Task 2: Reliability and Mean Time to Failure

Introduction

The purpose of this Task is to get an understanding of dependability measures such as the reliability and the mean time to failure (MTTF). Furthermore reliability modeling using reliability block diagrams (RBD) is practiced. As a preparation, read the third chapter of the lecture and the given material, which is taken from the book *Fault-Tolerant Systems* (Koren and Krishna). Assume in all subtasks that failures of components occur independently and are always permanent.

Task 2.1

Assume a processor with a constant failure rate λ and a MTTF of two years. The processor is composed of one million transistors and fails as soon as one transistor fails. Each transistor has the same constant failure rate λ_T.

1. What is the MTTF of a single transistor?
2. What is the probability that the processor will fail in its first year of operation?
3. How long does the processor work with a reliability of more than 50%? (This is typically defined as the *mission time* $\mathrm{MT}(r)$ for $r = 0.5$)
4. Given the information that the processor failed sometime in the interval between 4 and 8 years, what is the conditional probability that it failed before it was 5 years old?
5. What is the maximum value of λ_T such that the processor has a reliability of 50% after two years? What is the MTTF of the processor in this case?

Task 2.2

Plot the reliability function of the following *series* and *parallel systems*:

(a) A system consisting of a single component

(b) A series system consisting of two components

(c) A series system consisting of four components

(d) A parallel system consisting of two components

(e) A parallel system consisting of four components

Each component has the same constant failure rate $\lambda = 2/year$. Plot the reliability functions in two ways:

1. The x-axis is the reliability of a single component in the range from 1 to 0.
2. The x-axis is the time from 0 to 2 years.

In both cases, the five reliability functions may be drawn into the same chart.

Task 2.3

Different series and parallel systems are shown in Figure 1(a), 1(b) and 1(c). All components $(A - E)$ have the same reliability R. Express the reliability of each system as a function of R!

Task 2.4

Figure 2 shows a more complex system that cannot simply be described as a parallel or series system. Again it is assumed that all components have the same reliability R. Derive a lower and upper bound for the system's reliability as well as an exact formula! Plot all three functions into a single chart as described in task 2.2.1!

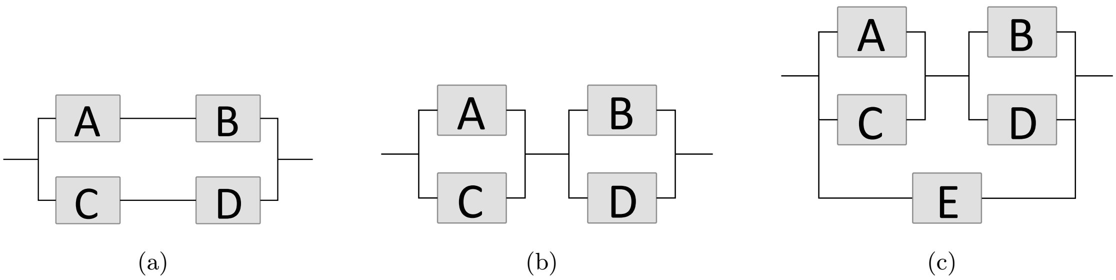

Figure 1: Series and parallel systems

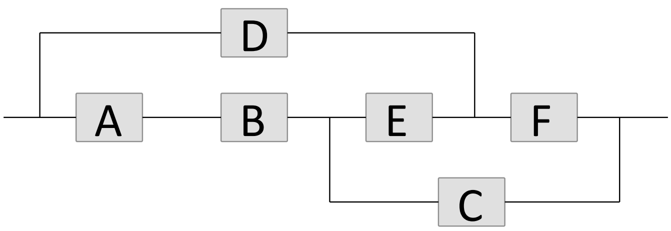

Figure 2: A non-series/parallel system

Task 2.5

1. Derive a formula for the reliability of a Triple Modular Redundancy (TMR) system!

2. Plot the reliability function of TMR into the charts of task 2.2! Assume that the functional components have the same failure rate as in task 2.2 and that the voter never fails. When does the TMR system become less reliable than the original system, consisting of a single component? Explain why!

3. Determine an upper bound for the failure rate of the voter such that:

   a) TMR provides a reliability improvement as long as the single component has a reliability of more than 90%.

   b) TMR provides a reliability improvement at all.

4. Would the formula from Task 2.5.1 still be correct, if component failures could also be temporary? Explain why or why not!