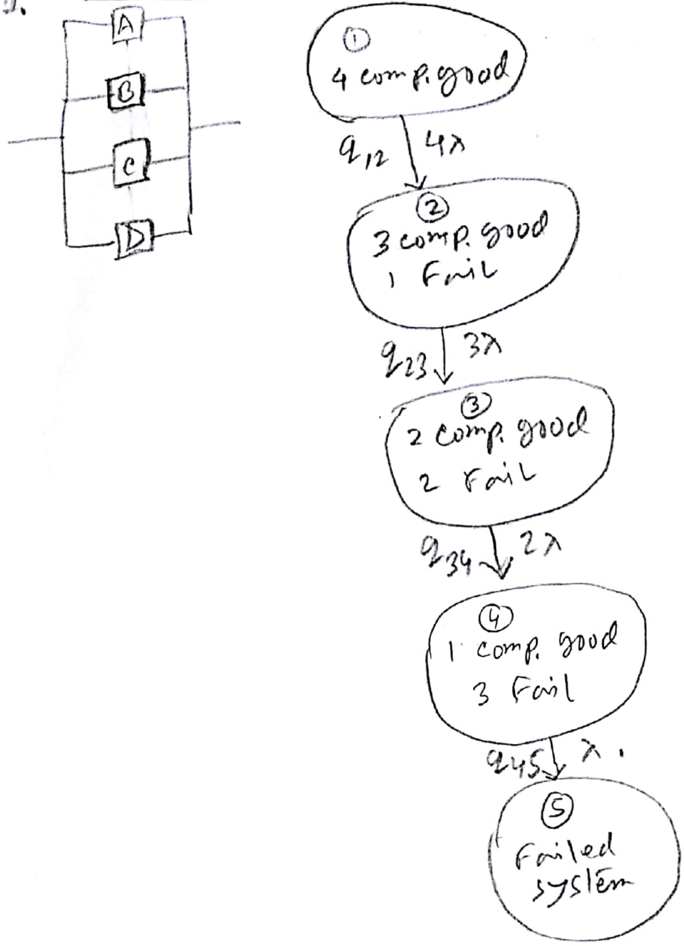
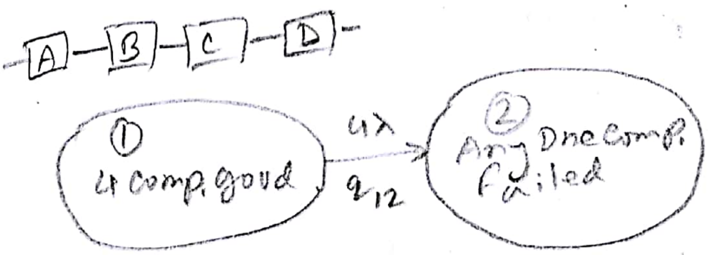
Task - 3.1

1. Figure (a)



$$Q = \begin{bmatrix} -4\lambda & 4\lambda & 0 & 0 & 0 \\ 0 & -3\lambda & 3\lambda & 0 & 0 \\ 0 & 0 & -2\lambda & 2\lambda & 0 \\ 0 & 0 & 0 & -\lambda & \lambda \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

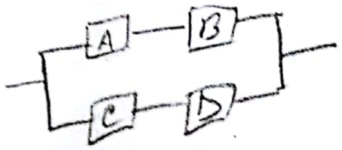
Figure (b)



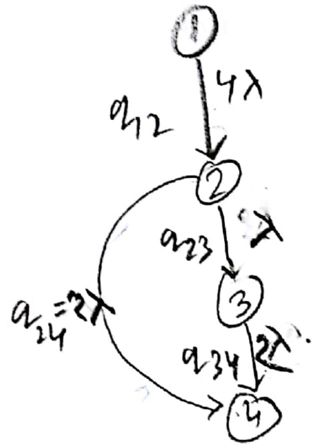
$$Q = \begin{bmatrix} -4\lambda & 4\lambda \\ 0 & 0 \end{bmatrix}$$

cont pg. 1(a).

Figure (e)



States	Remarks
1	A & B → works C, D → works
2	One component fails but Three others works.
3	Two in seriese fails Two in seriese works.
4	Failed system.



$$Q = \begin{bmatrix} -4\lambda & 4\lambda & 0 & 0 \\ 0 & -3\lambda & 2\lambda & 2\lambda \\ 0 & 0 & -2\lambda & 2\lambda \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

For verify:

$$R_{sys} = 2R^2 - R^4$$

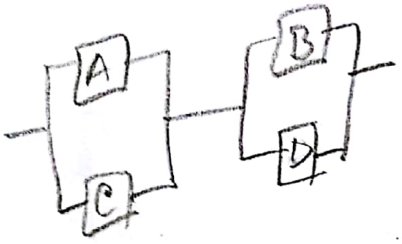
$$= 2(e^{-\lambda_i t})^2 - (e^{-\lambda_i t})^4$$

$$\boxed{R_{sys} = 2e^{-2\lambda_i t} - e^{-4\lambda_i t}}$$

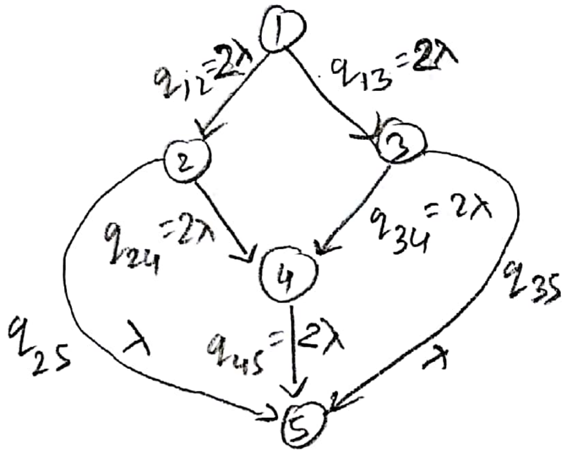
cont pg 2

Task 3.1

Figure (d)



States	Remarks.
1	A, C → works & B, D, work
2	¬A, C A, ¬C } works & B, D work.
3	A, C work & ¬B & D B & ¬D work
4	¬A // ¬C work (One failed) & ¬B // ¬D work (One failed)
5	¬A & ¬C work (Both failed) & ¬B & ¬D work. (Both failed)

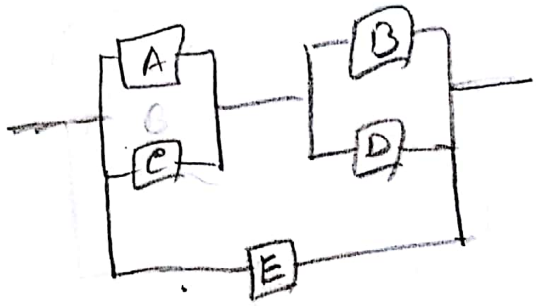


$$Q = \begin{bmatrix} -4\lambda & 2\lambda & 2\lambda & 0 & 0 \\ 0 & 0 & -3\lambda & 2\lambda & \lambda \\ 0 & 0 & -3\lambda & 2\lambda & \lambda \\ 0 & 0 & 0 & -2\lambda & 2\lambda \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

cont Pg 3.

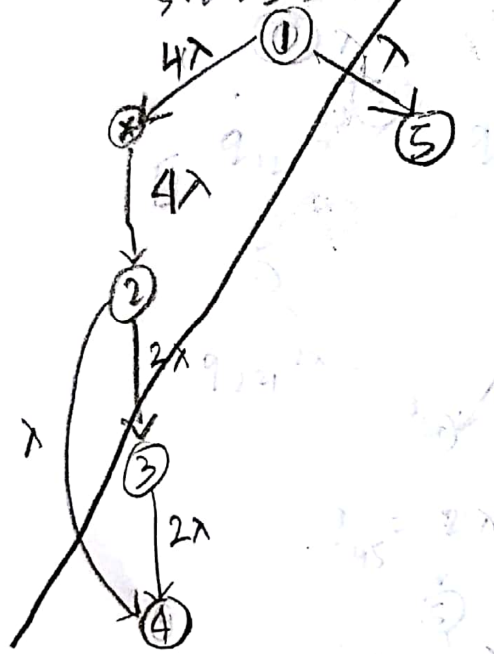
Task 3.1

Figure (e.)



States	Remarks.
1	A, C → works & B, D work.
2	¬A, C / A, ¬C } works & B, D work. E works
3	A, C works & ¬B & D / B & ¬D } work. E works.
4	¬A // ¬C work (one Failed) & ¬B // ¬D work (one Failed)
5	¬A & ¬C work (Both Failed) & ¬B & ¬D work (Both Failed)
6	System 1 & E Failed

Let,
System 1:
States = {1, 2, 3, 4, 5}.



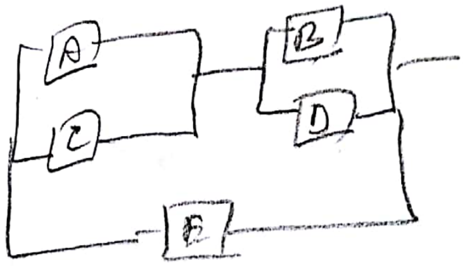
$$Q = \begin{bmatrix} -6\lambda & 3\lambda & 3\lambda & 0 & 0 & 0 \\ 0 & 0 & -2\lambda & 2\lambda & 0 & 0 \\ 0 & 0 & -2\lambda & 2\lambda & 0 & 0 \\ 0 & 0 & 0 & -2\lambda & 2\lambda & 0 \\ 0 & 0 & 0 & 0 & -\lambda & \lambda \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

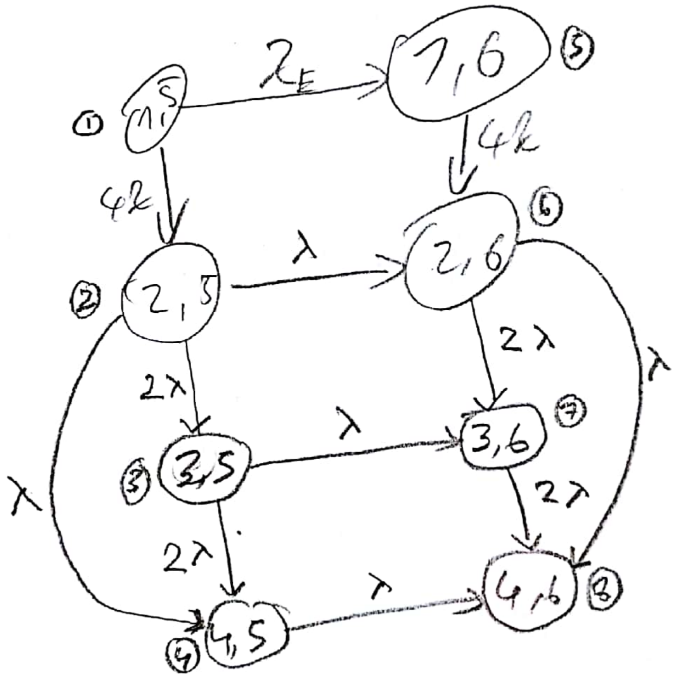
cont. pg 3(a).

Pg. 3.(a)

Task 3.1

Figure (e):





$$
Q = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \\ 8 \end{array}
\begin{array}{c}
\begin{array}{cccccccc} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \end{array} \\
\begin{bmatrix}
-5\lambda & 4\lambda & 0 & 0 & \lambda & 0 & 0 & 0 \\
0 & -4\lambda & 2\lambda & \lambda & 0 & \lambda & 0 & 0 \\
0 & 0 & -3\lambda & 2\lambda & 0 & 0 & \lambda & 0 \\
0 & 0 & 0 & -\lambda & 0 & 0 & 0 & \lambda \\
0 & 0 & 0 & 0 & -4\lambda & 4\lambda & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & -3\lambda & 2\lambda & \lambda \\
0 & 0 & 0 & 0 & 0 & 0 & -2\lambda & 2\lambda \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}
\end{array}
$$

cont. pg. 4.

3.

Differential Equation for System (b)

Here we use

$$\frac{dP_i(t)}{dt} = -\lambda_i P_i(t) + \sum_{j \neq i} \lambda_{ji} P_j(t)$$

$$\frac{dP_1(t)}{dt} = -4\lambda P_1(t)$$

$$\frac{dP_2(t)}{dt} = 4\lambda P_1(t)$$

$$① \xrightarrow[q_{12}]{4\lambda} ②$$

Differential Equation for System (c)

$$\frac{dP_1(t)}{dt} = -4\lambda P_1(t)$$

$$\frac{dP_2(t)}{dt} = 4\lambda P_1(t) - 3\lambda P_2(t)$$

$$\frac{dP_3(t)}{dt} = 1\lambda P_2(t) - 2\lambda P_3(t)$$

$$\frac{dP_4(t)}{dt} = 2\lambda P_3(t) + 2\lambda P_2(t)$$

cont. pg. 5.

Task 3.1

4. Relability formula for system (b):

$-\boxed{A}-\boxed{B}-\boxed{C}-\boxed{D}-$

(b)

We use: $\frac{dy(t)}{dt} = a(t)\cdot y(t) + g(t)$

&

$$y(t) = \frac{\int g(t)\cdot e^{-A(t)}\,dt + c}{e^{-A(t)}}$$

where $A(t) = \int a(t)\,dt.$

$$\frac{dP_1(t)}{dt} = \underset{a(t)}{-4\lambda}\, \underset{y(t)}{P_1(t)} + \underset{0}{g(t)}$$

we Have,

$$P_1(t) = \frac{\int g(t)\cdot e^{-A(t)}\,dt + c}{e^{-A(t)}}$$

where,

$A(t) = \int -4\lambda\, dt$ & $g(t) = 0$

$= -4\lambda t.$

$$P_1(t) = \frac{c}{e^{4\lambda t}}$$

$= e^{-4\lambda t}.c$

Now: for C, $t=0$

$1 = e^{-4\lambda 0}.c$

$c = 1.$

$$\boxed{P_1(t) = e^{-4\lambda t}}$$

$$\frac{dP_2(t)}{dt} = \underset{g(t)}{4\lambda P_1(t)} = 4\lambda\cdot e^{-4\lambda t}$$

$$P_2(t) = \frac{\int g(t)\cdot e^{-A(t)}\,dt + c}{e^{-A(t)}}$$

we have:

$$P_2(t) = \frac{\int 4\lambda\cdot e^{-4\lambda t}\cdot e^{0\lambda t}\,dt + c}{e^{0\lambda t}}$$

$= 4\lambda t.e^{-4\lambda t} + c.$

Now, for c, $t=0$

$1 = 4\lambda 0\cdot e^{-4\lambda\cdot 0} + c$

$\Rightarrow c = 1.$

$$\boxed{P_2(t) = 4\lambda t.e^{-4\lambda t}}$$

$4\lambda \int e^{\frac{-4\lambda t}{u}}\,dt$

$4\lambda \int e^{u}\,du$

$= 4\lambda\left(\frac{1}{-4\lambda}\right)\left(e^{-4\lambda t} + c\right)$

$$\boxed{P_2(t) = -e^{-4\lambda t} + c}$$

(Cont.)
4

$$\frac{d P_2(t)}{dt} = 4\lambda P_1(t) = 4\lambda \cdot e^{-4\lambda t}.$$

Formula is, $P_2(t) = \dfrac{\int g(t) \cdot e^{-A(t)} dt + c}{e^{-A(t)}}$

we have, $a(t) = 0$ by using formula;

$$\boxed{y(t) = \frac{\int g(t) \cdot e^{-A(t)} dt + c}{e^{-A(t)}}}$$

$P_2(t) = \int 4\lambda e^{-4\lambda t} dt + c;$

$= 4\lambda \left(\frac{1}{-4\lambda} e^{-4\lambda t} + c \right)$

$= -e^{-4\lambda t} + 4\lambda c$

Now, for c, $t = 0$.

$\Rightarrow 0 = -e^{0} + 4\lambda c$

$\Rightarrow 0 = -1 + 4\lambda c$

$\Rightarrow c = \frac{1}{4\lambda}$

$P_2(t) = -e^{-4\lambda t} + 4\lambda \cdot \frac{1}{4\lambda}.$

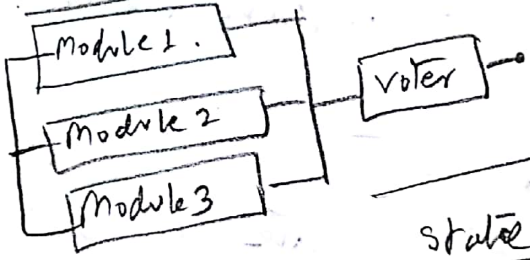
$$\boxed{P_2(t) = 1 - e^{-4\lambda t}.}$$

=0=

Task · 3·2

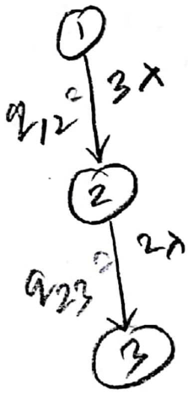
1. Markov model for TMR system:

voter never fail



Assume here voter never fail

state	Remarks.
1	3 components are working
2	2 components are working 1 fail.
3	2 components are failed 1 works means system failed



$$Q = \begin{bmatrix} -3\lambda & 3\lambda & 0 \\ 0 & -2\lambda & 2\lambda \\ 0 & 0 & 0 \end{bmatrix}$$

Formula

$$\boxed{\frac{dP_i(t)}{dt} = -\lambda_i P_i(t) + \sum_{j\neq i} \lambda_{ji} P_j(t)}$$

(2)

$$\boxed{\frac{dP_1(t)}{dt} = -3\lambda P_1(t)} \quad \text{(I)}$$

$$\boxed{\begin{aligned}\frac{dP_2(t)}{dt} &= -2\lambda P_2(t) + 3\lambda P_1(t) \\ &= 3\lambda P_1(t) - 2\lambda P_2(t).\end{aligned}} \quad \text{(II)}$$

$$\boxed{\frac{dP_3(t)}{dt} = 2\lambda P_2(t)} \quad \text{(III)}$$

cont pg. 8.

Task 3.2

(cont.) (2)

Our Formula is:

$$\frac{dy(t)}{dt} = a(t)\cdot y(t) + g(t) \quad \& \quad y(t) = \frac{\int g(t) e^{-A(t)} dt + c}{e^{-A(t)}}$$

where $A(t) = \int a(t)\, dt$.

For equation (i):

$$\frac{dP_1(t)}{dt} = -3\lambda P_1(t).$$

$y(t) = P_1(t)$

$a(t) = -3\lambda$

$A(t) = \int a(t)\, dt$

$= \int -3\lambda\, dt$

$= -3\lambda t + c$

So, $P_1(t) = \frac{c}{e^{-(-3\lambda t)}}$

$= \frac{c}{e^{(3\lambda t)}}$

$\boxed{P_1(t) = e^{-3\lambda t} c}$

Again, for c, t = 0

$1 = e^{-3\lambda t} c$

$\boxed{c = 1.}$

$\boxed{P_1(t) = e^{-3\lambda t}.}$

For equation (ii)

$$\frac{dP_2(t)}{dt} = 3\lambda P_1(t) - 2\lambda P_2(t)$$

$= 3\lambda e^{-3\lambda t} - 2\lambda P_2(t)$ [substitute $P_1(t)$]

Here, $g(t) = 3\lambda e^{-3\lambda t}$ | $A(t) = \int -2\lambda\, dt$

$a(t) = -2\lambda$ | $= -2\lambda t + c$

So $P_2(t) = \frac{\int 3\lambda e^{-3\lambda t} \cdot e^{2\lambda t} dt + c}{e^{2\lambda t}}$

$= \frac{3\lambda \left(\frac{e^{-\lambda t}}{-\lambda} + c \right)}{e^{2\lambda t}}$

$P_2(t) = -3\cancel{\lambda}e^{-3\lambda t} + 3\lambda c\, e^{-2\lambda t}$

Now for c, t = 0 $P_2(0) = 0$

$0 = -3 \times 1 + 3\lambda c \times 1$

$\Rightarrow c = + \frac{1}{\lambda}$

So $\boxed{\cancel{P_2(t) = 3\lambda e^{-3\lambda t} - 3\lambda e^{-2\lambda t}}}$

So $P_2(t) = 3e^{-2\lambda t} - 3e^{-3\lambda t}$

For equation (iii)

$$\frac{dP_3(t)}{dt} = 2\lambda P_2(t)$$

$= 2\lambda [3 e^{-2\lambda t} - 3 e^{-3\lambda t}]$

$= 6\lambda e^{-2\lambda t} - 6\lambda e^{-3\lambda t}$

Here $a(t) = 0$,

So $P_3(t) = \int (6\lambda (e^{-2\lambda t} - 6\lambda e^{-3\lambda t}) dt + c$

cont. Pg. 9.

$$P_3(t) = \cancel{6}^{3}\lambda \frac{e^{-2\lambda t}}{-\cancel{2}\lambda} - \cancel{6}^{2}\lambda \frac{e^{-3\lambda t}}{-\cancel{3}\lambda} + 6\lambda c$$

$$= 2e^{-3\lambda t} - 3e^{-2\lambda t} + 6\lambda c$$

Now for c, $t = 0$, $P_3(0) = 0$

$$\Rightarrow 0 = \cancel{2}\times 1 - 3\times 1 + 6\lambda c$$

$$\Rightarrow c = +\frac{1}{6\lambda}$$

$$\boxed{P_3(t) = 2e^{-3\lambda t} - 3e^{-2\lambda t} + 1.}$$

$$R_{system}(t) = 1 - P_3(t)$$

$$= 1 - (2e^{-3\lambda} - 3e^{-2\lambda t} + 1).$$

$$= 3e^{-2\lambda t} - 2e^{-3\lambda t}$$

$$\boxed{R_{system} = 3e^{-2\lambda t} - 2e^{-3\lambda t}.}$$

where as active system

$$\boxed{\text{Reliability } R_{TMR}(t) = 3R^2(t) - 2R^3(t)}$$

cont Pg. 10.

(3) If voter could fail at rate $\lambda_v > 0$?

Then the system become less reliable & it will become a single unit failure TMR system.

Again the TMR system reliability will decrease as the failure rate of λ_v will increase.

Markov Model:- Subtask 1.

$$
\begin{tikzcd}
1 \arrow[d, "3\lambda"] \arrow[dd, bend left=60, "\lambda_v"] \\
2 \arrow[d, "2\lambda"] \arrow[d, bend right=60, "\lambda_v"'] \\
3
\end{tikzcd}
$$

For subtask 2:

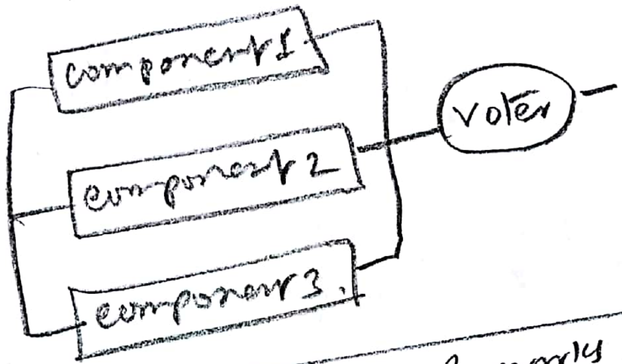
$$\boxed{R_{system} = e^{-\lambda_v t}\left(3e^{-2\lambda t} - 2e^{-3\lambda t}\right)}$$

=0=

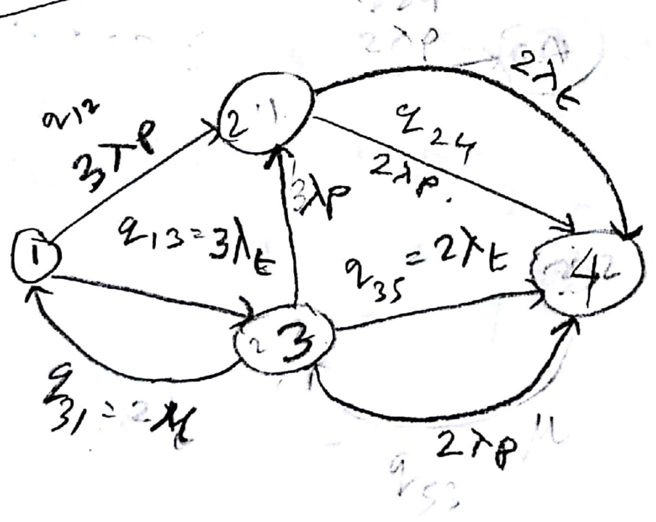
Task 3.3

1. Permanent & temporary unit failures:

Assume, voter never fail



States	Remarks.
1	3 components are working
2'1	2 component are working 1 " Permanent fail
3'2	2 component are working 1 " temporary fail
4'1	2 component are failed 1 " is working.
~~5~~	~~1 component is Permanent failed~~ ~~is Temporarily "~~. ~~2 "~~



λ_P = Permanent failure rate

λ_T = Temporary failure rate

μ = Repair Rate.

cont. Pg. 12

Task 3.3

1.

$$Q = \begin{array}{c|cccc} & 1 & 2 & 3 & 4 \\ \hline 1 & -(3\lambda_p+3\lambda_t) & 3\lambda_p & 3\lambda_t & 0 \\ 2 & 0 & -(2\lambda_p+2\lambda_t) & 0 & 2\lambda_p+2\lambda_t \\ 3 & \mu & \lambda_p & -(3\lambda_p+2\lambda_t+\mu) & 2\lambda_p+2\lambda_t \\ 4 & 0 & 0 & 0 & 0 \end{array}$$

2. Here, total failure rate $\lambda = 2$

So $\lambda_p + \lambda_t = 2$.

Again: $k \in \{0, \frac{1}{4}, \frac{1}{2}, \frac{3}{4}, 1\}$.

where, k of these failures is permanent (λ_p)

& the remain failures are temporary (λ_t).

$\mu = 100$.

$\lambda_p = 0 \cdot 0.5, 1, 1.5, 2$

$\lambda_t = 2, 1.5, 1, 0.5, 0$

Result:

graph ploted in MATLAB.

(3) All the above cases are same except

Part 1, $\mu = 100000$ &

Part 2, $\mu = 1$

Result:

2(Two) graph ploted in MATLAB.

cont. Pg. 13

4/

Failure Rate:	Permanent failure λ_p	Temporary failure λ_T
	0	2
	0.5	1.5
	1	1
	1.5	0.5
	2	0.

Here $k = \{0, \frac{1}{4}, \frac{1}{2}, \frac{3}{4}, 1\}$.

(i) Observation For:-
$\mu = 100$ & different combination of λ_p & λ_T.
the reliability is higher when the λ_T & μ is high than the λ_p. But increasing the λ_p with compare to λ_T the reliability goes down against single component failure.

(ii) Observation for:
$\mu = 100{,}000$ & different combination of λ_p & λ_T.
As the repair rate is very high compare to λ_p & λ_T, So it is more likely probable to repair the component after a failure happen. So μ makes the system more reliable.

(iii) Observation for:-
$\mu = 1$, & different combination of λ_p & λ_T.
As the repair rate is very low and even it is lower that λ_T, or λ_p and adding together λ_T & λ_p value, so it makes the whole system less reliable. As the system couldn't possible to repair at certain point of time so for all the combination of λ_p & λ_T the curve are very close to each other.

= 0 =

End of Task 3.