DES, Triple-DES and AES

Software Security

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Objectives of today's exercise

- ➜ Getting to know the meaning of *confusion* and *diffusion*, and how this concepts are implemented
- ➜ Understanding the advantages of a *Feistel network* and the basic principles of *DES* and *Triple-DES*
- ➜ Learning how to perform *attacks* against symmetric encryption systems, like DES and Triple-DES
- ➜ Understanding how *AES* works

DES/AES: Symmetric-key Encryption

System type / Security level		Concelation: sym. — sym. concelation system	Concelation: asym. — asym. concelation system	Authentikation: sym. — sym. authentication system	Authentikation: asym. — digital signature system
information theoretical		Vernam-Chiffre (one-time pad)		Authentication codes	
crypto-graphi-cally strong against...	active attack	Pseudo-one-time-pad with s^2-mod-n-Generator			GMR
	passive attack		System with s^2-mod-n-Generator		
well re-searched	mathe-matical		RSA		RSA
	chaos	DES/AES		DES/AES	

Source: *Andreas Pfitzmann: Security in IT-Networks*, 2012

DES - Data Encryption Standard
– Symmetric-key Concelation and Authentication –

History of DES

- ➜ At the beginning of the 1970s, an encryption standard was required for general use
- ➜ A public call for proposals was initiated by the NBS (*National Bureau of Standard*, USA), today NIST
- ➜ Important requirement: The strength of the encryption algorithm should be based only on the secrecy of the key and not on the secrecy of the algorithm (*Kerckhoffs's principle*)

Calls

1972 NBS publishes a first request for a standard encryption algorithm without success

1974 Second request was successful, IBM submited a new version of the *Lucifer* algorithm

1977 Ongoing development with support from the NSA, DES was defined as the encryption standard

What are the criteria for evaluating chaos-based encryption systems?

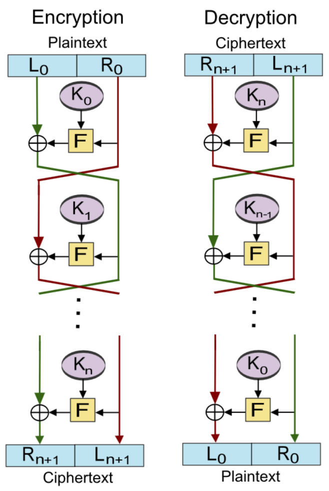
Diffusion

- Effectiveness of changing an input bit to as many output bits as possible, also known as an *avalanche effect*
- Implementation by *permutation* (e.g. using P-boxes), often also expansion permutation used

Confusion

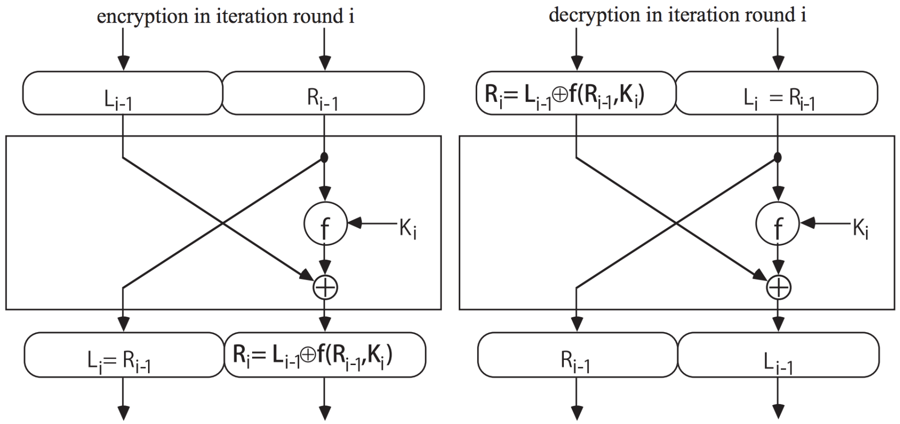
- Hiding the link between plain and ciphertext
- Implementation by *Substitution* (e.g. using S-boxes)

Basics of Feistel networks



- **Idea:** For encryption and decryption the same network can be used, only the order of the round keys has to be inverted
- Function *F* has *not to be inverted* for decryption
- DES based on a Feistel network (16 rounds)

Decryption Principle of DES



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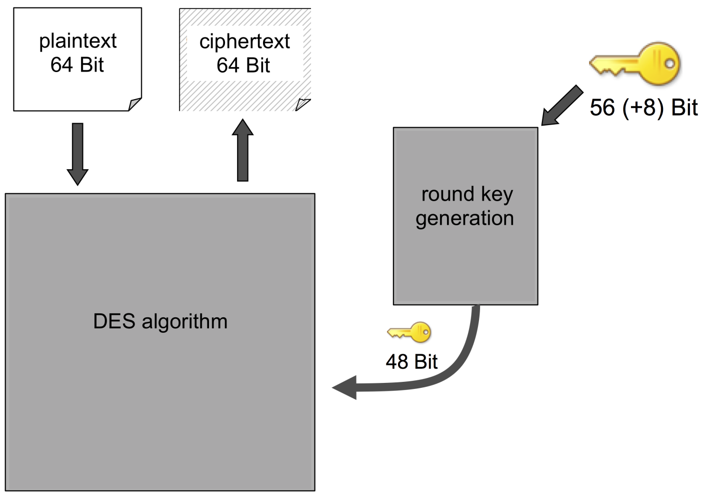
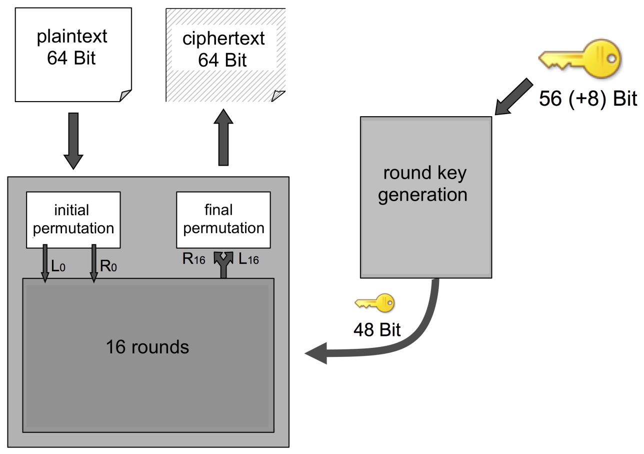
An Abstract View on the DES Algorithm (1)



An Abstract View on the DES Algorithm (2)



➜ Note: Initial and final permutation have no cryptographic significance!

Initial and Final Permutation

➜ The input block

$$X = (x_1, x_2, \ldots, x_{64})$$

is transformed to a permuted input block

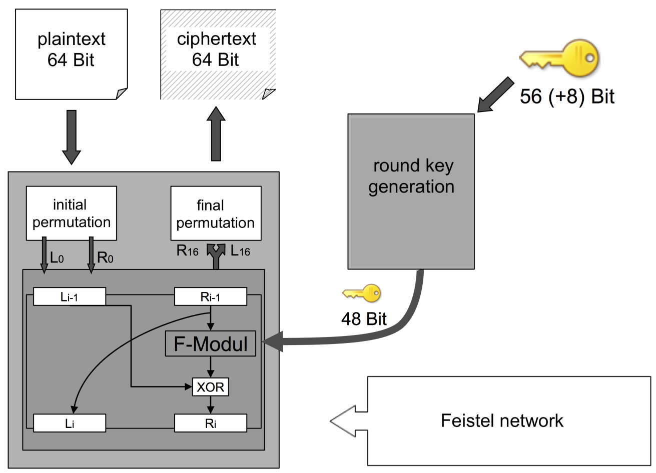
$$IP(X) = (x_{58}, x_{50}, \ldots, x_7)$$

according to the following rule

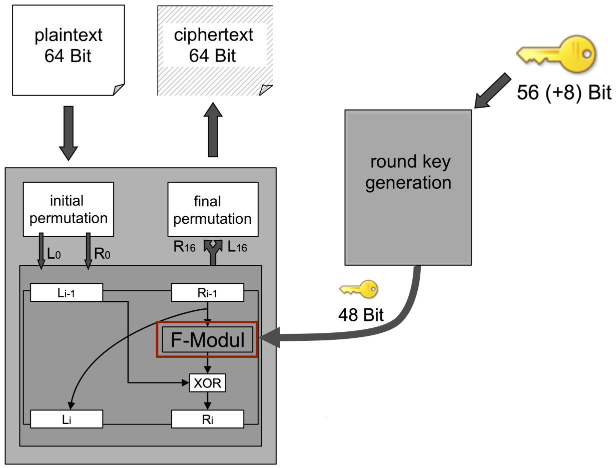
58	50	42	34	26	18	10	2
60	52	44	36	28	20	12	4
62	54	46	38	30	22	14	6
64	56	48	40	32	24	16	8
57	49	41	33	25	17	9	1
59	51	43	35	27	19	11	3
61	53	45	37	29	21	13	5
63	55	47	39	31	23	15	7

➜ The final permutation is defined by the inverted inital permutation IP^{-1}!

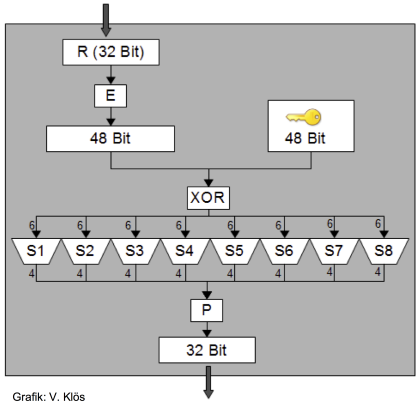
Feistel Network within the DES Algorithm



F-Module within the Feistel Network



Components of the F-Module



Grafik: V. Klös

- E: Expansion permutation
- S_i: Substitution boxes (S-boxes)
- P: Permutation for the next round

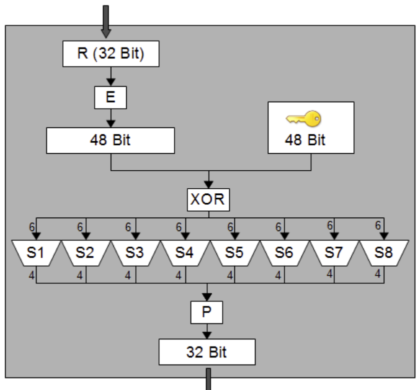
Expansion Permutation

➜ How to expand a 32-bit input block to an 48-bit block?

32	1	2	3	4	5
4	5	6	7	8	9
8	9	10	11	12	13
12	13	14	15	16	17
16	17	18	19	20	21
20	21	22	23	24	25
24	25	26	27	28	29
28	29	30	31	32	1

➜ Note: All fields highlighted in gray are added as duplicates of the input block by expansion

Components of the F-Module



Grafik: V. Klös

- E: Expansion permutation
- S_i: Substitution boxes (S-boxes)
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Substitution using S-Box S_1

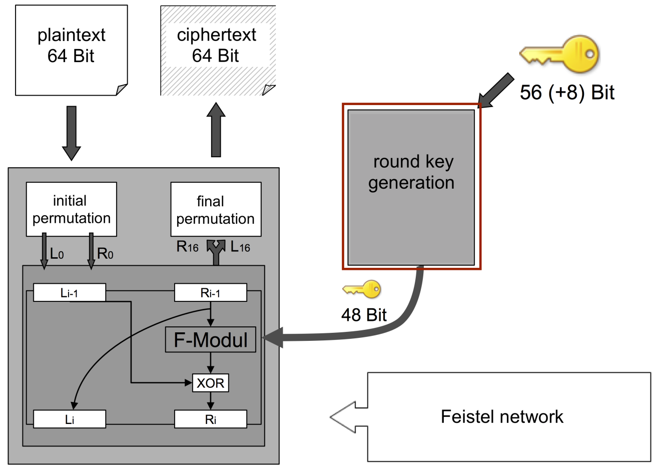
➜ Note, substitution is not bitwise, but according to the following procedure

1. Split the input: 100110
2. Calculate the row number: $10 \mathrel{\widehat{=}} 2$
3. Calculate the column number: $0011 \mathrel{\widehat{=}} 3$
4. Find the result using the s-box: $8 \mathrel{\widehat{=}} 1000$

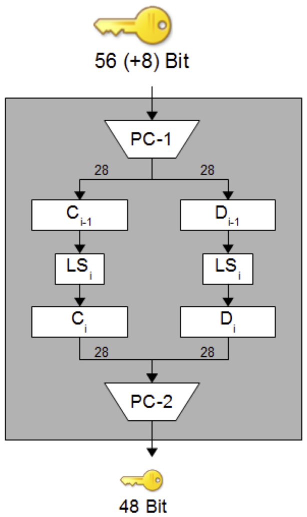
	0	1	2	3	4	5	6	7	8	9	10	11	12	13
0	14	4	13	1	2	15	11	8	3	10	6	12	5	9
1	0	15	7	4	14	2	13	1	10	6	12	11	9	5
2	4	1	14	8	13	6	2	11	15	12	9	7	3	10
3	15	12	8	2	4	9	1	7	5	11	3	14	10	0

➜ The substitution results in a compression of the bit block (reduction from 6 bits to 4 bits)

How to generate round keys?

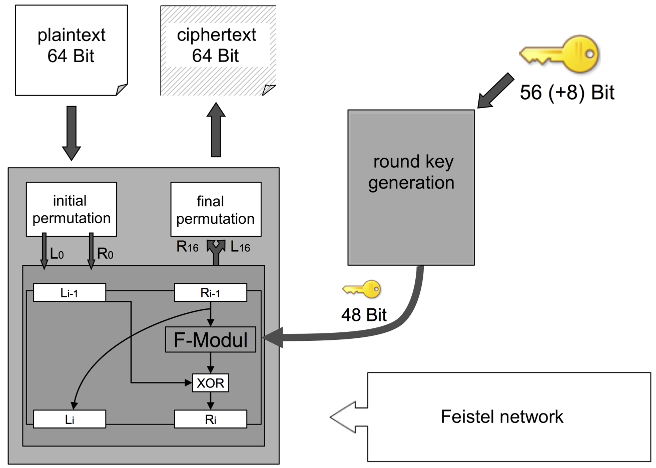


How to generate round keys?



- $PC-1$: Permuted choice (56 bits from 64)
- $PC-2$: Permuted choice (48 bits from 56)
- LS_i: Cyclical left-shift operation (by 1 or 2 bits)

Decryption using DES



➜ For decrypting, the same algorithm is used, but with the round keys in reverse order!

DES - Attacks

– Brute-Force Attack –

Brute-Force Attack for DES

➜ Brute-force attack for DES *can be easily implemented* today, since the size of the key space can be tested with 2^{56} in a practicable time

Procedure for Calculating the Key

1. Select a ciphertext C with the corresponding plaintext P
   ➜ Assumption is that you have such a pair, or
   P can be checked for plausibility in some other way
2. Check for each key Ki of the key space, whether
   $DES_{Ki}(P) = C$ or $DES_{Ki}^{-1}(C) = P$ holds
   ➜ If you succeed, the key is found

Conclusion

➜ *DES is considered insecure today and should not be used!*

How to use the complementarity property of DES?

➜ Given are two plaintexts P and $\overline{P}$, furthermore the attacker knows the corresponding ciphertexts $C1$ and $C2$ with
$C1 = DES_K(P)$ and $C2 = DES_K(\overline{P})$

Procedure for Calculating the Key

1. Select a $K1$ of the key space and calculate
   $C = DES_{K1}(P)$
   ➜ If $C = C1$ then $K1$ is the key you are looking for
   ➜ If $C = \overline{C2}$ then $\overline{K1}$ is the key you are looking for
2. If $C \neq C1$ and $C \neq \overline{C2}$ than we exclude two keys ($K1$ and $\overline{K1}$) from the key space and try Step 1 using another key

The *cost* of a brute-force attack using the complementarity property is *half* that of a normal brute-force attack, i.e. it is reduced to 2^{55}!

Background: Complementarity Property of DES

Why is the following statement correct?

Assuming ...
1. $C1 = DES_K(P)$ and $C2 = DES_K(\overline{P})$ and
2. for a test key $K1$ holds $\overline{C2} = DES_{K1}(P)$

... than the key you are looking for is $K = \overline{K1}$

Complementarity Property

$$DES_K(P) = \overline{DES_{\overline{K}}(\overline{P})}$$

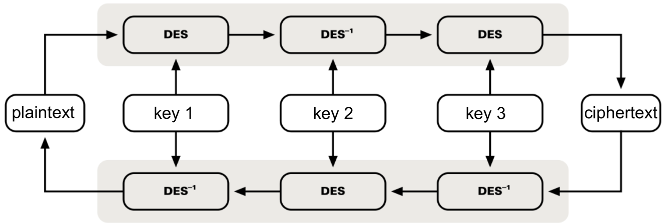
Proof

$$C2 = DES_K(\overline{P}) = \overline{DES_{\overline{K}}(\overline{\overline{P}})} = \overline{DES_{\overline{K}}(P)}$$

$$\Leftrightarrow \overline{C2} = DES_{\overline{K}}(P)$$

➜ If $\overline{C2} = DES_{K1}(P)$, than holds $K1 = \overline{K}$ and $K = \overline{K1}$

How much better is Triple-DES?



Using two different keys

- Order of the used keys: K_1-K_2-K_1
- Size of the key space: $2^{56} * 2^{56} = 2^{112}$

Using three different keys

- Order of the used keys: K_1-K_2-K_3
- Size of the key space: $2^{56} * 2^{56} + 2^{56} = 2^{112} + 2^{56}$
  - ➜ *Meet-in-the-Middle Attack*

Meet-in-the-Middle Attack for Triple-DES

➜ Let P a plaintext and C a ciphertext with
$C = DES_{K3}(DES_{K2}^{-1}(DES_{K1}(P)))$

Procedure

1. Calculate forward
   $\forall K1 \bullet C1 = DES_{K1}(P)$ ➜ 2^{56}
2. Calculate forward
   $\forall C1, K2 \bullet DES_{K2}^{-1}(C1) = P'$ ➜ $2^{56} * 2^{56}$
3. Calculate backward and check, whether $P' = P''$ holds
   $\forall K3 \bullet DES_{K3}^{-1}(C) = P''$ ➜ $2^{56} * 2^{56} + 2^{56}$

Note

➜ The attack requires disk space for 2^{56} blocks, i.e. for a block length of 64 bits you need 576.460 *terabyte*!

AES – Advanced Encryption Standard

– Symmetric-key Concelation and Authentication –

History of AES

➜ Call for proposals in 1997 initiated by the NIST (*National Institute of Standards and Technology*) for the development of a modern symmetric-key encryption standard

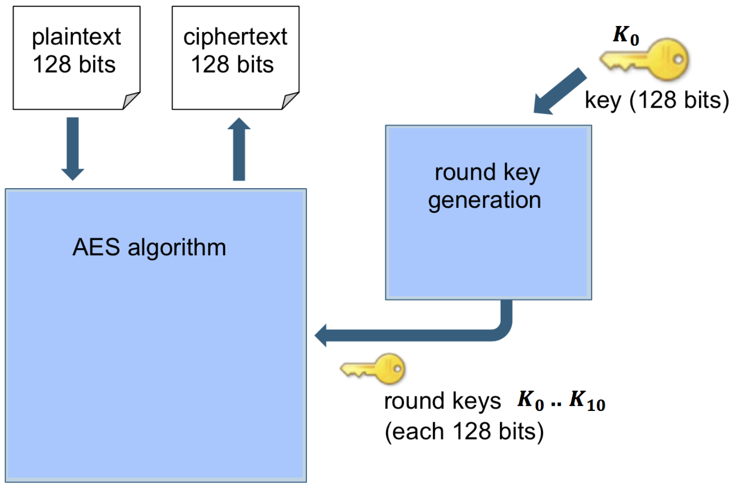
➜ In a first step, 5 promising candidates were selected from 15 submissions
- MARS (IBM)
- RC6 (USA)
- Rijndael (Belgium)
- Serpent (Europa)
- Twofish (USA)

➜ In 2000, the winner was awarded and the Rijndael algorithm was officially named AES

Characteristics of AES (as contrast to DES)

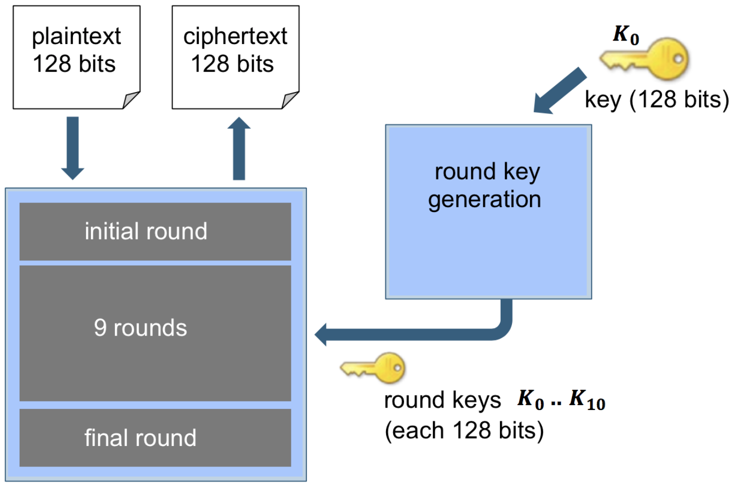
- Block cipher that can process input blocks with a length of 128 bits
- 3 variants with different key lengths
  ➜ 128, 192 or 256 bits
- Is based on a substitution-permutation network, not a Feistel network, i.e. encryption function must be inverted for decryption
- Number of rounds depends on key length
  ➜ 10, 12 or 14 rounds
- Calculations are based on polynomial arithmetic (addition & multiplication) in the galois field $GF(2^8)$

Abstract View on the 128-AES Algorithm



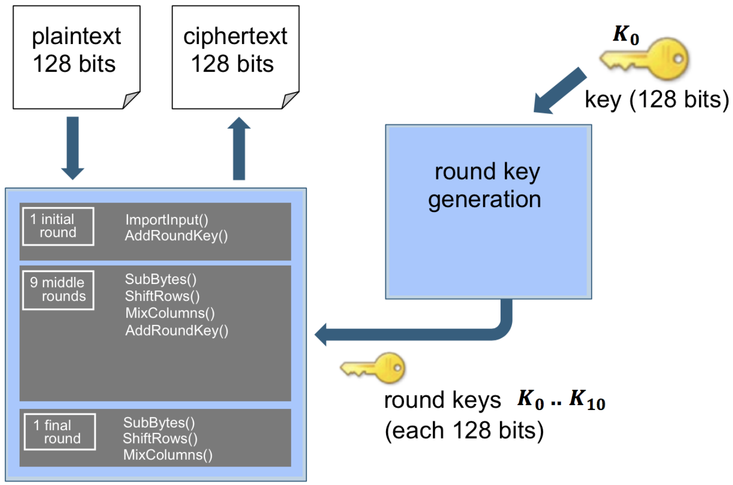
Grafik: B. Ortmann

Refined View on the AES-128 Algorithm



Grafik: B. Ortmann

Which Operations have to be implemented for AES?

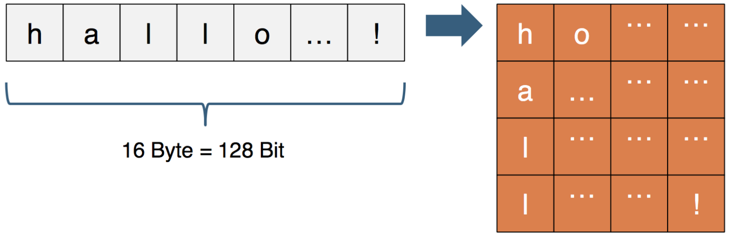


Grafik: B. Ortmann

How to import the input block?

Operation *ImportInput()*

- Internal state consists of a 4 x 4 matrix
- Each field of the matrix represents one byte (also often represented as hex)
- Plaintext of 128 bits is imported column by column



Grafik: B. Ortmann

AES-Operations

Operation *AddRoundKey()*

- Based on *polynomial addition*
- Column-by-column addition of state matrix and round key
- Only this operation depends on the key

Operation *SubBytes()*

- Performs confusion
- Byte-by-byte substitution using a predefined S-Box

Operation *ShiftRows()*

- Performs permutations
- Line-by-line cyclic left shifts (each line one shift more)

Operation *MixColumns()*

- Based on *polynomial multiplication*
- Column-by-column multiplication using a predefined matrix

Fundamentals: Polynomial Calculation

How to represent bytes using polynomials?

- Operations of AES are defined for the *Galois field* $GF(2^8)$
- i.e. each byte of the state matrix is interpreted as a polynomial

polynomial	binary	hex
0	00000000	00
1	00000001	01
x	00000010	02
$x+1$	00000011	03
x^2	00000100	04
x^2+1	00000101	05
...	...	...
$x^5+x^3+x^2+x+1$	00101111	$2F$
...	...	...

Polynomial Arithmetic for $GF(2^8)$

General Remarks

- Similar to residue classes of the number theory Galois fields are based on *finite field aritmetic*
- For $GF(2^8)$ we allow only polynomials of the form $c \cdot x^e$ with $c \in \{0,1\}$ and $e \in \{0,\dots,7\}$

Examples

- Polynomial $(x^3 + x + 1)$ is in $GF(2^8)$
- However the polynomials $(x^8 + x + 1)$ and $(2 \cdot x^3 + x + 1)$ are out of range

Calculations

- Calculation results that are out of range have to be reduced by a suitable modulo operation
- For addition and substraction we reduce the result using $\bmod 2$
- For multiplication we use an *irreducible reducing polynomial*

Polynomial Arithmetic for $GF(2^8)$

Example: Addition

- Addition results have to be reduced by $\bmod 2$

$$
\begin{array}{rl}
 & (x^3 + x + 1) \\
\oplus & (x^3 + x^2) \\
\hline
 & (2\cdot x^3 + x^2 + x + 1) \\
 & (0\cdot x^3 + x^2 + x + 1) \quad \bmod 2 \\
\hline
 & (x^2 + x + 1)
\end{array}
$$

- Addition in $GF(2^8)$ corresponds to XOR-operation

$$
\begin{array}{rlrl}
 & (x^3 + 1) & & (00001001) \\
\oplus & (x^3 + x^2 + x) & \oplus & (00001110) \\
\hline
 & (x^2 + x + 1) & & (00000111)
\end{array}
$$

- Addition and subtraction can be implemented identically

Polynomial Arithmetic for $GF(2^8)$

Example: Multiplication

- Multiplication results have to be reduced by an *irreducible polynomial*, for AES polynomial $(x^8 + x^4 + x^3 + x + 1)$ is used

$$
\begin{aligned}
p &= (x^6 + x^4 + x^2 + x + 1) \cdot (x^7 + x + 1) \\
&= (x^{13} + x^{11} + x^9 + x^8 + x^6 + x^5 + x^4 + x^3 + 1)
\end{aligned}
$$

- Polynomial p is not in $GF(2^8)$ ➜ we have to reduce it

$$(x^{13}+x^{11}+x^9+x^8+x^6+x^5+x^4+x^3+1) \bmod (x^8+x^4+x^3+x+1)$$

- Modulo $(x^8 + x^4 + x^3 + x + 1)$ is calculated by division

$$
\begin{array}{rll}
 & (x^{13}+x^{11}+x^9+x^8+x^6+x^5+x^4+x^3+1) & : (x^8+x^4+x^3+x+1) \\
\oplus & (x^{13}+x^9+x^8+x^6+x^5) & = x^5 + x^3 \\
\hline
 & (x^{11}+x^4+x^3+1) & \\
\oplus & (x^{11}+x^7+x^6+x^4+x^3) & \\
\hline
 & (x^7+x^6+1) &
\end{array}
$$

➜ We obtain for the reduced polynomial $(x^7 + x^6 + 1)$

Polynomial Arithmetic for $GF(2^8)$

Remarks

➜ Note, the AES algorithm requires only multiplications with the constants 1, 2 and 3

➜ This special cases can be implemented more efficiently

Efficient Multiplications in $GF(2^8)$ for AES

$$1 \cdot b := b$$

$$2 \cdot b := \begin{cases} (\text{left shift of } b)^1 & \text{if } b < 128 \\ (\text{left shift of } b) \oplus 00011011 & \text{else} \end{cases}$$

$$3 \cdot b := (2 \cdot b) \oplus b$$

[1] Note that this is not a cyclical left shift, but a normal left shift where the right side is filled with zeros

Example: AES Multiplication for *MixColumns*()

$$\begin{pmatrix} 02 & 03 & 01 & 01 \\ 01 & 02 & 03 & 01 \\ 01 & 01 & 02 & 03 \\ 03 & 01 & 01 & 02 \end{pmatrix} \cdot \begin{pmatrix} d4 \\ bf \\ 5d \\ 30 \end{pmatrix} = \begin{pmatrix} r_1 \\ r_2 \\ r_3 \\ r_4 \end{pmatrix} = \begin{pmatrix} 04 \\ 66 \\ 81 \\ e5 \end{pmatrix}$$

hex	bin	dec
d4	11010100	212
bf	10111111	191
5d	01011101	93
30	00110000	48

How to calculate r_1?

$$\boxed{r_1 = 2 \cdot d4 \oplus 3 \cdot bf \oplus 1 \cdot 5d \oplus 1 \cdot 30}$$

$$\begin{aligned} ir_1 &= 2 \cdot d4 = 10101000 \oplus 00011011 = 10110011 \\ ir_2 &= 3 \cdot bf \ = 01111110 \oplus 00011011 \oplus bf \ = 01100101 \oplus 10111111 = 11011010 \\ ir_3 &= 1 \cdot 5d \ = 01011101 \\ ir_4 &= 1 \cdot 30 \ = 00110000 \end{aligned}$$

$$\begin{aligned} r_1 &= ir_1 \oplus ir_2 \oplus ir_3 \oplus ir_4 \\ &= 10110011 \oplus 11011010 \oplus 01011101 \oplus 00110000 = 00000100 \mathrel{\widehat{=}} 04 \end{aligned}$$

The complete AES algorithm – including round key generation

➜ Animation to illustrate the AES algorithm

https://www.youtube.com/watch?v=mlzxpkdXP58

http://www.formaestudio.com/rijndaelinspector/archivos/Rijndael_Animation_v4_eng.swf