

Example 1

Watch presentation: [here]. <https://www.youtube.com/watch?v=pHES8eNor6k>

Let's select:

$P=11$ $Q=3$ [Link] <http://asecuritysite.com/encryption/rsa?val=11%2C3%2C3%2C4>

The calculation of n and Φ is:

$$n = P \times Q = 11 \times 3 = 33$$

$$\Phi = (p-1) \times (q-1) = 20$$

The factors of Φ are 1, 2, 4, 5, 10 and 20. Next the public exponent e is generated so that the greatest common divisor of e and Φ is 1 (e is relatively prime with Φ). Thus, the smallest value for e is:

$$e = 3$$

Next we can calculate d from:

$$(3 \times d) \bmod (20) = 1$$
 [Link]

Thus the smallest value of d will be:

$$d = 7$$

Encryption key [33,3]

Decryption key [33,7]

Then, with a message of 4, we get:

$$\text{Cipher} = (m)^e \bmod n$$

$$\text{Cipher} = (4)^3 \bmod 33 = 31$$

$$\text{Decoded} = (\text{cipher})^d \bmod n$$

$$\text{Decoded} = 31^7 \bmod 33 = 4$$

Example 2

Let's select (using the same P and Q , but we'll pick a different e value):

$P=11$ $Q=3$ [Link] <http://asecuritysite.com/encryption/rsa?val=11,3,7,2>

The calculation of n and Φ is:

$$n = P \times Q = 11 \times 3 = 33$$

$$\Phi = (p-1)(q-1) = 20$$

We can select e as:

$$e = 7$$

Next we can calculate d from:

$$7 \times d \bmod (20) = 1 \text{ [Link]}$$

$$d = 3$$

Encryption key [33,7]

Decryption key [33,3]

Then, with a message of 2, we get:

$$\text{Cipher} = (2)^7 \bmod 33 = 29$$

$$\text{Decoded} = 29^3 \bmod 33 = 2$$

Example 3

Let's select:

$$P = 13 \quad Q = 11 \text{ [Link]} \quad \text{http://asecuritysite.com/encryption/rsa?val=13\%2C11\%2C7\%2C7}$$

The calculation of n and PHI is:

$$n = P \times Q = 13 \times 11 = 143$$

$$\text{PHI} = (p-1)(q-1) = 120$$

We can select e as:

$$e = 7$$

Next we can calculate d from:

$$(7 \times d) \bmod (120) = 1 \text{ [Link]}$$

$$d = 103$$

Encryption key [143,7]

Decryption key [143,103]

Then, with a message of 7, we get:

$$\text{Cipher} = (7)^7 \bmod 143 = 6$$

$$\text{Decoded} = (6)^{103} \bmod 143 = 7$$

Example 4

Let's select:

$$P = 47 \quad Q = 71 \text{ [Link]} \quad \text{http://asecuritysite.com/encryption/rsa?val=47\%2C71\%2C79\%2C688}$$

The calculation of n and PHI is:

$$n = P \times Q = 13 \times 11 = 3337$$

$$\text{PHI} = (p-1)(q-1) = 3220$$

We can select e as:

$$e = 79$$

Next we can calculate d from:

$$(79 \times d) \bmod 3220 = 1 \text{ [Link]}$$

$$d = 1019$$

Encryption key [3337,79]

Decryption key [3337,1019]

Then, with a message of 688, we get:

$$\text{Cipher} = (688)^{79} \bmod 3337 = 1570$$

$$\text{Decoded} = (1570)^{1019} \bmod 3337 = 688$$

Example 5

Let's select:

$$P=23 \ Q=41 \text{ [Link]} \ <http://asecuritysite.com/encryption/rsa?val=23\%2C41\%2C7\%2C35>$$

The calculation of n and PHI is:

$$n = P \times Q = 23 \times 41 = 943$$

$$\text{PHI} = (p-1)(q-1) = 880$$

We can select e as:

$$e = 7$$

Next we can calculate d from:

$$(7 \times d) \bmod 880 = 1 \text{ [Link]}$$

$$d = 503$$

Encryption key [943,7]

Decryption key [943,503]

Then, with a message of 35, we get:

$$\text{Cipher} = (35)^7 \bmod 943 = 545$$

$$\text{Decoded} = 545^{503} \bmod 943 = 35$$

Example 6

Let's select:

P=61, Q=53 [Link] <http://asecuritysite.com/encryption/rsa?val=61,53,17,65>

The calculation of n and PHI is:

$$N = 61 \times 53 = 3233$$

$$\text{PHI} = (P-1)(Q-1) = 3120$$

We can select e as:

$$e = 17$$

Next we can calculate d from:

$$(d \times 17) \bmod (3120) = 1$$

$$d = 2753$$

Encryption key [3233,17]

Decryption key [3233,2753]

Then with a message of 65, we get:

$$\text{Cipher} = (65)^{17} \bmod 3233 = 2790$$

$$\text{Decoded} = 2790^{2753} \bmod 3233 = 65$$

Example 7

Let's select:

P=7, Q=13 [Link] <http://asecuritysite.com/encryption/rsa?val=7,13,5,10>

The calculation of n and PHI is:

$$N = 7 \times 13 = 91$$

$$\text{PHI} = (P-1)(Q-1) = 72$$

We can select e as:

$$e = 5$$

Next we can calculate d from:

$$(d \times 5) \bmod (72) = 1$$

$$d = 29$$

Encryption key [91,5]

Decryption key [91,29]

Then with a message of 10, we get:

$$\text{Cipher} = (10)^5 \bmod 91 = 82$$

$$\text{Decoded} = 82^{29} \bmod 91 = 10$$

■ Prof Bill Buchanan, 2015