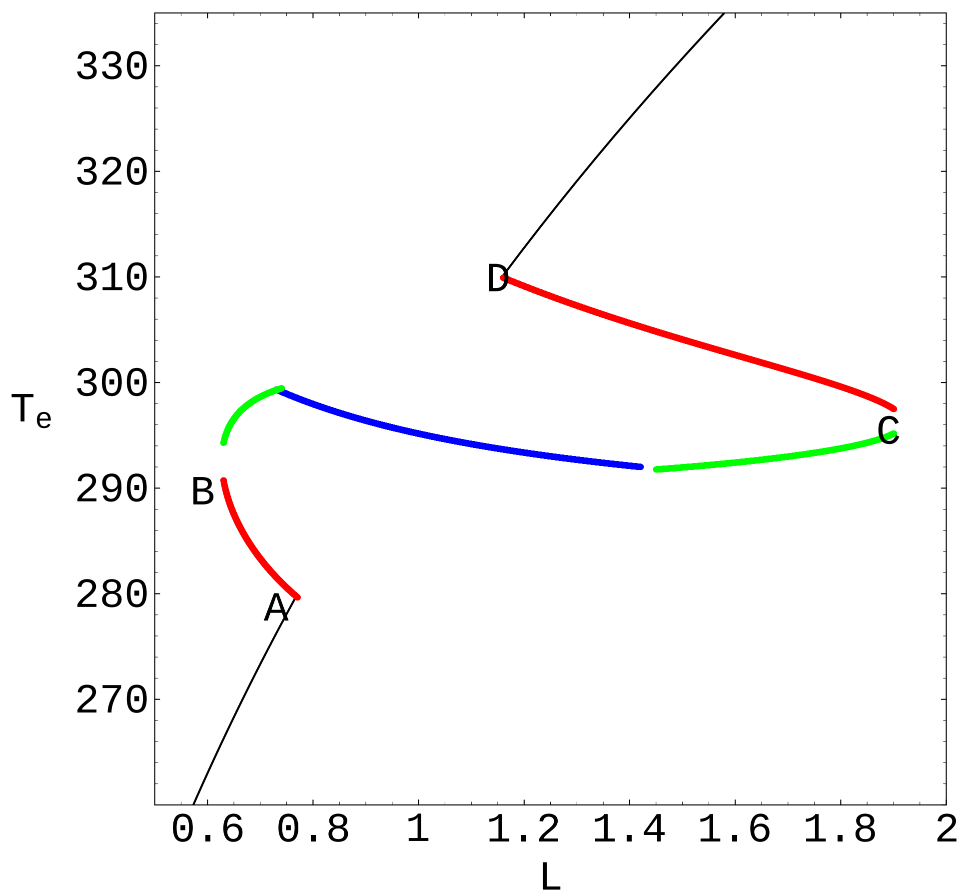

Figure 3.9: Mean Planetary Temperature in a Two Species Model. Dark portion of the graph corresponds to the state in which no life exists on the planet. The colored portion of the graph (red, green, blue) corresponds to the state in which life exists on the planet. The red portion shows unstable equilibria and the green portion corresponds to stable equilibria. The red and green portions of the curve connecting to Point B correspond to the state in which only black daisies exist on the planet, and those connecting to Points C correspond to the state in which only white daisies exist on the planet.

Table 3.3: Constant Growth Rate, Variable Death Rate. Planet Inhabited by Both Species. Stable States for increasing Solar Luminosity.

	L	α_b	α_w	T_e
Only Black Daisies Exist	None	None	None	None
Both Species Coexist	[0.77, 1.45)	[0.00, 0.66]	[0.08, 0. 75]	(291.86, 298.5]
Only White Daisies Exist	[1.45, 1.92]	[0.00, 0.00]	[0.75, 0.99]	[291.86, 296.27]

3.2.2 Comparisons

Let us now compare the model developed in Chapter 3 with the original Daisyworld model presented in Chapter 2. In order to make this comparison, mean planetary temperatures for both these models when both species may coexist are plotted on the same graph. The graph for the two species models includes graphs from the conditions when only black and white daisies exist, and hence comparing only the two species condition gives us complete insight into how the two models compare.

In Figure 3.10, the red curve represents planetary temperature for Daisyword with growth rate of daisies constant, and death rate variable, and the blue curve corresponds to mean planetary temperature in the original Daisyworld model. We see that both models have similar qualitative features; however the modified Daisyworld models has tolerance for a wider range of luminosity. It is also visible that when black daisies first appear, mean planetary temperature in the newer model is greater than that in the original model, and when white daisies appear, mean planetary temperature in the new model is lower. Both these conditions correspond to greater proportion of black and white daisies in the modified model, which is to be expected since maximum arable area in the one species conditions of the modified model is 99.9 percent, as compared to 67.3 percent in the original model. Temperature regulation when both species are alive is identical in the two models.

We have now completed the first step in exploring the role of death rate in the environmental feedback. There is little qualitative difference in the two models we have studied so far. We are now ready to investigate the impact of making both death and growth rates functionally dependent on local temperature on the system.

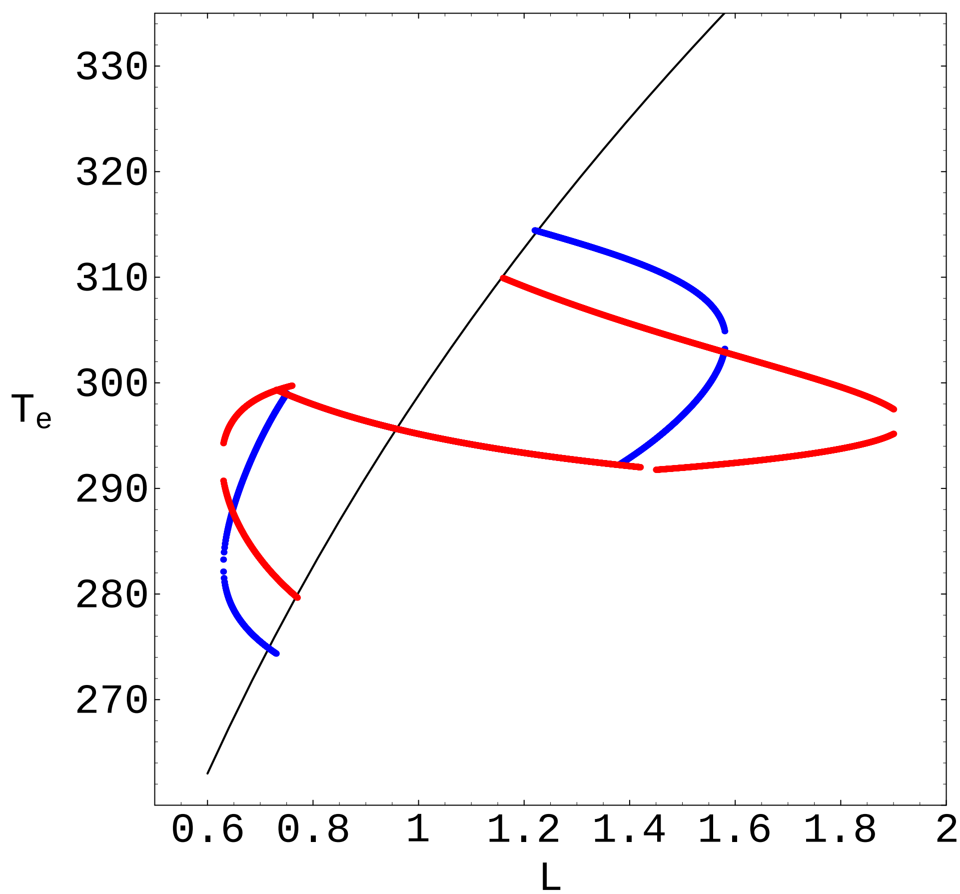

Figure 3.10: Comparison of models presented in Chapters 2 and 3. Blue curve shows mean planetary temperature in the original model, and the red curve shows temperature regulation with constant growth rate and variable death rate.

Chapter 4

VARIABLE GROWTH RATE; VARIABLE DEATH RATE

Local temperature should not affect the growth rate or the death rate of daisies in a mutually exclusive fashion because it seems logical that in favorable climate conditions daisies will be growing faster and dying slower, and vice versa. Thus, the feedback loop between life and the environment is stronger when temperature affects both death and growth rate of the daisies.

In this chapter, the temperature regulation capacity of Daisyworld is investigated when both growth and death rate of daisies are dependent on local temperature. This analysis will give us deeper insight into the role of death rate in the planet's capacity for self-regulation of its climate. The growth rate is the same as that used by Lovelock and Watson in their original paper [12], and the death rate has the same functional relationship to local temperature as was given by Equation 3.2 in Chapter 3.

4.1 One Species Models

We first consider the case where the ground contains seeds for only one species of daisy. First, some comments are made about the general condition which applies to either species of daisy.

When both growth and death rate of daisies are functionally dependent on temperature, we know from Equation 2.3 that the growth rate β_i is defined by$\beta_i = 1 - 0.003265(295.5 - T_i)^2$ and death rate g_i is defined by $g_i = 0.003262(295.5 - T_i)^2 + 0.001$ (from Equation 3.2).

We know that at equilibrium $\frac{d\alpha_i}{dt} = \alpha_i(x\beta_i - g_i) = 0$. It is reasonable to assume

$\alpha_i \neq 0$, because if daisies exist, they occupy some planteary area. It follows that $x = 1 - \alpha_i = \frac{g_i}{\beta_i}$, and thus,

$$\alpha_i = 1 - \frac{0.003262(295.5 - T_i)^2 + 0.001}{1 - 0.003265(295.5 - T_i)^2}.$$

The previous equation can be simplified to obtain

$$\alpha_i = \frac{1.9909(T_i - 307.87)(T_i - 283.13)}{(T_i - 313)(T_i - 278)}. \tag{4.1}$$

The maximum fractional area of the planet that is covered when only one species exists can be obtained from Equation 4.1 using elementary calculus. It is found that 99.9 per cent of the planet is covered when the local temperature is 295.5 K. This fact applies to either daisy species, as long as the ground contains seeds for only one species. When $0 \leq \alpha_i \leq 0.999$, then $283.13 \leq T_i \leq 307.87$, implying that daisies only exist when their local temperature is between 283.13 K and 307.87 K. The range of local temperatures for which daisies are alive in this case is less than that seen in Chapter 2, but greater than the range seen in Chapter 3.

Figure 4.1 shows the fractional area occupied by daisies as a function of local temperature. The shape of the graph is very similar to the shapes seen in Chapters 2 and 3.

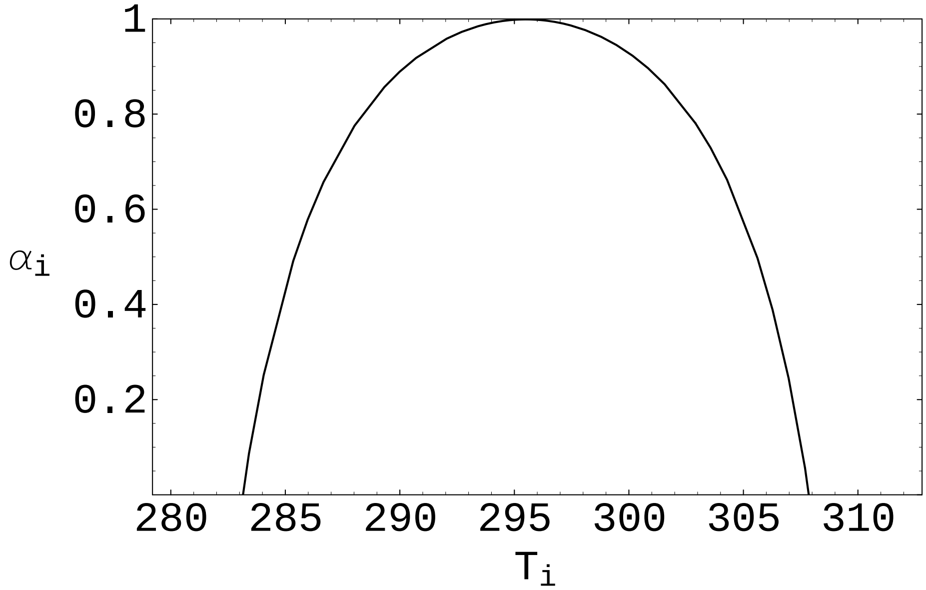

Figure 4.1: Fractional Area Occupied By Daisies in One Species Model when both growth and death rate of daisies are variable. α_i represents fractional area occupied by daisies, and T_i represents local temperature of daisies.

Now we can investigate the two separate conditions where the ground contains seeds for black daisies exclusively and white daisies exclusively.

4.1.1 Black Daisies Exclusively

Now let us consider the condition where the ground contains seeds for black daisies only. The equations required to investigate the temperature regulation capacity of the planet are shown below and their derivations are explained in Section C.1.1 of Appendix C.

The algebraic function describing functional relationship between fractional area occupied by daisies (α_b) and relative solar luminosity (L) is

$$\alpha_b = \frac{1.9909(T_b - 307.87)(T_b - 283.13)}{(T_b - 313)(T_b - 278)}. \tag{4.2}$$

Equation 4.2 is obtained from Equation 4.1 by replacing the subscript i with the subscript b, since Equation 4.1 applies to both black and white species (as long as the ground contains seeds only for one species). The algebraic relationship between local temperature T_b and relative solar luminosity L is

$$T_b^4 = 0.25q + \frac{0.5SL}{\sigma} + \alpha_b \left(\frac{0.25SL}{\sigma} - 0.25q \right). \tag{4.3}$$

The algebraic relationship between mean planetary temperature T_e and relative solar luminosity L is

$$T_e^4 = \frac{SL}{\sigma} \left(0.5 + 0.25\alpha_b \right). \tag{4.4}$$

Figures 4.2 and 4.3 demonstrate temperature regulation of the planet under the conditions we are studying. In Figure 4.3, portion of red curve between Points A and X corresponds to unstable states and portion of curve between Points B and X corresponds to stable states. The portions of the red and blue curves that lie to the right of their respective intersection points with the black curve are physically meaningless for reasons explained in Section 2.3.1.

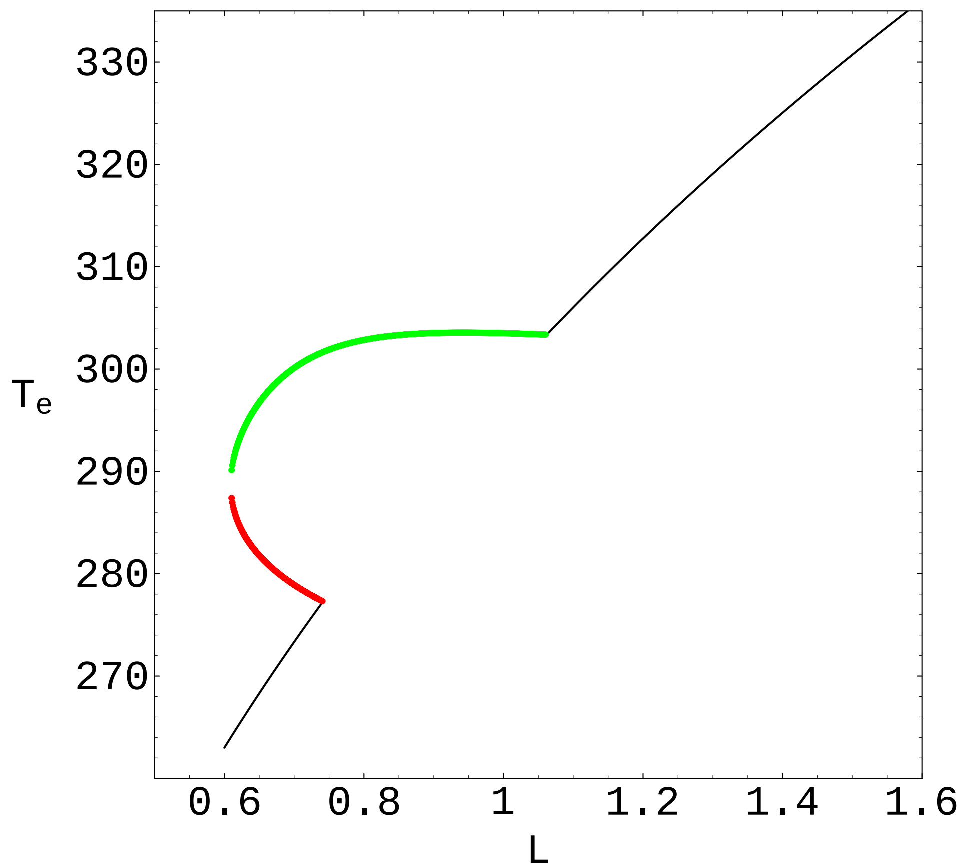

Figure 4.2: Black Daisies Exclusively: Mean Planetary Temperature as a function of Relative Solar Luminosity. Dark portion of the graph corresponds to the state in which no life exists on the planet. The red and green portions of the graph correspond to the state in which black daisies are alive on the planet. The green portion corresponds to stable equilibria, while the red portion corresponds to unstable equilibria.

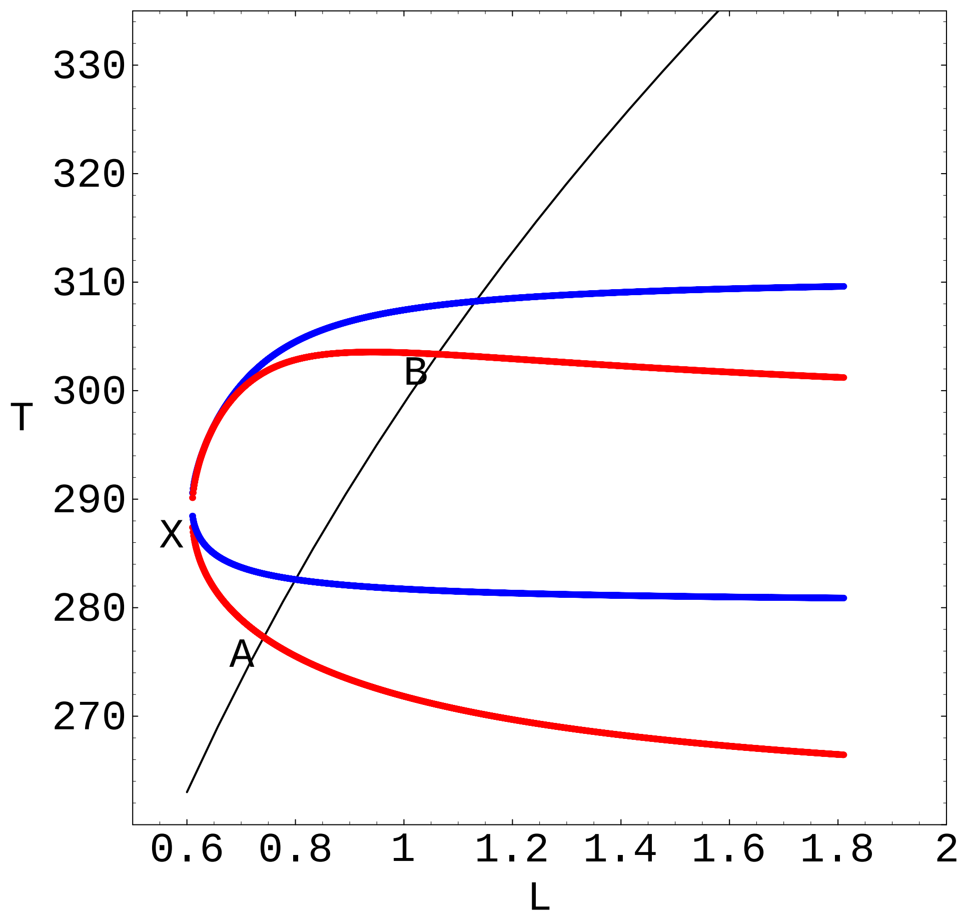

Figure 4.3: Black Daisies Exculsively. Mean Planetary Temperature and Local Temperatures as functions of Relative Solar Luminosity. Red curve corresponds to mean planetary temperature, blue curve corresponds to local temperature.

Temperature regulation on the planet shown by Figures 4.2 and 4.3 is very similar to the regulation seen in the previous two models.

As in Section 2.3.1 of Chapter 2, in order to find the unstable states, we need to find the domain in which local temperature decreases with increasing solar luminosity and vice versa. Hence, as in Sections 2.3.1 and 3.1.1 we need to calculate regions where $\frac{dL}{dT_b} < 0$. Therefore, we define an algebraic relationship between luminosity and local temperature. The desired relationship is obtained from Equation 4.3 and is given by

$$L = \sigma \frac{T_b^4 - 0.25q + 0.25\alpha_b q}{0.5S + 0.25\alpha_b S}. \tag{4.5}$$

Solving the inequality $\frac{dL}{dT_b} < 0$ yields $T_b < 289.47K$ in the admissible range. The range of stability in this model is narrower than the range of stability for the equivalent case in the original model, and broader than the range of stability in the model seen in Chapter 3. Figure 4.4 illustrates the relationship between luminosity and local temperature, and makes it easy to see on which range of values these parameters have an inverse relationship.

Table 4.1 summarizes the numerical ranges of stable and unstable states for this model. All values of luminosity that correspond to $T_b < 289.47$ represent unstable states for Daisyworld. These values of luminosity are represented by the red portion of the curve in Figure 4.2.

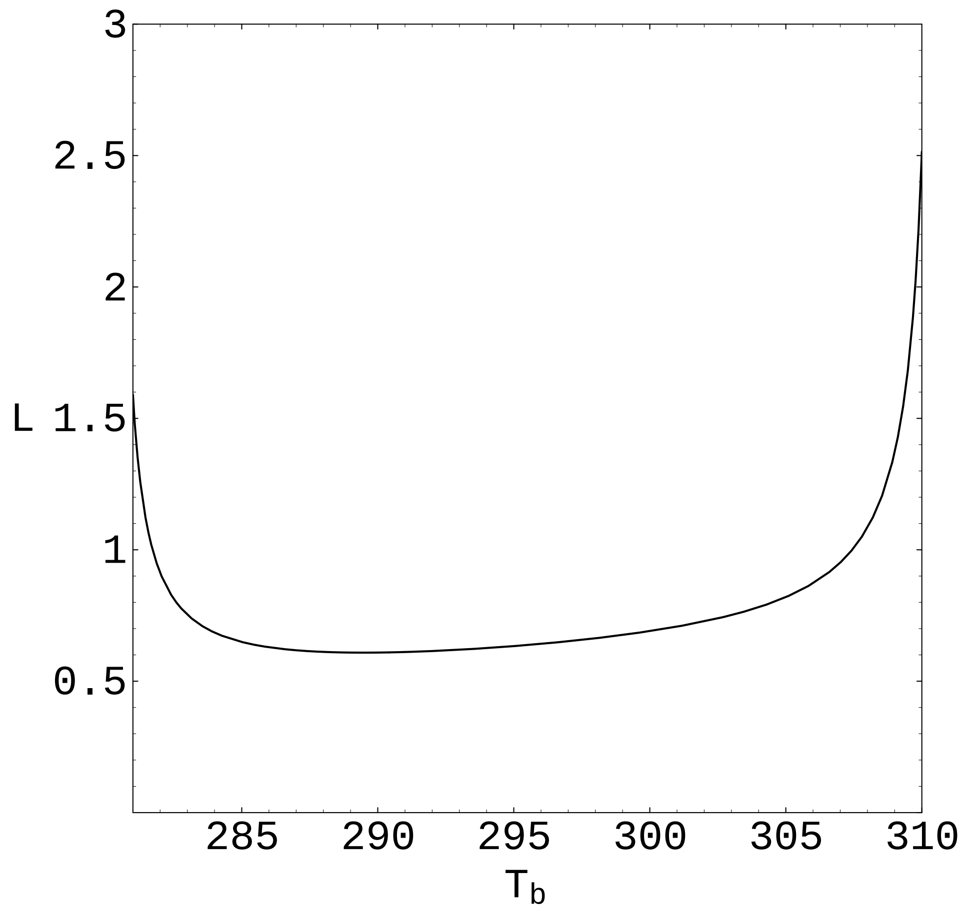

Figure 4.4: The relationship between relative solar luminosity and local temperature for planet occupied by only black daisies, when growth rate and death rate are variable, L is relative solar luminosity and T_b is local temperature of black daisies.

Table 4.1: Variable Growth Rate, Variable Death Rate. Planet Inhabited by Black Daisies Only. Stable and Unstable States.

	L	α_b	T_b	T_e
Unstable States	(0.61, 0.74]	(0.0, 0.86]	[283.13, 289.46)	[277.28, 288.43)
Stable States	[0.61, 1.06]	(0.00, 0.86]	(289.46, 307.87]	(288.43, 303.61]

Let us now analyze the various states for the condition where only black daisies may exist on the planet. Initially there are no daisies. As solar luminosity increases, black daisies first appear when $L = 0.74$. The local and mean planetary temperatures now are 283.13K and 277.28 K respectively. However, since this is an unstable state, the temperature of the daisies changes so that the population of black daisies jumps to a value in the stable region. This stable temperature is calculated by solving $L(283.13) = L(x)$ where x is the local temperature in the stable region and 285.44K is the value of local temperature in the unstable region. Solving this equation yields $x = 302.59K$. Thus the local temperature is now 302.59 K, and the mean planetary temperature at this point is 301.46 K. The daisies then survive until relative solar luminosity reaches 1.06, at which point their local temperature is 307.87 K and mean planetary temperature is 303.36 K. At any higher luminosity, the planet remains barren.

If solar luminosity were decreasing, daisies would first appear at a luminosity of 1.06 and then sustain until the luminosity reduced to 0.63. Any further reduction in luminosity would lead to an unstable state, and the planet would become barren.

4.1.2 White Daisies Exclusively

Now let us consider the case where only white daisies can exist on the planet. The equations required to study temperature regulation are given below and derived in Section C.1.2 of Appendix C.

The functional relationship between fractional area occupied by white daisies and local temperature is

$$\alpha_w = \frac{1.9909(T_w - 307.87)(T_w - 283.13)}{(T_w - 313)(T_w - 278)} \tag{4.6}$$

The algebraic relationship between local temperature of white daisies T_w and relative solar luminosity L is

$$T_w^4 = -0.25q + \frac{0.5SL}{\sigma} + \alpha_w \left(0.25q - \frac{0.25SL}{\sigma}\right). \tag{4.7}$$

The algebraic relationship between mean planetary temperature T_e and relative solar luminosity L is

$$T_e^4 = \frac{SL}{\sigma}\left(0.5 - 0.25\alpha_w\right). \tag{4.8}$$

Finally, the algebraic relationship between local temperature of white daisies relative solar luminosity L and local temperature of white daisies T_w is

$$L = \sigma\frac{T_w^4 + 0.25q - 0.25q\alpha_w}{0.5S - 0.25\alpha_w S}. \tag{4.9}$$

Figures 4.5 and 4.6 display temperature regulation in Daisyworld under the conditions we are studying. As in Section 2.3.2 of Chapter 2, in order to find the unstable states, we solve the inequality $\frac{dL}{dT_w} < 0$. Solving this inequality yields $T_w > 297.67$. The range of stability for white daisies in the current model is broader than the range seen for the model in Chapter 2, and narrower than the range seen in Chapter 3. This trend is the opposite of the trend observed with black daisies.

In Figure 4.6, portion of curve between Points A and X corresponds to stable states, and portion of curve between Points B and X corresponds to unstable states. The portions of the red and blue curves that lie to the left of their respective intersection points with the black curve are physically meaningless for reasons explained in Section 2.3.2.

Table 4.2 shows the numerical ranges of stability and instability for the model we are studying.

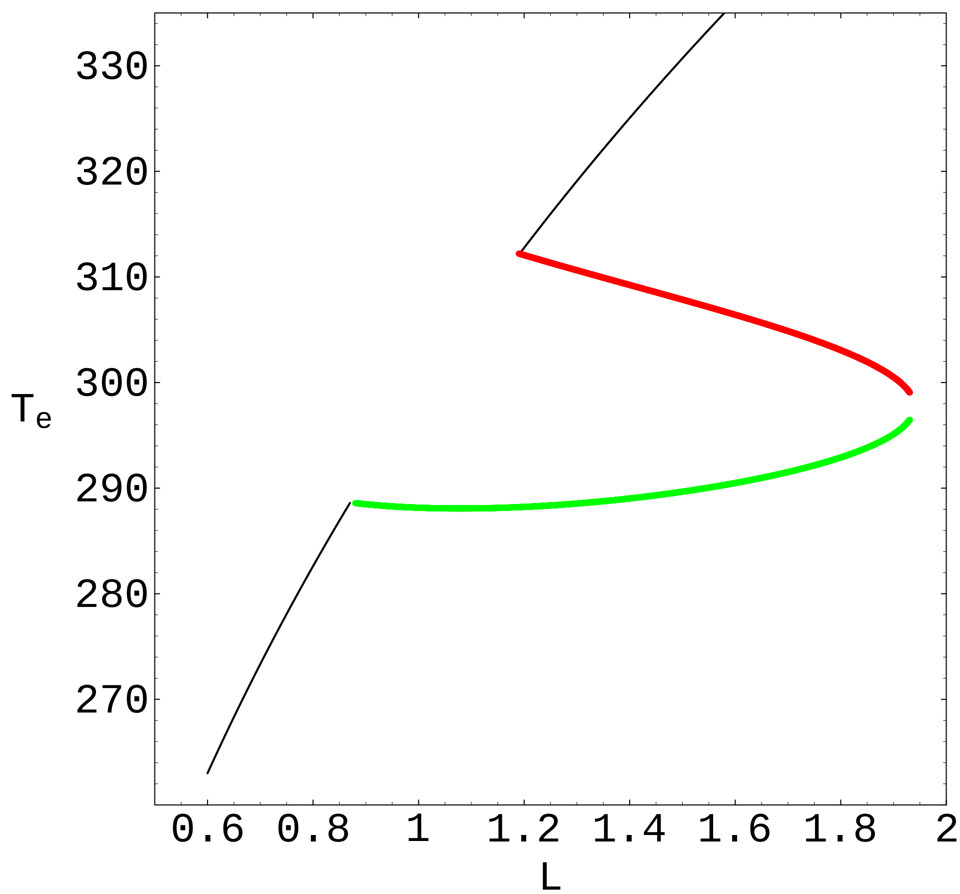

Figure 4.5: White Daisies Exclusively. Mean Planetary Temperature as a function of Relative Solar Luminosity. Both daisy growth and death rates are variable. Dark portion of the graph corresponds to the state in which no life exists on the planet. The red and green portions of the graph correspond to the state in which white daisies exist on the planet. The green portion shows stable equilibria, while the red portion corresponds to unstable equilibria.

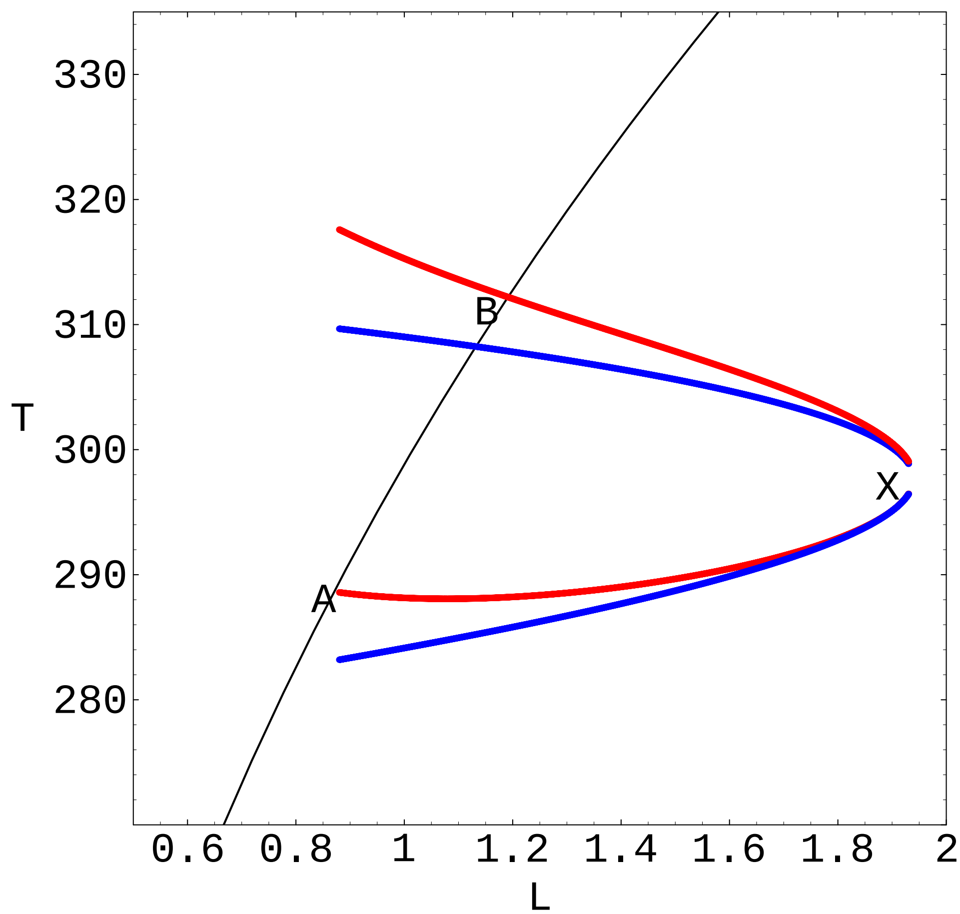

Figure 4.6: White Daisies Exculsively. Mean Planetary Temperature and Local Temperatures as functions of Relative Solar Luminosity. Red curve corresponds to mean planetary temperature, blue curve corresponds to local temperature.

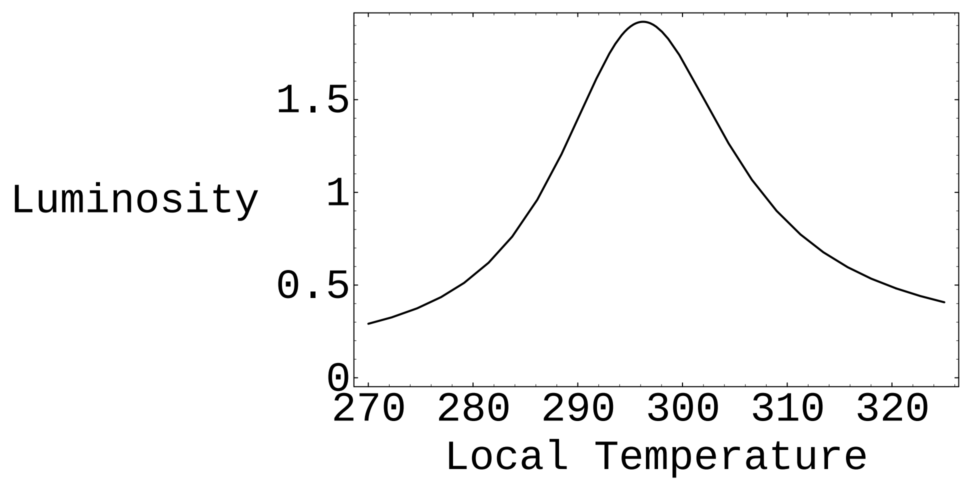

Figure 4.7: The relationship between Relative Solar Luminosity and Local Temperature for planet inhabited by white daisies only. The regions where $\frac{dL}{T_w} < 0$ correspond to unstable states. Thus, luminosities corresponding to $T_w > 297.67$ represent unstable states, shown in red in Figure 4.5.

Table 4.2: Variable Growth Rate, Variable Death Rate. Planet Inhabited by White Daisies Only. Stable and Unstable States.

	L	α_w	T_w	T_e
Stable States	[0.87, 1.94]	[0.00, 0.98]	[283.13, 297.67]	[288.64, 297.75]
Unstable States	[1.19, 1.94)	[0.00, 0.98]	(297.67, 307.87]	(297.75, 312.19)

From Table 4.2, as relative solar luminosity is increasing, daisies appear when the luminosity reaches 0.87. The daisies survive until luminosity reaches 1.93 and the local and mean planetary temperatures are respectively 297.67 K and 297.75 K. Any further increase in luminosity leads to an unstable state, and the daisies become extinct.

However, if the solar luminosity is decreasing, the daisies appear at a luminosity of 1.19 (the point X in Figure 4.6). However, since this is an unstable point, the mean planetary temperature would reduce to 312.19K, which corresponds to a local temperature of 307.87 K. Thus, if the sun's luminosity is decreasing, the daisies appear at a much lower solar luminosity (1.19) than the luminosity up to which they would survive if the sun was getting brighter (1.94).

4.2 Both Species Coexisting

4.2.1 The Two Species Model

In this section, the case where the ground contains seeds for both species is considered. The important equations for this condition are given below. These equations are derived in Section C.2 of Appendix C.

Equation 4.10 implies that 91.1 percent of the planet is occupied only when both daisy species coexist. However, when only one species is in existence, the fractional area occupied of the planet that is occupied can be different. As we saw in the one species models, when only one species in existence 99.9 per cent of the planet can be occupied and this relationship is expressed as

$$\alpha_b + \alpha_w = 0.91. \tag{4.10}$$

The functional relationship between fractional area occupied by black daisies α_b and relative solar luminosity L when both species coexist is

$$\alpha_b = \frac{0.764}{L - 0.129} - 0.544. \tag{4.11}$$

Mean planetary temperature T_e as a function of relative solar luminosity L when both species coexist is given by

$$T_e^4 = \frac{SL}{\sigma}\left(0.272 + 0.5\alpha_b\right). \tag{4.12}$$

In order to find the critical points in this model, we use the same techniques that were used in Sections 2.4 and 3.2.1. From Equation 4.10, we know that when both species coexist, 91.1 per cent of the planet is covered. Hence, solving the inequality $0 < \alpha_b < 0.911$ tells us the range of luminosity for which both species coexist. This inequality yields $0.69 < L < 1.45$. However, from Section 4.1.1 we know that in this model black daisies do not grow until the luminosity reaches 0.74. Hence, the true range of luminosity for which both species coexist is 0.74 to 1.65.

The graph in Figure 4.8 shows temperature regulation under condition we are currently studying. When both species coexist, mean planetary temperature decreases with increasing luminosity. However, this region is stable because in this region (i.e. when both species coexist) daisy population is maximum.

Table 4.2.1 below summarizes the numerical ranges of stability for this model.

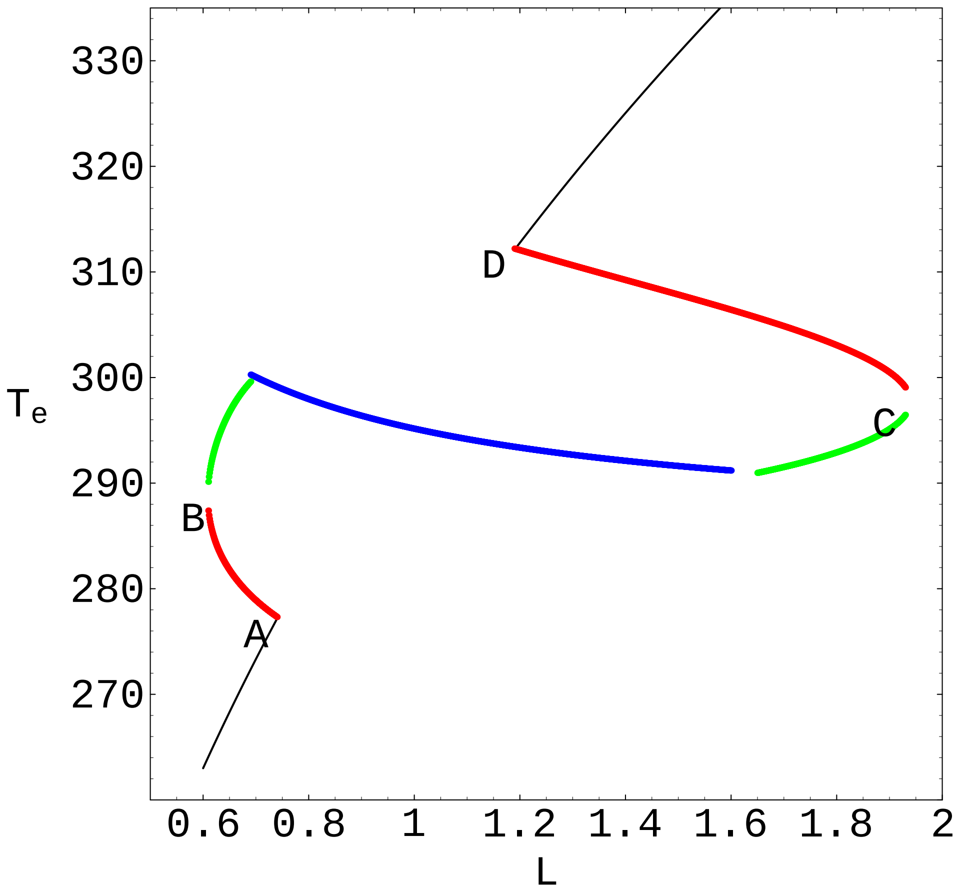

Figure 4.8: Mean Planetary Temperature in a Two Species Model. Dark portion of the graph corresponds to the state where no life exists on the planet. The colored portion of the graph (red, green, blue) corresponds to the state in which life exists on the planet. The red portion shows unstable equilibria and the green portion corresponds to stable equilibria. The red and green portions of the curve connecting to Point B correspond to the state in which only black daisies exist on the planet, and those connecting to Points C correspond to the state in which only white daisies exist on the planet.

Table 4.3: Variable Growth Rate, Variable Death Rate. Planet Inhabited by Both Species. Stable States for Increasing Solar Luminosity.

	L	α_b	α_w	T_e
Only Black Daisies Exist	None	None	None	None
Both Species Coexist	[0.74, 1.65)	[0.00, 0.81]	[0.10, 0.91]	(291.00, 299.13]
Only White Daisies Exist	[1.65, 1.94]	[0.00, 0.00]	[0.91, 0.99]	[291.00, 297.75]

If solar luminosity is increasing, the planet is initially barren. When solar luminosity reaches 0.74, black daisies sprout. At this luminosity, Daisyworld is in an unstable state, hence the population of daisies jumps to a stable value. In this model, white daisies can theoretically exist in the luminosity range 0.69 to 1.65. Black Daisies sprout before white daisies in Daisyworld if solar luminosity is increasing because they absorb more heat that white daisies. From Section 2.3.1 we know that black daisies do not sprout until the luminosity reaches 0.74, and since white daisies cannot sprout before black daisies, the luminosity range for which both species coexist is 0.74 to 1.65. Thus, when Daisyworld makes the jump from the unstable state at $L = 0.74$ to a stable state at the same luminosity, white daisies are present in that stable state. Both species coexist until the luminosity reaches 1.65, at which point black daisies become extinct. White daisies exist by themselves until the luminosity reaches 1.94, and then the planet becomes barren.

4.2.2 Comparisons

We now compare the model developed in Chapter 4 with the original Daisyworld model presented in Chapter 2. As in Section 3.2.2, in order to make this comparison, mean planetary temperatures for both these models when both species may coexist are plotted on the same graph.

Please see Figure 4.9. In this figure, the green curve represents planetary temperature for Daisyword with both growth and death rates of daisies variable, and the blue curve corresponds to mean planetary temperature in the original Daisyworld model. It is seen that again both models have similar qualitative features; however the modified Daisyworld models has tolerance for a wider range of luminosity. It is also visible that when black daisies first appear, mean planetary temperature in the newer model is greater (comparable to the observation made in the comparison in Section 3.2.2) than that in the original model, and when white daisies appear, mean planetary temperature in the new model is lower. Both these conditions correspond to greater proportion of black and white daisies in the modified model, which is to be expected since maximum arable area in the one species conditions of the modified model is 99.9 percent, as compared to 67.3 percent in the original model. Temperature regulation when both species are alive is identical in the two models.

Figure 4.10 compares mean planetary temperature in all three models for the the condition where both species coexist. The blue curve corresponds to mean planetary temperature in the original Daisyworld model, the red curve corresponds to the condition where growth rate of daisies is constant and death rate is variable, and the green curve illustrates the condition where both growth and death rates are variable. The two modified models both have tolerance for greater ranges of luminosity than the original model; the qualitative features of all three models, however, are the same.

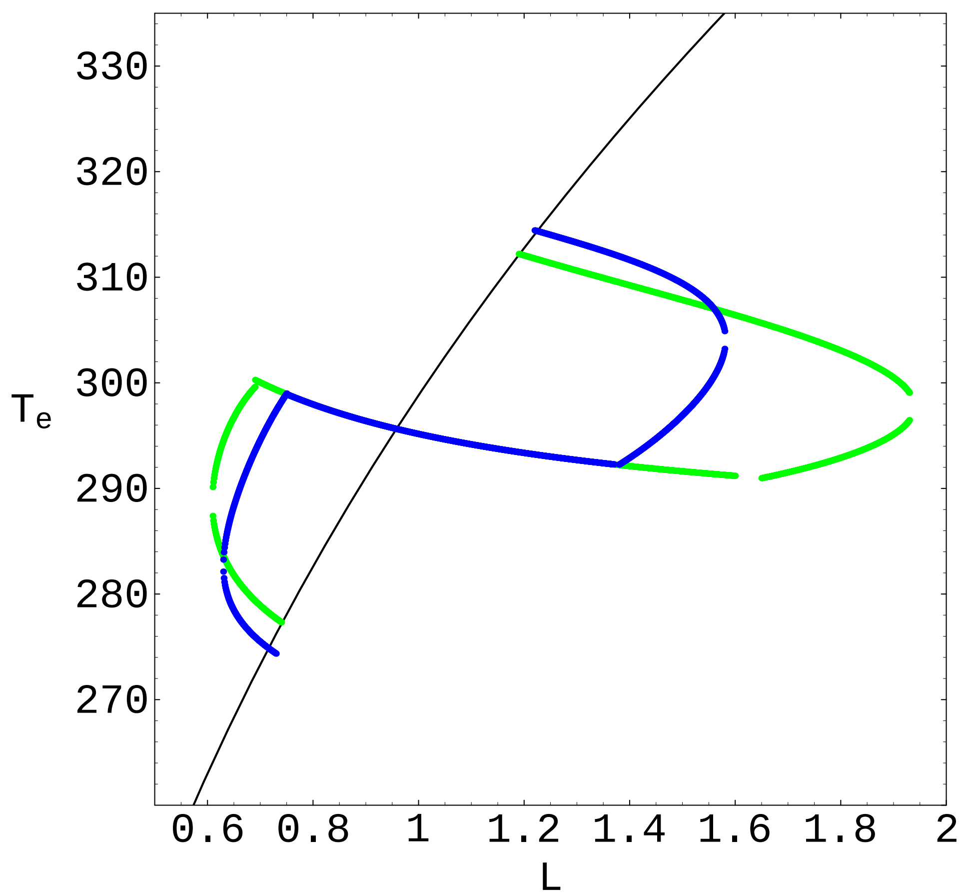

Figure 4.9: Comparison of models presented in Chapters 2 and 4. Blue curve shows mean planetary temperature in the original model, and the green curve shows temperature regulation with variable growth and death rates.

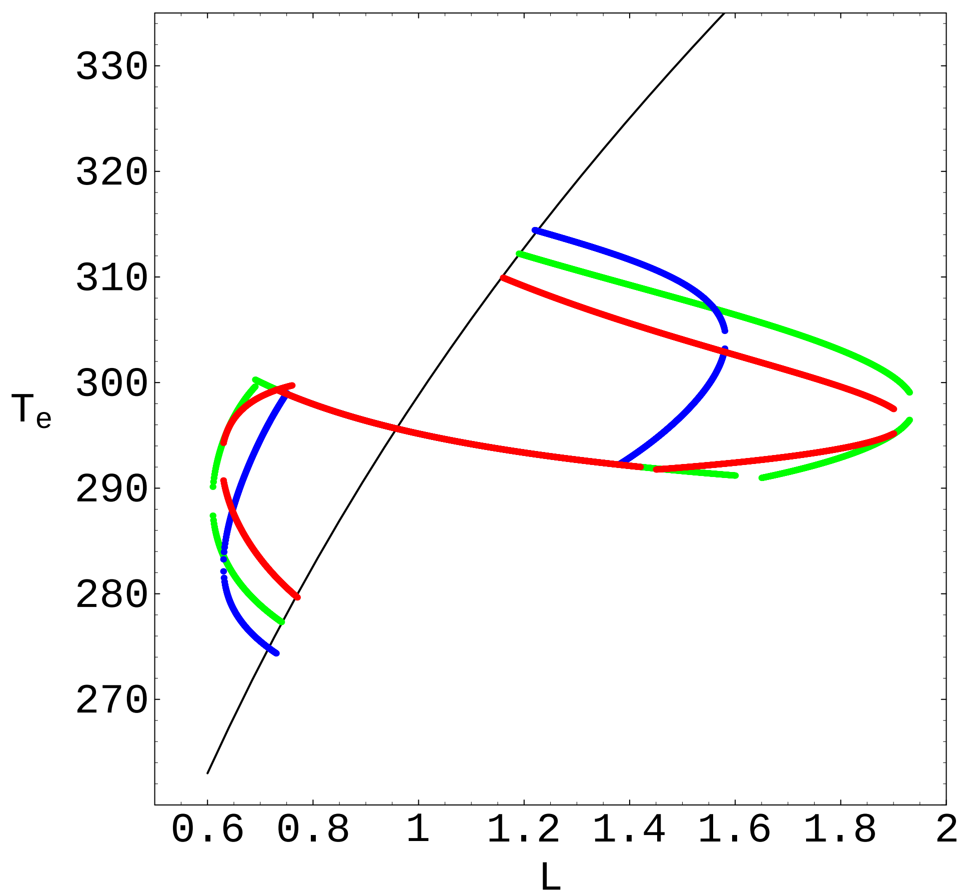

Figure 4.10: Comparison of all three models. Blue curve shows mean planetary temperature in the original model, the red curve corresponds to the condition where growth rate is constant and death rate is variable, and the green curve shows temperature regulation with both growth and death rates variable.

Chapter 5

DISCUSSIONS AND CONCLUSIONS

Daisyworld's beauty lies in its simple illustration of how a planet might self-regulate its climate. The model provides a strong mathematical foundation to the idea that living systems are strongly connected to the Earth's physical environment. This idea is especially significant in contemporary society, since as a species we humans have come to largely believe that we are superior to other forms of life, and the physical world. Daisyworld cleary illustrates the idea of symbiosis, not only between different living species, but also between the living and non-living world.

Daisyworld has also become an important means for theoretically studying various ecological processes, primarily because of its simplicity, and the relative ease with which it can be modified. The numerous versions that have been proposed have provided a simple tool for gaining insight into issues as varied as the impact of habitat fragmentation on ecosystems and the importance of biological diversity in their healthy functioning.

Hence, it seems that it is generally accepted that the basic assumptions on which Daisyworld is based are good, solid assumptions. But are they? We have seen that the original model contains two species of daisy, whose growth rate is dependent on temperature, yet the death rate remains constant. When we first studied the original model, it seemed counter-intuitive that the environment affects life in a selective fashion. It also seemed logical that under extreme environmental circumstances, daisies would have a harder time surviving. Thus, it appeared that the environmental feedback idea on Daisyworld was possibly flawed, and therefore changing the nature of this feedback could change certain qualitative features of the model. If the basic qualitative features of the model changed, then the conclusions based on recent work in theoretical ecology that are based on Daisyworld become suspect. Therefore, it seems prudent to test the effect of modifying environmental feedback in a manner that seems more consistent with biology on the predictions that Daisyworld makes.

The two new models we studied were qualitatively very similar to the original Daisyworld model. Varying the death rate had no significant impact on the essential properties of the system that demonstrate the planet's capacity for self regulation of its climate. Since changing the nature of feedback did not change the essential properties of the system, the robustness of the model is now revealed.

The two models that we developed in this paper have a discrepancy when compared to the original paper. In the original paper, the maximum limit on area occupied by one species models is 70 per cent of total planetary area, while the maximum area that could be occupied with both species coexistent is 67.3 per cent of total planetary area. It seems that the maximum limit on total occupied area should be same, regardless of how many species occupy the planet. On the other hand, in the original model, the maximum arable area is not appreciably different.

However, in the models I developed, the maximum limit on area in the two cases is more disparate. When the growth rate was constant (Chapter 3), the fertile area in the one species case was 99.9 percent, while in the two species case it was only 75 percent. When both growth and death rate were varied (Chapter 4), the maximum productive area in the one species model was again 99.9 percent, and in the two species model it was 91.1 per cent. While these inconsistencies in arable area might not affect the qualitative features of the system, they are certainly undersirable, because the fertile area in the one species and two species models should not be so different.

Lovelock and Watson's demonstration of a self regulating planet was clearly successful because while a lifeless planet was incapable of temperature regulation, the existence of life clearly regulated mean planetary temperature as solar luminosity changed. In fact, in all three models we studied, the two species models regulated temperature better than either of the one species models. The existence of two species resulted in a net decrease in mean planetary temperature, which neither species could singularly produce. Based on these investigations, it might seem logical to conjecture that more biological diversity is better for the overall health of an ecosystem.

Daisyworld also accurately models the physiological response of organisms to temperature; the daisies survive in a certain temperature range, and grow best at an optimum temperature, However, Daisyworld has limitations for accurately modeling the Earth. Daisyworld neither rotates about an axis, nor revolves about its sun. Hence, it does not experience night or seasonal variations. Daisyworld always radiates heat back to space in the same way; at any given time, however, parts of the Earth experience night and these parts radiate certain long waves which Daisworld does not account for.

The current work demonstrates that changing the death rate does not change the qualitative features of the system, but it does not explain why. The most fundamental question that can be drawn from this paper is why the variation in death rate rate does not change these basic qualitative features. Answering this question will provide considerable insight into how the feedback in Daisyworld actually works. Also, now that we are convinced about the robustness of the qualitative features of the model with regards to the nature of environmental feedback, it is interesting to investigate the impact of seasonal and diurnal variations on the system. Even though the time scales we are considering are extremely long (because we are talking about changing states with changing solar luminosity, and it takes a long time for solar luminosity to change), we cannot just take it for granted that these variations (night and day, seasons) will not have an impact on the system. Moreover, the Earth's axis is tilted, and a more realistic version of Daisyworld should account for rotational and axial effects.

We also tried to verify that Daisyworld does not violate the second law of thermodynamics. However, we could not develop a method to account for the change in entropy in the system. Ackland et. al. have suggested that the variation in clump sizes of each species of daisy makes it difficult to quantify the total entropy of the system, because “the entropy associated with variation in clump size is neglected” [2]. Verifying that Daisyworld does not violate the second law of thermodynamics will further add to its credibility.

In conclusion, I sincerely hope that Daisyworld accurately describes the mutualism between the biotic and the abiotic worlds. Daisyworld can go a long way in convincing the human race of our humble, not superior, place in the overall functioning of the Earth, and teaching us to be stewards, rather than conquerors.

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Appendix A

THE ORIGINAL MODEL

In Appendix A, derivations of important equations in Chapter 2 are shown. The code for producing graphs is written in Mathematica, but is not shown in the paper. The four cases where the planet is lifeless, covered by black daisies only, covered by white daisies only, and with both daisy species coexisting, are each considered separately.

A.1 Lifeless planet

In Section A.1, Equation 2.7 which describes mean planetary temperature as a function of relative solar luminosity is derived. Figure 2.2 illustrates mean planetary temperature as a function of relative solar luminosity.

Since the planet is lifeless, the area of bare ground is equal to the planetary area, implying $\alpha_g = 1$. From Equation 2.5 we know that $A = \alpha_g A_g + \alpha_b A_b + \alpha_w A_w$. But when the planet is lifeless, $\alpha_b = \alpha_w = 0$. Recall that $A_g = 0.5$. Hence, $A = 0.5$.

From Equation 2.4 we know that $\sigma T_e^4 = SL(1 - A)$. Since $A = 0.5$, $T_e^4 = \frac{SL}{2\sigma}$, resulting in Equation 2.7.

A.2 One Species Models

Now let us consider the relationshipss for the condition where only one species of plants can grow on the planet. We will first derive the equations for the general case, that apply to either species of daisy, and then consider the black and white daisies separately.

From Equation 2.3, we know the relationship between growth rate β_i and local temperature T_i is $\beta_i = 1 - 0.003265(295.5 - T_i)^2$. Equation 2.1 thus describes the the change in planetary area occupied by the daisies $\frac{d\alpha_i}{dt} = \alpha_i(x\beta_i - g)$. At equilibrium, the area occupied by daisies does not change; therefore, $\frac{d\alpha_i}{dt} = 0$. It makes sense to assume $\alpha_i \neq 0$ because we are studying the condition where one species of plants, either black or white, exists on the planet, and if the species is alive, it will occupy some area. Hence, $\beta_i = \frac{g}{x}$. When the ground contains seeds for only one species of daisy, $x = 1 - \alpha_i$. Hence, $\beta_i = \frac{g}{1-\alpha_i}$. Substituting $\beta_i = 1 - 0.003265(295.5 - T_i)^2$ and the constant value of death rate per unit time ($g = 0.3$) in this relation results in

$$\beta_i = 1 - 0.003265(295.5 - T_i)^2 = \frac{g}{1 - \alpha_i}$$

Therefore,

$$T_i = 295.5 \pm 17.5 \left(\sqrt{\frac{\alpha_i + g - 1}{\alpha_i - 1}} \right).$$

Squaring both sides,

$$\left(\pm 17.5 \sqrt{\frac{\alpha_i + g - 1}{\alpha_i - 1}} \right)^2 = (T_i - 295.5)^2,$$

$$\frac{\alpha_i + g - 1}{\alpha_i - 1} = \frac{(T_i - 295.5)^2}{17.5^2},$$

$$17.5^2(\alpha_i + g - 1) = (T_i - 295.5)^2(\alpha_i - 1).$$

Simplifying further,

$$17.5^2\alpha_i + 17.5^2(g - 1) = (T_i - 295.5)^2(\alpha_i - 1),$$

$$17.5^2\alpha_i - (T_i - 295.5)^2(\alpha_i - 1) = 17.5^2(1 - g),$$

$$\alpha_i(17.5^2 - (T_i - 295.5)^2) = 17.5^2(1 - g) - (T_i - 295.5)^2,$$

$$\alpha_i = \frac{17.5^2(1 - g) - (T_i - 295.5)^2}{17.5^2 - (T_i - 295.5)^2}.$$

Thus Equation 2.8 is obtained.

A.2.1 Black Daisies Exclusively

Now let us derive the equations that were used to study temperature regulation in the original daisyworld model, when only black daisies exist on the planet.

Equation 2.8 describes the relationship between fractional planetary area occupied by daisies (when only one species can exist on the planet) and local temperature. This relationship holds for both species of daisy. Hence, replacing the subscript i by the subscript b in Equation 2.8 yields the equation for fractional area occupied by black daisies as a function of local temperature, described by Equation 2.9.

$$\alpha_b = \frac{17.5^2(1-g) - (T_b - 295.5)^2}{17.5^2 - (T_b - 295.5)^2}.$$

Thus Equation 2.9 is derived.

When only black daisies are able to exist on the planet, the sum of the areas occupied by black daisies and that of bare ground are equal to the total planetary area, implying $\alpha_g + \alpha_b = 1$. Hence, the mean albedo of the planet is $A = \alpha_g A_g + \alpha_b A_b$. Recall that $A_b = 0.25$; therefore, $A = 0.5 - 0.25(\alpha_b)$.

Substituting $A = 0.5 - 0.25\alpha_b$ in Equation 2.4, yields

$$T_e^4 = \frac{SL}{\sigma}\left(0.5 + 0.25\alpha_b\right).$$

Thus Equation 2.11 is obtained.

Let us derive Equation 2.10 now, which describes the relationship between local temperature of black daisies and relative solar luminosity. Equation 2.6 tells us that $T_i^4 = q(A - A_i) + T_e^4$. When only black daisies are in existence, this equation is written as $T_b^4 = q(A - A_b) + T_e^4$. Substituting for T_e from Equation 2.11, $A = \alpha_g A_g + \alpha_b A_b$ and $A_b = 0.25$ we obtain $T_b^4 = 0.25q(1 - \alpha_b) + T_e^4$. Simplification of this relation yields

$$T_b^4 = 0.25q + \frac{0.5SL}{\sigma} + \alpha_b\left(\frac{0.25SL}{\sigma} - 0.25q\right).$$

Equation 2.10 is thus derived.

For any particular luminosity, the corresponding local temperature of black daisies can be obtained from Equation 2.10. Substituting this value of local temperature in Equation 2.9 gives us the fractional area of the planet occupied by daisies at that luminosity. Substituting both these values in Equation 2.11 then tells us the mean planetary temperature at a particular luminosity.

A.2.2 White Daisies Exclusively

We now derive the equations required to study temperature regulation in a planet occupied exclusively by white daisies.

The mean weighted albedo A of the planet when only white daisies can exist is $A = \alpha_w A_w + \alpha_g A_g$. Since, the albedo of white daisies (A_w) is 0.75, and $\alpha_g = 1 - \alpha_w$, therefore, $A = 0.5 + 0.25\alpha_w$. The condition where only black daisies exist on the planet, and the condition where only white daisies exist on the planet, differ only in the mean weighted albedo A of the planet. Besides the new mean weighted albedo A, the derivation for equations describing fractional area occupied by daisies as function of local temperature (Equation 2.13), the relationship between local temperature and relative solar luminosity (Equation 2.14), and the relationship between mean planetary temperature and relative solar luminosity (Equation 2.15), when only white daisies exist on the planet, is identical to the derivation of these equations when only black daisies exist on the planet. Thus, Equations 2.13, 2.14 and 2.15 are derived using the same techniques that were used in Appendix A.2.1.

A.3 Both Species Coexisting

Let us now derive the equations that were used to study temperature regulation when the ground contains seeds for both species of daisy, and hence both species coexist.

Equation 2.1 tells us that $\frac{d\alpha_i}{dt} = \alpha_i(x\beta_i - g)$. At equilibrium, $\frac{d\alpha_i}{dt} = 0$, hence $\frac{d\alpha_b}{dt} = \frac{d\alpha_w}{dt} = 0$. Therefore, $x\beta_b - g = (x\beta_w - g)$. Solving this equation yields $\beta_b = \beta_w$, implying that at equilibrium, growth rate of black and white daisies is the same.

We also know from 2.3 that $\beta_i = 1 - 0.003265(295.5 - T_i)^2$. Thus, $1 - 0.003265(295.5 - T_b)^2 = 1 - 0.003265(295.5 - T_v)^2$, implying $(295.5 - T_b)^2 = (295.5 - T_w)^2$. So, $T_b + T_w = 591$. Consequently, at equilibrium, the sum of the local temperatures of black and white daisies is 591 K.

From Equation 2.6 we understand that $T_i^4 = q(A - A_i) + T_e^4$. Hence, $T_b^4 = q(A - A_b) + T_e^4$ and $T_w^4 = q(A - A_w) + T_e^4$. Since $A_b = 0.25$ and $A_w = 0.75$, it follows that

$$T_b > T_w.$$

Also, $T_b^4 - T_w^4 = q(A_w - A_b)$. So, $T_b^4 - (591 - T_b^4) = q(A_w - A_b) = 0.5q$. This equation yields $T_b = 300.5K$ and $T_w = 290.5K$. since $T_b > T_w$ Thus, when both species of daisy potentially exist, the local temperature of each species is constant. In the one species models, local temperature of each species varied with varying solar luminosity.

We have established that at equilibrium $\frac{d\alpha_b}{dt} = \alpha_b(x\beta_b - g) = 0$. Therefore, $x = \frac{g}{\beta_i} = \frac{g}{1-0.003265(295.5-T_b)^2}$. Substituting for β_i and $g = 0.3$, $x = \frac{0.3}{1-0.003265(295.5-T_b)^2} = 0.327$. Consequently, when both species are in coexistence, 32.7 per cent of the planet is unoccupied. As a result, $\alpha_b + \alpha_w = 0.673$. Therefore, when both species coexist, 67.3 per cent of the planet's area is always occupied.

For a two species model, $\alpha_b + \alpha_w + \alpha_g = 1$ and the mean weighted albedo A of the planet is $A = \alpha_g A_g + \alpha_b A_b + \alpha_w A_w$. Hence, $A = 0.5 - 0.25(\alpha_b - \alpha_w)$. We earlier established that $\alpha_w = 0.673 - \alpha_b$. Therefore, $A = 0.5 - 0.25(\alpha_b - (0.673 - \alpha_b))$. Further simplification yields $A = 0.668 - 0.5\alpha_b$. Substituting $A = 0.668 - 0.5\alpha_b$ in Equation 2.4, results in

$$T_e^4 = \frac{SL}{\sigma}(0.332 + 0.5\alpha_b).$$

Thus, Equation 2.19 is derived.

From Equation 2.6 we obtain $T_b^4 = q(A - A_b) + T_e^4$. However, $A = 0.668 - 0.5\alpha_b$,

$A_b = 0.25$, and $T_e^4 = \frac{SL}{\sigma}(0.332 + 0.5\alpha_b)$, therefore $T_b^4 = q(0.418 - 0.5\alpha_b) + T_e^4$. Simplification results in $T_b^4 = q(0.418 - 0.5\alpha_b) + \frac{SL}{\sigma}(0.332 + 0.5\alpha_b)$. Substituting $T_b = 300.5$ in this equation and solving for α_b yields

$$\alpha_b = \frac{0.829}{L - 0.129} - 0.664$$

. Thus, Equation 2.18 is derived.

Equation 2.18 is equivalent to the expression for α_b given in the paper by P.T. Saunders, $\alpha_b = \frac{12.2}{14.7L-1.9} - 0.663$ [10].

Appendix B

CONSTANT GROWTH RATE; VARIABLE DEATH RATE

Let us now consider the equations for Chapter 3, where the growth rate of daisies is constant and the death rate is functionally dependent on local temperature. Recall that constant value of growth rate is the average value of the growth rate function β_i used in Chapter 2. The average value of growth rate $\overline{\beta_i}$ is 0.33. The functional relationship between death rate of daisies and local temperature is given by Equation 3.2 as

$$g_i = 0.00326204(295.5 - T_i)^2 + 0.001.$$

Recall also that we made an attempt to choose the function of g_i so the average value of this function would be as close to the constant value of death rate used by Lovelock and Watson in their original paper[12], and in several subsequent papers. However, if the average value of g_i is made equal to 0.3, then at optimum temperature, planetary area occupied by daises does not change, implying no daisies are dying. This consequence is not realistic, since even at the temperature that is best for daisy survival, some daisies should be dying.

B.1 One Species Models

We now consider the condition where only one species of daisy may exist on the planet. First, we study the general results that apply to both daisy species, and subsequently the condition where black and white daisies occupy the planet separately.

We first derive Equation 3.5, that describes the fractional area of daisies as a function of local temperature. The change in daisy area when the growth rate of daisies is constant and the death rate is variable is given by Equation 3.3 so that $\frac{d\alpha_i}{dt} = \alpha_i(x\overline{\beta_i} - g_i)$. At equilibrium $\frac{d\alpha_i}{dt} = 0$, and it is reasonable to assume that $\alpha_i \neq 0$ because if $\alpha_i = 0$, then the planet is barren, and in this condition we are studying the condition where one species is alive. Hence, $g_i = x\overline{\beta_i}$. Substituting $x = 1 - \alpha_i$ yields $0.00326204(295.5 - T_i)^2 + 0.001 = 0.33(1 - \alpha_i)$. Solving for T_i results in $T_i = 295.5 \pm \sqrt{\frac{0.33(1-\alpha_i)-0.001}{0.00326024}}$. Subsequently, solving for α_i in terms of T_i we obtain Equation 3.5 which is

$$\alpha_i = 0.999 - \frac{0.0032604}{\beta}(T_i - 295.5)^2.$$

B.1.1 Black Daisies Exclusively

We know that Equation 3.5 describes the proportional surface area of daisies with respect to their local temperature, and this equation applies to both species. Hence, the fractional area occupied by black daisies as a function of their local temperature can be obtained by replacing the subscript i by the subscript b in Equation 3.5; making this substitution yields

$$\alpha_b = 0.999 - \frac{0.0032604}{\beta}(T_b - 295.5)^2$$

which is Equation 3.6.

The derivation of the equations describing the relationship between local temperature of black daisies and relative solar luminosity (Equation 3.7) and mean planetary temperature as a function of relative solar luminosity (Equation 3.8) follows the same procedure that was used for for Equations 2.10 and 2.11 respectively in Section A.2.1 of Appendix A. The only differing parameter in these models is the relationship between α_b and T_b.

B.1.2 White Daisies Exclusively

We now explain derivations of equations that are required to study temperature regulation on a planet that is inhabited exclusively by white daisies. These derivations follow the same techniques that were used in Section B.1.1 of Appendix B and Section A.2.2 of Appendix A. Equations 3.10, 3.11 and 3.12 that describe the relationships between fractional daisy area and local temperature, local temperature and relative solar luminosity, and mean planetary temperature and relative solar luminosity, respectively, are derived by using the techniques that were used for Equations 2.13, 2.14 and 2.15.

B.2 Both Species Coexisting

Let us now study the derivation of equations that were used to study temperature regulation in a planet where both species coexist, and the only the death rate of daisies is affected by local temperature.

We first use the techniques that are used in Section A.3 of Appendix A to calculate what fractional area of the planet is covered when both species are alive. These calculations tell us that $\alpha_b + \alpha_w = 0.75$, implying that when both species are alive, 75 per cent of the planet is always covered.

Thus, by using the relation $\alpha_b + \alpha_w = 0.75$, and once again using techniques described in Section A.3, we obtain $\alpha_b = \frac{0.829}{L-0.129} - 0.625$ and $T_e^4 = \frac{SL}{\sigma}(0.313 + 0.5\alpha_b)$, which are Equations 3.15 and 3.16 respectively. Equations 3.15 and 3.16 are derived by using the same process that was used to derive Equations 2.18 and 2.19 respectively.

Appendix C

VARIABLE GROWTH RATE; VARIABLE DEATH RATE

We now see how the equations to study temperature regulation in a planet occupied by daisies whose growth and death rates are functionally dependent on local temperature are derived. Recall that the relationship between growth rate and temperature is the same as was used by Lovelock and Watson in their original paper[12] and several subsequent papers, and is explained in Chapter 2, i.e. $\beta_i = 1 - 0.003265(295.5 - T_i)^2$. The relationship between death rate and local temperature is $g_i = 0.003262(295.5 - T_i)^2 + 0.001$ as given by Equation 3.2 in Chapter 3.

C.1 One Species Models

Let us now consider the case where the planet can only support one species of daisy. We first take a look at some general equations that apply to both daisy species, and then proceed to analyze black and white daisies separately.

In this model, $\frac{d\alpha_i}{dt} = \alpha_i(x\beta_i - g_i)$. Since, at equilibrium $\frac{d\alpha_i}{dt} = 0$, and it is reasonable to assume that $\alpha_i \neq 0$, since $\alpha_i = 0$ implies the planet is lifeless, and we are considering the case where one daisy species exists and is alive. It follows that $x = 1 - \alpha_i = \frac{g_i}{\beta_i}$, hence

$$\alpha_i = 1 - \frac{0.003262(295.5 - T_i)^2 + 0.001}{1 - 0.003265(295.5 - T_i)^2}.$$

Simplifying the above equation yields

$$\alpha_i = \frac{1.9909(T_i - 307.87)(T_i - 283.13)}{(T_i - 313)(T_i - 278).}$$

Equation 4.1 is derived.

C.1.1 *Black Daisies Exclusively*

We now derive the equations that describe the relationships between relative solar luminosity and local temperature (Equation 4.3) and planetary temperature (Equation 4.4). In order to derive these equations, it is important to first obtain the equation that describes the functional relationship between fractional area of black daisies and their local temperature (Equation 4.2). This equation is obtained by replacing the subscript i by the subscript b in Equation 4.1 since Equation 4.1 applies to both black and white species of daisy.

Once Equation 4.2 is obtained, we follow the same procedure that is outlined in Section A.2.1 in Appendix A to derive Equations 4.3 and Equation 4.4. Equations 4.2, 4.3 and 4.4 are derived by using the same process that was used to derive Equations 2.9, 2.10 and 2.11 respectively.

C.1.2 *White Daisies Exclusively*

We now move on to examine the derivations of equations that describe temperature regulation when the planet is only inhabited by white daisies. The techniques used to derive equations that describe the relationship between relative solar luminosity and local temperature (Equation 4.7) and relative solar luminosity and mean planetary temperature (Equation 4.8) are identical to those shown for Equations 2.13, 2.14 and 2.15respectively.

C.2 Both Species Coexisting

Let us now consider the case where both black and white daisies coexist on the planet. We first follow the techniques outlined in Section A.3 to calculate the fractional area of the planet that is occupied when both species are simultaneously alive, i.e. $\alpha_w + \alpha_b = 0.911$. By using this fact, along with the techniques described in Section A.3, we obtain $\alpha_b = \frac{0.764}{L-0.129} - 0.544$ and $T_e^4 = \frac{SL}{\sigma}(0.272 + 0.5\alpha_b)$, which are Equation 4.11

and Equation 4.12 respectively. The techniques used for deriving Equation 4.11 are identical to those used to derive Equation 2.18, and the techniques used to derive Equation 4.12 are identical to those used to derive Equation 2.19.

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